Estimating Freeway Origin-Destination Patterns Using Automation Traffic Counts

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To enable the efficient use of existing roadway capacity, researchers and practitioners are developing advanced traffic management systems (ATMS), which has led to an increased interest in problems connected to the estimation of origin-destination (O-D) flows using information provided by freeway surveillance and control systems. A number of methods based on a linear traffic assignment model have been applied successfully to single intersections, and some of these estimators were extended to a section of freeway. The results from Monte Carlo simulation suggest that ordinary least squares (OLS) and expectation-maximization approaches were either biased or inefficient. A nonlinear least squares (NLS) estimator that eliminated model specification error was introduced, and it performed better in terms of statistical efficiency and lack of bias. This implies that accurate O-D estimation may require an accurate traffic flow model and that actual implementation may require joint estimation of O-D patterns and traffic flow model parameters. On the other hand, a constrained approximate maximum likelihood estimator performed better than OLS but somewhat worse than NLS, showing some potential for providing a simple and yet plausibly accurate approach.

Traffic congestion is an increasingly serious problem for many of the world's urban areas, but fiscal, social, and environmental constraints prohibit large increases of highway capacity. Thus the advanced traffic management systems (ATMS) and advanced driver information systems (ADIS) initiatives in the United States, and similar programs in other nations, have as one of their major objectives the efficient use of existing highway capacity. This is to be achieved by an increased availability of high-quality real-time information about traffic conditions, along with a more intimate linking of traffic control with travel demand management tactics. The success of such an approach will depend heavily on the availability of practical models describing the interaction between travel demand and traffic flow phenomena, models that can give real-time predictions of the effects of proposed traffic management actions. Most traffic models use some form of an origin-destination (O-D) matrix as the basic description of the demand for travel, which has led to an interest in using the data collected by traffic surveillance systems, especially traffic counts, to generate real-time estimates of O-D matrices.

In particular, it is hoped that the availability of time-series data of traffic counts will permit development of O-D estimators that have desirable statistical properties, such as consistency, efficiency, and lack of bias, and that will be able to track changes in the O-D patterns. For general networks, constructing such O-D estimators can be a difficult task, because of the possibility that many routes may connect any given O-D pair [Davis (*1*)], but the problem is simplified somewhat when one considers simple "linear" networks, such as single intersections, transit routes, and freeway segments, where each origin and destination are connected by at most one route (2).

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Since urban freeways carry a large fraction of total urban travel, it is not surprising that estimation of freeway O-D patterns has been receiving increased attention (*3–6*); one has available time-series data of on-ramp, off-ramp, and mainline traffic counts, and can infer the O-D pattern that generated them. The freeway O-D problem is similar to that of estimating turning movement volumes from entering and exiting counts at single intersections, a problem that has been treated extensively during the past decade (*7–11*). Particularly relevant here is the paper by Nihan and Davis (*10*) that described a Monte Carlo study comparing several variants of ordinary least squares (OLS) estimators of turning movement proportions. Nihan and Davis found that although the OLS-based estimators tended to be consistent and unbiased, data from 50 to 60 time points were needed before the standard error of estimate could be reduced to a usefully low level. This finding indicated a possible bound on the ability of time-varying implementations of OLS to track within-day changes in the O-D pattern, so that, even if a recursive estimator is consistent (i.e., converges eventually to the true values of the unknown parameters), when the rate at which its standard error of estimate goes to zero is slow compared with the time variation of the underlying parameter, the estimated values are not likely to be close to the (unknown) true values. This in turn suggests that a naive embrace of recursive estimation procedures without due consideration of their convergence properties is as likely to inject error into travel demand modeling as it is to inject truth.

When applied to freeway segments, the simple linear traffic assignment model takes the form

$$\hat{y}_j(t) = \Sigma_i\, b_{ij}\, q_i(t) \qquad (1)$$

where

$\hat{y}_j(t)$ = predicted traffic count at off-ramp j during time interval t, $j = 1, \ldots, n$;
$q_i(t)$ = actual traffic count at on-ramp i during time interval t, $i = 1, \ldots, m$; and
b_{ij} = probability that a vehicle entering at i is destined for j.

Traffic conservation considerations require that

$$0 \le b_{ij} \le 1 \qquad i = 1, \ldots, m, j = 1, \ldots, n \qquad (2a)$$

$$\Sigma_j\, b_{ij} = 1.0 \qquad i = 1, \ldots, m \qquad (2b)$$

Generally, on-ramp counts $q_i(t)$ and off-ramp counts $y_j(t)$ will be available from a freeway's surveillance system, and unconstrained OLS estimates of the unknown b_{ij} can be computed by minimizing the sum of squares function

$$S = \Sigma_t \Sigma_j\, [y_j(t) - \hat{y}_j(t)]^2 \qquad (3)$$

while constrained OLS would minimize Equation 3 subject to Equations 2a and 2b. Unfortunately, even though unconstrained OLS and its variants give plausible estimates when applied to simple intersection counts, they tend to fail when applied to counts obtained from freeway on- and off-ramps. Table 1 presents unconstrained OLS estimates for a short section of Interstate I-35W with four on-ramps and two off-ramps, where by convention the upstream mainline boundary is denoted as On-Ramp 1 while the downstream mainline boundary is denoted as Off-Ramp 2. The estimated proportions given in Table 1 were obtained by minimizing Equation 3 using 5-min on- and off-ramp counts. Since this section was about 1 mi (1.7 km) long, most of the vehicles entering during a 5-min interval will have exited during that same interval, and one would expect that time-varying travel times would not be a factor.

Clearly, these estimates show serious violations of the conservation conditions in Equations 2a and 2b, and although it would still be possible to minimize Equation 3 subject to Equations 2a and 2b, a usefully consistent or unbiased estimator should be able to produce reasonably close estimates without such devices. Thus it appears that when applied to freeway data, OLS estimators can lose the consistency and unbiasedness properties shown when applied to single intersections, and it has been the authors' experience that such results are the norm rather than the exception when using OLS and the linear traffic assignment model to estimate freeway O-D proportions. This situation is unfortunate because from a practical standpoint, recursive versions of OLS are very easy to implement and tend to be computationally fast *(10)*.

This discussion has identified two basic statistical issues with regard to freeway O-D estimators. The first concerns whether an estimator is unbiased or consistent, that is, whether on the average or in the long run the estimated O-D parameters will equal the true underlying values. A primary cause of bias or inconsistency is model specification error, in which the model that is assumed to generate the data differs substantially from the process actually generating the data. The linear traffic assignment model just described neglects the fact that the travel time between O-D pairs will differ both as a function of the distance separating the origin from the destination and as a function of the intervening traffic conditions. Such specification error may be responsible for the apparently biased estimates generated by OLS. But even if two estimators are unbiased, they may differ in efficiency, measured by the standard errors of the estimates as functions of sample size. The estimator with the lower standard error of estimate is more likely to generate estimates that are "close" to the true values when finite data sets are used. For example, in a linear regression model with heteroscedastic, normally distributed errors, simple OLS remains an unbiased estimator of the regression coefficients but is no longer efficient, the corresponding maximum likelihood estimator having smaller standard error.

This paper describes a Monte Carlo evaluation of four different approaches to estimating freeway O-D proportions b_{ij}, the objective being to decide which of the methods, under practically useful conditions, tend to be unbiased and to assess their relative statistical efficiency. Attention is restricted to off-line estimates of time-invariant parameters because the algorithms used to track time-varying O-D patterns are, for the most part, simply recursive versions of their off-line counterparts *(12)*. For instance, the extended Kalman filter approach described by Chang and Wu *(5)* can be viewed as a recursive implementation of a nonlinear weighted least-squares approach, whereas the Kalman filter method tested by Ashok and Ben-Akiva *(6)* is a recursive implementation of a linear, multilag least-squares approach. A biased or inefficient estimator will not lose these properties when implemented recursively, but a good off-line estimator is a good candidate for recursive implementation. In particular, there is a natural connection between the efficiency of an off-line estimator and the convergence rate of its recursive counterpart, in that the standard error of estimate for the off-line estimator obtained with a sample of size N is a lower bound for the standard error of the recursive estimator after N iterations.

TABLE 1 OLS Estimates for Typical Freeway Data

	Off-Ramp	
On-Ramp	1	2
1	0.0	0.79
2	−0.10	2.22
3	0.34	2.43
4	0.0	1.35

Of the four candidate estimators considered here, three are based on the simple linear assignment model—and hence are subject to specification error—but differ as to the optimization criterion used to compute the estimates. The fourth minimizes the same least-squares criterion used by OLS, but the predicted off-ramp volumes are computed by a nonlinear model that eliminates specification error, which is possible because simulated data are being used. The objective is to determine if the computational simplicity of the linear model can be retained by shifting to a different optimization criterion or whether its inherent specification error is so serious as to make it unusable.

The authors first describe the simulation model used to generate the Monte Carlo sample, then describe the four estimation procedures. Results of the estimators' performance on the simulated data are presented next, and the paper ends with a discussion of these results.

STOCHASTIC FREEWAY TRAFFIC SIMULATION MODEL

As noted earlier, the objective of this study is to assess the statistical properties of several candidate procedures for estimating freeway O-D parameters. The primary method of assessment is Monte Carlo simulation, in which a sample of simulated freeway on-ramp and off-ramp counts is generated, and then each candidate estimation procedure used to compute estimates from each simulated data set. This produces a pseudorandom sample of estimates for each procedure, and these samples are used to determine the presence or absence of desirable statistical properties. To produce simulated data that preserve both the random assignment of vehicles to off-ramps and the general features of traffic flow, the authors developed the STOMAC (stochastic macroscopic) simulation model, which is described in the following.

Before the model is introduced, it is necessary to clarify the following notation and terms. N time intervals (e.g., 5 min each) are assumed during the period of interest; let $t = 1, \ldots, N$ index these intervals. Each of the N intervals is in turn divided into T subintervals, each of duration Δ. Let these subintervals be indexed by $\tau = 1, \ldots, NT$. The intervals represent the level of aggregation at which count data is available, and the subintervals are the basic time unit of the simulation model.

Figure 1 shows a section of freeway with m on-ramps (including the upstream boundary of the section of freeway) and n off-ramps

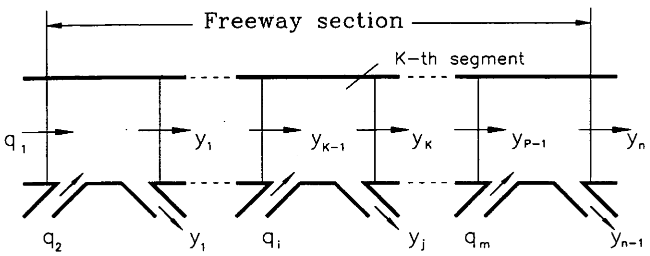

FIGURE 1 Freeway section with *m* on-ramps and *n* off-ramps, divided into *p* segments.

(including the downstream boundary), listed as $i = 1, \ldots, m$ and $j = 1, \ldots, n$, respectively. The upstream boundary will be treated as the first on-ramp and the downstream boundary will be the last off-ramp. The section of freeway has been divided into p segments, indexed by $k = 1, \ldots, p$, such that on-ramps are located only at the upstream boundary of a segment and off-ramps leave only at the downstream boundaries of segments. A further division of segments may be necessary to ensure that geometric features are constant within the segments.

The notation is defined as follows:

- $q_i(t)$ = traffic entering at on-ramp i during time interval t
- $\mathbf{q}(t)$ = m-dimensional vector whose elements are $q_i(t)$
- $\hat{y}_j(t)$ = traffic exiting at off-ramp j during time interval t
- $y_j(t)$ = forecast of traffic exiting at off-ramp j, during t
- $x_{ij}(t)$ = traffic entering on-ramp i and destined for off-ramp j, during time interval t
- b_{ij} = probability that a vehicle entering from on-ramp i is destined for off-ramp j,
- $\mathbf{B}$ = $m \times n$ dimensional matrix whose elements are b_{ij}
- $\mathbf{B}^T$ = transpose of $\mathbf{B}$
- $\mathbf{b}_i$ = m-dimensional vector containing $b_{ij}, j = 1, \ldots, n$
- $\mathbf{y}(t)$ = n-dimensional vector containing $y_j(t), j = 1, \ldots, n$
- $\mathbf{V}(t)$ = $n \times n$ covariance matrix of $\mathbf{y}(t)$, given $\mathbf{q}(t)$
- $P_k(\tau)$ = probability that a vehicle in segment k exits during subinterval τ
- $z_{kj}(\tau)$ = number of vehicles in segment k destined for j at beginning of subinterval τ
- $z_k(\tau) = \Sigma_j z_{kj}(\tau)$
- $q_{ij}(\tau)$ = number of vehicles entering at on-ramp i and destined for j during subinterval τ
- $q_i(\tau) = \Sigma_j q_{ij}(\tau)$
- $y_{kj}(\tau)$ = number of vehicles exiting from segment k and heading for j during subinterval τ
- $y_k(\tau) = \Sigma_j y_{kj}(\tau)$
- L_k = length of segment k
- M_k = number of lanes in segment k
- $r_k(\tau)$ = traffic density in segment $k = z_k(\tau)/(L_k * M_k)$
- $\overline{U}_e(r)$ = equilibrium speed and density function

The basic idea was to treat traffic flow on a freeway as the outcome of a type of stochastic process known as a Markov compartment process (*13*). In this model, each segment of the section of the freeway was treated as a Markovian compartment, from which vehicles exit with probability $P_k(\tau)$. Given the size of the compartment population at τ, each vehicle makes its exit independently of the others, so that the number of vehicles exiting is a binomial random variable with parameters $z_k(\tau)$ and $P_k(\tau)$. To derive plausible forms for the exit probabilities $P_k(\tau)$, imagine that vehicle l in segment k at the beginning of τ has a speed u_{kl} and a location s_{kl} that denotes the distance from vehicle l to the end of the downstream boundary of segment k. Also assume that the speeds u_{kl} and s_{kl} are assigned to the vehicles as independent, identically distributed random variables with density functions $f_k(u)$ and $g_k(s)$, respectively. Since vehicle l will exit segment k only if $s_{kl} < u_{kl} * \Delta$, the exiting probability $P_k(\tau)$ is

$$P_k = \text{prob}[s_{kl} < \Delta u_{kl}] = \int_0^\infty \int_0^{\Delta u_{kl}} g_k(s) f_k(u)\, ds\, du \tag{4}$$

and if it is assumed that the locations of vehicles are uniformly distributed, so that $g_k(s) = 1/L_k$, this double integral can be easily evaluated to produce

$$P_k = \frac{\Delta \overline{U}_k(\tau)}{L_k} \tag{5}$$

Here $\overline{U}_k(\tau)$ denotes the space mean speed of vehicles in segment k at the beginning of τ. This connection between the exit probability for a segment and its space-mean speed implies that a stochastic version can be formulated for any traffic flow model that describes space-mean speed. More detailed discussion of the ideas underlying Equation 5 can be found elsewhere (*14,15*).

In STOMAC, the state variables are $z_{kj}(\tau)$, the number of vehicles in segment k destined for off-ramp j at the beginning of the subinterval, and $\overline{U}_k(\tau)$, the space-mean speed of the vehicles in segment k. Assume that the random arrivals at on-ramps follow Poisson distributions and that the random exits from segments follow binomial distributions. That is,

$$y_{kj}(\tau) = \text{binomial } [z_{kj}(\tau), \Delta \overline{U}_k(\tau)/L_k] \tag{6}$$

$$q_{ij}(\tau) = \text{Poisson } [b_{ij} * q_i(\tau)] \tag{7}$$

In each segment, the number of vehicles satisfies the conservation equation

$$z_{kj}(\tau + 1) = z_{kj}(\tau) + y_{k-1,j}(\tau) - y_{kj}(\tau) + \Sigma_i w_{ik} q_{ij}(\tau) \tag{8}$$

where $w_{ik} = 1$ if on-ramp i joins segment k, and 0 otherwise.

Finally, Payne's discretized momentum equation (*16*) describes the evolution of $\overline{U}_k(\tau)$,

$$\overline{U}_k(\tau + 1) = \overline{U}_k(\tau) + \Delta \overline{U}_k(\tau) \frac{\overline{U}_{k-1}(\tau) - \overline{U}_k(\tau)}{L_k} + \frac{\Delta}{\Gamma} \{\overline{U}_e[r_k(\tau)] - \overline{U}_k(\tau)\} - \nu \Delta \frac{d_k r_{k+1}(\tau) - r_k(\tau)}{L_k \Gamma [r_k(\tau) + \kappa M_k]} \tag{9}$$

where $d_k = M_k/M_{k+1}$ and Γ, κ, ν are momentum equation parameters, which generally must be estimated.

STOMAC can be used to generate a series of simulated on-ramp volumes, distribute these volumes to off-ramps, and then propagate these destination-specific subflows. These simulated data make it possible to investigate the statistical properties of estimators for the O-D parameters, **B**. FORTRAN source listings for STOMAC and other computer programs used in this study can be found elsewhere (*17*).

DESCRIPTION OF ESTIMATION APPROACHES

Ordinary Least Squares

As pointed out earlier, the problem of estimating freeway O-D patterns is analogous to the problem of estimating the turning movement proportions for single intersections, where methods based on OLS can give useful estimates. The basic idea behind this approach is that from the standpoint of the traffic manager, the actual destinations selected by the vehicles arriving at an on-ramp are unknown, and if all that is known are the O-D proportions b_{ij} and the arrival volumes $q_i(t)$, the O-D demands $x_{ij}(t)$ can be viewed as generated by multinomial random outcomes. Ignoring travel time lags, the expected values of the off-ramp volumes are then as given in Equation 1 and the OLS estimates of the b_{ij} are found by minimizing Equation 3. This problem is solved easily using standard linear regression software.

Expectation-Maximization

Under reasonably general conditions, maximum likelihood (ML) estimates tend to be asymptotically efficient, which suggests that ML estimates may be more effective in tracking time-varying O-D patterns. Since, under the linear model, the off-ramp counts are simply sums of independent multinomial outcomes, the likelihood function of the off-ramp counts is theoretically available; in practice, however, since it will have the form of a multidimensional convolution, it will be very difficult to compute. The expectation-maximization (EM) algorithm has been recommended for problems of this type (*18*), and its basic idea is as follows. If one were able to observe the individual O-D-specific traffic flows $x_{ij}(t)$, the ML estimator for the O-D parameters would simply be

$$\hat{b}_{ij} = \frac{\Sigma_t x_{ij}(t)}{\Sigma_t q_i(t)} \qquad i = 1, \ldots, m, j = 1, \ldots, n \tag{10}$$

The practical problem, however, is to estimate b_{ij} when no $x_{ij}(t)$ can be observed directly and only the entering counts $q_i(t)$ and the exiting counts $y_j(t)$ are known from the freeway surveillance and control systems. Note that since

$$y_j(t) = \Sigma_i x_{ij}(t) \qquad j = 1, \ldots, n$$

this is an incomplete data problem, in which the sufficient statistics $\Sigma_i x_{ij}(t)$ are not observed directly. The authors' implementation of the EM algorithm begins with an initial estimate $\mathbf{B}_o$ and then estimates the conditional expectations of the $x_{ij}(t)$ using normal distribution methods:

$$\Sigma_t x_{ij}(t) = E\{\Sigma_t x_{ij}(t) \mid \mathbf{B}, \mathbf{y}(t), t = 1, \ldots, N\} \tag{11}$$

The $\mathbf{B}_o$ is then reestimated by substituting $\Sigma_t x_{ij}(t)$ in Equation 10 for $\Sigma_t x_{ij}(t)$. The process iterates between Equations 10 and 11 until a convergence criterion is satisfied. For single intersections, where the travel time differences between each O-D pair can be ignored, this EM estimator tends to give O-D estimates with considerably less sampling variability than does the OLS estimator. More detailed presentation of the EM formulas can be found elsewhere (*3,10*).

Constrained Approximate Maximum Likelihood

The EM algorithm was based on a multinomial likelihood function but used a normal approximation for the probability distribution of the $x_{ij}(t)$. Alternatively, a normal approximation could be used for the $y_j(t)$ and approximate ML estimates could be computed via the resulting likelihood function. A description of this estimation approach, called constrained approximate ML (CAML), is presented in the following.

Given the on-ramp observations $\mathbf{q}(t)$, the expected value of $\mathbf{y}(t)$ is

$$\hat{\mathbf{y}}(t) = E[\mathbf{y}(t) \mid \mathbf{q}(t)] = \mathbf{B}^T * \mathbf{q}(t) \tag{12}$$

where the covariance matrix of $\hat{\mathbf{y}}(t)$ can be obtained as

$$\mathbf{V}(t) = \mathrm{cov}[\hat{\mathbf{y}}(t) \mid \mathbf{q}(t)] = \Sigma_i q_i(t)[\mathrm{diag}\{\mathbf{b}_i\} - \mathbf{b}_i * \mathbf{b}_i^T] \tag{13}$$

Since $\mathbf{y}(t)$ is the sum of multinomial random vectors, for large values of $q_i(t)$ it will be approximately normally distributed, with approximate likelihood function

$$L = \Pi_t[(2\pi)^{m-1} * \mid \mathbf{V}(t) \mid]^{-0.5} * \exp\{-0.5 * [\mathbf{y}(t) - \mathbf{B}^T * \mathbf{q}(t)]^T * \mathbf{V}^{-1}(t)[\mathbf{y}(t) - \mathbf{B}^T * \mathbf{q}(t)]\} \tag{14}$$

Taking the logarithm of Equation 14 and simplifying results in the final objective function: Minimize

$$LL = \Sigma_t\{\log \mid \mathbf{V}(t) + [\mathbf{y}(t) - \mathbf{B}^T * \mathbf{q}(t)]^T * \mathbf{V}^{-1}(t)[\mathbf{y}(t) - \mathbf{B}^T * \mathbf{q}(t)]\} \tag{15}$$

subject to the constraints of Equation 2.

The $\mathbf{B}$ matrix that solves this problem will be the CAML estimates for the O-D parameters. Both the EM and CAML estimators can be viewed as constrained quasi-ML methods in which the underlying data generation process is approximated by the simple linear assignment model. Thus they can be viewed as attempts to preserve the simplicity of the linear model on the assumption that inefficiency rather than bias is responsible for the poor performance of OLS in Table 1.

Nonlinear Least Squares

One of the major dissimilarities between traffic flow at a single intersection and that on freeways is that the travel times between each freeway O-D pair will vary depending on the intervening traffic conditions. The three estimation procedures described earlier achieve computational simplicity by ignoring this travel time variability. As a benchmark, it was desirable to have an estimator that was not subject to specification error but still optimized with respect to the least-squares criterion. This led to the following nonlinear least-squares (NLS) approach.

Given a current estimate of $\mathbf{B}$, forecasted off-ramp counts were computed by performing the STOMAC recursion with the Poisson and binomial random numbers replaced by their expected values. Forecasted off-ramp counts for the subintervals were aggregated to produce forecasted 5-min counts $\hat{y}_j(t)$, and the sum-of-square function was computed as

$$SS = \Sigma_t\Sigma_j [y_j(t) - \hat{y}_j(t)]^2 \tag{16}$$

A final **B** matrix that minimized Equation 16 was computed iteratively by the nonlinear optimization routine E04JAF, which is contained in the NAG Workstation Library. By comparing the performance of NLS with that of EM and CAML, it should be possible to separate the effects of specification error versus an inefficient optimization criterion on O-D estimator performance. Software implementing STOMAC and the four estimators was written in FORTRAN, and all computations were carried out on a Sun Sparcstation 1+.

EVALUATION AND COMPARISON OF PARAMETER ESTIMATORS

Generation of Simulated Data Sets

So that the statistical properties of these estimators could be evaluated under plausible conditions, it was decided to calibrate STOMAC to an existing section of freeway rather than to construct a hypothetical example. Figure 2 depicts a seven-origin, four-destination section of northbound Interstate I-35W in south Minneapolis, Minnesota. The section is 2.5 mi (4.2 km) long and has a somewhat complicated O-D pattern. A sequence of 36 five-minute counts was obtained from the Minnesota Department of Transportation (MNDOT) for a typical morning weekday peak period from 6:00 to 9:00 a.m.

For STOMAC to be used to simulate traffic flow on this freeway section, two sets of model parameters must be determined. The first set governs the traffic flow properties of the freeway and consists of estimates of capacity and free-flow speed, needed for the equilibrium speed-density relationship, and the momentum equation parameters Γ, κ, ν. The second set of parameters consists of the O-D proportions b_{ij}. For this example, a capacity of 2,000 vehicles per lane per hour and free-flow speed of 65 mph (108 km/hr) were used, and values for the momentum equation parameters were taken from the paper by Cremer and May (*19*). Given these values, the O-D proportions were estimated using the 5-min count data provided by MNDOT via the NLS procedure described earlier; as a rough check of the plausibility of this model, Figure 3 displays the actual 5-min traffic counts for the downstream boundary of this freeway segment, along with the predicted values obtained using the parameterized model. The predicted values track the actual ones reasonably well. These estimates were then used in STOMAC to generate 50 data sets, each consisting of a simulated 3-hr sequence of 5-min on- and off-ramp counts, with the mean value of the Poisson arrivals being set equal to the actual 5-min on-ramp counts.

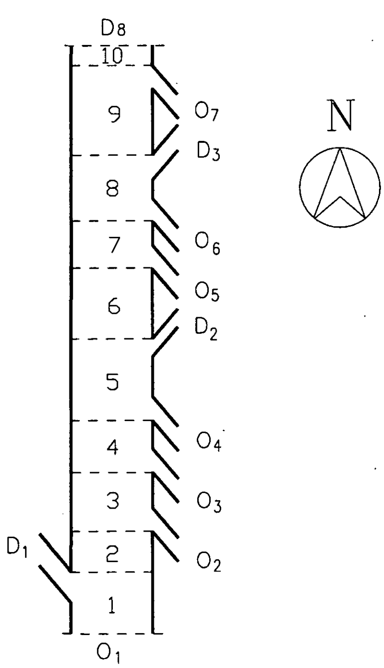

FIGURE 2 Freeway section with seven on-ramps and four off-ramps on I-35W, northbound.

Comparison and Evaluation of Results

By running each estimator mentioned earlier on each of the 50 data sets, the authors obtained samples of the estimators' behavior. From these samples, the sample means and standard deviations were computed in order to evaluate the statistical properties of unbiasedness and efficiency. Efficiency of an estimator is defined here in term of its sampling variance, which estimates the standard error. That is, the standard deviation of estimated parameters from the sample should be small in order for the estimator to be recognized as efficient. An unbiased estimator should be able to produce estimates that on the average equal the "true" parameter values. These results are displayed in Table 2. As an aggregate measure of the joint effect of bias and inefficiency, Table 2 also presents the root mean square (RMS) error between the true value and the estimates, which is computed by

$$RMS_{ij} = \sqrt{(\bar{b}_{ij} - b_{ij})^2 + var(\hat{b}_{ij})}$$

and can be interpreted as the average distance separating an estimate from the true value. The average CPU times needed to compute estimates for one data set are given here:

Estimator	*Average CPU Time (sec)*
OLS	0.18
NLS	2394.4
EM	60.1
CAML	67.0

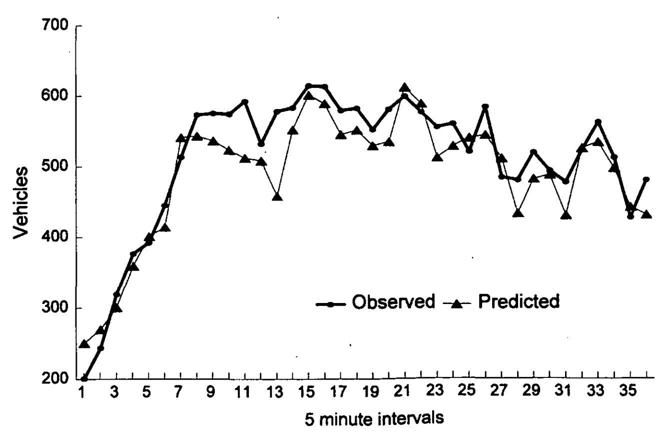

FIGURE 3 Comparison between observed and predicted mainline traffic volumes.

TABLE 2 Performance of Freeway O-D Parameter Estimators

	True	OLS Mean	OLS S.Dev	OLS RMS	NLS Mean	NLS S.Dev	NLS RMS
b11	.056	.055	.003	.003	.055	.004	.004
b12	.134	.133	.041	.041	.136	.022	.022
b13	.019	.025	.056	.056	.020	.030	.030
b14	.790	.671	.123	.171	.798	.042	.043
b22	.291	.388	.321	.335	.303	.055	.056
b23	.290	.250	.346	.348	.295	.080	.080
b24	.419	.993	.902	1.07	.409	.062	.063
b32	.223	.225	.055	.055	.234	.034	.036
b33	.148	.140	.047	.048	.145	.028	.028
b34	.629	.618	.146	.146	.619	.045	.046
b42	.194	.203	.129	.129	.195	.038	.038
b43	.240	.208	.138	.142	.232	.055	.056
b44	.566	.646	.354	.363	.574	.059	.060
b53	.263	.248	.077	.078	.260	.047	.047
b54	.737	.825	.191	.210	.743	.054	.054
b63	.284	.297	.376	.376	.281	.029	.029
b64	.716	.841	1.07	1.08	.715	.058	.058
		EM			**CAML**		
b11	.056	.056	.002	.002	.057	.005	.005
b12	.134	.176	.012	.044	.133	.031	.031
b13	.019	.137	.022	.120	.032	.031	.034
b14	.790	.630	.031	.163	.778	.043	.045
b22	.291	.193	.041	.106	.246	.121	.129
b23	.290	.167	.064	.139	.293	.134	.134
b24	.419	.639	.074	.232	.461	.151	.157
b32	.223	.182	.018	.045	.219	.043	.043
b33	.148	.134	.043	.045	.138	.041	.042
b34	.629	.685	.042	.070	.644	.043	.046
b42	.194	.201	.071	.071	.266	.158	.174
b43	.240	.220	.092	.094	.263	.138	.140
b44	.566	.579	.099	.100	.470	.181	.205
b53	.263	.105	.031	.161	.254	.054	.055
b54	.737	.895	.031	.161	.746	.054	.055
b63	.284	.123	.084	.182	.329	.175	.181
b64	.716	.877	.084	.182	.671	.175	.181

Table 2 indicates that the NLS estimator, for which specification error was not present, appears to be unbiased and is more efficient than the other approaches. Although the OLS approach seems to produce unbiased estimates, the high sampling variability makes even substantial differences between the sample average and the true value appear statistically insignificant. For practical purposes, the OLS estimates are essentially useless. For example, the "true" value of b_{44} is 0.566 and the mean and the standard deviation of OLS estimate are 0.646 and 0.354, respectively. The means that, appealing to the approximate normality of the OLS estimates, 95 percent of the estimates for b_{44} would fall in the interval [−0.62, 1.35], an interval that includes the true value; but, practically speaking, 100 percent of the estimates should fall in the interval [0,1].

For the EM estimator, the efficiency is comparable to NLS, but EM tended to give highly biased estimates, a tendency that Nihan and Davis also reported in their intersection study. One interesting finding is that the CAML estimator appears to be a useful compromise between the accurate, but computationally demanding, NLS procedure and the computationally simple, but inaccurate, OLS procedure. For the proportions corresponding to On-Ramps 1, 3, and 5, CAML has an efficiency of the same order of magnitude as NLS. This suggests that, at least for fairly short freeway segments, switching to an approximate ML approach can partly compensate for the effects of specification error.

The results given in Table 2 can also be interpreted as giving bounds on the expected accuracy of recursive, tracking algorithms. For example, after processing 3 hr worth of 5-min observations, one could expect an NLS estimate of b_{22} to be in the interval [0.29 − 0.11, 0.29 + 0.11] = [0.18, 0.40]. If b_{22} had changed during this period, then any estimate of these time-varying values, being based on fewer observations, would be less accurate than this.

CONCLUSIONS

This paper began by pointing out the importance of parameter estimation for ATMS practice, with attention to the fact that uncertainty in model parameter estimates will affect the effectiveness of control policies and their potential benefits. As with all simulation studies, these results should be considered illustrative rather than definitive. Certainly one can imagine constructing examples for which the outcome might be different. However, the simulation example used here was based on an existing section of freeway and on a traffic flow model that most would regard as plausible, if not conclusive, so there is good reason to expect that these results are more likely to be typical rather than anomalous.

Probably the most challenging aspect of these results is that no matter what O-D estimation procedure is used, a nontrivial amount of uncertainty concerning the actual parameter value will remain after processing 3 hr of data, and if the O-D parameters are in fact time-varying, this residual uncertainty will only increase. This calls into question the common practice of "certainty equivalent" prediction and control, in which parameter estimates are treated as known constants rather than as the uncertain quantities that they are and suggests that forecasting and control models that explicitly treat parameter uncertainty may improve on current practice.

ACKNOWLEDGMENTS

This research was supported in part by the Minnesota Department of Transportation and Center for Transportation Studies and in part by the National Science Foundation.

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All opinions and conclusions expressed here are solely the responsibilities of the authors.

Publication of this paper sponsored by Committee on Traffic Flow Theory and Characteristics.