Using Neural Networks To Synthesize Origin-Destination Flows in a Traffic Circle

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The traffic circle is a classic transportation problem for traffic engineers. Although it is easy to determine the volume of vehicles entering and exiting the circle at all points, it is difficult to determine the actual flow pattern of these vehicles. In other words, although it is easy to determine how many vehicles enter the circle from a given street, it is difficult to determine how many of those vehicles will leave the circle at each possible exit point. Currently, the only method of accurately determining this traffic flow is to visually track each vehicle as it enters and exits the circle, a laborious method of collecting data. However, emerging neural network technologies give researchers another approach. The capability of neural networks to handle subtle or contradictory information by organizing and capturing complex relationships, optimizing and generating analytical models, and learning and adapting the model when new data become available has made them increasingly popular in transportation and traffic flow models. The objective is to describe the development of a neural network model for generating origin-destination (O-D) information for traffic circles based on observed flow volumes on approaching and exiting legs. The quality of the model is evaluated with respect to the different methods used to train the model. Observations about the synthesized O-D matrices and the corresponding errors generated by the neural network model are also described.

The traffic circle is a classic transportation problem for traffic engineers. It may be easy to determine the volume of vehicles entering and exiting the circle at all points, but it is difficult to determine the actual flow pattern of the vehicles entering and exiting the circle. In other words, although it is easy to determine how many vehicles enter the circle from a given street, it is difficult to determine how many of those vehicles will leave the circle at each possible exit point. Currently, the only method of accurately determining this traffic flow is to visually track each vehicle as it enters and exits the circle, a laborious method of collecting data. However, emerging neural network technologies give researchers an alternative approach. The capability of neural networks to handle subtle or contradictory information by organizing and capturing complex relationships, optimizing and generating analytical models, and learning and adapting the model when new data become available has made them increasingly popular in transportation and traffic flow models.

The objective of this paper is to describe the development of a neural network model for generating origin-destination (O-D) information for traffic circles based on observed flow volumes on approaching and exiting legs. The major emphasis of this paper is to evaluate the quality of the model with respect to the different methods used to train the model. Observations regarding the synthesized O-D matrices and the corresponding errors generated by the neural network model are also described.

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NEURAL NETWORKS

Neural network models are algorithms for cognitive tasks, such as learning and optimization, that are loosely based on concepts derived from research into the nature of the brain. This paper adapts a formal definition of the neural network model as given by Müller and Reinhardt (*1*). In mathematical terms, a neural network model is defined as a directed graph with the following properties:

- A state variable s_i is associated with each node (neuron) i;
- A real-value weight w_{ij} (also known as coupling strength, synaptic strength, or synaptic efficacy) is associated with each link (synapse) (i,j) between two nodes i and j;
- A real-value bias (activation threshold) Φ_i is associated with each node (neuron) i; and
- A transfer function $f_i[s_i, w_{ij}, \Phi_i, (j \neq i)]$ or $f(\Sigma_j w_{ij} s_i - \Phi_i)$ is defined for node i that determines the state of the node as a function of its bias, the weights of its incoming links, and the states of the nodes connected to it by these links. The transfer function is often either a discontinuous step function or a smoothly increasing generalization known as a sigmoidal function.

Nodes without links leading toward them are called input nodes, and nodes without links leading away from them are called output nodes. A feed-forward network is a neural network that admits no closed path. A simple, multilayered, feed-forward neural network model is presented in Figure 1.

BACKPROPAGATION

Multilayered, feed-forward neural network models have recently been applied to many fields because of the development of an efficient method for determining the synaptic coupling strengths of such models. This method, called error backpropagation, is a supervised learning algorithm that iteratively adjusts the synaptic strengths w_{ij} so that the output signal differs as little as possible from the desired target. This is achieved by applying the gradient method, which yields the required modification Δw_{ij}. Since the operation of the network corresponds to a highly nonlinear mapping between the input and output—the transfer function is nonlinear—the method must be applied many times until convergence is reached.

Before training, initial synaptic strengths are applied to all node connections, and activation thresholds, which change over the training process, are set for each node. A global activation function calculates the output value of each node as the sum of the synaptic strengths multiplied by the corresponding values of the previous layer's nodes. An error function is defined that is the sum of the squares of the difference between the desired output and the actual

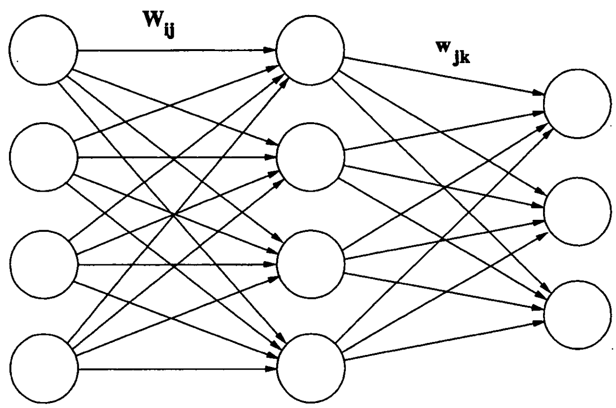

FIGURE 1 Example of multilayer, feed-forward neural network model.

output from the network. The backpropagation algorithm prescribes a method to minimize this error function by taking gradient with respect to the synaptic strengths and decides on the amount of incremental strength adjustment. Training is a series of running input-output pairs over the network and making incremental adjustments to the synaptic strength values. The backpropagation mechanism is described in more detail by Hertz et al. (*2*) and Müller and Reinhardt (*1*).

PREVIOUS RESEARCH

Yang et al. (*3*) have adopted a feed-forward neural network model for synthesizing O-D flows for both a four-way intersection and a short freeway segment. A two-layered, feed-forward neural network was built to model a four-way intersection. This network has four nodes in the input layer for modeling the entrances and four nodes in the output layer for modeling exits. A sigmoidal function serves as the transfer function. The optimization on a squared-error function was based on the error backpropagation method. After the training is completed, the weights of the connections from the input to the output layers are interpreted as the turning movement ratios. On the basis of the training data, the trained weights essentially summarize the traffic coming into and going out from the intersection in terms of ratios. From simulation results, the Yang et al. model has shown that a method based on backpropagation can estimate turning movement ratios with high tracking ability and stability.

NEURAL NETWORK TRAFFIC CIRCLE O-D MODEL

Physical Network

The network modeled in this study, Church Circle, is a traffic circle in the historic district of Annapolis, Maryland. Church Circle is the primary focal point for traffic entering and leaving downtown Annapolis, so large volumes of traffic flow through it. The circle connects to eight streets: College Avenue, School Street, Main Street, Duke of Gloucester Street, South Street, Franklin Street, West Street, and Northwest Street (Figure 2). Main, Franklin, West, School, and Northwest Streets and College Avenue contain lanes that enter the circle; Duke of Gloucester, South, Franklin, West, and School Streets and College Avenue contain lanes that exit the circle. It should also be noted that Main, Northwest, South, and Duke of Gloucester are one-way streets.

Traffic Flow Data

The data used for this model were extracted from an O-D license plate survey conducted during morning (7:00 to 9:00 a.m.), noon (11:30 a.m. to 2:30 p.m.), and afternoon (3:30 to 6:00 p.m.) peak periods to provide actual O-D volumes. These O-D traffic volumes were collected for 15-min intervals. Therefore, the data are comprised of 8 sets of O-D matrices for the morning peak period, 10 sets for the noon peak period, and 10 sets for the afternoon peak period.

On the basis of the data collected from the Church Circle site, the traffic patterns were significantly different for the morning, noon, and afternoon periods (Figure 3). In the morning, the O-D volume from Main to College was the highest, and the O-D traffic from Main to West was the second highest. Traffic volumes from all origins to Duke of Gloucester were significantly higher than those to South, Franklin, and School. During the noon period, the two highest O-D volumes were from School to College and from West to Duke of Gloucester. This is significantly different from the morning traffic pattern. On the other hand, traffic volumes from all origins to Duke of Gloucester are higher than during the morning period, but they are still significantly higher than those from all origins to South, Franklin, and School. During the afternoon period, the two highest O-D volumes were from School to College and from School to West. However, the overall traffic pattern is similar to that of the noon period except that Duke of Gloucester ceases to be a significant destination in the afternoon.

The standard deviations for the actual traffic flow during each of the three periods are shown in Figure 4. The standard deviations were calculated so that their effects with regard to the synthesized results could be determined. These standard deviations were calculated as follows:

$$Std\ dev_{odp} = \sqrt{\frac{\Sigma_{l=1,Lp}\,(t_{odl} - \bar{t}_{od})^2}{L_p}} \qquad (1)$$

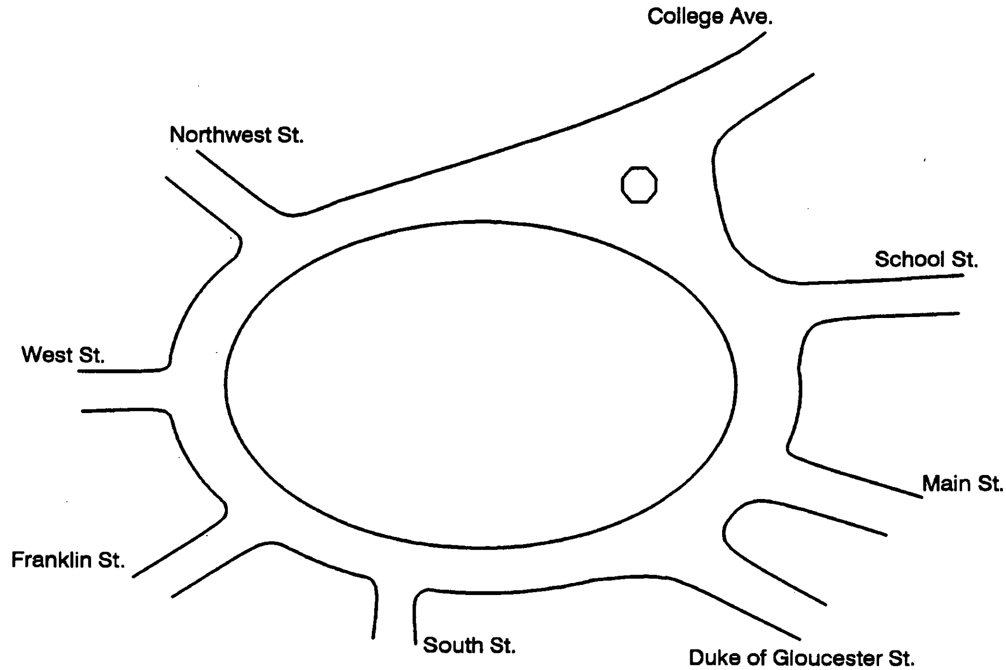

FIGURE 2 Church Circle in Annapolis, Md.

where

o = origin street,
d = destination street,
p = period (morning, noon, afternoon),
l = 15-min interval (1–8 for morning, 1–10 for noon and afternoon),
t = traffic volume, and
L_p = number of 15-min intervals in period p.

The pooled total standard deviation for a period p is defined as follows:

$$Pooled\ total\ std\ dev_p = \sqrt{\frac{\Sigma_{o=1,6}\ \Sigma_{d=1,6}\ std\ dev^2_{odp}}{6 \times 6}} \qquad (2)$$

The pooled total standard deviations based on actual O-D traffic flows were 10.64, 13.88, and 9.20 for morning, noon, and afternoon periods, respectively.

Model Formulation

A multilayer, feed-forward neural network model was formulated to synthesize the O-D matrix based on the traffic entering and exiting the circle (Figure 5). Prior experiences with neural networks and other documented sources have indicated that feed-forward neural networks with multiple hidden layers and many nodes do not necessarily produce better results. Models with one, two, and three hidden layers with 5 nodes were tested, as was a model with one hidden layer with 20 nodes. There was no significant difference in the results produced by these models. Thus, for the sake of simplicity, a basic three-layered model with a single five-node hidden layer was used. The primary reason for using the model presented in Figure 5 is that such a model has the potential to "learn" from rigorous "training" and to synthesize O-D traffic flows for any traffic circle intelligently.

There are two significant differences between the model presented in this paper and the model of Yang et al. (3). The first is the physical network to which each model is applied. The Yang et al. model synthesizes traffic turning movements for an intersection, whereas the model presented in this paper attempts to synthesize O-D traffic flows for a traffic circle. The traffic flow for the traffic circle is much more complex than the traffic flows for a typical four-way intersection, and the effort required to manually determine these movements is much more involved. The second difference is in the configuration of the neural network model itself. The Yang et al. model uses a two-layer network with no hidden layer. The entering counts are fed into the input layer, and the exiting counts are fed into the output layer. The resulting synaptic strengths between these layers are the turning movement ratios. Thus, Yang et al. merely use the neural network framework to model the intersection traffic turning movements. The neural network model based on field data is applicable only for that particular set of data, and the backpropagation algorithm would have to be used to reestimate the synaptic strengths every time a new set of traffic volumes was collected.

The model presented in this paper uses the neural network in the "conventional" manner. It has a hidden layer. The entering and exiting traffic volumes are fed into the input layer and the desired, or target, O-D matrix is fed into the output layer to adjust the synaptic strengths (i.e., train the model). There is no constraint on the synaptic strengths associated with each connection, and all synaptic strengths estimated by the backpropagation algorithm are retained as an integral part of the neural network model. After training, entering and exiting volumes can be input into the model to generate an O-D matrix. Using this modeling approach, the backpropagation algorithm does not have to be reapplied to estimate the synaptic strengths for the model every time a new set of traffic volumes is collected.

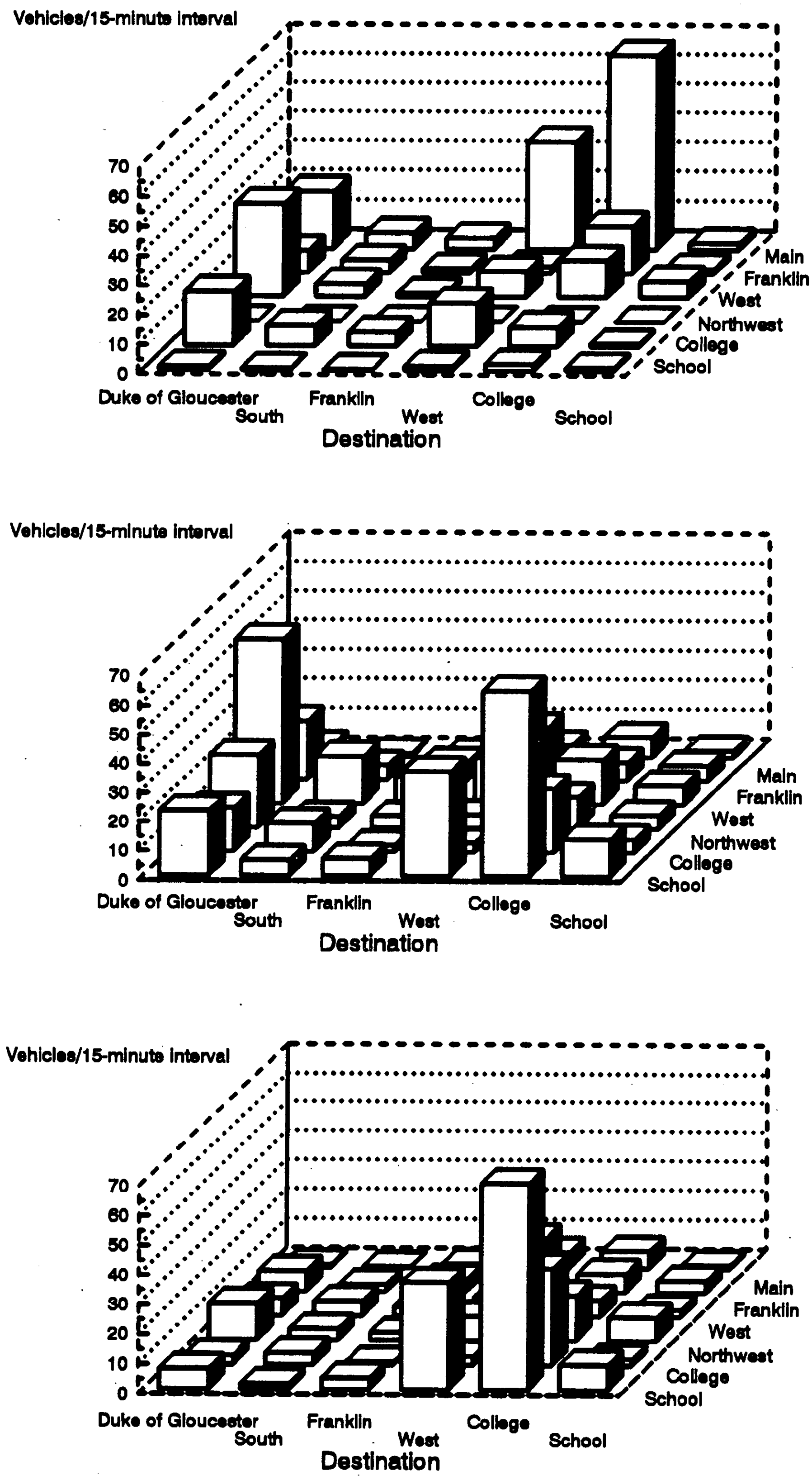

FIGURE 3 Average O-D volumes for morning (*top*), noon (*middle*), and afternoon (*bottom*) peak periods.

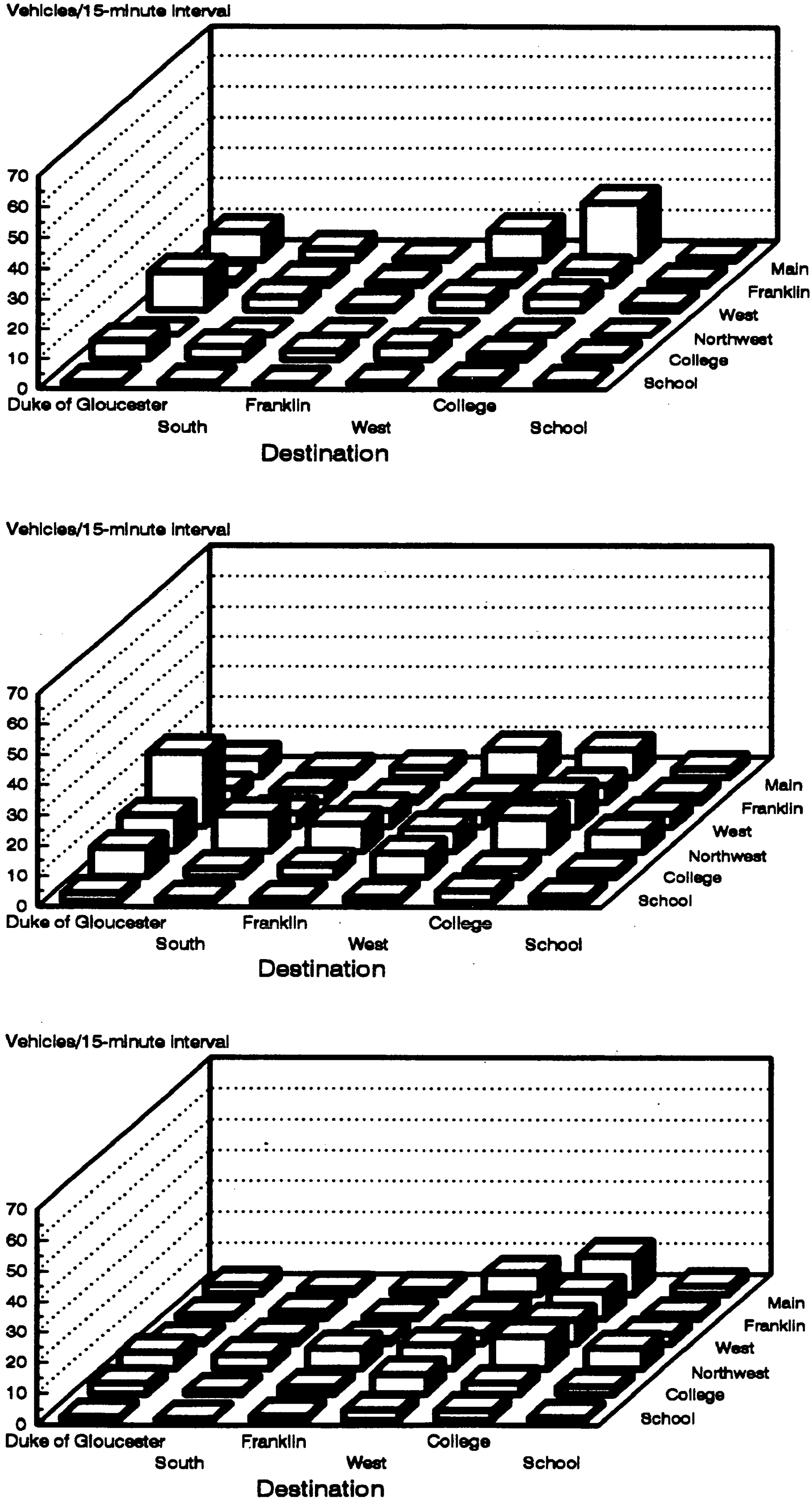

FIGURE 4 Standard deviations for morning (*top*), noon (*middle*), and afternoon (*bottom*) volumes.

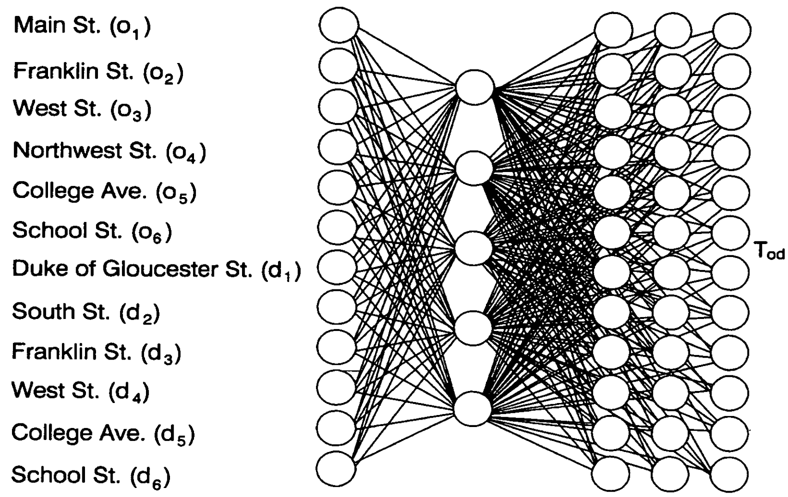

FIGURE 5 Neural network O-D flow model for Church Circle.

EXPERIMENTAL PROCEDURE

The experimental procedures for all of the trials described in this paper consist of the following three steps:

1. *Training the model.* The training set was extracted from the O-D license plate survey. This was accomplished by using the traffic volumes from the approaching and exiting streets as the input and the O-D volumes as the target output for each corresponding time interval. A time-interval parameter was included so that the model could develop a relationship between time periods and traffic patterns.
2. *Synthesizing O-D matrices.* The traffic volumes for approaching and exiting streets were used as input for the model to synthesize an O-D matrix for each corresponding time interval. As in training, the time interval for each set of volumes was specified so that it could be considered by the model.
3. *Evaluating the synthesized O-D matrices.* The synthesized O-D matrices produced by the model were evaluated by comparing them to the actual O-D matrices determined by the survey.

However, rather than comparing the synthesized and actual matrices for each time interval, the mean value for each O-D pair was calculated across all time periods. Mathematically, the mean of each O-D matrix was calculated using the following equation:

$$\bar{t}_{odp} = \Sigma_{l=1,L_p} \frac{t_{odl}}{L_p} \tag{3}$$

The averaged actual O-D matrices for the morning, noon, and afternoon periods were also derived using similar formulas. Then, the mean synthesized O-D flows were plotted against the averaged actual O-D flows for each corresponding cell with the matrix for morning, noon, and afternoon periods.

The results of the different trials are presented in Figures 6 through 9. These diagrams depict goodness-of-fit measures used to evaluate the neural network model's capability to synthesize O-D matrices based on the traffic flows from entering and exiting streets. If the neural network model can "learn" the O-D travel pattern on the basis of such a small data set, then the synthesized O-D flows should be close to the actual O-D flows. If the synthesized O-D flows are close to the actual O-D flows, then the points on these diagrams should be very close to the diagonal line. Thus, a diagram depicting the results of a good neural network model will have all or most points on or close to the diagonal line.

Two other goodness-of-fit measures are used to evaluate the synthesized O-D matrices generated by the model: mean absolute error and mean absolute average error. The formulations for these measures are described here:

$$\textit{Mean absolute error}_p = \frac{\Sigma_{o=1,6}\,\Sigma_{d=1,6}\,\Sigma_{l=1,L_p}\,|\hat{t}_{odl} - t_{odl}|}{36 \times L_p} \tag{4}$$

$$\textit{Mean absolute average error}_p = \frac{\Sigma_{o=1,6}\,\Sigma_{d=1,6}\left|\dfrac{\Sigma_{l=1,L_p}\hat{t}_{odl}}{L_p} - \dfrac{\Sigma_{l=1,L_p} t_{odl}}{L_p}\right|}{36} \tag{5}$$

where $\hat{t}_{odl}$ is the synthesized traffic from origin street o to destination street d during 15-min interval l, and t_{odl} is the actual traffic from origin street o to destination street d during 15-min interval l.

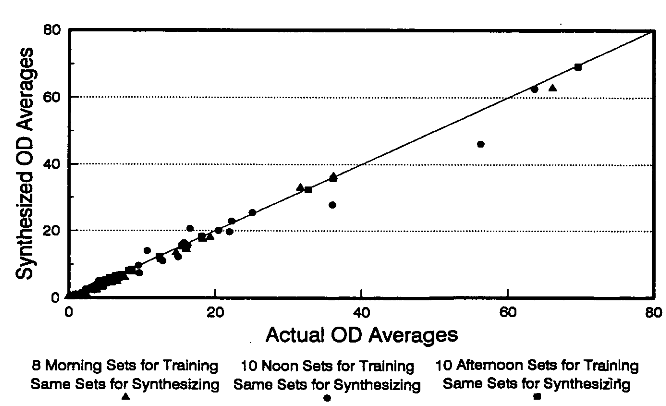

FIGURE 6 Comparison of O-D volumes synthesized by Method 1 and actual O-D volumes.

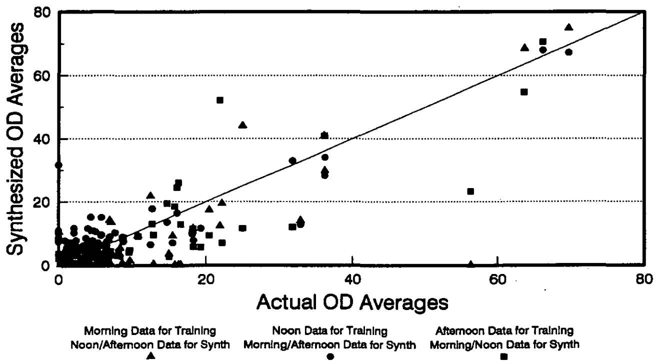

FIGURE 7 Comparison of O-D volumes synthesized by Method 2 and actual O-D volumes.

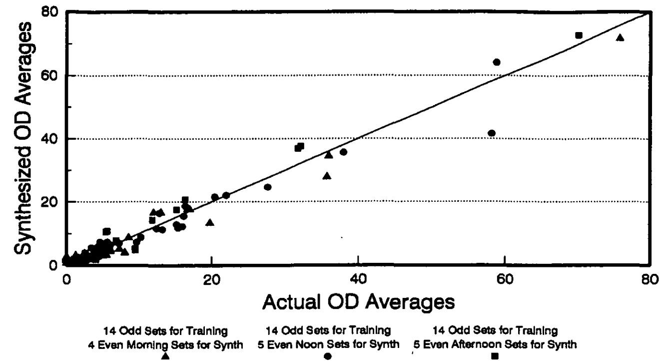

FIGURE 9 Comparison of O-D volumes synthesized by Method 4 and actual O-D volumes.

Training Model and Synthesizing O-D Matrices

Using the backpropagation method, various training methods were tested. O-D matrices were generated for each training method used. The various training schemes are described in the following sections.

Method 1

In Method 1, all traffic volume sets and all O-D matrices for each time period were used to train the model, and the same volume sets were used to synthesize O-D matrices.

In Trial 1, eight sets of morning traffic volume data (one set for each 15-min interval) from six entering streets and six exiting streets were used as input, and 36 O-D volumes were used as the output to train the model. The same eight sets of morning traffic volume data were input into the model to generate eight 36-element O-D matrices. These eight synthesized O-D matrices were then compared with the actual origin and destination traffic flows. This training scheme was used merely to generate some goodness-of-fit measures for the developed neural network model. Since the data used as input to synthesize O-D matrices were identical to those used to train the model, it is possible that the model "memorized" the training data and provided biased results.

Trial 2 was performed similarly to Trial 1, except that the noon traffic volume data were used to train the model and synthesize O-D matrices. The same procedure was followed in Trial 3 using afternoon training data and input.

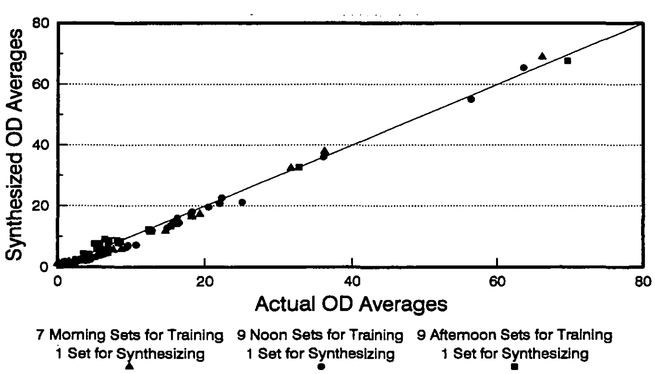

FIGURE 8 Comparison of O-D volumes synthesized by Method 3 and actual O-D volumes.

Method 2

In Method 2, traffic volume and O-D volume data from one time period were used to train the model, and traffic volume data from the other two time periods were used to synthesize O-D matrices for those periods.

For Trial 1, eight sets of morning traffic volume data (one set for each 15-min interval) from six entering streets and six exiting streets were used as input, and eight sets of 36-element O-D matrices were used as the output to train the model. The 10 sets of noon and afternoon traffic volumes (20 sets in all) were used as input for the model trained with morning traffic information. This input was used to synthesize ten 36-element O-D matrices for the noon peak period and ten for the afternoon period.

Trial 2 was conducted similarly to Trial 1, except that noon traffic volumes and O-D matrices were used to train the model, and morning and afternoon volumes were used to synthesize O-D matrices. In Trial 3, afternoon traffic volumes and O-D matrices were used to train the model, and morning and noon volumes were used to synthesize O-D matrices.

Method 3

In Method 3, all but one set of traffic volumes and flow matrices from one peak period were used to train the model, and the remaining set of traffic volumes were used to synthesize O-D matrices for that period.

For Trial 1, seven of the eight sets of morning volumes were used as input and the corresponding seven O-D matrices were used as output to train the model. The remaining set of morning volumes was used as input for the model to synthesize a 36-element O-D matrix.

Trial 2 was conducted similarly to Trial 1, except that nine sets of noon volumes and their corresponding matrices were used to train the model, and the remaining set of noon volumes was used as input for the model to synthesize a 36-element O-D matrix. In Trial 3, nine sets of afternoon volumes and their corresponding matrices were used to train the model, and the remaining set of afternoon volumes was used as input for the model to synthesize a 36-element O-D matrix.

This training scheme allows the model output to be compared with actual data not used to train the model. Thus, goodness-of-fit measures can be developed, ensuring that the output was not the result of the network memorizing the training set. This procedure was repeated using a different combination of sets to train the model and synthesize an O-D matrix each time. Since the developed neural network model used different information in estimating the model parameters, there is little chance that the model "memorized" the training data; thus, the model results should be unbiased.

Method 4

In Method 4, the odd sets of morning, noon, and afternoon traffic volumes and flow matrices were used to train the model, and the even sets of morning, noon, and afternoon traffic volumes were used to synthesize morning, noon, and afternoon flow matrices.

To really evaluate the ability of the neural network model to synthesize O-D traffic flows based on general traffic conditions, a training scheme was designed that would not isolate data from each time period. Under this scheme, all 28 sets of traffic volumes were grouped together and numbered from 1 to 28 depending on the time that the data were collected. Of these 28 sets, the 14 odd-numbered sets were used as input and the 36 O-D traffic volumes were used as target output to train the model. The remaining fourteen sets were used as input for the model to synthesize fourteen 36-element O-D matrices. These 14 synthesized matrices were then compared with the actual O-D traffic flows for morning, noon, and afternoon separately. This training scheme allows the model output to be compared with actual data not used to train the model. Thus, goodness-of-fit measures can be developed, ensuring that the output is not the result of the network memorizing the training set. Since the developed neural network model used different information for training the model, there is little chance that the model "memorized" the training data; thus, the model results should be unbiased.

PRELIMINARY RESULTS AND CONCLUSIONS

On the basis of the results presented in Table 1 and Figures 6 through 9, the following preliminary conclusions have been reached:

- The backpropagation technique was used to estimate the weights w_{ij} used in all transfer functions $f_i[s_i, w_{ij}, \Phi_i, (j \neq i)]$ or $f(\Sigma_j w_{ij} n_j - \Phi_i)$. The synthesized O-D traffic volumes were compared with the actual O-D traffic volumes. As shown in Figure 6 and Table 1, the synthesized O-D volume averages are very close to the actual averages (all points lie very close to the diagonal line). This indicates that the backpropagation technique did find a set of weights w_{ij} used in the applied transfer functions such that the discrepancies between the synthesized data and actual data have been minimized to a satisfactory level. From the results presented in Figure 6 and Table 1, the backpropagation technique generated good neural network models for morning, noon, and afternoon periods for average origin and destination traffic flows.
- As Figure 7 and Table 1 indicate, the errors induced by the second method are much larger than those generated by the other three methods. However, these errors were expected since the traffic patterns are quite different among the three periods. As these results demonstrate, a neural network model cannot produce acceptable results for situations—in this case, variations in traffic patterns—for which it has not been trained. Consequently, a model trained with traffic data from one period cannot be used to synthesize O-D volumes for other periods. This is also true for modeling other traffic circles. If the traffic circle has flow or network configuration characteristics that the model has not "seen," it theoretically cannot produce acceptable results.
- The errors generated by Method 3 are only slightly larger than those generated by the first method. From the results presented in Figures 6 and 8, it can be concluded that the neural network models do not particularly "memorize" patterns in the training data sets. In Method 3, 28 similar neural network models were developed. However, the synthesized O-D traffic volumes from these 28 neural network models were grouped and averaged for morning, noon, and afternoon periods. The average synthesized O-D volumes were very close to the actual O-D traffic volume averages.
- Figure 9 and Table 1 contain results generated by Method 4. The O-D volumes synthesized by the fourth method were grouped and averaged according to time period and were compared with the corresponding actual average O-D volumes. The errors generated by Method 4 are greater than those generated by the first and third

TABLE 1 Mean Absolute Error and Mean Absolute Average Error for Four Training Methods

Training Method	Time Period Used: For Training	Time Period Used: For Synthesizing Matrices	Error: Mean Absolute	Error: Mean Absolute Average
Method 1	Morning	Morning	2.94	0.93
	Noon	Noon	5.09	1.46
	Afternoon	Afternoon	2.62	0.19
Method 2	Morning	Noon	10.99	7.32
		Afternoon	5.18	4.03
	Noon	Morning	7.32	5.45
		Afternoon	5.32	3.72
	Afternoon	Morning	4.72	3.39
		Noon	9.24	5.79
Method 3	Morning	Morning	3.81	1.25
	Noon	Noon	6.64	1.54
	Afternoon	Afternoon	4.10	0.75
Method 4	Morning	Morning	3.41	1.92
	Noon	Noon	7.33	2.04
	Afternoon	Afternoon	3.37	1.70

methods, but these results are still quite acceptable. This indicates that a neural network model having the general form presented in Figure 5 is able to recognize variations among the morning, noon, and afternoon O-D traffic flows.

- According to the data in Table 1, the neural network models presented in this paper generated larger errors during the noon period. In other words, the neural network models were consistently less accurate in synthesizing O-D volumes for the noon period than for the morning and afternoon periods. This is probably due to the fact that the O-D traffic flow patterns have more variations during the noon period (as demonstrated earlier in this paper) than during the morning and afternoon periods, which consist mostly of work-related traffic.
- Figure 9 clearly indicates that a neural network model having the general form presented in Figure 5 can recognize traffic pattern variations among the morning, noon, and afternoon peak periods. Thus, the final conclusion is that such a neural network model can synthesize adequate O-D traffic volumes as long as the model has been trained with O-D traffic volumes that cover all the anticipated traffic patterns. In other words, as long as the model has "seen" similar data sets, it can recognize variations in traffic patterns and synthesize reasonable O-D traffic volumes.

SUMMARY

The neural network model developed using the backpropagation method can be used to synthesize O-D traffic flows for traffic circles. However, this conclusion has been reached by analyzing results of a few neural network models based on traffic data collected from one traffic circle for 7 hr. More data from this and other traffic circles are needed to verify further the findings of this study.

It should be noted that although this study concentrated on synthesizing O-D flows for traffic circles, the proposed model formulation easily could apply to synthesizing O-D information on linear freeway sections where on-ramp, off-ramp, and link traffic volumes are readily available. The model formulation could also apply to synthesizing O-D matrices for an urban street network, on the basis of the observed link traffic volumes. The effectiveness of neural network models in synthesizing O-D matrices based on the observed link traffic volume, however, might depend on the geometry of the network and the availability of link traffic volumes.

Traffic engineers have two ways to apply similar neural network models to synthesize O-D flows based on traffic volumes from entering and exiting streets. One is the procedure presented in this summary. Traffic engineers could collect actual O-D traffic flow information for morning, noon, and afternoon periods. Then a single model for all periods or multiple neural network models for individual periods could be generated. Next, any future O-D traffic flow data could be synthesized on the basis of traffic flow information collected at the entrances and exits of a traffic circle.

Neural network models can be used to synthesize origin and destination information for traffic circles in a second way. As discussed, neural network models are unable to synthesize traffic flow data for patterns for which they have not been trained. Also, one combined model can adequately synthesize O-D traffic flow as long as the model has "seen" the given pattern before. Thus, it is conceivable that one can devise a procedure that simulates a set of O-D traffic flow conditions that will cover all actual traffic patterns. The neural network model based on training using such simulated data should be able to synthesize O-D traffic flows as long as the simulated training data sets cover actual traffic patterns. Thus, the proposed neural network model should be able to "learn" traffic circle traffic patterns based on the simulated information and reduce the data collection task. If the model could "learn" the "rules" from simulated data and make inferences about actual information collected from streets, the neural network model presented in this paper would be an actual intelligent model.

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Publication of this paper sponsored by Committee on Traffic Flow Theory and Characteristics.