Estimating Destination-Specific Traffic Densities on Urban Freeways for Advanced Traffic Management

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A continuous-time Markov compartment model of freeway traffic flow is presented and tested using simulated and real data. By using the method of large population approximation, the underlying stochastic process is approximated by the sum of a nonlinear deterministic process and a linear, time-varying Gaussian stochastic process. With this approximation a Kalman filter that tracks the density of a freeway section, broken down by destination, was derived. The filter was then tested using simulated data and actual freeway data obtained from Interstate 35W.

Advanced traffic management systems (ATMS) seek to combine an understanding of traveler route selection with improved real-time monitoring of traffic networks in order to alleviate the effects of traffic congestion without requiring substantial new roadway capacity. In particular, driver information and route guidance systems attempt to maximize existing roadway capacity by informing drivers of under- and overused routes or of temporary reductions in capacity.

The effective use of route guidance and driver information, however, requires the ability to forecast driver reactions, their tendencies to select new routes, departure times, modes, and so on in response to information; a number of researchers have developed models aimed at producing such forecasts. In principle, route diversion can be forecast using route selection models common in traffic assignment, but unlike traditional traffic assignment, for short-term (i.e., within-peak) forecasts, it generally will be insufficient to know the origin-destination (O-D) pattern of the traveling public. This is because a substantial component of the traffic, say, 15 min into the future will be composed of vehicles that were already en route when the information or guidance was made available.

Since route selection behavior depends on the particular origin and destination between which a driver is traveling, real-time diversion forecasting will require knowing the breakdown, by O-D pair, of the number of vehicles on each link of a roadway network. A simplification occurs when drivers can be assumed to follow a Markovian routing rule, in which one's future path depends only on one's destination and current location in the network. This condition occurs when modeling simple freeway sections or when route choice follows a logit assignment principle. In this case, knowing the number of vehicles and their distribution across destinations on each link of the road network is necessary for forecasting future route selection activities (*1,2*). For demand forecasting purposes, a vector containing these destination-specific vehicle counts can be considered the state of the traffic system.

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Unfortunately, almost all traffic sensors provide data, such as the traffic volumes and lane occupancies provided by magnetic loop detectors, that are aggregated across the network's O-D-specific subflows, so that the traffic state must be estimated rather than measured directly. This is a filtering problem, which can be solved using the results of modern systems theory if one has at hand a unified, real-time model of traffic flow and assignment. Such models can be constructed using a class of stochastic process models called Markov compartment models (*1,3*). This paper describes the development and testing of such a model for traffic flow on a segment of urban freeway.

MARKOV COMPARTMENT MODEL OF FREEWAY TRAFFIC FLOW

A compartmental system is defined as "a system which is made up of a finite number of macroscopic subsystems, called compartments, each of which is well mixed, and the compartments interact by exchanging materials. There may be inputs from the environment into one or more of the compartments, and there may be outputs from one or more of the compartments into the environment" (*4*). Karmeshu and Pathria (*5*) proposed a Markov compartment model for highway traffic and provided an asymptotic analysis using a diffusion approximation. Here the material is composed of vehicles, and the stochastic nature of material transfer is caused by the random movement of vehicles according to a continuous-time Markov process. Now imagine that a segment of freeway has been divided into sections, such that on-ramps join the freeway only at the upstream boundaries of sections, off-ramps diverge from the freeway only at the downstream boundaries of sections, and mainline detectors are located at the downstream boundaries of sections. In addition, the number of lanes, grade, and other geometric characteristics are constant within the section.

Assume that the freeway has m origins, indexed by $i = 1, \ldots, m$; s destinations, indexed by $j = 1, \ldots, s$; and n sections, indexed by $k = 1, \ldots, n$. By convention, origin 1 is taken to be the upstream mainline boundary of the original freeway segment, while destination s is taken to be the downstream mainline boundary. Next, define the following variables:

$x_{oi}(t)$ = total remaining vehicles at origin i at time t,
$x_{dj}(t)$ = total vehicles that have exited the segment at destination j by time t,
$x_{kj}(t)$ = vehicles in section k destined for j at time t,
$y_l(t)$ = total vehicles counted at counter l up to time t.

Assume that the total number of vehicles in the system is fixed, so that

$$N = \Sigma_i x_{oi}(t) + \Sigma_k \Sigma_j x_{kj}(t) + \Sigma_j x_{dj}(t)$$

is constant at all times t. Let

$$\mathbf{x}(t)\ [x_{o1}(t), \ldots, x_{om}(t), x_{11}(t), x_{12}(t), \ldots, x_{ns}(t), x_{d1}(t), \ldots, x_{ds}(t)]^T$$

be a column vector containing the various compartment populations, and

$$\mathbf{y}(t) = [y_1(t), \ldots, y_p(t)]^T$$

be a column vector containing the count totals. Letting $\mathbf{e}_g$ denote a column vector with all elements equal to 0 except for position g, which is 1, and letting g, h index arbitrary elements of the vector $\mathbf{x}$, it will be assumed that over a very short time interval of length Δ, transitions of the form

$$\begin{bmatrix} \mathbf{x}(t+\Delta) \\ \mathbf{y}(t+\Delta) \end{bmatrix} - \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{y}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{e}_h - \mathbf{e}_g \\ \mathbf{H}\mathbf{e}_g \end{bmatrix} \tag{1}$$

occur with probability $x_g q_{g,h}[\mathbf{x}(t)]\Delta + o(\Delta)$, transitions with

$$[\mathbf{x}(t+\Delta)^T, \mathbf{y}(t+\Delta)^T]^T - [\mathbf{x}(t)^T, \mathbf{y}(t)^T]^T = \mathbf{0}$$

occur with probability $1 - \Sigma_{h \neq g} x_g q_{g,h}[\mathbf{x}(t)]\Delta + o(\Delta)$, and all other transitions have a probability that is $o(\Delta)$. Note that a $\mathbf{x}(t+\Delta) - \mathbf{x}(t) = \mathbf{e}_h - \mathbf{e}_g$ corresponds to the transition of a vehicle from compartment g to compartment h. By defining

$$\mathbf{H}_{lg} \begin{cases} = 1 \text{ if counter } l \text{ registers departure from } g \\ = 0 \text{ otherwise} \end{cases}$$

$\mathbf{y}(t+\Delta) - \mathbf{y}(t) = \mathbf{H}\mathbf{e}_g$ corresponds to an increment in the counter registering departures from g. The vehicle movements follow a closed, continuous-time Markov compartment model (or, equivalently, a nonlinear birth and death process), with the state vector augmented to include vehicle counts. The problem then is to use the counts at time t to produce estimates of the unobserved segment populations $x_{kj}(t)$. When the transition intensities $q_{g,h}[\mathbf{x}(t)]$ are not constant, the resulting filtering problem will be nonlinear and often intractable. Fortunately, given reasonable conditions on the functions $q_{g,h}[\mathbf{x}(t)]$, Lehoczky's argument (*6*) can be adapted to this case to show that as N, the total number of vehicles in the system, becomes large, the stochastic evolution of the random vectors $[\mathbf{x}(t)^T, \mathbf{y}(t)^T]^T$ can be approximated by the sum of a nonlinear deterministic process and a linear, time-varying Gaussian stochastic process. That is,

$$\begin{bmatrix} \mathbf{x}(t) \\ \mathbf{y}(t) \end{bmatrix} \approx \begin{bmatrix} \bar{\mathbf{x}}(t) \\ \bar{\mathbf{y}}(t) \end{bmatrix} + \mathbf{z}(t) \tag{2}$$

where the deterministic, mean value process satisfies the ordinary differential equation

$$\frac{d\bar{x}_g(t)}{dt} = \Sigma_h \bar{x}_h(t) q_{h,g}[\bar{\mathbf{x}}(t)]$$
$$\frac{d\bar{y}_l(t)}{dt} = \Sigma_g H_{lg} \bar{x}_g(t) \Sigma_{u \neq g} q_{g,u}[\bar{\mathbf{x}}(t)] \tag{3}$$

and $\mathbf{z}(t)$ is a zero-mean, Gaussian random vector with covariance matrix $\mathbf{P}(t)$, which evolves according to the Ricatti equation

$$\frac{d\mathbf{P}(t)}{dt} = F[\bar{x}(t)]\, P(t) + P(t)F[\bar{x}(t)]^T + G[\bar{x}(t)] \tag{4}$$

Here $\mathbf{F}(.)$ denotes the Jacobian matrix of the right-hand side of Equation 3 with respect to $\mathbf{x}(t)$, while $\mathbf{G}(.)$ is a covariance term that depends only on $\mathbf{x}(t)$.

Given initial estimates $\mathbf{x}(0)$, $\mathbf{P}(0)$, $\mathbf{y}(0) = \mathbf{0}$, Equations 3 and 4 can be solved numerically to give approximate expected compartment totals and cumulative counts, along with variances and covariances for any future time t. When actual counts become available at some time T_k, the standard formulas for the Kalman filter (*7*) can be used to give a measurement update of compartment totals and their covariance terms. Equations 2 and 3 can then be restarted with $\mathbf{x}(0) = \mathbf{x}(T_k)$, $\mathbf{P}(0) = \mathbf{P}(T_k)$ and $\mathbf{y}(0) = \mathbf{0}$, and the recursion continued until the next count becomes available.

DETERMINING TRANSITION RATES

Implementation of the filter requires that appropriate functions are selected for the transition intensities. For the transitions from the origin sources to mainline sections, it is convenient to use transition intensities of the form $q_{oi}b_{ij}$, where q_{oi} equals the constant arrival intensity from on-ramp i, and b_{ij} is the probability that a vehicle is destined for off-ramp j, given it arrives at on-ramp i.

If the origin populations $x_{oi}(t)$ are large enough so that the number of total arrivals during the time period of interest is a small proportion of the original total, the quantity $x_{oi}(t)q_{oi}$ can be taken as a constant λ_{oi}, giving Poisson arrival rates at the freeway origins.

To obtain functions giving the transition rates within the mainline sections, assume that at time t the vehicles in section k have speeds assigned as independent, identically distributed random outcomes from a common speed distribution, and that distances from the downstream boundary of section k are assigned as independent, identically distributed outcomes from a uniform random variable with probability density $1/L_k$, where L_k is the length of section k. It is then straightforward to show that the probability of a randomly selected vehicle exiting section k during a short interval of length Δ is simply $\bar{u}_k(t)\Delta/L_k$, where $\bar{u}_k(t)$ gives the space-mean speed for section k at time t. The formulation can then be closed by requiring the space-mean speeds $\bar{u}_k$ to depend directly on $\mathbf{x}(t)$ via a form for the equilibrium speed-density relations of traffic flow theory, giving a version of the simple continuum model. As formulated though, this model will tend to lock up when the densities in a section rise above the critical density (*8*).

Although Markov traffic models can be extended to produce analogs of higher-order continuum models (*3,9*), a simpler solution is to use a device originally attributable to Szeto and Gazis (*10*) and allow the flow across the boundary of two sections to depend on both the upstream and downstream densities. The two-dimensional per lane flow-density relation (transition rate) used in this paper takes the form

$$Q(d_k, d_{k+1}) = d_k u_0 e^{-1/2(d_k/d_c)2} \left[1 - \left(\frac{d_{k+1}}{d_j}\right)^r\right] \qquad d_k \leq d_c$$
$$Q_0\left[1 - \left(\frac{d_{k+1}}{d_j}\right)^r\right] \qquad d_k > d_c \tag{5}$$

where

u_o = free-flow speed,
d_c = critical density,
Q_0 = capacity flow, and
d_j = jam density.

For a constant downstream density d_{k+1}, Equation 5 gives an increasing cross-boundary flow as the upstream density d_k increases, up to the point at which d_k equals the critical density. The cross-boundary flow then remains constant, thus modeling the upstream section as (approximately) an oversaturated finite-server queue. As the downstream density d_{k+1} approaches the jam density d_j, the cross-boundary flow goes to 0, with the sensitivity of this effect being governed by the exponent r. Figure 1 displays a plot of Equation 5 as calibrated for an actual segment of freeway.

The continuous-time Markov compartment (MARCOM) freeway traffic flow simulation model incorporating transition intensities can be expressed as the following simple recursive process, well-suited for computer simulation:

- *Step 0*: Given O-D splitting probabilities b_{ij} and destination-specific variables $x_{kj}(0)$, let $t = 0$; $i = 1, \ldots, m$; $k = 1, \ldots, n$; $j = 1, \ldots, s$.
- *Step 1*: Generate the next arrival time at origin i destined for j, Δ_{ij}, as an exponential outcome with parameter $\lambda_{oi} b_{ij}$, λ_{oi} = arrival rate at on-ramp i.
- *Step 2*: Calculate the mainline transition rates $x_{kj} q_{k,h}(t)$ from Equation 5, $q_{k,h}(t)$ = mainline transition intensity.
- *Step 3*: Generate the next transition time at each compartment k destined for j, Δ_{kj}, as an exponential outcome with parameter $x_{kj} q_{k,h}$.
- *Step 4*: Pick a minimum next arrival time Δ_{min} among (Δ_{ij}, Δ_{kj}).
- *Step 5*: Let $t = t + \Delta_{min}$, update state variable $x_{kj}(t)$:

$x_{kj}(t + \Delta_{min}) = x_{kj}(t) + 1$, if it is a birth compartment;
$x_{kj}(t + \Delta_{min}) = x_{kj}(t) - 1$, if it is a death compartment; and
$y_{lg}(t + \Delta_{min}) = y_{lg}(t) + 1$, if detector l register departures, from g.

- *Step 6*: Go to Step 1.

PRELIMINARY TESTING OF MARCOM MODEL OF FREEWAY TRAFFIC FLOW

Behavior of MARCOM at a Lane Drop Bottleneck

Although the basic idea behind Equation 5 is not new, the traffic flow model that results is still somewhat novel, and it was first desired to see if Equation 5 could produce reasonable behavior at bottlenecks. To this end, a computer program implementing MARCOM was written and used to generate simulated flows for the hypothetical 3.5-mi freeway section shown in Figure 2. Here, the number of lanes is reduced from three to two behind the fifth of 12 subsections (the length of subsection was uniformly chosen to 1,500 ft). A 60-min simulation started with demand of 3,000 vehicles per hour (vph) and then increased to 4,800 vph, which exceeds the capacity of the two-lane section by approximately 20 percent, and finally decreased to 1,200 vph. The simulation results of this hypothetical case are depicted in Figures 3 and 4, which show the volume and density trajectories of the bottleneck section at 5-min intervals. As illustrated in these figures, the MARCOM provides a reasonable

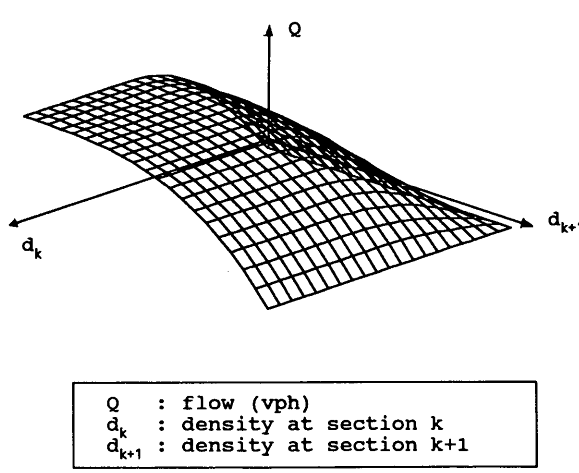

FIGURE 1 Two-dimensional flow-density relationship.

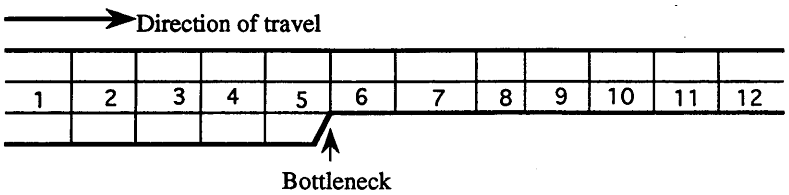

FIGURE 2 Geometrics of freeway section with bottleneck.

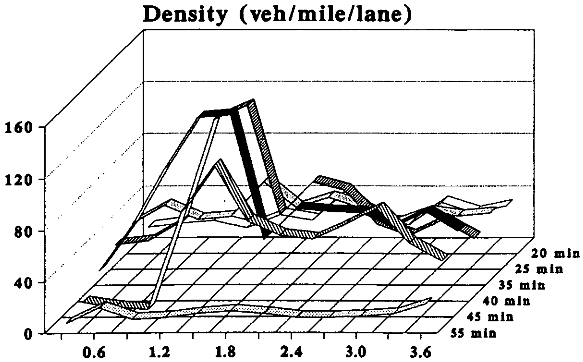

FIGURE 3 Density trajectories at a bottleneck.

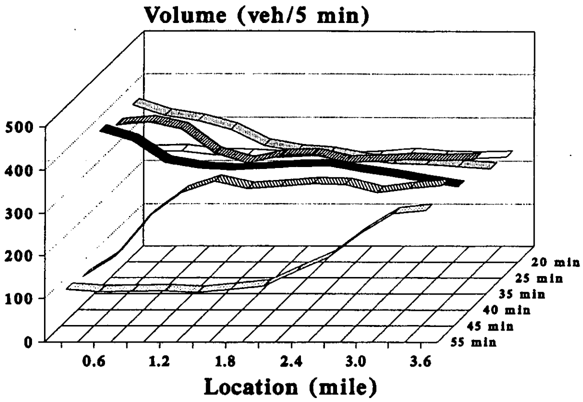

FIGURE 4 Volume trajectories at a bottleneck.

description of queue build-up and dissipation in that (a) congestion starts in front of the bottleneck and moves in upstream direction, while the density within the bottleneck section is around critical density; and (b) the volumes in the bottleneck section are limited to the capacity during congestion building and dissipation.

Calibration and Verification of MARCOM

As stated earlier, the ultimate objective of this research is to estimate destination-specific traffic densities on freeways. The solution strategy was to describe a Markovian traffic model, approximate the Markovian model with a linear stochastic model, and apply the theory of Kalman filtering to the linear model in order to estimate the destination-specific densities. Three questions then arise concerning this approach: (a) how reasonable is the underlying Markovian traffic model? (b) how accurate is the linear approximation? and (c) how well does the resulting Kalman filter perform? Since destination-specific densities are almost impossible to observe in practice, the accuracy of the Kalman filter must be assessed using simulated data. To this end, the MARCOM simulation program just described was calibrated using real data and run for model verification. Figure 5 depicts a seven-origin, four-destination segment of northbound Interstate highway I-35W that is 4.0 km (2.5 mi) long. Five-minute cumulative volume and lane occupancy measurements during a 3-hr morning peak period (6:00 to 9:00 a.m.) for mainline, on-ramp, and off-ramp stations were obtained from the Minnesota Department of Transportation (MNDOT).

To run the stochastic simulation model, MARCOM, it is necessary to know the on-ramp arrival rates λ_{oi}, the O-D splitting probabilities b_{ij}, and the parameters governing the flow-density relation in Equation 5. The arrival rates can simply be estimated as those values that reproduced the corresponding 5-min arrival counts allowing the arrival rates to vary for each 5-min interval.

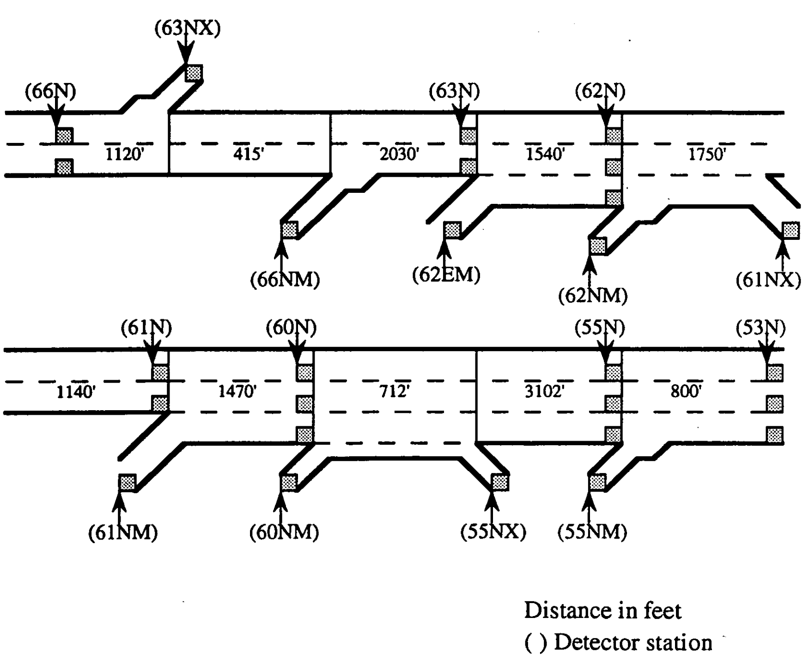

FIGURE 5 Geometrics of test section (I-35W northbound).

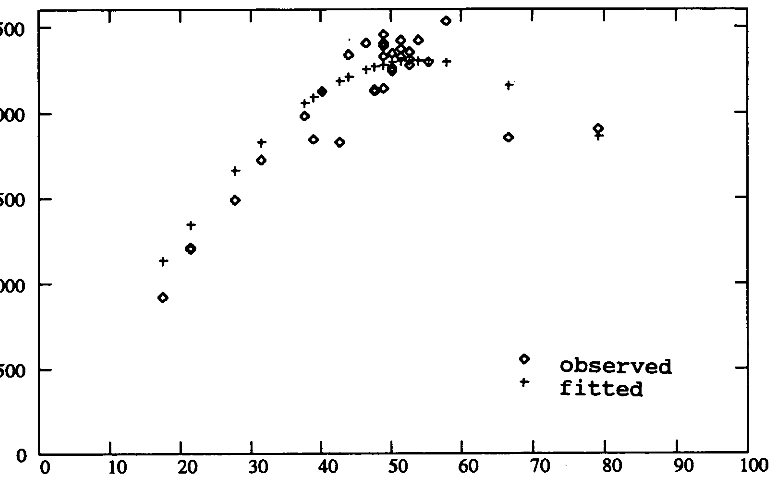

FIGURE 6 Fitted and observed steady-state flow versus density.

To determine the parameters for Equation 5, the lane occupancy measurements were converted to approximate density values and the parameters u_0, d_c, d_j, and r in Equation 5 were estimated using nonlinear least squares by setting $d_k = d_{k+1}$, corresponding to the notion of approximate homogeneous flow. Figure 6 shows the observed and fitted flow-density curve obtained for the estimates u_0 = 66.6 mph, d_c = 64.5 veh/lane/mi, d_j = 120 veh/lane/mi, and r = 3. Finally, the splitting probabilities b_{ij} were estimated by using the estimated traffic flow parameters to numerically solve the mean value Equation 3 given a trial set of b_{ij} values. For a given set of origin counts, this produced estimated destination counts, and those b_{ij} values that minimized the sum of squared errors between forecast and actual counts were obtained by embedding this routine in a nonlinear optimization program. This method produced reliable O-D parameter estimates in a recent research (*3*) when incorporating an accurate traffic flow model. These estimates were then used as inputs to a MARCOM that simulated the Markov compartment process described earlier to generate simulated traffic counts for various time intervals as well as destination-specific section populations, $x_{kj}(t)$.

The resulting comparisons of volume and density generally indicated good agreement between simulated and actual data. In order to evaluate the model performance quantitatively, two error measurements (mean absolute percentage difference and mean absolute error) are calculated. As indicated by the error measures in Table 1, MARCOM provided a reasonable reproduction of traffic flows.

TABLE 1 Mean Error of Simulated Volume Results (6:00 to 9:00 a.m.)

Detector Station	63N	62N	61N	55N	53N
MAPD[a]	2.2	2.0	2.0	2.1	2.2
MAE[b]	13	21	18	27	28

[a]Mean Absolute Percentage Difference (%) = $\sum_{k=1,N}(100*(\text{Measured-Simulated})_k/\text{Measured}_k)/N$

[b]Mean Absolute Error (veh/5 min) = $\sum_{k=1,N}(\text{Measured-Simulated})_k/N$ where N is the Number of Measured Points

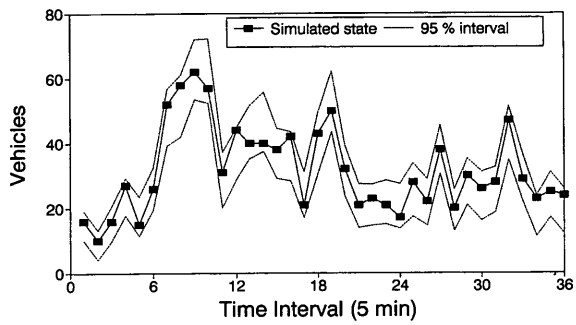

FIGURE 7 Simulated state and confidence interval (state variable: ×54).

ESTIMATING DESTINATION-SPECIFIC VEHICLES USING SIMULATED COUNT DATA

Next, the estimated parameter values were used to implement the density-tracking Kalman filter for the segment of I-35W depicted in Figure 5. First, instantaneous destination-specific volume counts at the end of every 5-min interval and 5-min cumulative volume counts at designated detectors were generated by MARCOM. Next, the Kalman filter was used to estimate destination-specific densities using simulated volume counts.

Figures 7 and 8 show the simulated destination-specific traffic densities along with the approximate 95 percent confidence produced by the Kalman filter. The two dotted curves in Figures 7 through 10 indicate the two-standard deviation envelope produced by the Kalman filter. In each case the estimation range tracks the simulated values reasonably well, with the larger volume flows being tracked somewhat better. This indicates that the filter is performing properly, although it is an approximation of the original process.

Finally, as an additional test of the model's accuracy, the Kalman filter was used to generate predicted mainline and off-ramp counts when fed by actual on-ramp counts. Figure 9 shows actual mainline counts along with the 95 percent prediction range for one of the detector stations, and Figure 10 compares actual ramp counts along

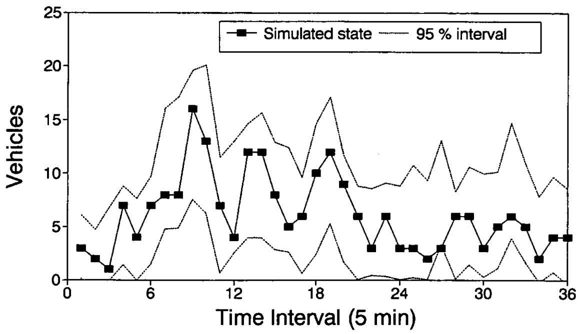

FIGURE 8 Simulated state and confidence interval (state variable: ×52).

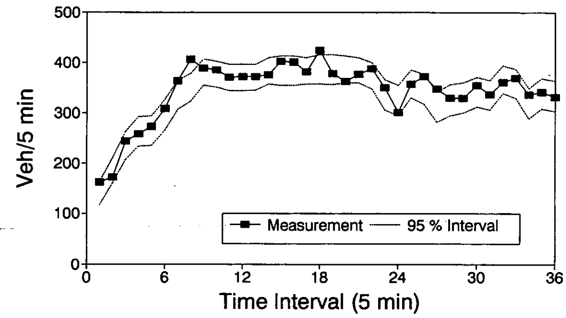

FIGURE 9 Actual volume and confidence interval (mainline: Station 61N).

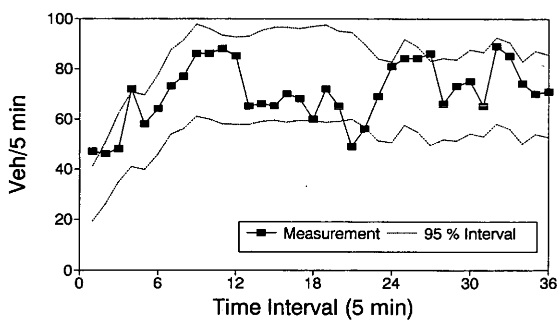

FIGURE 10 Actual volume and confidence interval (off-ramp: Station 61NX).

with the 95 percent prediction range. The mainline volumes are tracked reasonably well, and the Kalman filter appears to predict the mean value of the off-ramp count with accuracy but not its fluctuations as well as desired.

CONCLUSION

This paper began by arguing that a destination-specific breakdown of the traffic currently on a road network is an essential input to any route diversion forecasting method but that such information cannot be measured directly by existing surveillance systems. For the more tractable case of freeway segments, a Kalman filter that could produce such estimates was derived from a Markov compartment model of traffic flow and tested using data from an existing segment of freeway. Generally, the Markov model provided a reasonable description of freeway traffic flow, and the Kalman filter produced reasonable estimates of the destination-specific volumes, although for the lower-volume subflows, the proportion of error was somewhat greater.

Overall, it appears that this Kalman filtering approach provides the information needed for real-time diversion forecasting, at least for freeway segments. One obvious line of improvement would be to develop an adaptive filter by replacing the off-line parameter estimation procedures with their recursive equivalents. This would permit the tracking of slowly varying changes in the O-D pattern and possibly improve the accuracy shown in Figure 10. The main challenge, however, is to extend this filtering method to general traffic networks; this effort is currently under investigation.

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