Development and Comparative Evaluation of High-Order Traffic Flow Models

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Five high-order continuum traffic flow models are compared: Payne's original model, Papageorgiou's improved model, the semiviscous model and the viscous model, a proposed high-order model, and the simple continuum model based on the pipeline cases. The stability of the high-order models is analyzed and the shock structure investigated in all models. In addition, the importance of the proper choice of finite-difference method is addressed. For this reason, three explicit finite-difference methods for numerical implementation—the Lax method, the explicit Euler method, and the upwind scheme with flux vector splitting—are discussed. The test with hypothetical data and the comparison of numerical results with field data suggest that high-order models implemented through the upwind method are more accurate than the simple continuum model. For congested cases, the proposed high-order model appears to be more accurate than the other high-order models for all cases tested.

Since Lighthill and Whitham (*1*) first applied a simple continuum model to describe the characteristics of traffic flow in 1955, much progress has been made in the development and application of macroscopic continuum traffic flow models, especially with the introduction of the high-order continuum models. For example, since 1985, Michalopoulos et al. (*2–4*) have developed a microcomputer simulation program, KRONOS, based on the simple continuum model. KRONOS has been used by the Minnesota Department of Transportation for simulating freeway traffic. In 1971 Payne developed a high-order continuum model that includes the effects of the drivers' reaction and acceleration (*5*). Later he applied this high-order model to the computer simulation program, FREFLO (*6*). Since then, researchers in traffic flow theory have developed a few new high-order continuum models. Examples are Papageorgiou's improved high-order model (*7,8*), the semiviscous and viscous high-order models (*9,10*), and others (*11–13*).

As is well known, high-order continuum models are more sophisticated than the simple continuum model because the simple continuum model is based only on the conservation equation, but high-order models include not only the conservation but also the momentum equation, which accounts for the dynamic effects of inertia and acceleration of traffic mass. However, it is unknown whether in practice high-order continuum models produce better results than those of the simple continuum model.

Although it is well understood that a finite-difference method can affect the computational accuracy of continuum traffic flow models, the importance of the proper choice of finite-difference method was not addressed properly in the past, and some improper finite-difference methods were applied to the continuum models. Only recently have other finite-difference methods been applied to the continuum models. For example, some implicit methods for the simple continuum model and the semiviscous model are discussed by Chronopoulos et al. (*14,15*). Leo and Pretty (*16*) used an upwind method for the simple continuum model and Payne's model. In addition, Lyrintzis et al. surveyed the application of upwind methods including the total variation diminishing (TVD) method to the simple continuum model (*17*). Although the implicit first-order upwind scheme is strongly recommended for the simple continuum model, it is not clear which finite-difference method should be used with the high-order continuum models to achieve a higher computational accuracy. The purpose of this paper is to address these questions.

The five high-order continuum models have been investigated and compared with the simple continuum model for the pipeline cases. These five high-order models are Payne's high-order model (*5,6*), Papageorgiou's improved high-order model (*7,8*), the semiviscous model, and the viscous model (*9,10*) as well as a new high-order model developed here. The stability of the high-order models is analyzed and the shock structure investigated in all models. Three explicit finite-difference methods—the Lax method (*18*), the explicit Euler method, and the upwind scheme with flux vector splitting (*19,20*)—are discussed. Through mathematical analysis, testing with hypothetical data, and comparison of numerical results with field data, it is demonstrated that high-order models implemented through the upwind scheme with flux vector splitting can perform better than the simple continuum model. Furthermore, the proposed high-order model appears to be more accurate than the other high-order models.

CONTINUUM TRAFFIC FLOW MODELS: AN OVERVIEW

Simple Continuum Model

The simple continuum model proposed by Lighthill and Whitham (*1*) consists essentially of a conservation equation for the pipeline case

$$\frac{\partial k}{\partial t} + \frac{\partial q}{\partial x} = 0 \tag{1a}$$

supplemented by the definition of the flow rate

$$q = uk \tag{1b}$$

and a speed-density (u-k) relationship

$$u = u_e(k) \tag{1c}$$

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where

k = traffic density (veh/km),
q = flow rate of traffic stream (veh/hr),
u = space-mean speed (km/hr),
t = time,
x = space, and
$u_e(k)$ = equilibrium relationship between speed and traffic density.

It is well-known that Equation 1a is nonlinear and dominated by convective effects. Therefore, the simple continuum model (Equation 1) always leads to discontinuous solutions so that a smooth solution can exist only for a finite time, even when the initial condition is arbitrarily smooth. However, actual traffic flow changes smoothly. This means that, from a theoretical point of view, the simple continuum model does not accurately describe the traffic dynamics.

It should be noted that the numerical treatment of the simple continuum model can introduce numerical dissipation to smooth the discontinuous solution. For example, the Lax method (*18*) can be used because it introduces a strong dissipation effect to the simple continuum model (*2,19*). Although the dissipation effect might be introduced to the simple continuum model by using the other finite-difference methods, there is still another drawback to the simple continuum model. That is, changes in speed in the simple continuum model occur instantaneously and fluctuations of speed about equilibrium values are not allowed. This problem cannot be overcome by using a finite-difference method. Nevertheless, the simple continuum model usually captures the basic shock wave structure and gives reliable results for various test cases and geometries (*4*).

Original High-Order Model

Payne proposed a more attractive high-order continuum traffic flow model in which a momentum equation is included (*5*). This model is called the original high-order model. The momentum equation in this model was derived from car-following theory. The state equations of the original high-order model for the pipeline case are

$$\frac{\partial k}{\partial t} + \frac{\partial q}{\partial x} = 0 \tag{2a}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{1}{T}\left\{u_e(k) - u - \frac{\nu}{k}\frac{\partial k}{\partial x}\right\} \tag{2b}$$

$$q = uk \tag{2c}$$

where T is the constant reaction time and ν is an anticipation coefficient that is the function of the density with the following form

$$\nu = -\frac{1}{2}\frac{du_e}{dk} \tag{3}$$

It should be noted that a constant anticipation coefficient was later suggested by Payne (*6*).

Since the momentum equation (Equation 2b) is included in the original high-order model, some new features emerge. First, by using the linearized theory to the original high-order model, it can be seen that an equilibrium state $[k_0, u_0 = u_e(k_0)]$ exists in the original high-order model if the following condition holds (*21*):

$$u_0 + \sqrt{\frac{\nu}{T}} > c_0 > u_0 - \sqrt{\frac{\nu}{T}} \tag{4}$$

where c_0 is the kinematic wave speed and

$$c_0 = u_0 + k_0\left(\frac{du_e}{dk}\right)_{k_0} \tag{5}$$

Another new feature is that there is a smooth shock. That is, the shock can be represented by

$$\xi = x - Ut \tag{6}$$

where U is the constant speed of the smooth shock. To see this, substituting Equation 6 into Equation 2 (for the pipeline case) yields a single equation for the density

$$[\nu - T(u-U)^2]\frac{dk}{d\xi} = k[u_e(k) - u] \tag{7}$$

It has been proved (*21*) that Equation 7 has a unique solution if and only if

$$\nu > T(u-U)^2$$

that is

$$u - \sqrt{\frac{\nu}{T}} < U < u + \sqrt{\frac{\nu}{T}} \tag{9}$$

In other words, Equation 6 does represent the smooth shock for the original high-order model if the condition (Equation 8) holds. Moreover, the coefficient $[\nu - T(u-U)^2]$ determines the shock thickness that represents the space containing the shock. The larger the value of the coefficient, the thicker the shock, and vice versa. It should be pointed out that if the condition (Equation 8) does not hold, the smooth shock does not exist but a discontinuity occurs.

From this discussion, it can be seen that the original high-order model is superior to the simple continuum model conceptually. Unfortunately, because the explicit Euler-like finite-difference method was applied to the original high-order model (*5*), application of this model does not show the superiority. Indeed, applying the explicit Euler-like method to the original high-order model (Equation 2) yields

$$k_j^{n+1} = k_j^n + \frac{\Delta t}{\Delta x}[q_j^{n+1} + q_{j+1}^{n+1}] \tag{9a}$$

$$u_j^{n+1} = u_j^n - \frac{\Delta t}{\Delta x}u_j^n[u_j^n - u_{j-1}^n] + \frac{\Delta t}{T}\left\{u_e(k_j^n) - u_j^n - \frac{\nu}{k_j^n\Delta x}[k_{j+1}^n - k_j^n]\right\} \tag{9b}$$

$$q_j^{n+1} = \frac{1}{4}[k_{j-1}^n + k_j^n][u_{j-1}^n + u_j^n] \tag{9c}$$

From this discretized form, it is evident that the original high-order model cannot work at the smaller values of the density because of the term $\nu[k_{j+1}^n - k_j^n]/k_j^n\Delta x$. Since this discretized form does not come from the conservation form of the system, it cannot produce the correct shock intensities (*22*). Moreover, this discretized form is unstable from the computational point of view. To see this, investigate the truncation error associated with Equation 9a. Here only those terms that involve a second-space derivative of the density k are needed, since they are the only ones that contribute to a diffusion. The effective diffusion coefficient for Equation 9a, through terms of order Δt^2 and Δx^2, is

$$-\frac{\Delta t}{2}u^2\left(1-\frac{\Delta t}{T}\right)-\frac{\Delta t}{2}\frac{\nu}{T}\left(1-\frac{\Delta t}{3T}\right)+\left\{\frac{\Delta t^2}{6}\left(9u^2+\frac{7\nu}{T}\right)-\frac{\Delta x^2}{4}\right\}\frac{\partial u}{\partial x}-\frac{\Delta t^2}{6}\frac{u}{T}\left(5k\frac{du_e}{dk}+3u_e\right) \quad (10)$$

It has been proved that instabilities can occur wherever a diffusion coefficient is negative (*23*). From Equation 10, it is easy to see that the first two terms are always negative; the third term will be negative when traffic becomes congested. Thus, under congested flow, the computed solutions provided by this discretized form of the original high-order model become unstable. Therefore, in order to implement the original high-order model effectively, an alternative finite-difference method is needed.

Although another finite-difference method can be applied to the original high-order model to improve its performance, there is still a problem in the model: reaction time. As car-following theory suggests, the reaction time is the time measured from the time at which the lead driver initiates a stop until the second driver initiates his or her own stopping maneuver. After such a time, the velocities of the two vehicles are assumed equal (*24*). This would mean that the second vehicle has a jump in speed, but this is not the case. In fact, after the reaction time, there is still a process of adjusting speed for the second vehicle, which is called the relaxation process. Such a relaxation process is not included in the original high-order model because only the reaction time is taken into account. For this reason, the authors propose the new high-order continuum model presented in a later section.

Improved High-Order Model

On the basis of the original high-order model, Papageorgiou (*7,8*) proposed an improved high-order continuum model. The equations of this improved high-order model for the pipeline case are

$$\frac{\partial k}{\partial t}+\frac{\partial q}{\partial x}=0 \quad (11a)$$

$$\frac{\partial u}{\partial t}+u\zeta\frac{\partial u}{\partial x}=\frac{1}{T}\left\{[u_e(k)-u]-\frac{\nu}{k+\kappa}\frac{\partial k}{\partial x}\right\} \quad (11b)$$

$$q=uk \quad (11c)$$

where κ and ζ are constants. The improved high-order model was based on the Euler-like discretized form of the original high-order model. So to improve the computational effect of the original high-order model, κ was added to keep the third term on the right-hand side of Equation 9b limited when the density k becomes small; ζ was added only for the numerical computation of the model.

To see the difference between the improved and the original high-order models, linearize the improved high-order model (Equation 11) for small perturbations about the state $[k_0, u_0 = u_e(k_0)]$. Thus it can be seen that the state $[k_0, u_0 = u_e(k_0)]$ is equilibrium if the following condition holds:

$$u_0+\sqrt{\frac{k_0\nu}{k_0+\kappa T}+\frac{1}{4}u_0^2(1-\zeta)^2}-\frac{1}{2}u_0(1-\zeta)>c_0>u_0-\sqrt{\frac{k_0\nu}{k_0+\kappa T}+\frac{1}{4}u_0^2(1-\zeta)^2}-\frac{1}{2}u_0(1-\zeta) \quad (12)$$

Comparing with the stable condition (Equation 4) of the original high-order model, when $\zeta = 1$ and $\kappa \neq 0$, the range of stability of Equation 12 is less than that of Equation 4; when $\zeta > 1$ or $\zeta < 1$, the range of stability of Equation 12 shifts right or left relative to the range of stability of Equation 4.

Now the shock structure of the improved high-order model will be investigated. Using the same method as in the original high-order model,

$$\left[\frac{k\nu}{k+\kappa}-T(u\zeta-U)(u-U)\right]\frac{dk}{d\xi}=k[u_e(k)-u] \quad (13)$$

Thus, the smooth shock exists if and only if

$$\frac{k\nu}{k+\kappa}>T(u\zeta-U)(u-U) \quad (14)$$

Moreover, comparing Equation 14 with Equation 7, the shock thickness of the improved high-order model is less than or equal to that of the original high-order model because the coefficient of the left term of Equation 14 is less than or equal to that of the left term of Equation 7.

From this discussion, the improved high-order model might have more accurate computational results than the original high-order model. However, this conclusion depends on the choice of the parameters κ and ζ. Since the improved high-order model was developed on the basis of the Euler-like discretized form of the original high-order model, the discretized form of the improved high-order model still suffers the same instability problem. Moreover, the upwind scheme cannot be used with flux vector splitting (see next section) to overcome the instability problem of this model because the Jacobian is not homogeneous.

Semiviscous Model

Michalopoulos et al. (*9,10*) proposed two high-order continuum models: the semiviscous model and the viscous model. The equations of the semiviscous model for the pipeline case are

$$\frac{\partial k}{\partial t}+\frac{\partial q}{\partial x}=0 \quad (15a)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=\frac{1}{T(k)}[u_f(k)-u]-\alpha k^{\beta}\frac{\partial k}{\partial x} \quad (15b)$$

$$q=uk \quad (15c)$$

where

$u_f(x)$ = free-flow speed,
α = positive constant (and $\sqrt{\alpha}$ has the dimension of velocity), and
β = parameter, usually chosen as -1.

Note that the first term on the right side of Equation 15b represents relaxation, which is the process whereby drivers adjust their speeds to the free-flow speeds. Thus $T(k)$ is the relaxation time, which is a function of density and is given as

$$T(k) = T_0\left[1 + \frac{\gamma k}{k_{\text{jam}} - \gamma k}\right] \tag{16}$$

where $T_0 > 0$ and $0 < \gamma < 1$ are constants and k_{jam} is the jam density. It should be noted that this relaxation term can contribute to Equation 15b only when $u_f(x)$ is changed from one section of the roadway to another.

In comparison with the previous models, the main feature of the semiviscous model is that it does not require an explicit equilibrium speed-density relationship. The semiviscous model appears to be more appealing for field applications, but, because of the simplification, new problems occur. First consider a pipeline case with a fixed free-flow speed where the relaxation term has disappeared. The semiviscous model is reduced to the perfect gas dynamic model:

$$\frac{\partial k}{\partial t} + \frac{\partial (uk)}{\partial x} = 0 \tag{17a}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{\alpha}{k}\frac{\partial k}{\partial x} \tag{17b}$$

where the value of β is chosen as -1. It has been shown that for an originally continuous compression wave, the system described by Equation 17 always yields a discontinuity (*25*). In fact, Equation 17b is Greenberg's one-dimensional fluid state equation (*26*). Thus, when the free-flow speed is fixed for the pipeline, the semiviscous model produces the same results as the simple continuum model.

Next, consider a pipeline with two free-flow speeds. In this case, one must use the the full form of the semiviscous model to describe traffic flow. If the free-flow speeds are decreasingly distributed on the pipeline, then the contribution of the first term on the right side of Equation 15b to the upstream always represents acceleration. Clearly, this is not the case. This means that the relaxation process in which the free-flow speed serves as the desired state for the adjustment of speed is incorrect. Hence, some modifications to the semiviscous model are needed.

Nevertheless, when combined with the upwind scheme with flux vector splitting (which will be referred as the "upwind method") (*19,20*), the semiviscous model appears to be working more effectively than the simple continuum model. This is because the upwind method introduces physical propagation properties in the discretization process of the semiviscous model. That is, a forward difference is used for an upstream moving wave and a backward difference for a downstream moving wave. Moreover, the upwind method still introduces a numerical viscosity into the discretized form so that shocks can be smeared out. It should be pointed out that the semiviscous model should be modified when the free-flow speed is not constant.

Viscous Model

The viscous model discussed here was proposed by Michalopoulos et al. (*9*). The equations of the viscous model for the pipeline case are

$$\frac{\partial k}{\partial t} + \frac{\partial q}{\partial x} = 0 \tag{18a}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\alpha k^{\beta}\frac{\partial k}{\partial x} + \eta k^{\rho}\frac{\partial^2 k}{\partial x^2} \tag{18b}$$

$$q = uk \tag{18c}$$

where η is the viscous parameter and ρ is a dimensionless constant. The first term on the right-hand side of Equation 18b represents anticipation. The second term on the right side of Equation 18b is the viscosity term, which is used to address traffic friction. It should be noted that the viscous term always exists in the model regardless of the geometry of the freeway. In addition, the viscous model does not use the equilibrium speed-density relationship.

When the semiviscous model is compared with the viscous model, the viscous model can be derived from the semiviscous model if the relaxation term is replaced by the viscous term for the pipeline case. Indeed, both relaxation and viscosity have the same effect—smearing out of the shock. However, from gas dynamics it is known that only when the relaxation time is small can the effect of the relaxation be replaced by a corresponding bulk viscosity (*27*). As will be seen, the relaxation time in the congested traffic flow is small, whereas the relaxation time in the uncongested traffic flow is large. Therefore, the relaxation process cannot be totally replaced by viscosity. Hence, the viscous model could lead to inaccuracies.

Finally, since the Euler method was used with the viscous model, the discretized form of the viscous model is unstable because this discretized form lacks a positive mass diffusion, even though there is a viscous term in the momentum equation.

PROPOSED HIGH-ORDER MODEL

As mentioned earlier, the original high-order model considers only the reaction time and ignores the relaxation time. A question that may arise is whether the relaxation property does in fact exist in a macroscopic sense. Clearly, from the microscopic point of view, there is a process of adjusting speed for the second vehicle after the reaction time. Moreover, it has been suggested that drivers have different behavior at different density levels. For example, at low densities, interaction between drivers becomes negligible, but at high densities, the interaction becomes strong. Hence, from the macroscopic point of view, the process of adjusting speed can be considered as the process of relaxation of drivers' speed to the equilibrium speed, and the relaxation time at a high density level should be shorter than that at a low density level in order to avoid a collision. Therefore, the author proposes the following high-order continuum model for the pipeline case:

$$\frac{\partial k}{\partial t} + \frac{\partial q}{\partial x} = 0 \tag{19a}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{1}{T(k)}\left[u_e(k) - u\right] - \sigma k^{\beta}\frac{\partial k}{\partial x} \tag{19b}$$

$$q = uk \tag{19c}$$

where β is a parameter. To use the upwind scheme with flux vector splitting for this model, β is chosen as -1 in order to make the Jacobian homogeneous. Thus σ is an anticipation constant and $\sqrt{\sigma}$ has the dimension of velocity. $T(k)$ is the relaxation time, which should be a function of density k. Since the relaxation time at a high density level is shorter than that at a low density level, the following general function for $T(k)$ is suggested:

$$T(k) = T_0\left[1 + \frac{1}{\left(\frac{k}{k_m}\right)^{\theta}}\right] \tag{20}$$

where

k_m = critical density,
T_0 = constant reaction time, and
$\theta > 0$.

Thus, when $k \to 0$, $T(k) \to \infty$; when $k > k_m$, $T(k) \to T_0$. This means that at high density levels, the relaxation time is equal to the driver's reaction time. This formula of $T(k)$ is physically acceptable. Moreover, for simplicity, the following equilibrium speed can be used:

$$u_e(k) = u_f\left[1 - \left(\frac{k}{k_{\text{jam}}}\right)^{\psi}\right] \tag{21}$$

where ψ is a positive parameter. Other forms of the u-k relationship can also be used with this model (i.e., the model is independent of the choice of u-k relationship). The previous form of Equation 21 was selected for easy parameter calibration.

In comparison with the original high-order model, the proposed high-order model takes the relaxation process into account and treats the relaxation time as a function of the density. In addition, the relaxation time in the new model appears only at the first term on the right side of Equation 19b, making the new model more reasonable from the physical point of view and easier to be treated by sophisticated finite-difference methods.

A comparison of the proposed high-order model with the semiviscous model shows that the difference between them is that a different relaxation process is adopted by each model. When traffic becomes congested, the relaxation process adopted by the proposed high-order model does not produce the incorrect speed change that occurs in the semiviscous model.

To see a detailed difference between the proposed and the original high-order models, linearize the proposed high-order model (Equation 19) for small perturbations around the state $[k_0, u_0 = u_e(k_0)]$ when $\beta = -1$. Thus, the state $[k_0, u_0 = u_e(k_0)]$ is equilibrium if the following condition holds:

$$u_0 + \sqrt{\sigma} > c_0 > u_0 - \sqrt{\sigma} \tag{22}$$

Compared with the stable condition (Equation 4) of the original high-order model, if $\sigma = \nu/T$, then Equation 22 is equal to Equation 4; if $\sigma > \nu/T$, then the range of stability given by Equation 22 is larger than that given by Equation 4; if $\sigma < \nu/T$, then the range of stability given by Equation 22 is smaller than that given by Equation 4.

Now the shock structure of the proposed high-order model for the case of $\beta = -1$ will be investigated. Using the same method as for the original high-order model,

$$T(k)[\sigma - (u - U)^2]\frac{dk}{d\xi} = k[u_e(k) - u] \tag{23}$$

Thus, the smooth shock exists when the following condition holds:

$$\sigma > (u - U)^2 \tag{24}$$

In addition, comparing Equation 23 with Equation 8, it is seen that when traffic is uncongested, the shock thickness of the proposed high-order model is larger than that of the original high-order model when the value of $T(k)$ is large; when traffic is congested, the shock thickness of the proposed high-order model may be equal to that of the original high-order model because the value of $T(k)$ will approach the constant reaction time.

To implement the proposed high-order model, the upwind method is applied to the model [see details elsewhere (*28*)]. It should be noted that preliminary results show that a high-order TVD method is computationally very expensive and less accurate than the first-order upwind method used, because the former results in shock waves sharper than they really are. However, implicit methods have some merits (*15*) and should be investigated further.

TEST RESULTS

Testing with Hypothetical Data

The continuum models discussed earlier are investigated on the basis of a hypothetical case in order to find the model that can produce a reasonable description of traffic when an incident occurs downstream. For this reason, assume the hypothetical case (Case 1) described next. The freeway geometry for this case is a three-lane, 1,828-m-long pipeline section as shown in Figure 1 (*top*). The analysis period is 15 min. Traffic flow during the first 5 min is assumed to be uncongested. After the first 5 min congestion occurs at the downstream end and continues for 5 min. Then the incident is removed from the downstream end. Figure 1 (*bottom*) gives the flow patterns at the upstream and downstream boundaries. For comparing the results produced by the proposed high-order model, the simple continuum model and the original high-order model as implemented in the CORFLO (*29*) as well as the improved high-order model, the equilibrium speed-density relationship of KRONOS (*4*) was used for implementing these models. This relationship is

$$u_e(\text{km/hr}) = \begin{cases} 105 & \text{for } k \le 9 \text{ (veh/km)} \\ -\dfrac{1{,}155}{729}k + \dfrac{83{,}160}{729} + \dfrac{34{,}020}{729k} & \text{for } 9 \le k \le 36 \\ -\dfrac{21}{64}k + \dfrac{189}{8} + \dfrac{6{,}699}{4k} & \text{for } 36 \le k \le 116 \end{cases}$$

Now look at the simulation results of speed because speed is one of the variables that have the shock behavior. The simulation results of 5-min average speed at time interval [5,10] produced by the simple continuum model with the Lax method, the semiviscous model with the upwind method, and the proposed high-order model with the upwind method are shown in Figure 2. From Figure 2, it is clear that the simple continuum model, the semiviscous model, and the proposed high-order model capture the shock wave propagation. However, the proposed high-order model is more accurate in capturing the shock wave than the other two models because the shock wave produced by the proposed high-order model backward propagates the same as the theoretical value.

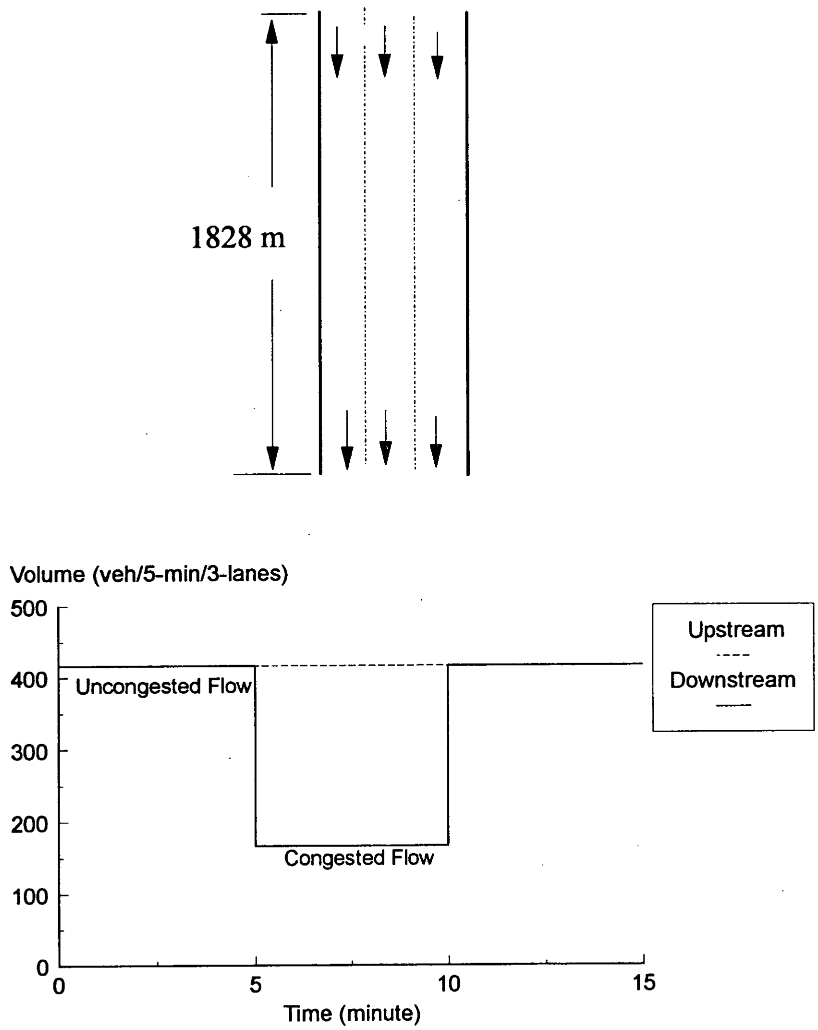

FIGURE 1 Geometry (*top*) and volume at upstream and downstream boundaries (*bottom*), Case 1.

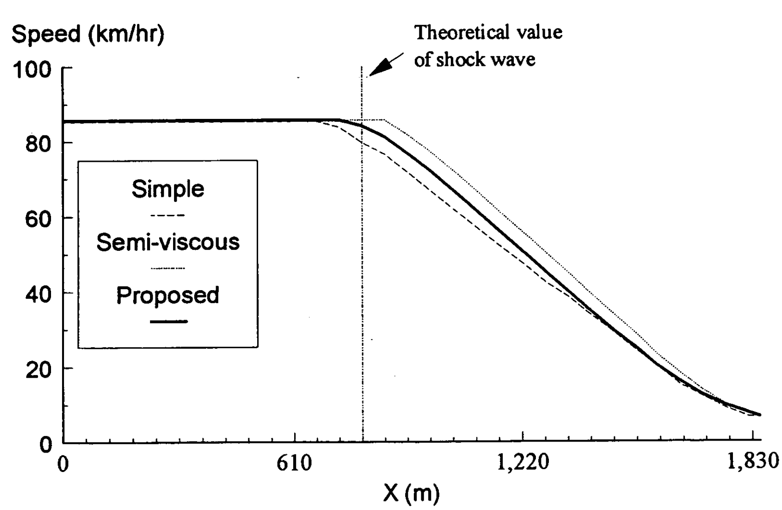

FIGURE 2 Five-minute average speed at time interval [5,10] produced by simple continuum model (Lax method), semiviscous model (upwind method), and proposed high-order model (upwind method), Case 1.

Unfortunately, when Case 1 was investigated by using the original high-order model (CORFLO), the improved high-order model, and the viscous model, these three models did not produce the correct shock wave as shown in Figure 2. This is because as shown earlier, the explicit Euler method is adopted by these three models. To demonstrate this assertion, the authors applied the upwind method to the original high-order model. The results showed that the upwind discretized form of the original high-order model did capture the shock wave propagation. Moreover, the original high-order model (CORFLO) and the improved high-order model for the Euler method appear to be very sensitive to the parameters chosen (*28*). Details of the outputs from this case as well as results from several other hypothetical cases can be found elsewhere (*28*).

Parameter Calibration

It has been shown that all the high-order continuum models include parameters. So before the models are used with field data, these parameters must be calibrated. In the past, parameters were calibrated by trial and error. Such a process is very time-consuming and requires great effort. To minimize the effort, the authors have developed a procedure that has been incorporated into their simulation program without user interface beyond the supply of field data.

This parameter calibration is an optimization problem in which the objective function is defined as follows:

$$\min f(x_1, x_2, \ldots, x_p) = \sum_{i=1}^{n} \{MSE_i(V) + MSE_i(S)\} \tag{25}$$

where

$x_j (j = 1, 2, \ldots, p)$ = parameters to be calibrated,
n = number of checking points, and

$$MSE(y) = \frac{1}{N} \sum_{i=1}^{N} [y_i^o - y_i^c] \tag{26}$$

where

y = volume or speed,
y^o = observed data,
y^c = computed result,
MSE = mean squared error, and
N = number of observations.

The optimization procedure is based on the Fletcher-Reeves conjugate method (*30*). The gradients in this method are evaluated by a finite-difference approximation in the procedure. Thus, by using this optimization procedure in parameter calibration, the minimization of the objective function, Equation 25, at least local minimization, yields an optimized set of parameters. Other more sophisticated optimization strategies (e.g., Monte Carlo methods) will be explored in the future.

Testing with Field Data

In this subsection, two test cases with field data are presented [others can be found elsewhere (*28*)]. Case 2 is based on a two-lane pipeline freeway of the Minneapolis I-35W between the 76th and 70th Streets. Traffic data used in Case 2 were the uncongested northbound traffic from 6:30 to 8:30 a.m. on November 7, 1989. The roadway geometry and arrival and departure traffic patterns for Case 2 are shown in Figure 3. The checking point is at the middle of the freeway. Case 3 is based on a four-lane pipeline freeway from I-35W close to downtown Minneapolis, starting from 26th Street and ending at 31st Street. Traffic data used by Case 3 were the congested southbound traffic from 4:00 to 6:40 p.m. on November 14, 1989. Congestion starts at 4:05 p.m. at the downstream boundary and lasts 2 hr 15 min. The geometry and arrival and departure patterns for Case 3 are shown in Figure 4. The checking point for Case 3 is also at the middle of the freeway.

To evaluate each model quantitatively, the following statistics are calculated to get the error indexes based on the deviations of simulation results from the field observations:

$$MAE = \frac{1}{N} \sum_{1}^{N} |\text{observed} - \text{computed}|$$

$$MPE = \frac{1}{N} \sum_{1}^{N} \frac{|\text{observed} - \text{computed}|}{\text{observed}}$$

$$MSE = \frac{1}{N} \sum_{1}^{N} (\text{observed} - \text{computed})^2$$

$$\text{Std. deviation} = \sqrt{\frac{1}{N-1} \sum_{1}^{N} (\text{observed} - \text{computed})^2}$$

where

MAE = mean absolute error,
MPE = mean percentage error, and
N = number of observations (i.e., the number of time intervals).

In these two cases, six models are investigated, namely, the simple continuum model, the original high-order model, the improved high-order model, the semiviscous model, the viscous model, and the proposed high-order model. Δx = 61 m and Δt = 1 sec are adopted for each model except CORFLO, in which the step size in space and time are determined internally (Δx = 31 m and Δt = 1 sec). The *u-k* curve wherever it applies was obtained from data collected from I-35W. CORFLO has built in three types of *u-k* curves to choose from; all three types have been tried and the best results are presented. Except for the simple continuum model and CORFLO, the parameters in the other four models were calibrated by using the optimization procedure mentioned earlier.

Results for the two test cases are presented in Tables 1 and 2. It can be seen that

1. When there is no downstream congestion (as in Case 2), all models including the simple continuum model performed at about the same error level except CORFLO.
2. When downstream congestion begins, different models produce different results. The simple continuum model gave good results that were better than CORFLO. Comparing the results produced by CORFLO and the improved high-order model, which are solved with the same numerical method, it is seen that the improved high-order model was more accurate than the original high-order model. It is clear that the proposed high-order model was the over-

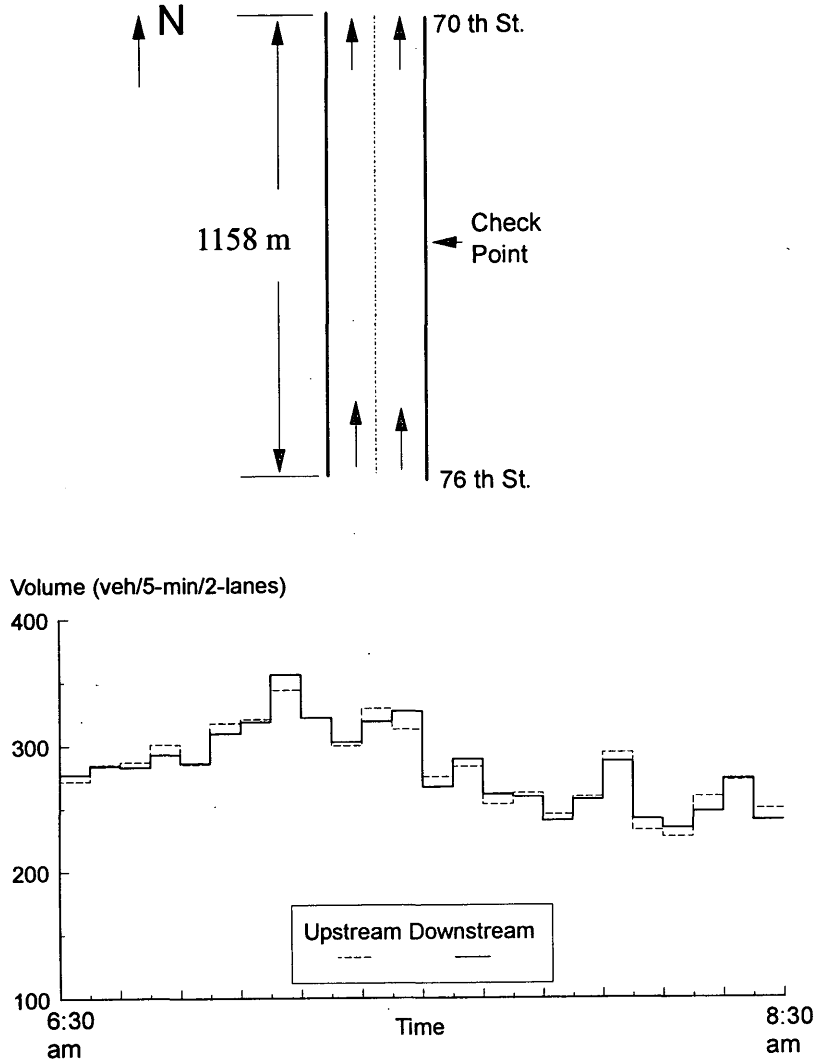

FIGURE 3 Geometry (*top*) and volume at upstream and downstream boundaries (*bottom*), Case 2.

all best in terms of accuracy, with an *MSE* of 290. The viscous model always produces a large error in *MSE* than the other high-order models (except CORFLO).

3. All the high-order models except the proposed high-order model use the different values of parameters for Cases 2 and 3 in order to get good results. This means that the proposed high-order model is the easiest to calibrate.

4. From the tested cases (*28*), the results from the simple continuum model appear to be very sensitive to the choice of the speed-density relationship. The proposed high-order model is less dependent on the choice of the speed-density relationship.

CONCLUSIONS

Five continuum models and a proposed high-order model have been reviewed. Merits and limitations of the various formulations were identified. Preliminary comparative testing of the models was also undertaken. From the hypothetical case, the simple continuum model, the semiviscous model, and the proposed high-order model properly capture the shock wave structure. The ability of CORFLO (the original high-order model), the improved high-order model, and the viscous model to capture shock waves accurately is limited.

In the authors' preliminary testing with field data, all models including the simple continuum model give reliable results. For uncongested cases tested, no apparent merit of high-order modeling versus simple continuum modeling was found. For congested cases tested most high-order models show some error reductions. For all the cases tested, the proposed high-order model produces a smaller error than the other models. Moreover, the proposed high-order model has the strong robust property of parameters for various cases. This property is very important for implementing the proposed high-order model in practice because one can use only a set of precalibrated parameters.

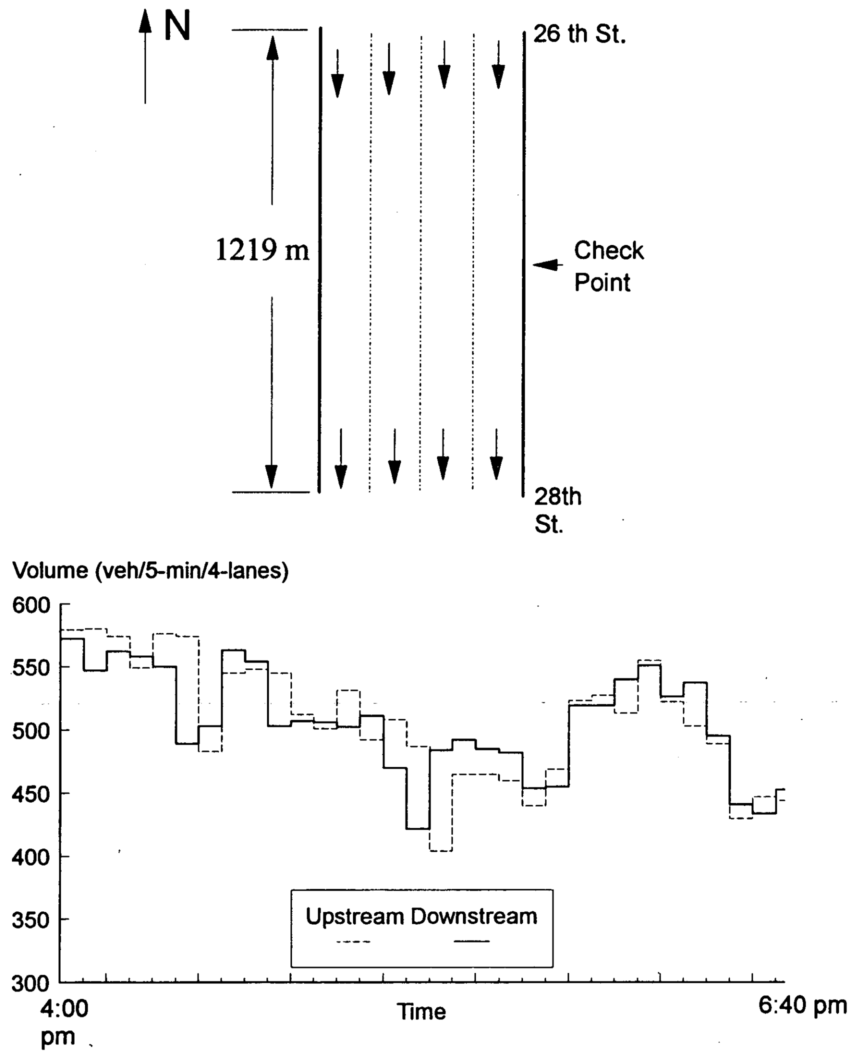

FIGURE 4 Geometry (*top*) and volume at upstream and downstream boundaries (*bottom*), Case 3.

TABLE 1 Error Indexes for Case 2

Models (method)	Simple continuum model (Lax)	Improved high-order model (Euler)	CORFLO (Euler)	Original high-order model (Upwind)	Semi-viscous model (Upwind)	Viscous model (Euler)	Proposed high-order model (Upwind)
MAE	4	4	8	4	4	4	4
MPE	2	2	2	2	2	2	2
MSE	27	27	52	27	22	27	22
Std.Dev	5	5	7	5	5	5	5

(1). MAE and Std.Dev: Veh/5-minutes;
(2). MSE: (Veh/5-minutes)^2.

For finite-difference methods, the Euler method is not good for the numerical implementation of traffic flow models. The upwind scheme with flux vector splitting is recommended for computational accuracy and efficiency.

Finally, the simple continuum model is very sensitive to the choice of the speed-density relationship, whereas most high-order models are less sensitive or not sensitive at all.

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TABLE 2 Error Indexes for Case 3

Models (method)	Simple continuum model (Lax)	Improved high-order model (Euler)	CORFLO (Euler)	Original high-order model (Upwind)	Semi-viscous model (Upwind)	Viscous model (Euler)	Proposed high-order model (Upwind)
MAE	18	15	24	17	19	15	15
MPE	4	3	5	3	4	3	3
MSE	522	416	999	370	508	511	290
Std.Dev	23	21	32	20	23	23	17

(1). MAE and Std.Dev: Veh/5-minutes;
(2). MSE: $(\text{Veh/5-minutes})^2$.

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