Prediction of Creep Effect in Segmental Concrete Bridge Construction

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Presented is a theoretically sound and practical approach for determining concrete creep effects on deflection and stress distributions in reinforced concrete bridges subject to structural system changes during construction. An approximate solution adopted by the American Concrete Institute (ACI) sometimes produces a significant error and considers only one structural system change. An exact solution would require calculation of an integral relaxation function. A simplified and accurate solution for determining concrete creep effects for any number of structural system changes is described. The proposed method is as simple to implement as the ACI method and is highly accurate.

Construction of reinforced concrete bridges presents the problem of structures with a variable structural system. Although it is well known that creep and imposed displacements can cause redistribution of the stresses and bending moments in structures, the significance of structural system changes on the long-term stress distribution in concrete structures is not well documented. Introduction of delayed restraints and the special characteristics of concrete as an aging viscoelastic material allow increased deflection under constant loading and, as a result, lead to redistribution of stresses. The effect was first considered in the design of the bridges over the Neva in (then) Leningrad, USSR (*1*) and the Vltava in Hlanda, Czech Republic (*2*).

In general, the problem requires the solution of a system of integral-type equations. A numerical solution may be obtained with a step-by-step method, taking into account particular sequences of construction. In that manner, a continuous beam built span by span has been analyzed in a work by Bažant and Ony (*3*).

In the case of homogeneous structures (i.e., structures in which the material properties are the same at all points), the exact solution in closed form was obtained by Kharlab (*1*) and independently (in less advanced form) by Dezi et al. (*4*). However, the solution requires calculation of an integral relaxation function. A simplified solution by the Age-Adjusted Effective Modulus Method (AEMM) was obtained by Bažant and Najjar (*2*), Bažant (*5*), and Kristtek and Bažant (*6*). The solution was adopted by American Concrete Institute (ACI) as a recommendation (*7*). In most cases, the method gives a good approximation, although sometimes its error is significant. Moreover, the ACI recommendation considers only the case of one structural system change.

The aim of this paper is to develop a simplified but accurate solution for any number of structural system changes by combining Kharlab's exact solution and Bažant's simplified solution.

CREEP OF CONCRETE

Different approaches to describing the constitutive relationship of concrete considering creep exist: models derived by ACI (*7*), by Bažant and Panula (*8*), and by Bažant and Prasannan (*9*), among others.

In the present study, a linear creep law is assumed. The assumption is acceptable for stresses less than 0.4 of the standard cylinder compressive strength. The relation between stress σ and strain ϵ may be written in the Stieltjes integral form, as follows:

$$\epsilon(t) - \epsilon_0(t) = \int_{t_0}^{t} J(t, \tau)\, d\sigma(\tau) \qquad (1)$$

where

t_0 = age when the stress and deformation first appear;
$\epsilon_0(t)$ = stress-independent inelastic strain, such as shrinkage strain or thermal dilatation; and
$J(t, \tau)$ = creep compliance, [i.e., strain at time t (including elastic strain) caused by a unit constant stress acting since time τ].

In terms of the creep coefficient $\phi(t, \tau)$, creep compliance is defined as

$$J(t, \tau) = \frac{1 + \phi(t, \tau)}{E(\tau)} \qquad (2)$$

where $E(\tau)$ is the instantaneous elastic modulus.

In general, a universally acceptable model does not exist. In this investigation, however, the ACI model has been chosen as the most widely used one. According to the ACI recommendation (*7*)

$$\phi(t, \tau) = \phi_u(\tau) f(t - \tau) \qquad (3)$$

where

$$f(t - \tau) = \frac{(t - \tau)^{0.6}}{10 + (1 - \tau)^{0.6}} \qquad (4)$$

$$\phi_u(\tau) = \phi(\infty, 7)\, 1.25\, \tau^{-0.118} \qquad (5)$$

$$E(\tau) = E(28)\sqrt{\frac{\tau}{4 + 0.85\,\tau}} \qquad (6)$$

where t, τ are given in days.

STRUCTURES SUBJECTED TO ONE STRUCTURAL SYSTEM CHANGE

A statically determinate or indeterminate homogeneous system V_0 is loaded by its constant dead load at age t_0. At time t_1, the system V_0 is changed to system $V_{(1)}$ by means of introducing a redundant

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rigid restraint without any sudden change of stresses. Following Kharlab (*1*), consider system V_1, which has the same configuration at time t_0 as system $V_{(1)}$ and has elastic modulus $E(t_0)$. Then, force variables (stresses, bending moments), $Y(t, x)$, and deformation variables (strains, deflections), $W(t, x)$, at time t and location x can be represented in terms of the corresponding elastic force variables, $Y_0^{el}(x)$ and $Y_1^{el}(x)$, and elastic deformation variables, $W_0^{el}(x)$ and $W_1^{el}(x)$, as follows:

$$Y(t, x) = Y_0^{el}(x)\,[1 - \zeta(t, t_0, t_1)] + Y_1^{el}(x)\,\zeta(t, t_0, t_1) \quad (7)$$

$$W(t, x) = W_0^{el}(x)[1 + \phi(t_1, t_0)] + W_1^{el}(x)\,[\phi(t, t_0) - \phi(t_1, t_0)] \quad (8)$$

$$\zeta(t, t_0, t_1) = \int_{t_1}^{t} R(t, \tau)\, dJ(t, t_0) \quad (9)$$

where $R(t, \tau)$ is the relaxation function, which is defined as the stress at age t caused by unit constant strain introduced at age τ.

Integral relaxation function, $\zeta\ (t, t_0, t_1)$, may be calculated numerically with high accuracy in a step-by-step manner (*4*), although such a process may at times be tedious. The simplified solution obtained using the AEMM (*2,5,6*) has the following form:

$$Y(t, x) = Y_0^{el}(x)\,[1 - r(t, t_0, t_1)] + Y_1^{el}(x)\,r(t, t_0, t_1) \quad (10)$$

$$W(t, x) = W_0^{el}(x)\,[1 + \phi(t_1, t_0)] + W_1^{el,l}(x)\,[\phi(t, t_0) - \phi(t_1, t_0)] \quad (11)$$

$$r(t, t_0, t_1) = \frac{\phi(t, t_0) - \phi(t_1, t_0)}{1 + \chi(t, t_1)\,\phi(t, t_1)} \quad (12)$$

where $W_1^{el}(x)$ is the elastic deformation variable in the elastic system V_1 based on elastic modulus $E(t_1)$, and $\chi(t, t_1)$ is the aging coefficient (*2,5,7*).

Comparison of the exact and simplified solutions for the force variable (Equations 7 and 10, respectively) shows that $r(t, t_0, t_1)$ can be considered as an approximation of the integral relaxation function $\zeta\ (t, t_0, t_1)$. The approximation is good enough if $t_0 \approx t_1$ or $t_0 \ll t_1$. Nevertheless, sometimes the error from that approximation is significant. The AEMM can be improved by introducing a delayed coefficient $\alpha(t_1, t_0)$ and recasting the solution for the force variables in the form

$$Y(t, x) = Y_0^{el}(x)\,[1 - \gamma(t, t_0, t_1)] + Y_1^{el}(x)\,\gamma(t, t_0, t_1) \quad (13)$$

where

$$\gamma(t, t_0, t_1) = \frac{\phi(t, t_0) - \phi(t_1, t_0)}{1 + \alpha(t_1, t_0)\,\chi(t, t_1)\,\phi(t, t_1)} \quad (14)$$

Values of coefficient $\alpha(t_1, t_0)$ have been obtained and are represented in Table 1. For interpolation in the table, it is better to assume linear dependence on log t_0 and log $(t_1 - t_0)$. For the sake of those who may be interested in a long-term solution only, values of $\gamma(t, t_0, t_1)$ have also been computed for $t - t_1 = 10^4$ days, and are presented in Table 2.

Comparison of the exact and simplified solutions for the deformation variable (Equations 8 and 11, respectively) shows that a discrepancy arises only in the determination of elastic variables; the exact solution requires calculation of them based on modulus $E(t_0)$, but the approximate solution requires use of modulus $E(t_1)$. Therefore, using the exact solution is not more complicated than the approximate one and, as a result, it is preferable.

NUMERICAL EXAMPLE

As an example, consider a pair of two simply supported beams, which are made continuous over the middle support after they have

TABLE 1 Delayed Coefficient α (t_1, t_0)

$t_1 - t_0$ days	$\phi(\infty, 7)$	t_0 days: 10^1	10^2	10^3	10^4
0	0.5	1.0	1.0	1.0	1.0
0	1.5	1.0	1.0	1.0	1.0
0	2.5	1.0	1.0	1.0	1.0
0	3.5	1.0	1.0	1.0	1.0
10	0.5	0.551	0.949	0.989	0.997
10	1.5	0.765	0.954	0.988	0.998
10	2.5	0.791	0.953	0.989	0.998
10	3.5	0.798	0.952	0.988	0.997
10^2	0.5	0.229	0.871	0.963	0.990
10^2	1.5	0.578	0.892	0.962	0.990
10^2	2.5	0.641	0.893	0.962	0.990
10^2	3.5	0.668	0.892	0.961	0.989
$\geqslant 10^3$	0.5	0.073	0.804	0.924	0.968
$\geqslant 10^3$	1.5	0.520	0.856	0.925	0.968
$\geqslant 10^3$	2.5	0.608	0.865	0.924	0.968
$\geqslant 10^3$	3.5	0.645	0.868	0.923	0.967

TABLE 2 Long-Term Values of Function $\gamma(t, t_0, t_1)(t - t_1 = 10^4$ days)

t_1-t_0 days	$f(\infty,7)$	t_0 days 10^1	10^2	10^3	10^4
10	0.5	0.282	0.190	0.149	0.119
10	1.5	0.550	0.383	0.317	0.267
10	2.5	0.675	0.479	0.408	0.356
10	3.5	0.745	0.536	0.465	0.414
10^2	0.5	0.155	0.101	0.077	0.061
10^2	1.5	0.320	0.209	0.166	0.138
10^2	2.5	0.405	0.266	0.215	0.184
10^2	3.5	0.457	0.301	0.246	0.215
$\geqslant 10^3$	0.5	0.047	0.030	0.023	0.018
$\geqslant 10^3$	1.5	0.102	0.066	0.050	0.040
$\geqslant 10^3$	2.5	0.133	0.086	0.066	0.054
$\geqslant 10^3$	3.5	0.153	0.099	0.076	0.063

already started carrying their own weight [Figure 1 (a) and (b)]. The spans are assumed to start carrying their dead load of 17.7 N/cm (10 lb/in.) at age $t_0 = 28$ days. At 60 days, the beams are joined monolithically above the middle support. The second moment of area of each beam is assumed to be 0.0405 m^4 (5 ft^4). Furthermore, $E(28) = 27.9$ MPa (4 Mpsi), and $\phi(\infty, 7) = 2.5$.

To evaluate the bending moment in the beam above the middle support at 100 days, it is necessary to find the following parameters:

- Bending moments in the corresponding elastic systems.
- Aging coefficient, $\chi(100, 60)$; and
- Delay coefficient, $\alpha(60, 28)$.

Elastic bending moment above the inner support in the system without a joint above this support is $M_0^{el} = 0.\ N - m$.

Elastic bending moment at the same location in the system with a joint is $M_1^{el} = 113.9$ kN-m (84,400 ft-lb).

According to ACI recommendations (*7*)

$\chi(20, 10) = 0.774$ $\quad\chi(110, 100) = 0.804$

$\chi(110, 10) = 0.842$ $\quad\chi(200, 100) = 0.935$

From linear interpolation, $\chi(100, 60) = 0.874$.

Similarly, from Table 1

$\alpha(20, 10) = 0.764$ $\quad\alpha(110, 100) = 0.952$

$\alpha(110, 10) = 0.641$ $\quad\alpha(200, 100) = 0.888$

and from linear interpolation $\alpha(60, 28) = 0.799$

Substitution of these values into Equation 13 gives $\gamma(100, 28, 60) = 0.155$.

Finally, from Equation 14, the magnitude of the bending moment in the beam above the middle support at age 100 days is $M(100) = 0\,(1-0.155) + 113.9\,(0.155) = 17.7$ kN-m (13080 ft-lb).

For determination of long-term bending moment at the same location, one can use Table 2

$\gamma(20, 10) = 0.675$ $\quad\gamma(110, 100) = 0.479$

$\gamma(110, 10) = 0.405$ $\quad\gamma(200, 100) = 0.266$

From interpolation, $\gamma = 0.464$; as a result, $M(10060) = 52.9$ kN-m (39160 ft-lb).

Figure 1 (c) presents the bending moment distribution at $t = 60$, 100, and 10,060 days. Table 3 shows a comparison of these results (proposed method) with the corresponding AEMM and exact solutions. One may conclude that the proposed method is as simple to implement as the AEMM at the same time it is highly accurate.

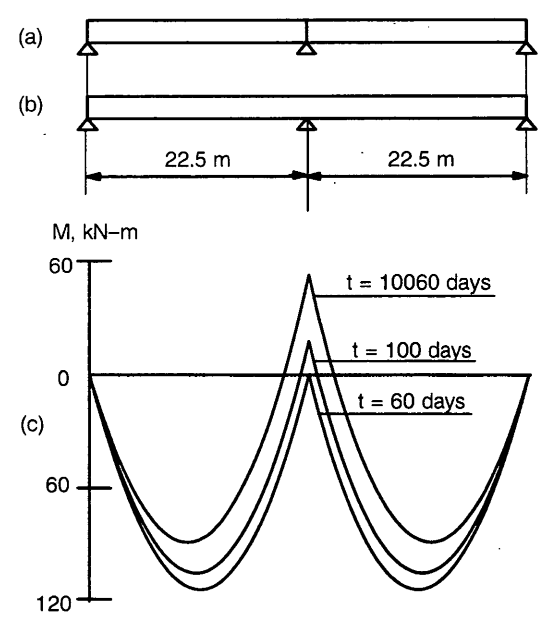

FIGURE 1 (a) Initial structural system; (b) structural system at 60 days; (c) bending moment distributions at 60, 100, and 10,060 days.

TABLE 3 Comparison of Exact and Simplified Methods

t, days	Bending moment over inner support (kN-m) Exact solution	Proposed Method	AEMM
60	0	0	0
100	17.7	17.7	16.1
10060	50.9	52.9	46.3

STRUCTURES WITH DELAYED RESTRAINTS INTRODUCED AT DIFFERENT SUCCESSIVE TIMES

For structures with delayed restraints introduced at different successive times, use of Kharlab's principle of independent restraint actions (*1*) is justified. It states that if the principle of superposition is satisfied, then the $(i + 1)$th delayed restraint acts on system $V_{(i)}$ as if it were its first and only restraint, (i.e., as if it were not introduced onto system $V_{(i)}$, which is subjected to i delayed restraints, but onto system V_i, for which all restraints were initial, and onto which no new restraints would be introduced subsequently. Application of this principle to the case of constant loads and discrete structural changes leads to the following expression for any force variables $Y_j(x, t)$ and deformation variables $W_j(x, t)$ in a system $V_{(j)}$ with j delayed restraints, in terms of the corresponding force and deformation variables, $Y_i^{el}(x)$ and $W_i^{el}(x)$, in the elastic system V_i based on elastic modulus $E(t_0)$

$$Y_j(t, x) = Y_0^{el}(x)\,[1 - \zeta(t, t_0, t_1)] + \sum_{i=1}^{j-1} [Y_i^{el}(x)\,[\zeta(t, t_0, t_i) - \zeta(t, t_0, t_{i+1})] + Y_j^{el}(x)\,\zeta(t, t_0, t_j) \tag{15}$$

$$W_j(t,x) = W_0^{el}(x)\,[1 + \phi(t_1, t_0)] + \sum_{i=1}^{j-1} [W_i^{el}(x)\,[\phi(t_{i+1}, t_0) - \phi(t_i, t_0)] + W_j^{el}[\phi(t, t_0) - \phi(t_j, t_0)] \tag{16}$$

$$\zeta(t, t_0, t_j) = \int_{t_j}^{t} R(t, \tau)\, dJ(t, t_0) \tag{17}$$

where t_i is the time when the restraint was introduced.

In a manner similar to the case involving one delayed restraint, the integral relaxation functions $\zeta(t, t_0, t_i)$ may be replaced by functions $\gamma(t, t_0, t_1)$. Therefore, a simplified solution for the force variables under multiple delayed restraints may be presented in the form

$$Y_j(t, x) = Y_0^{el}(x)\,[1 - \gamma(t, t_0, t_1)] + \sum_{i=1}^{j-1} [Y_i^{el}(x)\,\{\gamma(t, t_0, t_i) - \gamma(t, t_0, t_{i+1})]\} + Y_j^{el}(x)\,\gamma(t, t_0, t_j) \tag{18}$$

For the deformation variables, as in the case with one delayed restraint, the exact solution is recommended.

CONCLUSIONS

The study highlights an important consideration for time-dependent analysis of structures with a variable structural system. Concrete bridges are one example of structures subjected to repeated structural changes. The specific objective was to develop a simple but accurate method for evaluating stress and displacement redistribution in homogeneous structures.

The product of this study is a method obtained by modifying the AEMM method. It requires evaluation of one additional coefficient, the values of which are tabulated.

The proposed method was compared with the AEMM method and Kharlab's exact solution. It was found that the proposed method is as simple to implement as the AEMM and that it is highly accurate.

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