

Resilient Modulus of Subgrade Soils: Comparison of Two Constitutive Equations

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The concept of resilient modulus has been used to explain the nonlinear stress-strain characteristics of subgrade soils. During the past few decades, several constitutive models have been developed for the resilient modulus of subgrade soils. No stress or deformation analysis can be useful unless a correct constitutive equation that describes the actual behavior of material has been used in the analysis. When the correct form of constitutive equation is selected, there is a need for the accurate k parameters, which vary from soil to soil. Under a Georgia Department of Transportation research project, subgrade soil samples were tested in the laboratory using AASHTO T274-82 to determine their resilient moduli. Results were used to compare two widely used constitutive equations and to study the effect of material and physical properties of subgrade soils on the k values of these equations. Two well-known constitutive equations (bulk stress and universal model) are compared for their capability of modeling granular subgrade soils. This comparison shows that the resilient modulus of granular subgrade soils are better described by the universal model, where resilient modulus is a function of bulk stress and deviator stress. The universal model and the semi-log model, where the resilient modulus is a function of deviator stress, were selected to model granular and cohesive soils, respectively, to study the effect of material and physical properties of subgrade soils on their resilient modulus. Results show that the k parameters in the constitutive equations can be calculated using material and physical properties of the soil, and the values of k parameters vary within wide ranges for cohesive and granular subgrade soils.

In recent years highway engineers have devoted considerable effort to determining the nonlinear stress-strain characteristics of subgrade soils. During the past few decades several constitutive models have been developed and used by pavement design engineers. These developments have provided powerful tools for research and design engineers to conduct pavement analysis in a more realistic manner. However stress or deformation analysis cannot be useful unless a correct constitutive equation that describes the actual behavior of material has been used in the analysis.

Each time a load passes in a pavement structure, the pavement rebounds less than it was deflected under load. After repeated loading and unloading sequences, each layer accumulates only a small amount of permanent deformation, with recoverable or resilient deformation. To explain this behavior, researchers have used the concept of resilient modulus, which can be defined as

$$M_R = \frac{\sigma_d}{\epsilon_R} \quad (1)$$

where

M_R = resilient modulus,
 σ_d = repeated deviator stress ($\sigma_1 - \sigma_3$) as defined in Figure 1,

ϵ_R = recoverable axial strain in the direction of principal stress σ_1 , and

$\sigma_1, \sigma_2, \sigma_3$ = principal stresses as shown in Figure 1.

Soil samples collected from 35 test sites throughout Georgia were tested in the laboratory using AASHTO T274-82 to determine their resilient modulus. Four replicate samples were run for each soil. A set of these resilient moduli test data was used in this study.

The objectives of this study were to compare two widely used constitutive equations (bulk stress and universal model) and study the effect of material and physical properties of subgrade soils on k parameters of the constitutive equations.

BACKGROUND

Granular Soils

Research (1,2) has shown that the resilient modulus of granular materials increases with increasing confining stress. There are several relationships to describe the nonlinear stress-strain characteristics of granular materials. The following bulk stress model is currently used by most pavement design engineers (3,4):

$$M_R = k_1 P_a \left[\frac{\Theta}{P_a} \right]^{k_2} \quad (2)$$

where

M_R = resilient modulus of granular soils,

Θ = bulk stress or first stress invariant ($\sigma_1 + \sigma_2 + \sigma_3$),

$\sigma_1, \sigma_2, \sigma_3$ = principal stresses as shown in Figure 1,

k_1, k_2 = material and physical property parameters, and

P_a = atmospheric pressure, expressed in the same unit as M_R and Θ , used to make the constants independent of the units used.

The main disadvantage of this model is that it does not adequately model the effect of deviator stress. In 1981 May and Witczak (5) suggested the following equation to describe the resilient modulus of granular materials:

$$M_R = K_1 k_1 \Theta^{k_2} \quad (3)$$

where K_1 is a function of pavement structure, test load, and developed shear strain, and k_1, k_2 are constants.

In 1985 Uzan (6) demonstrated that Equation 2 cannot adequately describe the nonlinear behavior of granular soils. In 1988, he suggested Equation 4 to describe the nonlinear behavior found in repeated load triaxial tests, which was obtained from empirical ob-

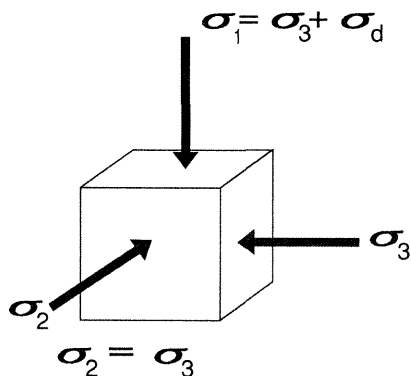


FIGURE 1 Illustration of principal stresses acting on a soil element.

servations. This model includes the influence of deviator stress on resilient modulus.

$$M_R = k_1 P_a \left[\frac{\Theta}{P_a} \right]^{k_2} \left[\frac{\sigma_d}{P_a} \right]^{k_3} \quad (4)$$

where σ_d is deviator stress ($\sigma_1 - \sigma_3$) as defined in Figure 1, and k_1 , k_2 , k_3 are material and physical property parameters.

In 1987 Lade and Nelson (7) presented a constitutive model that shows that modulus of granular materials is a function of the first stress invariant (bulk stress) and the second invariant of the stress deviator tensor. This development provides support for the validity of Equation 4 instead of Equation 2. Brown and Pappin's (8) non-linear stress-strain relationship also agrees with Equation 4 instead of Equation 2.

Cohesive Soils

Sneddon (9) conducted resilient modulus tests for sand and fine-grained soils. Results indicated that the resilient modulus of sandy soils is a function of the applied deviator stress and the confining stress. However the resilient modulus of fine-grained soils is mainly a function of the applied deviator stress, when single confining stress level is considered. In general the resilient modulus of these soils decreases with increasing deviator stress. In 1976 Thompson and Robnett (10) introduced an arithmetic model (Equation 5) to describe the resilient properties of fine-grained soils. This model was successfully used in the ILLI-PAVE (11,12) computer program.

$$M_R = k_2 + k_3 (k_1 - \sigma_d) \quad k_1 > \sigma_d$$

$$M_R = k_2 + k_4 (\sigma_d - k_1) \quad k_1 < \sigma_d \quad (5)$$

where

M_R = resilient modulus of the fine-grained soil,

σ_d = deviator stress ($\sigma_1 - \sigma_3$), and

k_1, k_2, k_3, k_4 = material and physical property parameters.

Another successful model often used to describe the behavior of cohesive soils is the semi-log model (Equation 6). This model has the advantage of having fewer material constants than the arithmetic model.

$$M_R = k_1 P_a \left[\frac{\sigma_d}{P_a} \right]^{k_3} \quad (6)$$

As mentioned by Uzan and Scullion (13), Equation 4 can be used as a universal model for all types of soils. For a constant modulus or linear elastic material, both k_2 and k_3 are set to zero (14). Also the bulk stress model (Equation 2) can be obtained by setting k_3 to zero. The semi-log model (Equation 6) can be obtained by setting k_2 to zero.

LABORATORY TESTS

Subgrade soil samples collected from different locations in Georgia were classified and separated into cohesive and granular categories according to the AASHTO soil classification. Each soil was subjected to laboratory tests such as sieve analysis, Atterberg limits, percent swell and shrinkage, optimum moisture, maximum dry unit weight, and California Bearing Ratio.

Resilient modulus tests were carried out according to the AASHTO T 274-82 (1986) test procedure. The tested samples had a diameter of 73 mm (2.875 in.) and a height of 142.2 mm (5.6 in.) and were statically compacted in three layers. A sample is placed in a triaxial device and subjected to the repetitive loads and stresses expected in a pavement system. These tests were performed on four replicate samples of each soil. One of these samples was compacted to 95 percent compaction; the other three were compacted to 100 percent. The ratio of sample dry unit weight to maximum dry unit weight of soil was taken as the measure of compaction. Low compacted samples had a moisture content of approximately 3 percent above the optimum moisture content. Two of the other three samples had moisture contents approximately 1.5 percent below and 1.5 percent above the optimum moisture. The moisture content of the fourth sample was kept close to optimum. Practical difficulties prevented obtaining the exact intended compactions and moisture contents. The results of the tests were recorded along with the other soil properties.

STUDY DATA

Data used in this study were obtained from a data base created from the aforementioned laboratory test results. To avoid inaccuracies, data were selected according to the following criteria, based on research findings and observations:

1. All soils, fine and coarse grained, that have decreasing resilient modulus with increasing deviator stress at least at lower deviator stresses (6-8,15,16).
2. All soils, fine and coarse grained, that have increasing resilient modulus with increasing confining stress (1,2).
3. All soils, fine and coarse grained, that have decreasing resilient modulus with increasing moisture content in the vicinity of optimum moisture (16) when other soil properties are kept constant.

Fourteen cohesive and 15 granular data sets that satisfied the criteria were used for the study. Some of these soils did not have resilient modulus test results for low-compacted, high-moisture samples. Therefore the resilient modulus test results of low-compacted, high-moisture samples were not included in this study.

DATA ANALYSIS

Granular Soils

The measured resilient moduli values corresponding to confining stresses of 6.89 and 34.45 kPa (1 and 5 psi) and deviator stresses of 13.78, 34.45, 51.68, 68.9, and 103.35 kPa (2, 5, 7.5, 10, and 15 psi) were used to obtain a close representation of stress conditions of the subgrade. These stress conditions and the measured resilient moduli were used to develop relationships for each soil sample in which the resilient modulus is a function of both the bulk stress and the deviator stress. The form of this relationship is given in Equation 4, the universal model, which was transformed to linear form as shown in Equation 7 to carry out linear regressions:

$$\text{Log}(M_R) = \text{Log}(k_1 P_a) + k_2 \text{Log}\left[\frac{\Theta}{P_a}\right] + k_3 \text{Log}\left[\frac{\sigma_d}{P_a}\right] \quad (7)$$

Linear regressions were performed for each set of data and k_1 , k_2 , and k_3 were found for each soil sample. These developed relationships and the stress conditions of soil samples were then used to back-calculate the resilient moduli for each stress condition of each soil sample. These resilient moduli values were referred to as the predicted resilient moduli. Atmospheric pressure used in these analyses was 101.3 kPa (14.7 psi). The same data were used to develop relationships for each soil sample in which the resilient modulus is a function of bulk stress. The form of this relationship is given in Equation 2, the bulk stress model. Equation 8 gives the linear form of the bulk stress model used for regression analysis.

$$\text{Log}(M_R) = \text{Log}(k_1 P_a) + k_2 \text{Log}\left[\frac{\Theta}{P_a}\right] \quad (8)$$

The predicted resilient moduli obtained from the bulk stress model gave a poor correlation to actual resilient moduli (Figure 2). Figure 3 shows a close correlation between predicted resilient moduli obtained from the universal model and the actual resilient moduli. Symbols A, B, C, and D in the figures refer to one, two, three, and four overlapping data points, respectively. The statistics given in Table 1 provide evidence of better predictability in describing behavior of granular soils using the universal model than using the bulk stress model. Fifty percent of the predicted resilient moduli obtained using the bulk stress model were within ± 16.6 percent of the actual values, whereas the predicted moduli were within ± 5 percent using the universal model. The universal model also provided better predictions than Equation 2 for the 25th, 75th, and 90th percentiles (Table 1). The universal model is therefore more suitable to represent the relationship between resilient modulus and stress levels of granular soils used in this study. The coefficients k_1 , k_2 , and k_3 obtained from least-square regressions of the universal model for each sample are listed in Table 2 with moisture content (MC), percent saturation (SATU), and compaction (COMP) of the sample. The values of these coefficients were used in further analyses. Table 2 also gives the optimum moisture content (MOIST) of each soil and the ratio of MC and MOIST (MCR).

A multiple regression analysis approach was used to obtain the relationships among k parameters (dependent variables) and other soil properties such as percent passing #40 sieve (S40), percent passing

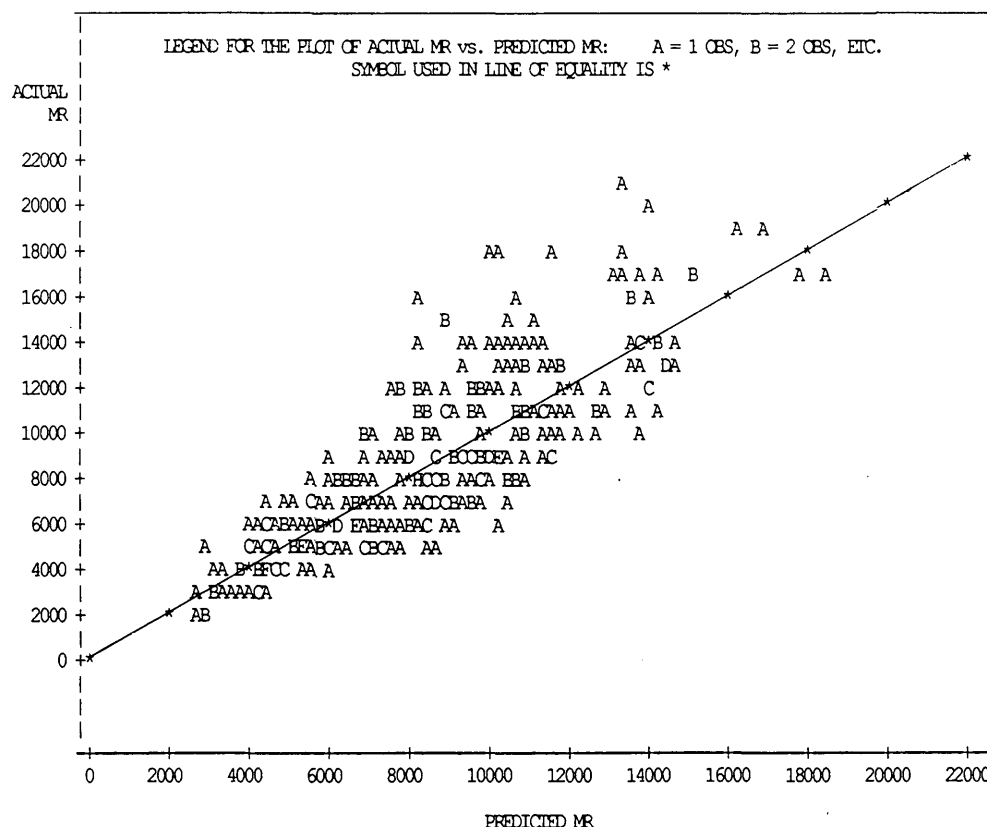


FIGURE 2 Actual resilient moduli versus predicted resilient moduli obtained from bulk stress model.

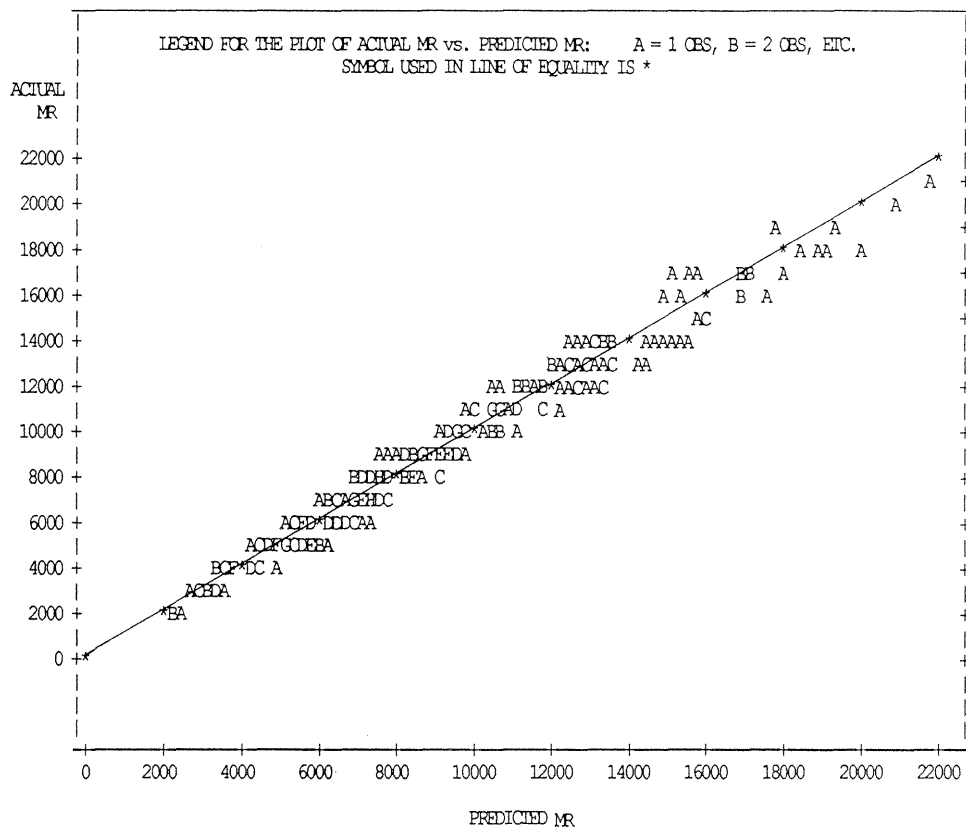


FIGURE 3 Actual resilient moduli versus predicted resilient moduli obtained from universal model.

#60 sieve (S60), percentage of clay (CLY), percentage of silt (SLT), percent swell (SW), percent shrinkage (SH), maximum dry unit weight (DEN), optimum moisture content (MOIST), California Bearing Ratio (CBR), sample moisture content (MC), sample compaction (COMP), and percent saturation (SATU). Table 3 gives the index properties of each soil used for the study. Three separate regression analyses were done for the three coefficients k_1 , k_2 , and k_3 .

To select the best subset of these variables and their interaction terms, The "PROC STEPWISE" procedure in SAS (17) was used with maximum R -square improvement technique (MAXR). This regression procedure does not settle on a single model, but tries to find the best one-variable model, the best two-variable model, and so forth. The stepwise procedure with the MAXR method begins by finding the one-variable model producing the highest R^2 . Then another variable, the one that yields the greatest increase in R^2 , is added. Once the two-variable model is obtained, each variable in the

model is compared to each variable not in the model. For each comparison, MAXR determines whether removing one variable and replacing it with the other variable increases R^2 . After comparing all possible switches, MAXR makes the switch that produces the largest increase in R^2 . By doing this MAXR finds the best two-variable model. Another variable is then added to the model and the comparing and switching process is repeated to find the best three-variable model, and so forth. The number of parameters that correspond to the lowest coefficient of performance value (C_p) was selected to describe the data (18). In each model developed, the multicollinearity effect of independent variables was evaluated and interaction terms were selected to minimize the multicollinearity effect.

The difference between the forward or backward stepwise technique and the MAXR technique is that all switches are evaluated before any switch is made in the MAXR method. In the forward or

TABLE 1 Difference in Actual Resilient Moduli and Predicted Resilient Moduli as a Percentage of Actual Resilient Moduli for Granular Soil Data

Percentile	25	50	75	90
Using Equation 2 (Bulk stress model)	9.0	16.6	27.6	35.4
Using Equation 4 (Universal model)	3.0	5.0	7.3	10.4

TABLE 2 Physical Properties of Granular Soil Samples and Their Least-Square Regression Results

ID	MOIST	MC	MCR	COMP	SATU	k ₁	k ₂	k ₃	R ²
1-I	12	11.2	0.93	1.00	80.3	392	0.291	-0.487	0.97
1-II	12	12.5	1.04	1.00	91.0	349	0.316	-0.531	0.89
1-III	12	9.7	0.81	1.00	70.1	543	0.268	-0.402	0.98
2-I	18	17.0	0.94	1.01	78.7	401	0.239	-0.484	0.98
2-II	18	19.5	1.08	0.98	82.6	326	0.328	-0.627	0.93
2-III	18	14.9	0.83	1.02	69.8	715	0.175	-0.330	0.94
3-I	16	16.0	1.00	1.00	91.0	451	0.301	-0.501	0.97
3-II	16	17.4	1.09	1.00	99.4	413	0.316	-0.574	0.94
3-III	16	14.8	0.93	1.00	83.8	642	0.199	-0.403	0.97
4-I	25	24.5	0.98	1.01	82.2	528	0.304	-0.364	0.94
4-II	25	25.9	1.04	1.01	87.0	403	0.318	-0.385	0.84
4-III	25	22.9	0.92	1.01	77.1	703	0.292	-0.259	0.93
5-I	16	15.2	0.95	1.01	78.3	356	0.285	-0.304	0.93
5-II	16	17.0	1.06	1.01	86.8	335	0.293	-0.369	0.90
5-III	16	13.8	0.86	1.01	70.9	547	0.203	-0.213	0.75
6-I	17	15.7	0.92	1.02	80.8	573	0.201	-0.272	0.82
6-II	17	17.2	1.01	1.02	88.5	423	0.250	-0.317	0.83
6-III	17	14.0	0.82	1.02	72.4	832	0.145	-0.152	0.68
7-I	19	19.3	1.02	1.00	81.5	214	0.404	-0.343	0.91
7-II	19	20.6	1.08	1.00	87.2	173	0.412	-0.403	0.98
7-III	19	17.1	0.90	1.03	73.3	299	0.319	-0.351	0.95
8-I	16	15.7	0.98	1.01	80.0	241	0.379	-0.319	0.95
8-II	16	17.0	1.06	1.01	86.7	211	0.441	-0.340	0.92
8-III	16	14.0	0.88	1.01	71.3	284	0.295	-0.292	0.91
9-I	11	13.4	1.22	0.99	69.3	280	0.328	-0.336	0.90
9-II	11	14.3	1.30	0.99	78.8	252	0.349	-0.322	0.94
9-III	11	11.1	1.01	1.00	61.6	324	0.267	-0.301	0.91
10-I	16	15.7	0.98	1.01	71.5	430	0.457	-0.340	0.95
10-II	16	16.6	1.04	1.01	76.8	338	0.479	-0.373	0.90
10-III	16	13.4	0.84	1.01	62.2	534	0.368	-0.298	0.91
11-I	14	13.1	0.94	1.01	70.2	458	0.401	-0.353	0.96
11-II	14	14.4	1.03	1.01	78.7	384	0.444	-0.385	0.89
11-III	14	11.6	0.83	1.02	63.0	573	0.345	-0.294	0.94
12-I	14	13.6	0.97	1.01	78.3	668	0.398	-0.302	0.94
12-II	14	15.2	1.09	1.00	87.2	494	0.469	-0.363	0.94
12-III	14	12.1	0.86	1.01	69.7	918	0.326	-0.159	0.93
13-I	12	11.5	0.96	0.99	66.6	354	0.484	-0.403	0.95
13-II	12	12.7	1.06	0.99	74.4	334	0.498	-0.459	0.90
13-III	12	9.8	0.82	0.99	57.3	446	0.436	-0.367	0.94
14-I	16	14.7	0.92	1.01	70.5	440	0.429	-0.382	0.95
14-II	16	16.3	1.02	1.01	78.1	346	0.454	-0.446	0.90
14-III	16	13.4	0.84	1.01	64.1	507	0.397	-0.330	0.96
15-I	20	19.8	0.99	1.01	76.4	183	0.400	-0.450	0.89
15-II	20	21.4	1.07	1.01	82.4	130	0.430	-0.451	0.90
15-III	20	18.2	0.91	1.01	70.3	201	0.342	-0.437	0.93

backward stepwise method, the worst variable may be removed without considering what adding the best remaining variable might accomplish. The MAXR method could require much more computer time than the stepwise method.

Cohesive Soils

The measured resilient moduli values corresponding to the confining stress of 20.67 kPa (3 psi) and the deviator stresses of 13.78,

27.56, 55.12, and 68.9 kPa (2, 4, 8, and 10 psi) were used to obtain a close representation of stress conditions of subgrade. The stress conditions and the measured resilient moduli were used to develop relationships in which the resilient modulus is a function of deviator stress. The form of this relationship is given in Equation 6, the deviator stress model, which was transformed into the following linear form to carry out linear regressions:

$$\text{Log}(M_R) = \text{Log}(k_1 P_a) + k_3 \text{Log}\left[\frac{\sigma_d}{P_a}\right] \quad (9)$$

TABLE 3 Index Properties of Granular Soils

ID	SLT	CLY	CBR	SW	SH	S40	DEN
1	8	22	2.5	4	4	72	121
2	9	16	4.4	13	2	50	108
3	8	13	8.7	19	2	38	113
4	19	52	8.1	7	4	96	92
5	13	23	2.5	15	7	69	108
6	9	32	4.7	7	10	75	108
7	15	23	4.3	19	2	82	103
8	12	17	9.8	24	1	68	110
9	8	15	2.8	20	4	68	114
10	28	27	4.7	18	2	97	106
11	21	26	8.1	17	2	99	111
12	12	22	2.5	4	2	71	112
13	13	19	13.5	13	3	69	114
14	19	24	13.0	12	4	98	107
15	3	16	32.4	1	0	50	123

Linear regressions were performed for each set of data and k_1 and k_3 were found for each cohesive soil sample. Atmospheric pressure of 101.3 kPa (14.7 psi) was also used. The coefficients k_1 and k_3 that were obtained from least-square regressions of the deviator stress model for each sample are listed in Table 4 with sample MC, SATU, and COMP of the samples.

The multiple regression analysis approach, described in the discussion of granular soils, was used to obtain relationships for k_1 and k_3 . In these regressions, the independent variable CBR, which was used in granular soil regressions, was replaced by liquid limit (LL) and plasticity index (PI). Table 5 gives the index properties of each soil used for the study.

RESULTS

Granular Soils

Table 6 presents the descriptive statistics of the k values. Mean and median of each k value were close. Mean, maximum, and minimum values of k_1 were 421, 918, and 130, respectively. These results show that the k values of granular soils are subject to a great degree of variability. Equations 10–12 represent the regression equations found for k_1 , k_2 , and k_3 , respectively, from the multiple regression procedure. Complete regression results, including analysis of variance tables, are given in the Georgia Department of Transportation Special Report 91001 (19).

$$\begin{aligned} \text{Log}(k_1) = & 3.479 - 0.07 * MC + 0.24 * MCR + 3.681 \\ & * COMP + 0.011 * SLT + 0.006 * CLY - 0.025 \\ & * SW - 0.039 * DEN + 0.004 * (SW^2/CLY) \\ & + 0.003 * (DEN^2/S40) \quad R^2 = 0.94 \end{aligned} \quad (10)$$

$$\begin{aligned} k_2 = & 6.044 - 0.053 * MOIST - 2.076 * COMP + 0.0053 \\ & * SATU - 0.0056 * CLY + 0.0088 * SW - 0.0069 * SH \\ & - 0.027 * DEN + 0.012 * CBR + 0.003 * \{(SW^2/CLY)\} \\ & - 0.31 * (SW + SH)/CLY \quad R^2 = 0.96 \end{aligned} \quad (11)$$

$$\begin{aligned} k_3 = & 3.752 - 0.068 * MC + 0.309 * MCR - 0.006 * SLT \\ & + 0.0053 * CLY + 0.026 * SH - 0.033 * DEN \\ & - 0.0009 * (SW^2/CLY) + 0.00004 * (SATU^2/SH) \\ & - 0.0026 * (CBR * SH) \quad R^2 = 0.87 \end{aligned} \quad (12)$$

The success of multiple regression analysis to obtain relationships for k_1 , k_2 , and k_3 is shown in Figures 4–6. Actual material and physical properties of soils (given in Tables 2 and 3) were used to calculate k_1 , k_2 , and k_3 from Equations 10–12, respectively. These k values were labeled as predicted k values. The plots of predicted k values against actual k values with the line of equality clearly indicate the good fit of the regressions. The coefficients of determination for each regression (also given in the figures) were 0.94, 0.96, and 0.87 for k_1 , k_2 , and k_3 , respectively.

As a measure of model evaluation, predicted k values and the stress levels were used to back-calculate the resilient moduli values using the universal model. These resilient moduli values were labeled as predicted resilient moduli values. These predicted resilient moduli values versus actual laboratory-observed resilient moduli values and the line of equality are shown in Figure 7. This plot shows a close correlation between actual and predicted resilient moduli values.

The descriptive statistics for the actual and predicted resilient moduli values show that 50 percent of the predicted resilient moduli values were within ± 8 percent of actual values. They also show that 25 percent, 75 percent, and 90 percent of the predicted values were within ± 4 percent, ± 15 percent, and ± 21 percent of the actual values, respectively. Thus these results show that for granular soils the k parameters of Equation 4 have a direct relationship to material and physical properties of each soil.

Cohesive Soils

Table 7 gives the descriptive statistics of the k values. Mean and median of each k value were very close. Mean, maximum, minimum, and standard deviation of k_1 were 645, 1263, 188, and 252, respectively. These results show that the k values of cohesive soils are also

TABLE 4 Physical Properties of Cohesive Soil Samples and Their Least-Square Regression Results

ID	MOIST	MC	MCR	COMP	SATU	k ₁	k ₃	R ²
1-I	20	19.2	0.96	1.01	89.3	382	-0.466	0.94
1-II	20	21.7	1.09	1.00	98.7	287	-0.478	0.97
1-III	20	18.4	0.92	1.01	84.3	574	-0.322	0.89
2-I	20	20.2	1.01	1.00	93.0	276	-0.511	0.98
2-II	20	20.8	1.04	1.01	97.7	188	-0.598	0.98
2-III	20	17.5	0.88	1.01	82.7	450	-0.368	0.96
3-I	20	19.9	1.00	1.00	86.3	657	-0.188	0.80
3-II	20	21.1	1.06	1.00	92.1	431	-0.261	0.80
3-III	20	18.1	0.91	1.00	79.0	745	-0.128	0.51
4-I	19	18.1	0.95	1.00	88.7	608	-0.264	0.95
4-II	19	19.9	1.05	1.00	96.8	423	-0.272	0.97
4-III	19	16.4	0.86	1.00	80.8	774	-0.251	0.88
5-I	20	19.2	0.96	1.00	89.5	641	-0.219	0.99
5-II	20	20.6	1.03	1.00	96.1	442	-0.312	0.90
5-III	20	17.7	0.89	1.00	82.6	657	-0.134	0.78
6-I	17	16.6	0.98	1.00	85.0	777	-0.169	0.71
6-II	17	18.2	1.07	1.00	93.1	473	-0.235	0.74
6-III	17	14.6	0.86	1.01	75.1	913	-0.079	0.70
7-I	19	18.4	0.97	1.01	84.8	651	-0.273	0.94
7-II	19	19.1	1.01	1.01	89.2	549	-0.260	0.90
7-III	19	16.6	0.87	1.01	76.6	943	-0.136	0.98
8-I	18	17.7	0.98	1.01	85.0	460	-0.323	0.90
8-II	18	18.6	1.03	1.01	90.7	299	-0.424	0.97
8-III	18	15.9	0.88	1.01	77.0	599	-0.177	0.78
9-I	15	14.2	0.95	1.00	78.1	650	-0.243	0.93
9-II	15	15.5	1.03	1.00	85.6	474	-0.366	0.97
9-III	15	12.2	0.81	1.01	68.0	823	-0.072	0.97
10-I	16	15.6	0.98	1.01	83.3	917	-0.204	0.98
10-II	16	16.3	1.02	1.01	89.0	685	-0.211	0.90
10-III	16	13.6	0.85	1.01	73.9	1169	-0.074	0.97
11-I	21	20.2	0.96	1.00	77.1	916	-0.184	0.95
11-II	21	21.5	1.02	1.00	90.3	748	-0.216	0.84
11-III	21	18.5	0.88	1.01	77.8	1263	-0.090	0.99
12-I	16	15.5	0.97	1.01	83.1	541	-0.414	0.89
12-II	16	17.1	1.07	1.01	91.3	310	-0.501	0.98
12-III	16	13.6	0.85	1.01	73.7	808	-0.274	0.86
13-I	18	19.1	1.06	0.99	83.0	967	-0.109	0.89
13-II	18	20.5	1.14	0.99	89.1	734	-0.176	0.82
13-III	18	17.7	0.98	0.99	76.8	1181	-0.068	0.98
14-I	22	19.9	0.90	1.02	82.5	560	-0.221	0.91
14-II	22	21.7	0.99	1.02	89.5	442	-0.262	0.93
14-III	22	18.5	0.84	1.02	76.7	691	-0.206	0.95

TABLE 5 Index Properties of Cohesive Soils

ID	SLT	CLY	LL	PI	SW	SH	S40	DEN
1	14	52	40.3	20.9	3	9	79	105
2	11	29	35	10.8	5	7	67	107
3	14	55	38.9	19.2	3	11	90	106
4	10	36	36	17.5	6	8	84	108
5	43	39	40.5	17.8	13	8	89	106
6	6	32	46.5	30.4	10	12	88	109
7	11	39	43	18	7	5	73	104
8	14	31	40	13.1	16	7	72	107
9	10	27	33	11.3	17	4	64	112
10	12	32	49	32	9	10	95	111
11	10	51	59	18	3	14	83	103
12	11	31	30	12	7	4	50	110
13	7	40	34	14	2	5	87	111
14	10	43	39	14	3	1	95	101

TABLE 6 Mean, Median, Standard Deviation, Maximum, and Minimum of k Values of Granular Soils

	Samples	Mean	Median	Std.Dev.	Maximum	Minimum
k_1	45	421	401	173	918	130
k_2	45	0.34	0.33	0.089	0.50	0.15
k_3	45	-0.37	-0.36	0.095	-0.15	-0.63

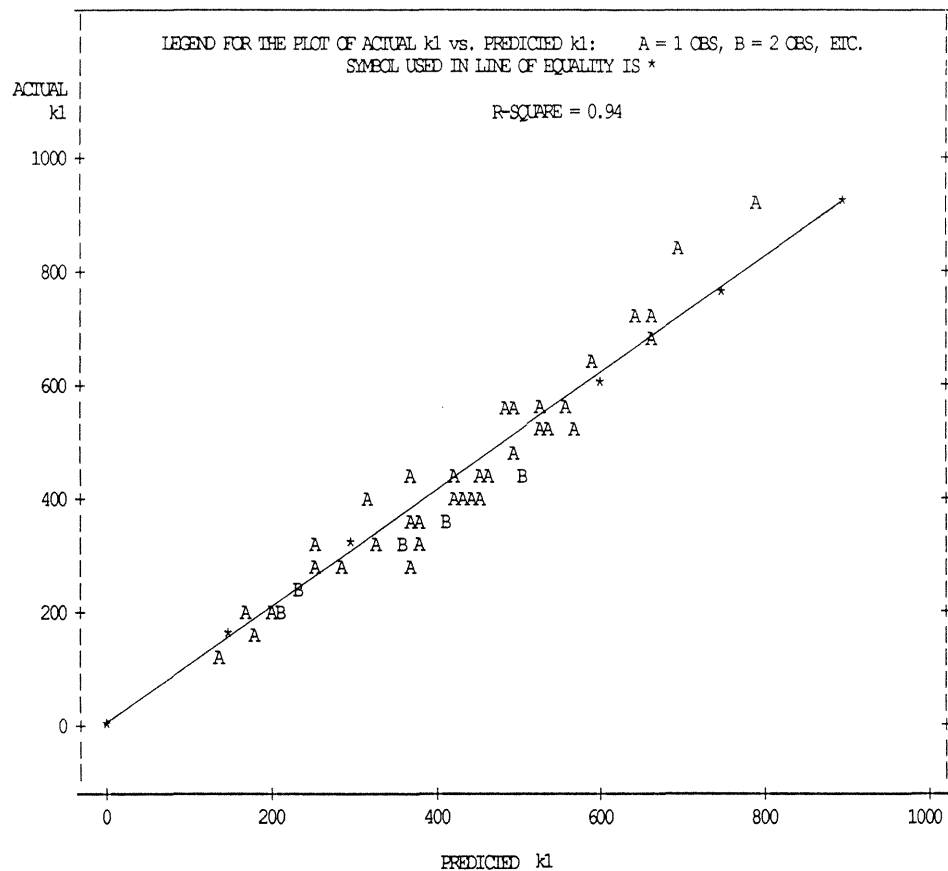


FIGURE 4 Actual k_1 versus predicted k_1 for granular soils.

subject to a great degree of variability. Equations 13 and 14 represent the regression equations found for k_1 and k_3 , respectively, from the multiple regression procedure. Complete regression results, including analysis of variance table are presented elsewhere (19). Results of an evaluation of the success of multiple regression analysis to obtain the relationships between k_1 and k_3 are shown in Figures 8 and 9. Actual material and physical properties of soils (given in Tables 4 and 5) were used to calculate k_1 and k_3 from Equations 13 and 14, respectively. These k values were labeled as predicted k values. The plots of predicted k values against actual k values show the effectiveness of the regressions. The coefficients of determination for each regression (also given in the figures) are 0.95 and 0.88 for k_1 and k_3 , respectively.

$$\begin{aligned} \text{Log}(k_1) = & 19.813 - 0.045 * \text{MOIST} - 0.131 * \text{MC} - 9.171 \\ & * \text{COMP} + 0037 * \text{SLT} + 0.015 * \text{LL} - 0.016 * \text{PI} \\ & - 0.021 * \text{SW} - 0.052 * \text{DEN} + 0.00001 \\ & * (\text{S40} * \text{SATU}) \quad R^2 = 0.95 \end{aligned} \quad (13)$$

$$\begin{aligned} k_3 = & 10.274 - 0.097 * \text{MOIST} - 1.06 * \text{MCR} - 3.471 \\ & * \text{COMP} + 0.0088 * \text{S40} - 0.0087 * \text{PI} + 0.014 * \text{SH} \\ & - 0.046 * \text{DEN} \quad R^2 = 0.88 \end{aligned} \quad (14)$$

As a measure of model evaluation, predicted k values and the stress levels of soil samples were used to back-calculate the resilient

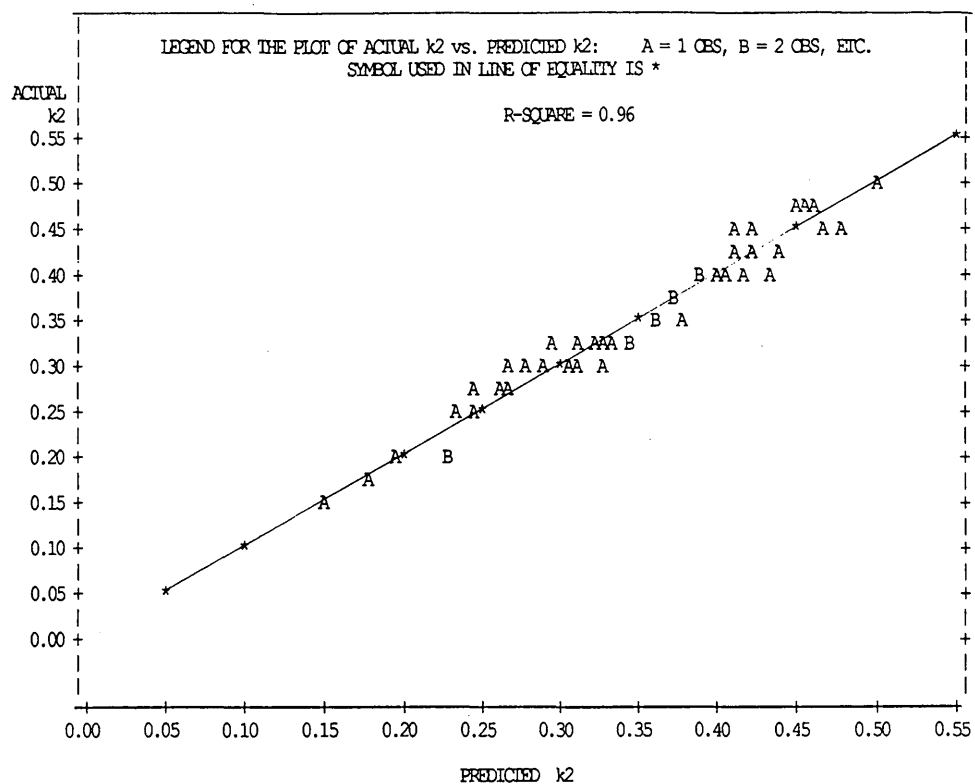


FIGURE 5 Actual k_2 versus predicted k_2 for granular soils.

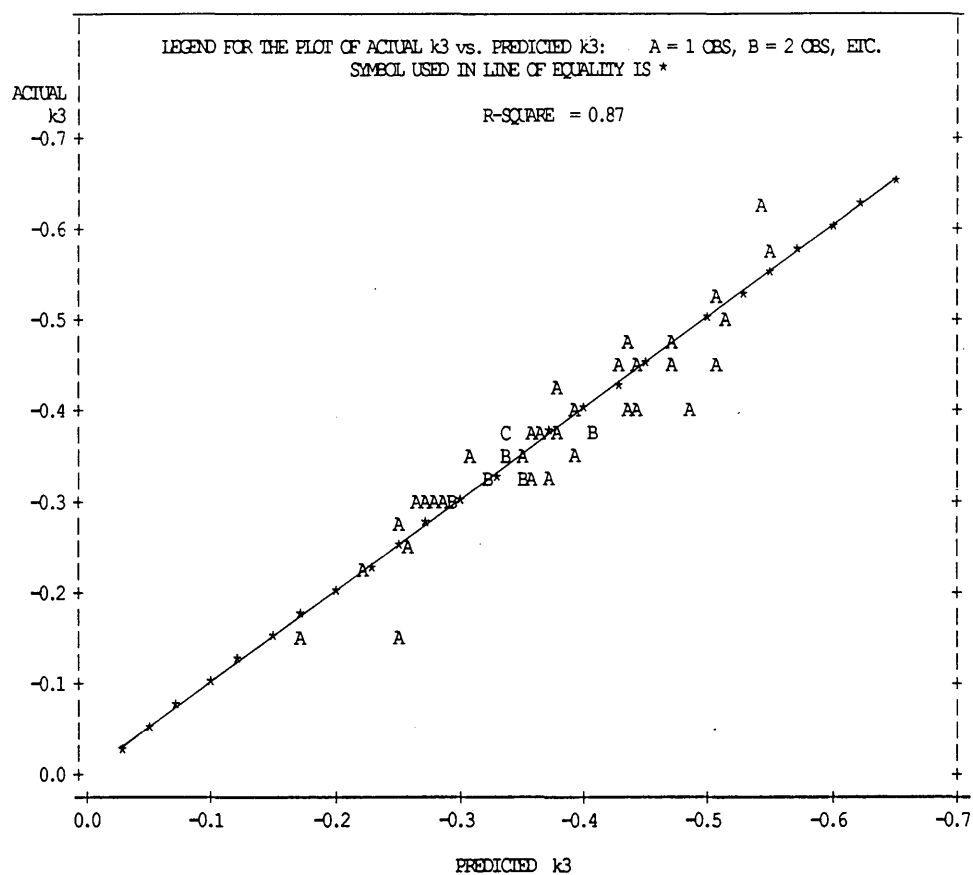


FIGURE 6 Actual k_3 versus predicted k_3 for granular soils.

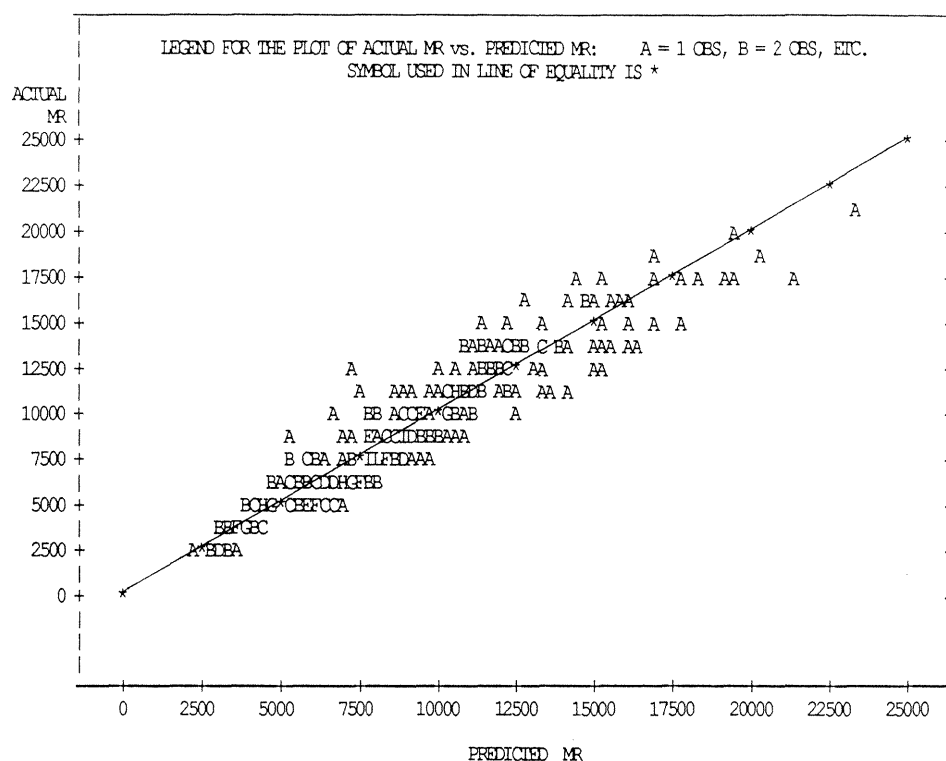


FIGURE 7 Actual resilient moduli versus predicted resilient moduli for granular soils.

moduli values using the deviator stress model. These values, named as predicted resilient moduli values, and the line of equality are shown in Figure 10. This plot shows a close correlation between actual and predicted resilient moduli values.

The descriptive statistics for actual and predicted resilient moduli values show that 50 percent of the predicted resilient moduli values were within ± 7 percent of actual values. They also show that 25 percent, 75 percent, and 90 percent of the predicted values were within ± 3 percent, ± 11 percent, and ± 18 percent of the actual values, respectively. Thus these results show that the k parameters of Equation 6 have a direct relation to the material and physical properties of each soil.

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

1. Equation 4 (the universal model) is capable of describing the behavior of granular soils better than Equation 2 (the bulk stress model).

2. Both granular and cohesive soils have a wide spread in their k parameters. These k parameters depend on the material and physical properties of soil.

Recommendations

1. When a complete data base of resilient modulus test results and material and physical properties of soils is available, it is possible to develop a regression model to predict resilient modulus.

2. The universal model is recommended for use with cohesive soils if the model development data have more than one confining stress level.

3. It is important to omit any incorrect data from the data base.

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TABLE 7 Mean, Median, Standard Deviation, Maximum, and Minimum k Values of Cohesive Soils

	Samples	Mean	Median	Std.Dev.	Maximum	Minimum
k_1	42	645	645	252	1263	188
k_3	42	-0.26	-0.24	0.13	-0.07	-0.60

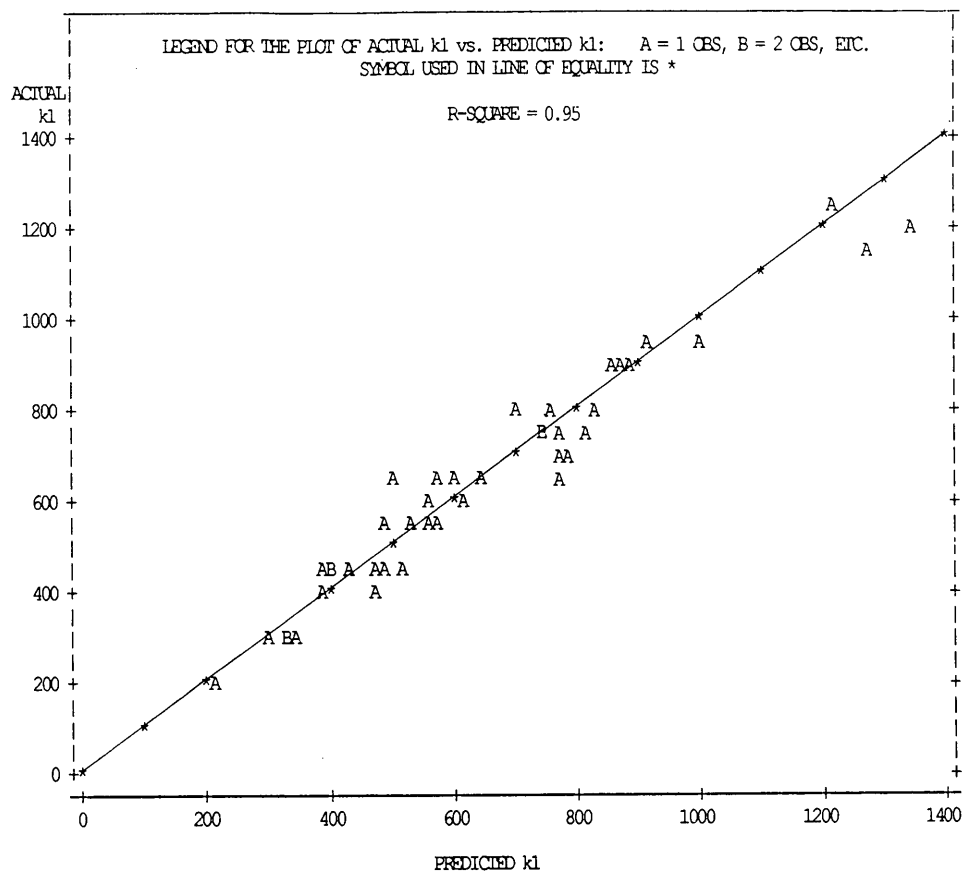


FIGURE 8 Actual k_1 versus predicted k_1 for cohesive soils.

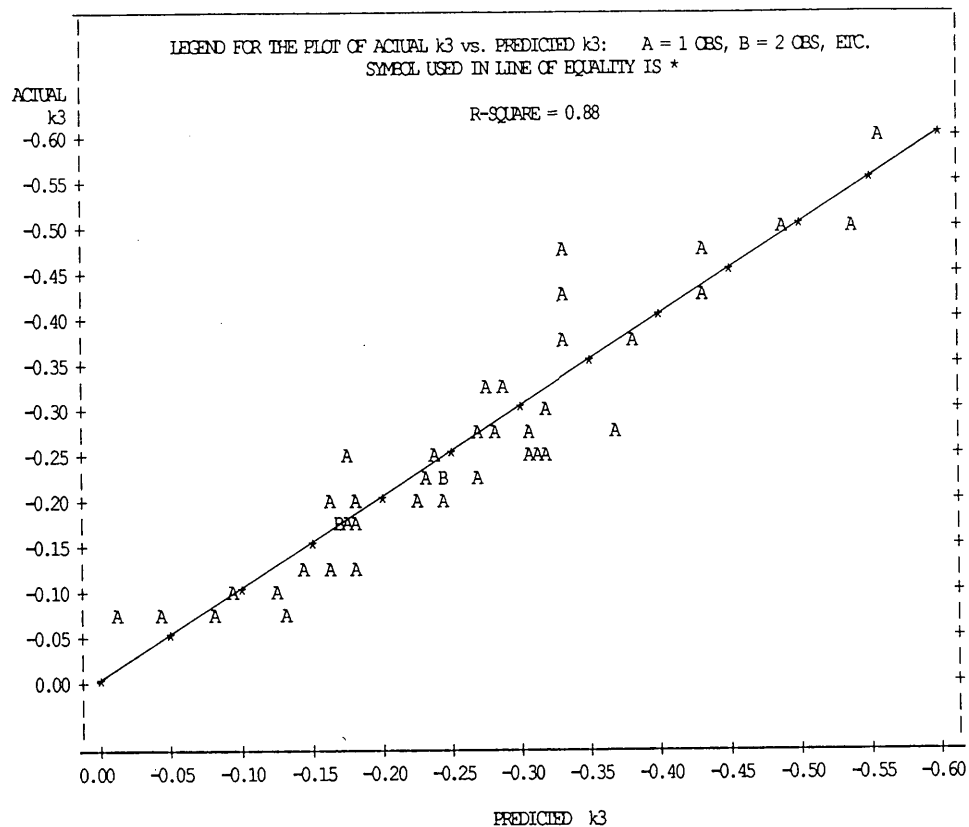


FIGURE 9 Actual k_3 versus predicted k_3 for cohesive soils.

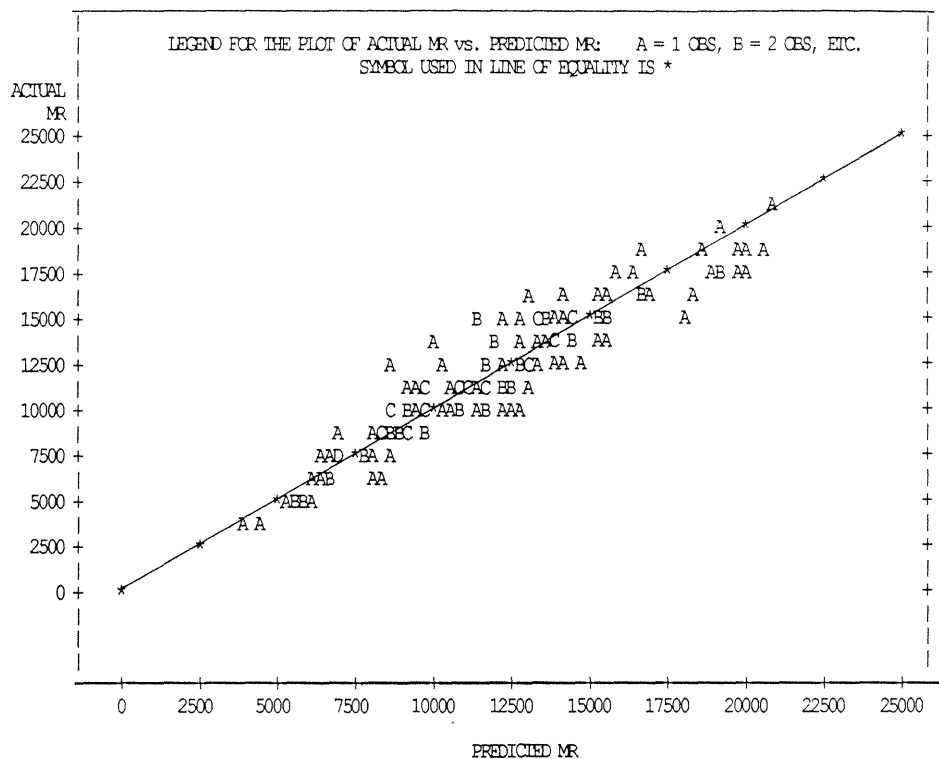


FIGURE 10 Actual resilient moduli versus predicted resilient moduli for cohesive soils.

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