Composite Pay Equations: General Approach

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Highway construction specifications involving the acceptance testing of several different quality characteristics are sometimes confusing and difficult to administer. A procedure is developed by which multiple quality measures may be combined in a rational manner in a single, composite pay equation. This approach is scientifically sound and may be applied to almost any construction specification for which a relationship between quality and performance is known or can be approximated. An example based on portland cement concrete pavement is presented to illustrate the practicality of this method.

Highway agencies use many construction specifications that award adjusted payment appropriate for the level of quality received. These specifications often involve acceptance testing of multiple quality characteristics—such as, thickness, strength, and riding quality of pavement—that must be combined in some way to arrive at the overall pay factor for each construction item or lot of material. Typically, extensive specification language is required to describe exactly how the individual quality measures are to be combined. A method to simplify this process is sought.

It has long been recognized that uniformity of materials and construction is an important quality characteristic. Although uniformity is desirable, it probably is more important to ensure that very little of the construction item is of such low quality that more than routine maintenance will be required. This has led to the widespread use of the percent defective, representing the portion of the lot falling outside specification limits (or its counterpart, percent within limits) as the statistical measure of quality. This measure is well suited as an acceptance parameter because it encourages simultaneous control of both the process mean and uniformity.

The example that follows illustrates how the measures of percent defective for separate quality characteristics can be combined into a single pay equation to develop a process that is easy to understand and administer.

EXAMPLE

The New Jersey Department of Transportation (NJDOT) currently uses five measures of quality for portland cement concrete pavement: slump, air entrainment, thickness, compressive strength, and smoothness (riding quality).

Because it is possible to measure the slump and air entrainment of plastic concrete before it is placed, it has been the practice of NJDOT to accept or reject the concrete on the basis of these tests as it is delivered to the job site. Because the other three quality characteristics cannot be measured until after the concrete has been placed and cured, tests for these characteristics are typically completed a month or more after placement. In this case, the acceptance decision usually takes the form of a pay adjustment, depending upon the level of quality that has been achieved. One possible pay equation that combines these three measures for individual acceptance lots is the following:

$$PF = 105 - 0.12\ PD_{THICKNESS} - 0.10\ PD_{STRENGTH} - 0.11\ (PD_{SMOOTHNESS})^2 \qquad (1)$$

where

PF = pay factor (percent),
$PD_{THICKNESS}$ = thickness percent defective,
$PD_{STRENGTH}$ = strength percent defective,
$PD_{SMOOTHNESS}$ = smoothness percent defective length.

An advantage of using percent defective (instead of percent within limits) is the clarity it provides in the pay equation. It can easily be seen that Equation 1 pays a maximum of 105 percent when all quality measures are at zero percent defective and that this value decreases as the percent defective of any of the individual quality measures increases.

ACCEPTABLE AND REJECTABLE QUALITY LEVELS

For any statistical construction specification, an acceptable quality level (AQL) must be defined. This selection is usually based on empirical observation of quality levels that have performed well in the past, although it may be based on other engineering considerations. Values around $PD = 10$ below some appropriate limit have typically been used, and this is believed to be suitable for thickness and strength in this example. For pavement smoothness, various research studies of NJDOT concrete pavement with expansion joints have suggested that $PD = 5$ is an appropriate AQL.

At the other extreme, as a safeguard against seriously defective work, it is customary to define a rejectable quality level (RQL) at which the agency reserves the option to require removal and replacement, corrective action, or assignment of a minimum pay factor. As a general rule, RQL values must be set at sufficiently low levels of quality that such drastic action is truly warranted. Because pavement failure does not pose a major safety hazard (such as the catastrophic failure of a bridge member, for example), the RQL limits for thickness and strength can be set at relatively high levels of percent defective. For pavement smoothness, however, studies by NJDOT researchers indicate that a percent defective length of 15, computed from the cumulative length of dye marks put down in the wheel paths by a 3.05-m (10-ft) rolling straightedge, provides a ride

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that is so rough that immediate corrective action is usually required. The following table of RQL limits and pay factors is proposed:

Quality Measure	*RQL PD*	*RQL Pay Factor (Percent)*
Thickness	70	65
Strength	70	65
Smoothness	15	65

The proposed RQL of PD = 15 for riding quality is almost the same as NJDOT has used in the past. The RQL value of PD = 70 recommended for thickness and strength may appear to be more lenient than past practices but in fact is not.

The current RQL for thickness is defined as an average value that is more than 12.7 mm (½ in.) less than the design value. Based on the typical standard deviation of 6.4 mm (¼ in.) for rigid pavement thickness, this corresponds to a percent defective value of nearly 98 percent. The proposed RQL of PD = 70 is more demanding and corresponds to an average thickness deficiency of only about 3.2 mm (⅛ in.). [This is still about 12.7 mm (½ in.) less than the desired average thickness necessary to achieve the AQL of PD = 10. In other words, to meet the thickness requirement of this specification, the contractor must set a target value of about 9.5 mm (⅜ in.) greater than the design thickness.]

The current RQL for concrete pavement compressive strength is 10 percent below the structural design strength (f_c') of 20,670 kPa (3000 psi). Based on a typical standard deviation of 2067 kPa (300 psi), this corresponds to approximately PD = 85 below the class design strength of 25,493 kPa (3700 psi) for rigid pavement. In this case, the proposed RQL of PD = 70 below the class design strength is just slightly more stringent than past practices. It is believed to be sufficiently effective and administratively simpler to base the definitions of both the AQL and the RQL on the class design strength.

HOW THE ACCEPTANCE PROCEDURE WORKS

For pavement of clearly outstanding quality (zero percent defective for all three measures), Equation 1 awards a maximum pay factor of 105 percent. It is believed that the highway agency receives more than comparable value in terms of the extended service life of pavement of this quality.

It is also necessary to have some degree of bonus provision with specifications of this type for them to perform fairly. The reason for this is intimately linked with statistical sampling theory. Because of the natural variability of any sampling and testing process, some samples will underestimate the quality and others will overestimate it. Unless the adjusted pay schedule is designed to allow bonuses and reductions to balance in a natural way, the average pay factor will be biased downward at the AQL, and acceptable work may be unfairly penalized. Fortunately, even a small bonus provision usually corrects this problem.

For pavement of varying levels of percent defective in the three quality measures, Equation 1 assigns pay factors that range from the maximum of 105 percent to a minimum of about 65 percent. This minimum occurs when all three measures approach their respective RQL values.

If the RQL value is reached on any one of the quality measures, the highway agency has the option to require removal and replacement or corrective action. If for practical reasons these options are not exercised, a minimum pay factor of PF = 65 is assigned. There is no need to make the computation with the pay equation in this case.

A distinctive feature of this acceptance procedure is that, provided none of the quality measures reaches the RQL value, it permits surpluses and deficiencies in thickness and strength to offset one another. This is consistent with the AASHTO design equation, which allows the same flexibility when the initial design for the pavement is being developed. Another feature is the use of the second power (square) of smoothness percent defective in Equation 1. As a result, this term tends to exert greater influence so that relatively high levels of riding quality must be achieved for the contractor to benefit appreciably from the bonus provision.

Procedures of this type have two distinct advantages. One is technical: the equation can be designed to assign pay factors that are directly related to the value of the construction item through a rational, scientifically based process. The other advantage is administrative. The worksheet in Figure 1 shows how easy it is for anyone with a minimal amount of training to apply the procedure. In contrast, current specifications usually require careful attention to several isolated sections of text and may be more confusing and prone to error or misinterpretation.

PERFORMANCE MODELS

To the extent possible, pay schedules should be based on models that relate the various quality measures to the performance or service life of the construction items to which they are applied. When performance models are lacking and insufficient information is available from which to develop them, it will be necessary to rely on the judgment of experienced engineers.

The desire of engineers to create an incentive to control both the mean level and the uniformity of the construction process dictates the use of the percent defective as the statistical measure of quality. As already noted, some agencies use the counterpart of this quality measure, the percent within limits, which is functionally equivalent but sacrifices something in terms of clarity of the pay equation.

For pavement thickness and strength, the AASHTO design procedure (*1*) provides a convenient way to relate as-built quality to service life. Although it may eventually be supplanted by mechanistic design methods, the AASHTO procedure is widely used and provides basic guidance on the relative importance of key quality measures. For portland cement concrete pavement, it can be demonstrated that thickness and strength are of primary (and nearly equal) importance and that an increase in one can offset a decrease in the other.

For riding quality, a relationship such as the AASHTO design procedure is not available. However, an approximate performance model can be developed on the basis of expectations of experienced engineers. This will at least provide useful guidance in determining the term in the pay equation that pertains to pavement smoothness. The following assumptions are believed to be realistic:

- Design life = 20 years,
- AQL: $PD_{SMOOTHNESS}$ = 5 yields 20-year life expectancy,
- RQL: $PD_{SMOOTHNESS}$ = 15 yields zero life expectancy,
- $PD_{SMOOTHNESS}$ = 0 adds 5 years to intended 20-year design life, and
- The performance curve should be *S* shaped and approach the *X*-axis asymptotically.

Given these conditions and assumptions, an exponential relationship such as that given by Equation 2 provides a reasonable es-

ROUTE 123 SECTION 10A LOT 21 DATE 6/1/93

START STATION 12 + 34 END STATION 22 + 34

TEST VALUES AND PD COMPUTATIONS

THICKNESS: 9.987, 10.623, 10.152, 10.229, 10.386

STRENGTH: 3825, 4230, 3870, 4520, 4015

SMOOTHNESS: 4, 3, 4, 2, 1, 3, 1, 4, 2, 4, 4, 3, 3, 4

THICKNESS	STRENGTH	SMOOTHNESS
Limit 10	Limit 3700	Total Length = 1000
N = 5	N = 5	Defective Length = 42
$\bar{X}$ = 10.275	$\bar{X}$ = 4092	PD = 4.20
S = 0.242	S = 286.7	
Q = 1.14	Q = 1.37	
PD = 12.37	PD = 6.56	NOTE: Q = ($\bar{X}$ - Limit)/S

RQL LIMITATIONS

THICKNESS	STRENGTH	SMOOTHNESS
PD < 70?	PD < 70?	PD < 15?
Yes ✓	Yes ✓	Yes ✓
No[a] __	No[a] __	No[a] __

(a) Removal and replacement, corrective action, or PF = 65. Skip pay factor computation.

PAY FACTOR COMPUTATION

$$PF = 105 - 0.12\ PD_{THICKNESS} - 0.10\ PD_{STRENGTH} - 0.11(PD_{SMOOTHNESS})^2$$

$$= 105 - 0.12(12.37) - 0.10(6.56) - 0.11(4.20)(4.20)$$

$$= 105 - 1.48 - 0.66 - 1.94 = 100.92$$

FINAL DISPOSITION OF LOT

Remove/Replace: Yes __ No ✓ Corrective Action: Yes __ No ✓

If both "No", enter pay factor: PF = 100.92

FIGURE 1 Worksheet for composite acceptance procedure.

timate of the expected years of service life ($L_{EXPECTED}$). Figure 2 shows that this equation satisfies the assumed conditions.

$$L_{EXPECTED} = 25\, e^{-0.001785(PD_{SMOOTHNESS})^3} \quad (2)$$

This performance relationship can then be combined with Equation 3 (*2*, p. 21) to develop a table of appropriate pay factors as a function of smoothness percent defective.

$$PF = 100\,[1 + (C_{OVERLAY}/C_{PAVEMENT})\,(R^{L_{DESIGN}} - R^{L_{EXPECTED}})/(1 - R^{L_{OVERLAY}})] \quad (3)$$

The terms and typical NJDOT values for rigid pavement with expansion joints are as follows:

PF = appropriate pay factor (percent) = dependent variable,
$L_{EXPECTED}$ = expected life of pavement (years) = independent variable,
$C_{PAVEMENT}$ = unit cost of pavement (bid item only) = \$47.85/m² (\$40/sy),
$C_{OVERLAY}$ = unit cost of overlay (total in-place cost) = \$11.96/m² (\$10/sy),
L_{DESIGN} = design life of pavement = 20 years,
$L_{OVERLAY}$ = expected life of overlay = 10 years,
$R = (1 + R_{INFLATION}/100)/(1 + R_{INTEREST}/100)$,
$R_{INFLATION}$ = long-term annual inflation rate = 4.0 percent, and
$R_{INTEREST}$ = long-term annual interest rate = 8.0 percent.

This expression can be applied to develop pay schedules for either rigid or flexible pavement. For the portland cement concrete pavement in this example typical results are as follows:

Smoothness Percent Defective	*Expected Life (Years)*	*Appropriate Pay Factor (Percent)*
0.0	25.0	106.4
2.5	24.3	105.6
5.0	20.0	100.0
7.5	11.8	86.4
10.0	4.2	69.5
12.5	0.8	60.1
15.0	0.1	58.0

Although these values must be regarded as approximate, they are the result of a rational model and procedure that relates riding qual-

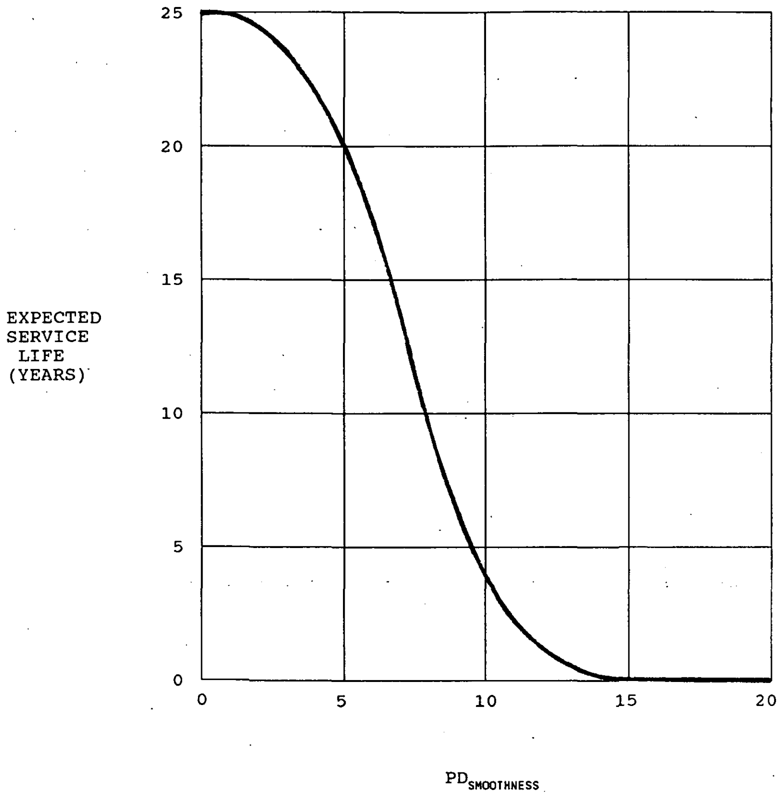

FIGURE 2 Approximate performance model for pavement smoothness.

ity to expected life and economic value of the as-constructed pavement. As such, they provide guidance useful in developing the pay schedule. Sensitivity tests have shown that these results are relatively stable over a wide range of input values. To be conservative, it was decided to set the maximum obtainable pay factor at a value somewhat less than 106 percent and the minimum pay factor for RQL work somewhat higher than 58 percent, as in Equation 1 and the in-text table of RQL limits.

DEVELOPING THE PAY EQUATION

Although there are an almost limitless number of forms the pay equation could assume, Equation 1 represents one of the simplest ways to translate the information in the performance models into a workable acceptance procedure. The coefficients of the terms in the pay equation must be chosen with care to be appropriate and defensible. A logical sequence of steps that works well with equations of this type follows:

1. Select the maximum (bonus) pay factor believed to be justified by superior quality. This is the intercept (constant term) of the pay equation.
2. Select the coefficients of the individual terms so that (a) the equation pays 100 percent when all quality measures are at their respective AQL values, (b) the magnitude of each coefficient reflects the relative importance of the corresponding quality measure, and (c) the amount of pay adjustment (bonus or reduction) is consistent with available performance models.
3. Select appropriate RQL values and the minimum pay factor to be assigned when the option to require removal and replacement is not exercised. This provision has considerable influence on how the average pay factor declines as quality decreases.
4. Check the operating characteristic (OC) curves for the complete acceptance procedure to ensure the procedure will perform as intended.

Step 1

In addition to the fairness issue discussed, both the AASHTO design equation and the approximate performance model for riding quality suggest that some degree of bonus for superior quality is warranted. On the basis of the potential savings associated with extended service life, it was decided that bonus pay factors up to a maximum of 105 percent are justified.

Step 2

The AQL values of percent defective for thickness, strength, and smoothness are 10, 10, and 5, respectively. Sensitivity tests (*3*) of the AASHTO design procedure indicate that thickness has slightly more effect than strength and accordingly warrants a slightly larger coefficient in the pay equation. The desire to make smoothness more dominant dictates a quadratic (squared) term for this part of the expression. The coefficients presented in Equation 1 were determined by trial and error to achieve the desired results.

Step 3

Another study (*4*) involving an engineering-economics analysis of portland cement concrete pavement concluded that a pay factor of approximately 60 percent is appropriate if the pavement is so poorly constructed that an immediate overlay is required. Equation 3, when combined with the approximate performance model for riding quality, produces essentially the same result. By using this information as a guide, a less stringent RQL pay factor of 65 percent has been recommended whenever the option to require removal and replacement is waived.

Step 4

As a final step, it is necessary to check the OC curves to determine how the complete acceptance procedure will perform. This is described in the following section.

SIMULATION TESTS

To determine how any statistical acceptance procedure will perform, it is necessary to examine the OC curves. A general discussion of this topic is contained in a recent TRB publication (*2*, p. 19). This step provides information in graphical or tabular form indicating the capability of the acceptance procedure to discriminate between acceptable and unacceptable work. It is through the study of such curves that the risks to both the highway agency and the contractor can be known and controlled at suitably low levels.

A computer program was developed to test thoroughly the acceptance procedure given by Equation 1 over a wide range of quality levels. Figure 3 gives an example of the output for a typical run. Figures 4 and 5 give several OC curves obtained from a series of runs with this program. It can be seen from Figure 4 that the theoretically appropriate pay relationship obtained with the tentative performance model for pavement smoothness is sloped more steeply than the current NJDOT stepped pay schedule, suggesting that somewhat larger pay reductions should be assessed for substandard quality and that some degree of bonus is warranted for superior quality. It can also be seen in this figure that, when thickness and strength are precisely at the AQL, both the pay schedule proposed as Equation 1 and the resulting OC curve lie about midway between the current stepped pay schedule and the theoretically appropriate pay levels. It is believed that such a compromise is a practical blend of effectiveness and defensibility, providing ample incentive to produce good quality pavements and at the same time adequately protecting the agency's interests. Note that, when all three characteristics are at their respective AQL levels (i.e., smoothness PD = 5 in Figure 4), the average pay factor is 100 percent, providing the contractor with full payment when acceptable work is delivered. As the smoothness percent defective approaches zero (extremely smooth riding quality), the expected pay factor exceeds 100 percent, approaching a maximum of about 103 percent.

To see what happens when thickness and strength are at other than their AQL values, a series of OC curves is plotted in Figure 5 on a more expanded scale. Curve B, for which thickness and strength are at their AQL values, is the same curve that appears in Figure 4. Note that almost the same curve is obtained when thickness PD = 0 and strength PD = 20, demonstrating that surpluses and deficiencies in these two quality measures can compensate for each other.

Curves C and D do not compensate in a similar manner to produce a single curve because the degree to which thickness and strength compensate is a function of the actual values involved. The negative effect of poor quality becomes greater as the quality decreases because there is a greater chance of triggering the RQL provision and having the minimum pay factor assigned. Curve D, because it represents a more extreme case of poor quality, indicates generally lower pay levels than Curve C. This may be a desirable effect because it tends to reward uniformity of quality.

Curve A was constructed under the assumption that both thickness and strength are at truly superior levels of zero percent defective. In this case, the curve approaches the maximum bonus pay factor of 105 percent as the pavement riding quality approaches an equally superior value. The other curves in this figure are labeled to

SIMULATION OF COMPOSITE ACCEPTANCE PROCEDURE FOR PCC PAVEMENT

REPLICATIONS = 1000

SEED NUMBER = 1234567

	SAMPLE SIZE	PERCENT DEFECTIVE	CV	RQL PD	RQL PAY FACTOR
THICKNESS	5	10	2.42	70	65
STRENGTH	5	10	7.34	70	65
SMOOTHNESS	1	VARIABLE	20.00	15	65

PF = 105 - .12 PD(THK) - .10 PD(STR) - .11 PD(SMOOTH)**2

PD(SMOOTH)	AVERAGE PF
.0	102.7
2.5	102.1
5.0	100.0
7.5	96.4
10.0	91.2
12.5	83.1
15.0	75.3
17.5	69.2
20.0	66.8

FIGURE 3 Typical output for computer simulation test.

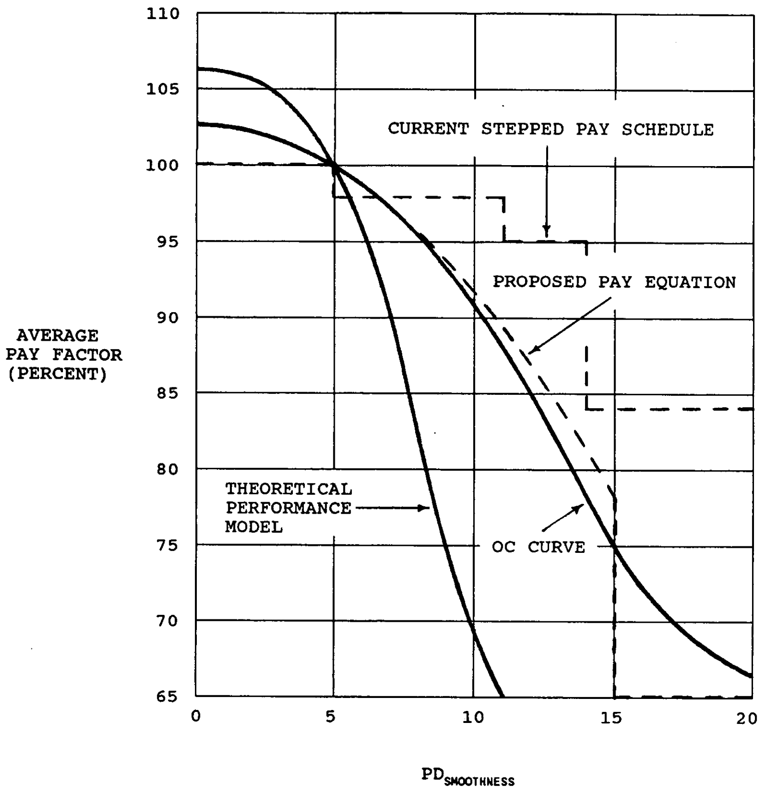

FIGURE 4 OC curve with thickness and strength held constant at AQL values.

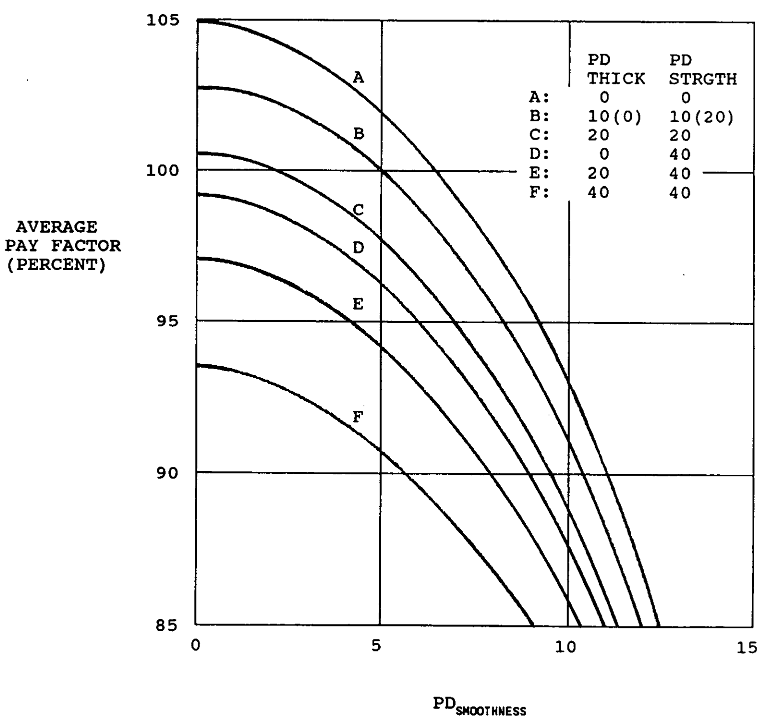

FIGURE 5 OC curves for selected combinations of quality levels of thickness and strength.

show various selected levels of quality control for thickness and strength. Curve C, for example, indicates that when thickness and strength are both marginally defective at a level of PD = 20 the pavement must be extremely smooth to obtain an average pay factor of 100 percent. Similarly, Curve A demonstrates that, if the smoothness percent defective exceeds PD = 6.5, the overall average pay factor will not exceed 100 percent no matter how thick or strong the pavement is.

OTHER PAY EQUATION FORMS

Although the linear form of pay equation may be the most practical for many applications, there may be situations in which other forms are more suitable. In Equation 1, the second power was used for the riding quality term to increase the effect of this term as the level of quality decreased. If the opposite effect had been desired, the square root (or some other fractional power) could have been used.

Another way that riding quality could have been made the dominant factor in Equation 1 would be to add the constraint that the maximum pay factor (PF_{MAX}) to be awarded is limited by a second expression, such as Equation 4. This is a more powerful restriction that was not thought to be necessary in this particular example.

$$PF_{MAX} = 111 - 2.0\ PD_{SMOOTHNESS} \tag{4}$$

There could be still other situations in which either the product or the average of the individual pay adjustments might be the most appropriate mathematical expression to use in the pay equation. In effect, Equation 1 makes use of a weighted average because the coefficients of the three terms are not identical. In certain special cases, either the largest or the smallest of a series of individual pay adjustments might be most nearly representative of the effect on the actual value of the construction item. In most cases, however, a linear expression similar to Equation 1 is likely to be satisfactory. In all cases, the construction of the OC curves will answer questions about the ultimate performance of the acceptance procedure.

SUMMARY AND CONCLUSIONS

A method was developed by which several acceptance requirements can be combined into a single, composite pay equation. The procedure is rational and scientifically based to the extent that performance models relating quality to service life exist or can be approximated. It is believed that this approach requires considerably less specification language to describe, is much simpler to understand and administer, and is less prone to error.

An example based on portland cement concrete pavement was presented. In essence, the relatively complex AASHTO design equation was combined with engineering-economics principles and restated in the form of a linear pay equation (except for one quadratic term). Of necessity, there is some loss of rigor in this simplification, but it is believed to be made up by the practicality of this approach. An extensive series of OC curves was developed to demonstrate that the procedure will perform as desired over a wide range of conditions.

This general approach can be applied to almost any construction specification involving any number of acceptance parameters. The linear form of the pay equation is probably the most practical, but other forms may offer certain advantages and should not be ruled out. The widespread interest in developing better performance relationships will continue to improve the models that support this approach. Where such models are not yet available, engineering knowledge and experience can bridge the gap to ensure that sensible results are obtained over the range of conditions likely to be encountered. A series of steps was presented that can be useful in developing the pay equation and associated RQL provisions. The final and perhaps most important step is the construction of the OC curves to verify that the acceptance procedure will function as intended.

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