Viscoelastic Analysis of Hot Mix Asphalt Pavement Structures

G. M. Rowe, S. F. Brown, M. J. Sharrock, and M. G. Bouldin

A new method of analysis for mechanistic pavement design has been developed based on improved material characteristics, which more accurately reflects fatigue cracking and permanent deformation performance. The key components of the method involve the (a) dissipated energy fatigue criterion in which the cumulative energy dissipated in asphaltic materials is directly proportional to fatigue damage, (b) viscoelastic materials characterization, and (c) pavement analysis using the finite-element method for determining response to repeated wheel loading. The changing properties of asphaltic materials with temperature are a key feature of the procedure with temperature conditions being provided by using a heat flow model. Output of the program includes predictions of percentage fatigue damage and rut depth at various numbers of load applications.

An improved method of pavement analysis has been developed that more accurately reflects field performance than many procedures used in the past. It uses a viscoelastic characterization of the asphaltic material along with input about traffic, geometry, and climate. The need for the development of a new procedure is evident from the lack of prediction capability with existing procedures, particularly when new or unfamiliar materials are used. For example, polymer modified asphalts have different characteristics from conventional asphalts, particularly with regard to their ability to exhibit elastic recovery at elevated temperatures. This can be considered by using a simple rheological model such as the Burgers model (Figure 1), which contains elastic, viscous, and viscoelastic elements. The combination of elastic and viscous elements in series is a Maxwell element, whereas arranging the viscous and elastic elements in parallel is a Kelvin element. The strain associated with the viscoelastic element is completely recovered at infinite time. The introduction of polymers into asphalt materials increases the proportion of strain that is recoverable after loading and reduces that associated with the viscous element. Current programs used for pavement design generally rely on elastic analysis, for example, of layered systems in the Shell Pavement Design Manual (1). Design to limit permanent deformation is often related to vertical elastic strain at the formation level, which is conceptually invalid and, hence, a semiempirical procedure. It is based on backanalysis of pavements with known performance. It has been shown (2) that the measured strain response in pavement experiments is not well predicted by linear elastic theory. Thus, the use of elastic analysis methods alone has limitations and can result in incorrect characterization of pavement performance.

G. M. Rowe, SWK Pavement Engineering, Inc., P.O. Box 211, Millington, N.J. S. F. Brown, University of Nottingham, University Park, Nottingham, England. M. J. Sharrock, SWK Pavement Engineering, Inc., 22 Andover Road North, Winchester, England. M. G. Bouldin, Applied Technology, 1339 Allston, Houston, Tex.

A finite-element (FE) program known as PACE has been developed to perform the computations using viscoelastic material characteristics defined by testing mix specimens with a repeated load creep test or conducting a frequency sweep test. Strain hardening is considered by describing the material properties obtained from the repeated load creep test as a function of temperature and shear strain expressed in terms of the second deviatoric strain invariant. Temperature depth profiles for any location are incorporated by using on FE heat flow model known as HiRoad.

This paper describes the computational functions of the program, the material models, traffic data, temperature predictions, and computation procedures.

FINITE-ELEMENT MODEL

The software consists of a core FE program that interacts with other programs and subroutines that provide information on material properties, pavement temperatures, and traffic conditions (Figure 2). The greater part of the existing finite-element code, initially drawn upon for program development, is based on work described by Owen and Hinton (3). A brief description of the main features of the code follows.

The displacement method was adopted, so the unknowns are translational displacements at each node in the x and y global directions. Eight-node isoparametric elements are used, so the same shape functions are used for the geometry of the element as for the variation of the unknowns. Three independent stress components are allowed at each integration point to cater for plane strain conditions.

Linear isotropic elasticity is assumed for modeling elastic behavior with Young's modulus and Poisson's ratio part of the input. Viscoelastic materials are treated as a special case of viscoelastic-plastic behavior, in which the yield stress in the plastic slider element, for the onset of viscoplasticity is reduced to zero (see Figure 3). Elastic material behavior is also obtained as a special case, by specifying a very large yield stress for the plastic element so that viscoplastic flow cannot occur.

Wheel loading is applied in increments. At each increment, an initial solution is obtained with the current *out-of-balance forces* and a set of *pseudoforces* generated to drive the viscous flow during a subsequent time-stepping process. Each increment of load therefore makes up an initial solution followed by further equation solutions at a number of time steps. The size of time step employed is related to the material properties.

The method by which the basic one-dimensional *rheological model* is used for analysis with the finite-element method is briefly outlined in the following. Where viscoplastic components are referred to, these would also relate to purely viscous components, since the yield stress for plasticity F_o can be zero, in view of the rheological model employed (as illustrated in Figure 3).

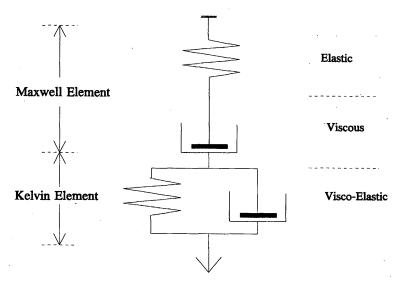


FIGURE 1 Burgers model.

The total strain is assumed to be separable into elastic and viscoplastic components and, hence, in terms of strain rates:

$$\dot{\mathbf{\epsilon}} = \dot{\mathbf{\epsilon}}_{e} + \dot{\mathbf{\epsilon}}_{vn} \tag{1}$$

The elastic component is assumed to experience the total stress acting, and therefore the total stress rate depends on the elastic strain rate by way of the elasticity matrix *D*:

$$\dot{\sigma} = D\dot{\epsilon}_e \tag{2}$$

Viscoplastic flow is assumed to occur when a scaler function of the stress (which is common to the elastic component and the viscoplastic component) and/or of the viscoplastic strain exceeds the scaler yield condition determined by the yield criterion. The yield condition may also be a function of a strain hardening parameter K. For flow to occur,

$$F(\sigma, \epsilon_{vp}) > \sigma_v(K)$$
 (3)

The viscoplastic strain rate is determined from the current *state of stress* by the relationship:

$$\dot{\epsilon}_{vp} = \gamma < \mathcal{F}(F) > \frac{\partial F}{\partial \sigma}$$
 (4)

In Equation 4, $\partial F/\partial \sigma$ represents a plastic potential and γ is a fluidity (reciprocal of viscosity). The yield function F is that of Von Mises and the function F(F) chosen is given by

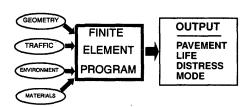


FIGURE 2 Finite-element program.

$$\mathcal{F}(F) = \frac{F - \sigma_{y}}{\sigma_{y}} \tag{5}$$

For the case of purely viscous flow, with zero yield stress for plasticity,

$$\mathcal{F}(F) = F \tag{6}$$

The resulting viscous strain increment is assumed to be entirely (shear) deviatoric (no volume change) and proportional to the current deviator stresses. The mixture viscosity is, consequentially, a shear viscosity in these circumstances. This is considered to lead to a reasonable approximation of viscous flow with viscous or viscoelastic material behavior. The time stepping is implemented by dividing the time into n intervals with the strain increment in a time interval of $\Delta t^n = t^{n+l} - t^n$, being written in the general form

$$\Delta \epsilon_{\nu p}^{n} = \Delta t^{n} \left[(1 - \theta) \,\dot{\epsilon}_{\nu p}^{n} + \theta \dot{\epsilon}_{\nu p}^{n+1} \right] \tag{7}$$

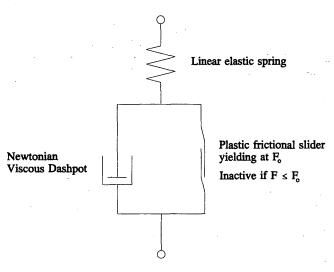


FIGURE 3 Rheological representation of model used.

The significance of this expression is that when $\theta=0$, the iteration scheme is the fully "explicit" (forward difference) time integration. The strain increment is completely determined from the conditions at the start of the time interval. When $\theta=1$, the iteration scheme is fully "implicit" (backward difference) time integration. If $\theta=0.5$, then the scheme becomes "implicit trapezoidal" and uses information from both ends of the time interval. The program currently uses the explicit scheme of iteration for the integration ($\theta=0$).

Equation 4 can be expressed as a linearized series as

$$\dot{\boldsymbol{\epsilon}}_{vp}^{n+1} \simeq \dot{\boldsymbol{\epsilon}}_{vp}^{n} + H^{n} \, \Delta \boldsymbol{\sigma}^{n} \tag{8}$$

where

$$H^{n} = \left(\frac{\partial \epsilon_{vp}}{\partial \sigma}\right)^{n} \equiv H^{n}\left(\sigma^{n}\right) \tag{9}$$

The general expression, Equation 7, for the strain increment results in

$$\Delta \epsilon_{vp}^{n} = \dot{\epsilon}_{vp}^{n} \, \Delta t^{n} + C^{n} \, \Delta \sigma^{n} \tag{10}$$

where

$$C^n = \theta \Delta t^n H^n \tag{11}$$

Equation 2 can be expressed in incremental form as

$$\Delta \sigma^n = D\Delta \epsilon_e^n = D(\Delta \epsilon^n - \Delta \epsilon_{vp}^n) \tag{12}$$

where

$$\Delta \epsilon^n = B^n \Delta d^n \tag{13}$$

The total strain increment is expressed in terms of the displacement increment as shown in Equation 13 and by substituting for $\Delta \varepsilon_{vp}^n$ from Equation 10, the stress increment becomes

$$\Delta \sigma^n = \hat{D}^n (B^n \Delta d^n - \dot{\epsilon}_{vp}^n \Delta t^n) \tag{14}$$

where

$$\hat{D}^n = (D^{-1} + C^n)^{-1} \tag{15}$$

To obtain a set of global equations in these circumstances, the conditions of equilibrium are next considered. Using Equations 10 and 14, the displacement increment during a time step is obtained from the following:

$$\Delta d^n = [K_T^n]^{-1} \Delta V^n \tag{16}$$

where

$$\Delta V^{n} = \int_{\Omega} [B^{n}]^{T} \hat{D}^{n} \, \dot{\epsilon}_{vp}^{n} \, \Delta t^{n} d\Omega + \Delta f^{n} \tag{17}$$

where Δf^n represents the increment of loading, assumed nonzero only at the start of time stepping during a load increment.

The parameter K_T^n is a tangential stiffness matrix defined by

$$K_T^n = \int_{\Omega} [B^n]^T \hat{D}^n B^n d\Omega \tag{18}$$

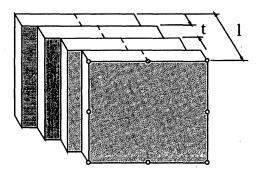
The algorithm shown in Equation 18 gives a linearized approximation to the incremental equilibrium, Equation 17, and, therefore, the total stresses accumulated from stress increments will not, in general, be quite correct. The technique adopted to reduce this error and to avoid using an iterative process in the solution, is as follows: a set of residual or out-of-balance forces ψ^{n+1} are determined for time n+1 from the relation in Equation 19. These residual forces are then added to the applied force increment at the next time step:

$$\psi^{n+1} = \int_{\Omega} [B^{n+1}]^T \sigma^{n+1} d\Omega + f^{n+1}$$
 (19)

After each load pulse is applied and values of deviatoric shear strain obtained, a loop in the program is made to the material file where, for the design temperatures, all the material properties are updated to be consistent with the second deviatoric strain invariant and the temperature. Thus, the model is considered to introduce nonlinear viscoelasticity into the material behavior (strain hardening) with successive load applications.

Experience has shown that real materials cannot be characterized by simple models as shown in Figure 3 but, in fact, require more complex models to explain their behavior. A method of obtaining more realistic material response, in the context of FE modeling, is to build up a composite action by using a number of different "overlays" of simpler materials, each with different characteristics. The material to be analyzed is assumed to be composed of several layers, each of which undergoes the same deformation (*strain compatibility*). The total stress field in the material is then obtained by a summation to which each part of the overlay contributes in proportion to the fractional weighting allocated in the total material. In a two-dimensional situation, the total thickness is taken to be unity and the weighting for each material simply equals its thickness in the overlay (4–6). This concept is illustrated in Figure 4.

This technique has been implemented to obtain a model that will allow input of parameters associated with a *generalized Maxwell model* (Figure 5). The initial version of the software uses properties associated with a *two-element Maxwell model with strain hardening* for permanent deformation calculations and a *four-element Maxwell model* for fatigue life calculations. With two elements, a reasonable description of the material is obtained but if 4 to 10 elements are used, an excellent fit results over a wider time range (7).



Unit thickness overlay composed of 4 materials, 2-D situation FIGURE 4 Overlay model.

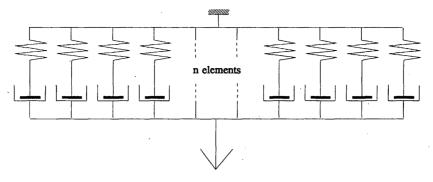


FIGURE 5 Generalized Maxwell model.

CLIMATE

Climatic factors play a dominating role in asphalt pavement design procedures since they affect both permanent deformation and fatigue cracking, as well as other modes of pavement distress. To enable a rigorous analysis of climatic effects to be introduced, an FE heat flow model HiRoad was used to generate 24 temperaturedepth profiles, one for each hour of the day. The calculations are repeated for 12 periods corresponding to each of the 12 months of the year. The model uses an energy balance calculation with boundary conditions that consist of a heat transfer coefficient used in conjunction with air temperature, together with a radiation flux, at the upper surface and a fixed temperature at a 1-m depth, equal to the average monthly air temperature. The computation is started at dawn, assuming a constant temperature with depth for simplicity. The heat flow equations are integrated by an explicit time-stepping procedure, while simultaneously, the heat transfer conditions at the surface are varied with time according to the predetermined patterns of air temperature together with direct and diffuse radiation. After 24 hr, dawn is again reached. The temperature-depth variation found at this time is treated as a new estimate of the starting conditions, and the time-stepping process is repeated for another 24 hr. In this way, successive approximations of the initial boundary conditions are obtained. When the initial and final states of the 24-hr period match closely, the desired solution for the period has been determined. The radiation at the surface is obtained as follows:

- \bullet Day—a mean value of solar constant of 1,362 W/m² is assumed, that is, the radiation intensity normal to the sun's direction above the earth's atmosphere. This is taken to vary seasonally by ± 3.5 percent due to the varying radius of the earth's orbit. Generally accepted published information on the proportion of the radiation reaching the ground is assumed, dependent on the elevation of the sun, the height above sea level, and cloud cover. An absorptivity of 0.9 is taken for the asphaltic materials.
- Night—A constant reradiation of 120 W/m² to space is assumed. This is developed and terminated linearly during the first hour and last hour of darkness, to give a pattern continuous with the daytime radiation input.

An approximate daily variation of air temperature for each month is constructed from average daily maximum and minimum temperatures, with an allowance of plus and minus a number of standard deviations to cover the required proportion of the extremes, varying linearly with maximum and minimum temperature over the year. Together with the computed surface temperature, this defines the

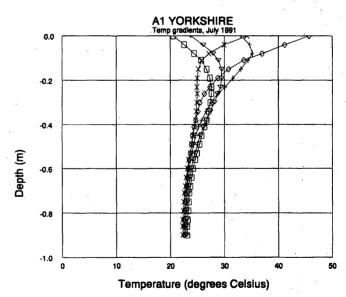
remaining surface heat transfer, using a heat transfer coefficient of 23 W/m²/°C.

The heat flow calculation is done iteratively, employing the previously mentioned FE method. Typical thermal properties are assumed for the asphaltic mixture as follows: conductivity (K_c) 1.5 W/m°K, mass density (ρ) 2 400 kg/m³, specific heat (Cp) 960 J/kg°K.

Those properties are used to obtain diffusivity:

$$\kappa = \frac{K_c}{(\rho \times Cp)} \tag{20}$$

Thus, the default value used in the FE heat flow calculations for diffusivity is 6.51e-7 m²/s. Typical examples for computed temperature depth profiles are given in Figure 6. These are then used to determine temperature and damage weighted material properties for design. The weighting methods are discussed with the performance models.



KEY:

- ☐ Sunrise + 0 hours
- × Sunrise + 5 hours
- ♦ Sunrise + 10 hours
- + Sunrise + 15 hours

Sunrise + 20 hours

FIGURE 6 Typical calculated temperature depth relationship.

TRAFFIC AND PAVEMENT GEOMETRY

Traffic Loading

Estimation of the number of cumulative equivalent single-axle loads (ESALs) is often required for design purposes. If known ESALs are used as input, however, the definition of an ESAL is a function of the analysis method, and although traditionally a fourth power law has been assumed, it is often not valid for the calculations. Consequently, the software allows the user to consider the effect of other axle loads or wheel configurations to obtain a conversion to ESALs. The latter is of particular interest because it allows the user to investigate the effect of different wheel load configurations with respect to the anticipated pavement damage (8-10).

Pavement Geometry

In most situations, a computer program for pavement design must have the capability to consider several bound and unbound layers. The program enables use of up to six asphalt bound layers (typically construction layers) and a further eight lower layers that could be either bound existing pavement layers, unbound granular layers (typically foundation layers), or soils. Generally, they consist of one granular base layer and up to seven foundation layers. The material model for the lower layers would consider the material to be elastic, whereas viscoelastic behavior is used for the upper asphalt bound layers. Nonlinear resilient response of the foundation is typically considered by increasing the foundation stiffness with depth. The user could use an elastic lower layer to describe an existing bound asphalt layer that is to be overlaid with a new asphalt layer and only model the upper layers (or layer) as viscoelastic layers.

PERFORMANCE MODELS

Two separate calculation procedures are performed, the first for fatigue cracking and the second for the permanent deformation. This approach is similar to existing procedures (1), and no interaction of the mechanisms of failure is currently considered.

Fatigue Cracking

Fatigue performance is predicted by considering the energy dissipated (work done) in asphaltic materials under loading, with the damage being proportional to the cumulative dissipated energy. This approach is similar to that used in the *Shell Pavement Design Manual* (1), with some additional features made possible by recent research, including a direct calculation of dissipated energy from the FE analysis and incorporation of an improved model for determining pavement life from consideration of dissipated energy. The calculation for fatigue life consists of the following steps:

- 1. Calculate dissipated energy contours in the pavement structure over a range of temperatures that cover the highest to lowest expected.
- 2. Calculate the fatigue life, from the relationships proposed in a work by Rowe (11), at each value of temperature considered in the program.
- 3. Calculate the damage under a single-axle load at a temperature, *T*, assuming Miners' linear damage rule to be valid:

$$D_T = \frac{1}{N} \tag{21}$$

4. For each month, using the relationship between damage and temperature, calculate the mean damage for the month. This is done by considering the traffic and temperature in 24 one-hr increments, that is.

$$D_{m} = \frac{\int_{1}^{24} \check{N}(t)D_{T}dt}{\int_{1}^{24} \check{N}(t)dt}$$
 (22)

where

$$\check{N}(t) = \frac{N(t)}{24}$$

$$\int_{1}^{\infty} N(t)dt$$
(23)

5. Assuming a uniform flow of traffic over the year, calculate the yearly traffic weighted mean damage from the following:

$$\bar{D} = \frac{1}{12} \int_{1}^{12} D_m dm \tag{24}$$

6. Using the relationship established earlier between temperature and damage under a standard axle load, it would be possible to equate the yearly traffic weighted mean damage to temperature and material properties. However, this is not necessary since life can be computed from

$$N = \frac{1}{\overline{D}} \tag{25}$$

To account for the effect of rest periods, lateral wander by traffic, and crack growth, the figures from Equation 25 are multiplied by factors of 10 or 14 for either 10 or 45 percent cracking, respectively (12). A typical contour plot of damage calculated by the FE analysis is shown in Figure 7, while Figure 8 indicates the prediction of cracking percentage against the number of axle loads.

Permanent Deformation

The permanent deformation performance calculated from the model uses developments of procedures in the *Shell Pavement Design Manual (1)* and those developed by Nunn (8). The major assumptions are

- Dilation is not allowed, so the effective Poisson's ratio associated with the permanent strain is less than 0.5;
- Uniaxial or shear properties are used to define a triaxial stress state; and
 - A yield condition is not currently implemented.

In addition to the foregoing assumptions, it is assumed that material properties change as the material undergoes permanent deformation and the viscoelastic properties are changed as a function of the permanent deviatoric strain. This introduces nonlinear SINGLE-WHEEL MODEL Refined 7x16 element mesh (PLANE STRAIN)
MINIMUM = 0.7 MAXIMUM = 38.5

CONTOURS OF ENERGY J/mcu

'92 RE-RUN

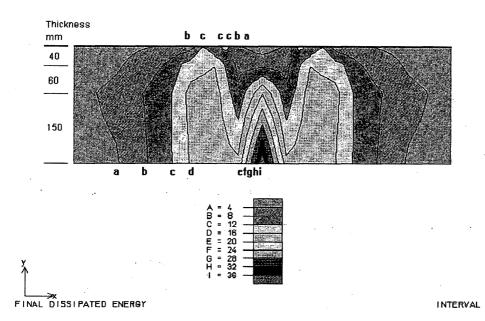


FIGURE 7 Fatigue damage contour.

(incrementally linear) properties and also a variation of properties within different elements of the pavement structure at a similar time increment. This aspect is different from the assumptions made in many design procedures (1,8) that use cumulative loading time to describe the nonlinear change in material properties. Temperature weighting is considered a function of the "inverse loss compliance," 1/D'', using the model presented in a work by Bouldin et al. (7).

Thus, the steps used to compute permanent deformation are as follows:

- 1. Calculate the traffic weighted pavement temperature using 1/D''.
- 2. For traffic Increment 1, calculate the permanent deformation for one axle and the permanent deviatoric strain.
- 3. Apply deformation for number of load passes considered in increment.

- 4. Obtain new material properties as a function of deviatoric shear strain.
- 5. Calculate permanent deformation for one axle pass in an increment.
- 6. Repeat Steps 3 to 5 until the design life or until a critical rut depth is obtained.

Typical examples of a rut development plot and deformed pavement shape are given in Figures 9 and 10, respectively.

CONCLUSIONS

A new method of pavement analysis based upon realistic material properties has been developed. Viscoelastic characterization of the material is used with appropriate criteria for fatigue damage and

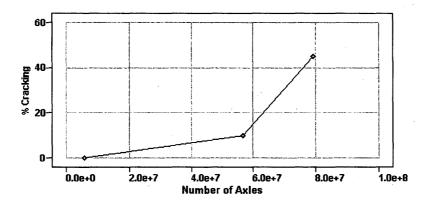


FIGURE 8 Percentage cracking versus life, damage development from fatigue calculation: crack initiation, 5.66e6 axles; 10 percent cracking, 5.66e7 axles; 45 percent cracking, 7.92e7 axles.

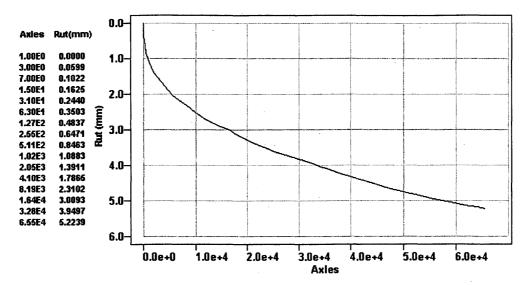


FIGURE 9 Computed rut development to 65535 wheel passes.

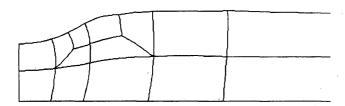


FIGURE 10 Deformed pavement shape (half of section), computed rut profile at 65536 wheel passes, deformations multiplied by factor 7.

permanent deformation. The key components of the method involve the following:

- 1. Dissipated energy fatigue criterion. This considers the energy dissipated (work) in asphaltic materials with the damage being directly proportional to the cumulative dissipated energy. This criterion replaces the "strain criterion" currently considered for fatigue damage.
- 2. Viscoelastic materials characterization. Repeated load creep testing allows the viscoelastic properties of an asphaltic material to be obtained. These parameters are directly related to rutting and enable predictions of deformation in the field and comparisons to be made between different materials.
- 3. Pavement modeling. A computerized method for determining stresses and strains associated with viscoelastic pavement materials has been developed. The computer program PACE uses a finite-element method and considers various inputs, such as pavement geometry, material properties, traffic, and environment. This method allows computation of pavement deformation and dissipated energy (as related to fatigue damage) to be performed.

NOTATION

English Symbols

 $B^n =$ Strain matrix at time station t^n (constant, when infinitesimal strain assumed)

 C_P = Specific heat

 $C^n = Matrix proportional to H^n$

d = Displacement vector

D = Elasticity matrix

D'' = Extensional loss compliance

 $D_m =$ Monthly traffic weighted mean damage

 D_T = Fatigue damage at a temperature T

D =Yearly traffic weighted mean damage

 \hat{D} = Elastic-viscoplastic matrix

e = Elastic

E =Elastic modulus

 f^n = Force vector (applied forces acting at time station n)

 $F(\sigma, \epsilon_{vp}) =$ Scaler yield function

FE = Finite element

 $H^n =$ Matrix dependent on yield criterion adopted, at time station n

J'' = Shear loss compliance

K = Tangential stiffness matrix

 $K_c = Conductivity$

n = Time station

N =Number of axles

 \check{N} = Normalized traffic distribution

Pi = Proportion of traffic in increment i

s = Specific heat

T = Transpose of vector or matrix

T = Temperature

vp = Viscoplastic

V = Viscosity

 V^n = Force vector of pseudoloads for time steps at time station n

Greek Symbols

 γ = Fluidity, reciprocal of viscosity (scaler)

 $\Delta = A$ small change in any quantity

 Δt^n = Time interval between t^n and t^{n+1}

 $\Delta \sigma^n = \text{Stress change occurring in time interval } \Delta t^n \text{ (vector)}$

 ε = Strain vector

 $\dot{\varepsilon} = Strain rate$

- θ = Time stepping algorithm parameter (scaler)
- $\kappa = Diffusivity$
- $\rho = Density$
- σ = Stress vector
- $\dot{\sigma} = \text{Stress rate}$
- $\sigma_v = \text{Uniaxial yield stress (scaler)}$
- $\Phi(F)$ = Scaler function of stress
 - Ψ = Out-of-balance force, or residual vector
 - Ω = Domain of problem

Other Symbols

- $\langle \rangle$ = Macaulay brackets
- $\partial F/\partial \sigma$ = "Plastic" potential for associated plasticity; enables strain rate to be defined as a function of stress gradient
 - $\mathcal{F} = Function$
 - Derivative with respect to time
- t^{n}, t^{n+1} = Two successive time stations

REFERENCES

- Shell Pavement Design Manual, Shell International Petroleum Company, London, England, 1978.
- Huhtala, M., R. Alkio, J. Pihljamaki, M. Pienmaki, and P. Halonan. Behavior of Bituminous Materials Under Moving Wheel Loads. *Journal of the Association of Asphalt Paving Technologists*, Vol. 59, 1990, pp. 422–442.

- 3. Owen, D. R. J., and E. Hinton. Finite Elements in Plasticity: Theory and Practice. Pineridge Press, Swansea, Wales, 1980.
- Zienkiewicz, O. C., G. C. Nayak, and D. R. J. Owen. Composite and Overlay Models in Numerical Analysis of Elasto-Plastic Continua. Presented at International Symposium on Foundations of Plasticity, Warsaw, Poland, 1972.
- Owen, D. R. J., A. Prakash, and O. C. Zienkiewicz. Finite Element Analysis of Non-Linear Composite Materials by Use of Overlay Systems. Computers and Structures, Vol. 4, 1974, pp. 1251–1267.
- Pande, G. N., D. R. J. Owen, and O. C. Zienkiewicz. Overlay Models in Time Dependent Nonlinear Material Analysis. *Computers and Structures*, Vol. 7, 1977, pp. 435–443.
- Bouldin, M. G., G. M. Rowe, J. B. Sousa, and M. J. Sharrock. Repetitive Creep as a Tool to Determine the Mix Rheology of Hot Mix Asphalt. *Journal of the Association of Asphalt Paving Technologists*, Vol. 63, 1994, pp. 182–223.
- 8. Nunn, M. E. Prediction of Permanent Deformation in Bituminous Pavement Layers. Research Report RR26. Transport and Road Research Laboratory, Crowthorne, England, 1986.
- Huhtala, M. The Effect of Different Trucks on Road Pavements. Proc., International Symposium on Heavy Vehicle Weights and Dimensions, Canada. 1986.
- Bell, C. A. and S. Randhawa. Truck Tire Issues: Evaluation of Impacts of High Pressure Tires and Single-Tired Axles in Oregon. Oregon State University, 1992.
- 11. Rowe, G. M. Performance of Asphalt Mixtures in the Trapezoidal Fatigue Test. *Journal of the Association of Asphalt Paving Technologists*, Vol. 62, 1993, pp. 344–384.
- Deacon, J. A., A. A. Tayebali, J. S. Coplantz, F. N. Finn, and C. L. Monismith. Fatigue Response of Asphalt—Aggregate Mixes: Part III Mix Design and Analysis. SHRP Project A-003A. Asphalt Research Program, Institute of Transportation Studies, University of California, Berkeley, 1992.

Publication of this paper sponsored by Committee on Flexible Pavement Design.