Analytical Procedures in Nondestructive Testing Pavement Evaluation

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An overview of typical procedures currently used for pavement evaluation using nondestructive testing deflection data is provided, focusing primarily on backcalculation. Some of the more typical problems encountered in these approaches are briefly discussed. Critical issues related to fundamental theoretical assumptions of static loading, as well as material continuity, homogeneity, and elastic behavior, are addressed, particularly in the context of validation of backcalculation results.

Structural evaluation of pavement deflection response using nondestructive test (NDT) data has been growing since the introduction of the Benkelman beam at the WASHO Road Test in the early 1950s. Developments in analytical techniques, coupled with improved deflection measurement capabilities, have resulted in the current so-called backcalculation techniques widely used in pavement evaluation. This paper provides an overview of existing techniques used for structural analysis of pavement NDT deflection data, discusses some of the issues and shortcomings of these procedures, and provides some conjecture on expected and possible future developments in the field.

DEFLECTION USES

Early use of deflection data typically involved consideration of maximum deflection directly under the load, relative to empirical standards. Usually some statistical measure of deflections on a pavement section is compared with a "tolerable" deflection level for that section under the expected traffic. If the measured value exceeds the tolerable deflection, an empirical procedure determines the corrective measure required—usually an overlay—to reduce the measured deflections to the tolerable level. Examples of this approach include the Asphalt Institute's MS-17 (*1*) and CalTrans's Test Method 356 (*2*). In some states, maximum deflections are monitored during spring thaw, and load restrictions are placed when the thawing pavement's deflection reaches a certain level. Empirical use of deflection basin data usually involves one of the basin parameters that combine some or all of the measured basin deflections into a single number.

With a trend toward mechanistic pavement analysis and design, which are based on fundamental engineering principles, the use of deflection data has become more sophisticated. Complete deflection basins are used, in a procedure known as backcalculation, to estimate in situ elastic moduli for each pavement layer. Knowledge of the existing layer thicknesses is typically necessary for this procedure. The backcalculated moduli themselves provide an indication of layer condition. They are also used in an elastic layer or finite-element program to calculate stresses and strains resulting from applied loads. These stresses and strains are used with fatigue or distress relationships to evaluate damage accumulation under traffic and predict pavement failure. They can also be used to evaluate corrective measures, such as overlays, rehabilitation, or reconstruction. It is these mechanistic analyses of pavement deflection that this paper is intended to address. Briefly, the backcalculation procedure involves calculation of theoretical deflections under the applied load using assumed pavement layer moduli. These theoretical deflections are compared with measured deflections and the assumed moduli are then adjusted in an iterative procedure until theoretical and measured deflection basins reach an acceptable match. The moduli derived in this way are considered representative of the pavement response to load and can be used to calculate stresses or strains in the pavement structure for analysis purposes.

Currently, calculation of theoretical deflections, and the subsequent stress or strain calculations, typically involve linear elastic theory. Elastic theory may be applied through the use of the following:

- Traditional layered elastic programs based on numerical integration procedures, such as ELSYM5, CHEVRON (various versions), BISAR, and WESLEA;
- The Odemark-Boussinesq transformed section approach instead of numerical integration;
- Finite-element programs, either those that have been specifically oriented toward pavement analysis, such as ILLI-PAVE or MICHPAVE, or general structural analysis programs such as SAP (various versions), ANSYS, ABACUS, ADINA, and so forth;
- Plate theory, such as the Westergaard solutions for portland cement concrete (PCC) pavements; and
- Neural networks trained to reproduce results that emulate one of the foregoing applications (*3,4*).

BACKCALCULATION

An in-depth summary of the historical developments of NDT, backcalculation, and theoretical considerations, as well as associated technology were provided in a state-of-the-art presentation in 1988 (*5*). Some of the concerns regarding the differences between backcalculated results using different backcalculation programs on the same deflection data were illustrated. These are typically technical problems but they are exacerbated by the continuing development of similar backcalculation programs. In many cases, new programs have little to differentiate them from existing software other than a name. In a description of the Strategic Highway Research Program (SHRP) backcalculation procedure software selection a table listing

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the most common backcalculation procedures in use at that time was originally included (*6*). This table, somewhat modified, is included here as Table 1 to illustrate similarities and differences between programs. In reviewing Table 1, one should keep in mind that the CHEVRON and ELSYM5 numerical integration routines are identical and, until recently, produced erroneous results under certain circumstances due to an error in the integration procedure. This error was corrected in 1992 by Irwin and verified by comparison with the BISAR program (L. H. Irwin, personal communication, 1993).

The programs included in Table 1 are by no means a comprehensive listing of backcalculation routines. Other programs in use today include COMDEF, 15BCONPAS, PROBE, ILLIBACK, LMBS, DEFMET, RPEDDI, PHONIX, PEACH, FALMAN, CLEVERCALC, EPLOPT, OAF, SEARCH, EFROMD, and more. Most of the programs rely on linear elastic layered theory, or a variation thereof, for the basic structural model. In comparing results from these programs, the primary criterion used for evaluating accuracy is based on the goodness of fit of computed deflections to measured deflections. As computing power has increased, so has the ability to improve the goodness of fit. In many cases, improving the goodness of fit does not necessarily mean that the theoretical model better represents actual pavement response. If an existing pavement structure is in such a condition that it clearly violates some of the fundamental assumptions of elastic theory, then a good fit between measured and calculated deflections should not be expected, and goodness-of-fit should not be the determining factor for deciding if a solution is realistic or not. This point is also made by Lytton (*5*), who discusses the need for experience in analysis, with materials and with deflections to ensure that the backcalculation process yields the most acceptable set of moduli for a given deflection basin. It should be noted that essentially all pavements violate the fundamental assumptions of linear elastic theory, albeit to differing degrees.

Also important, and related to the issue discussed previously, is the provision of a modulus value through backcalculation that is a layer parameter and not necessarily the layer material modulus, which can be measured using laboratory tests on a sample of the layer material. This is due to the geometry of typical deflection basin measurements that is on the order of a 1.8-m (6-ft) length so that the effect of horizontal layer and material variability over that dimension is included in the backcalculated moduli. This variability includes damage such as cracking, both on the macro- and microstructural level. Simply stated, the problem lies with the in situ modulus being unknown, so that backcalculated values cannot be validated directly.

PROBLEMS ENCOUNTERED IN BACKCALCULATION

Some of the more common problems encountered in backcalculation are briefly described in the following paragraphs. More detailed discussions are available elsewhere (*7*). In many cases, pavement deflection measurements include irregularities that are generally related to differences between measured pavement response and the theoretical models used to predict that response. These irregularities may result from a number of reasons, including pavement distress, variations in layer thickness, nonlinear material response, presence of bedrock or other stiff layers, moisture and temperature effects, and so forth. Anomalies within the pavement structure, such as culverts and utility ducts are not discussed here since they can be observed and are considered atypical. It should be pointed out that as backcalculation techniques mature, some of the problems are being addressed by software modification.

Input Data Effects

Input data effects include seed moduli, modulus limits, and layer thicknesses, as well as program controls, such as number of iterations and convergence criteria. Because of the nonuniqueness of the solution, it is possible to obtain different backcalculated moduli for a given deflection basin by using different seed moduli or limits. Many of these problems are being addressed by software development, such as using the measured deflection data to aid in selection of relevant input values.

Compensating Layer and Nonlinearity Effects

Compensating layer and nonlinearity effects essentially result from incorrect modeling of the pavement material response and the sequential nature of the backcalculation iterative procedure, as well as the geometry of a deflection basin test. A typical result may show, as an example, subgrade modulus that is significantly higher than expected for the material type, while the base layer modulus is far too low and the surfacing modulus is too high. This probably occurs most commonly for a significantly stress softening subgrade, where the subgrade stress level for the outer sensors in a falling weight deflectometer (FWD) test is very much lower than the subgrade stress level directly beneath the load plate. The apparent subgrade modulus for the outer sensor location is therefore higher than the apparent subgrade modulus directly beneath the load plate. If the subgrade is modeled as a linear elastic material, then—since most backcalculation routines first calculate subgrade modulus from the outer sensors—the higher modulus value is calculated and assumed to be constant throughout. At the next iteration, when the base modulus is being calculated, the too high subgrade modulus is compensated for by calculating a modulus that is too low for the base, to match the deflections measured in this region. In other words, alternating layers exhibit a high or low compensating effect. Ideally, correctly modeling nonlinear material response will avoid this type of error, and this is becoming more and more common (e.g., ELMOD, MODCOMP3, EVERCALC, and BOUSDEF can all use nonlinear material models). If an elastic subgrade is used, the inclusion of a stiff layer, or the use of a layered subgrade, can help alleviate the problem. This is at least partially the reason why some backcalculation routines include a stiff layer by default at some depth [usually approximately 6 m (20 ft)]. It is also worth noting that the effect of too rapidly decreasing deflections with distance is often due to the dynamic nature of the impulse load.

Subgrade Stiff Layer

For the purpose of a general definition, a "stiff" layer is one below which there is little or no apparent contribution to the measured surface deflections. Stiff layers can be real or apparent and are possibly the most common problem encountered during the evaluation of deflection basins.

TABLE 1 Partial List of Layer Moduli Backcalculation Programs *(6)*

Program Name	Developed By	Forward Calculation Method	Forward Calculation Subroutine	Back-calculation Method	Non-Linear Analysis	Rigid Layer Analysis	Layer Interface Analysis	Maximum Number of Layers	Seed Moduli	Range of Acceptable Modulus	Ability to Fix Modulus	Convergence Routine	Error Conver-gence Function
BISDEF	USACE-WES	Multi-Layer Elastic Theory	BISAR (Proprietary)	Iterative	No	Yes	Variable	Cannot Exceed No. of Deflec., Works Best For 3 Unknowns	Required	Required	Yes	Sum of Squares of Absolute Error	Yes
BOUSDEF	ZHOU, et.al. OREGON STATE UNIV.	Odemark-Boussinesq	Odemark-Boussinesq	Iterative	Yes	Yes	Fixed (Rough)	5, Works Best for 3 Unknowns	Required	Required	Yes	Sum of Percent Errors	Yes
CHEVDEF	USACE-WES	Multi-Layer Elastic Theory	CHEVRON	Iterative	No	Yes	Fixed (Rough)	Cannot Exceed No. of Deflec., Works Best For 3 Unknowns	Required	Required	Yes	Sum of Squares of Absolute Error	Yes
ELMOD/ ELCON	P. ULLIDTZ DYNATEST	Odemark-Boussinesq	Odemark-Boussinesq	Iterative	Yes (Sub-grade Only)	Yes (variable)	Fixed (Rough)	Up to 4, Exclusive of Rigid Layer	None	No	Yes	Relative Error on 5 Sensors	No
ELSDEF	TEXAS A&M UNIV., USACE-WES	Multi-Layer Elastic Theory	ELSYM5	Iterative	No	Yes	Fixed (Rough)	Cannot Exceed No. of Deflec., Works Best For 3 Unknowns	Required	Required	Yes	Sum of Squares of Absolute Error	Yes
EMOD	PCS/LAW	Multi-Layer Elastic Theory	CHEVRON	Iterative	Yes (Sub-grade Only)	No	Fixed (Rough)	3	Required	Required	Yes	Sum of Relative Squared Error	No
EVERCALC	J. MAHONEY, et.al.	Multi-Layer Elastic Theory	CHEVRON	Iterative	Yes	Yes	Fixed (Rough)	3 Exclusive of Rigid Layer	Required	Required	Yes	Sum of Absolute Error	No
FPEDDI	W. UDDIN	Multi-Layer Elastic Theory	BASINPT	Iterative	Yes	Yes (Variable)	Fixed (Rough)	Unknown	Program Gener-ated	Unknown	Unknown	Unknown	No
ISSEM4	R. STUBSTAD	Multi-Layer Elastic Theory	ELSYM5	Iterative	Yes (Finite Cylinder Concept)	No	Fixed (Rough)	4	Required	Required	Yes	Relative Deflec. Error	No
MODCOMP 3	L. IRWIN, SZEBENYI	Multi-Layer Elastic Theory	CHEVRON	Iterative	Yes	Yes	Fixed (Rough)	2 to 15 layers, Max 5 Unknown Layers	Required	Required	Yes	Relative Deflec. Error at Sensors	No
MODULUS	TEXAS TRANS. INSTITUTE	Multi-Layer Elastic Theory	WESLEA	Data Base	No	Yes (Variable)	Fixed?	Up to 4 Unknown plus Stiff Layer	Required	Required	Yes	Sum of Relative Squared Error	Yes
PADAL	S.F. BROWN, et. al.	Multi-Layer Elastic Theory	UNKNOWN	Iterative	Yes (Sub-grade Only)	Unknown	Fixed?	Unknown	Required	Unknown	Unknown	Sum of Relative Squared Error	Unknown
WESDEF	USACE-WES	Multi-Layer Elastic Theory	WESLEA	Iterative	No	Yes	Variable	Up to 5 Layers	Required	Required	Yes	Sum of Squares of Absolute Error	Yes
MICHBAK	MICHIGAN STATE	Multi-Layer Elastic Theory	CHEVRON	Iterative	No	Yes	Fixed	Up to 4 Unknown plus Stiff Layer	Required	Optional	Yes	Sum of Relative Squared Error	Yes

The stiff layer may, in fact, consist of rock or other stiff materials. However, the effect has also been observed where a water table is encountered near the surface. Possibly the most common phenomenon is due to the previously described subgrade nonlinearity effects resulting in an apparent stiff layer effect with backcalculated moduli exhibiting the compensating effect. For the case where an actual rigid layer exists, computer backcalculation programs such as MODULUS, BISDEF, and WESDEF have a rigid layer subroutine built in. Bedrock information can be obtained from geologic maps, by coring, or by penetration resistance measurement. Depth to the stiff layer can also be estimated from the deflection data as done in ELMOD and MODULUS. The best approach is to model the actual situation as closely as possible.

One approach for the "apparent" stiff layer problem—if a layered elastic backcalculation program is used—is to divide the subgrade into two or more layers, allowing the backcalculation program to assign modular ratios that achieve the best fit.

Pavement Layer Thickness Effects

As a result of limitations in the backcalculation software and the limited time available to perform backcalculation activities in a production environment, pavement layer thicknesses are generally assumed to be constant over the pavement section under test. This is seldom the case. Pavement layer thickness variations result from various construction and maintenance details, even under specially controlled conditions.

On Texas SHRP sections, it has been found that asphalt concrete thicknesses may vary up to 2 in. within 500 ft. Pavement layer thickness variations will produce variations in the deflections from point to point that are indistinguishable from layer moduli variations. The net result is that this variation manifests itself in the backcalculated moduli for the various layers. In some cases, these moduli variations are not significant. However it is desirable to use correct layer thicknesses, and various techniques, such as GPR, are improving the ability to obtain thickness data.

It should be noted that surface layer thicknesses of less than 75 mm (3 in.) cannot be reliably characterized with FWD data, primarily due to the geometry of the loading and measuring system. Moduli of thin layers are generally difficult to determine from FWD data.

Relative Layer Stiffness Effects

Backcalculaton can describe a pavement layer's stiffness only to the degree to which that layer affects the deflections. Thin layers contribute only a small portion to the overall deflection and as a result, the accuracy of their backcalculated values is reduced.

To some extent, the layer thickness discussion covers relative layer stiffness effects. However, the intent of this section is to emphasize that the layer stiffness (i.e., combination of thickness and modulus) needs to be relatively significant (compared with other pavement components) for it to influence the surface deflections. If this is not the case, then backcalculation approaches will not be successful in identifying the effect of the layer. As an example, consider a 200-mm-thick (8-in.) natural gravel base course. If this layer is placed on an average subgrade and surfaced with a chip seal, it is relatively stiff, and backcalculation will easily evaluate the difference in modulus between the base and subgrade. On the other hand, if this base material occurs beneath a 400-mm (16-in.) PCC slab, it is not relatively stiff and it is unlikely that the backcalculation process will be able to reliably separate the contribution of this layer from the subgrade effect.

Similar problems occur for many unbound base and subbase combinations. These materials often differ only in terms of gradation and indicator specifications, and their moduli are relatively similar, so that their contributions to the deflection response are difficult to separate. Similarly, if the surfacing consists of more than one asphalt concrete (AC) layer, they should be considered as a single layer. There is generally not enough difference between the response of an AC surfacing layer and an asphalt-treated base to evaluate these layers separately.

CRITICAL ISSUES IN BACKCALCULATION

When evaluating backcalculation procedures, it is important to be aware of the simplifications made in modeling the pavement structure. Most of the current procedures are based on the following assumptions:

- The loading is static,
- The materials are continuous and homogeneous, and
- The relationship between strain and stress follows Hooke's law, that is, linear elastic.

The Royal Dutch/Shell Laboratory in Amsterdam began studying pavement dynamics using a road vibration machine in 1951. Both dynamic deflections and wave propagation were used to determine the stiffness of different pavement layers (*8,9*). The work of Lamb (*10*) was used by the Laboratoire Central des Ponts et Chaussées in France (*11*) and more recently the work of Kausel and Peek (*12*) has been used by several researchers (*13,14*). Finite-element methods have also been used for dynamic analysis of road structures (*15,16*).

In spite of all the effort put into dynamic analysis, it is not widely used. One reason is the computational capacity required. Dynamic finite-element analysis, for example, requires a mainframe computer. More important, however, are the additional parameters needed to characterize the materials. In a dynamic analysis, the viscous and viscoelastic properties of the material should be considered; Poisson's ratio becomes more critical when using wave propagation; and the density of the different materials must also be known.

This leads to the second assumption: the materials are continuous or compatible. All of the above-mentioned methods are based on continuum mechanics, but few pavement materials are continuous. Most pavement materials are particulate in nature, and even in asphalt at normal temperature, the deformations from elastic compression of the grains are negligible compared with the deformations from sliding of the grains.

In well-compacted granular materials, volume expansion (dilation) often occurs under loading. In a paper on plasticity in soils, it was concluded,

> There has been a good deal of debate about unstable behavior that develops in association with volume expansions. Loading of such a soil is accompanied by local inhomogenities in the form of slip lines, shear bands, or "bifurcations," as they are now commonly called. . . . It occurs in real soils in nature very frequently, is the source of many soil engineering problems, and so far is not represented by a single soil

> model. At present, it is also difficult to see how a suitable model could be implemented in a finite element code, since each individual element must have the opportunity of developing shear bands as the loading progresses. Their position cannot be predicted in advance (*17*).

Since then, more widespread use has been made of the distinct-element method or micromechanical modeling based on the work of Cundall (*18*) and Strack and Cundall (*19*). This, however, puts an even larger strain on computing capacity and also requires knowledge of the grain-to-grain contact characteristics and on the influence of water or bitumen. Even though the distinct-element method cannot be used for backcalculation in the foreseeable future, it may still be used to study the distribution of stresses and strains in granular materials, and possibly to modify methods based on continuum mechanics.

From the foregoing, it is already clear that the use of Hooke's law for pavement materials is very much a simplification of reality, and even that the development of other constitutive equations considering viscosity, nonlinearity, or anisotropy may not be of much help.

In addition, it may be recalled that pavement response also depends on the distance from the pavement edge (or a joint) and on the degree of cohesion or friction between pavement layers. The material characteristics and layer thicknesses also vary along the length and width of the pavement and with the depth in the subgrade, as well as with the climatic conditions (temperature, temperature gradient, moisture content and distribution, frost, etc.).

Even with all these shortcomings, it is still necessary to use backcalculation procedures. The deterioration of pavements depends on the stresses and strains in the layers; to determine the critical stresses or strains, the stiffnesses of the layers must be known. Laboratory testing may be used for some materials, but are often expensive and not very reliable. A validation (and modification) of existing backcalculation procedures is needed. Some validation can be done by comparing moduli derived from backcalculation with moduli determined by laboratory testing, but only for a few bitumen or cement bound materials. A thorough validation must be based on a comparison of stresses and strains derived from backcalculation with stresses and strains measured in the pavement layers.

Validation through comparison of measured and calculated stresses and strains (or deflections at multiple depths) is not a simple matter. It has been attempted over a number of years at a number of locations, using a variety of instruments for measuring the in situ stresses and strains. In some cases, such as that reported in a work by Lenngren (*20*), very good correlation has been found. A very interesting international experiment on measuring strain in bituminous layers was conducted at Nardo, Italy, under the sponsorship of Organization for Economic Cooperation and Development (*21*). With the renewed interest in full-scale testing of instrumented pavements, similar international experiments could prove very useful.

SUMMARY AND CONCLUSION

- The use of backcalculated moduli is essential to the application of mechanistic principles to pavement evaluation. Backcalculation techniques and software have advanced greatly over the past decade. In spite of that advancement, many routine problems are still encountered that are handled in a variety of more or less satisfactory ways. However, the critical issues remain since they are the fundamental assumptions of the theoretical models typically used. Possibly the most "correct" approach in the future will involve use of stochastic, nonlinear, dynamic finite-element analysis applied at the particulate level, which would require the routine availability of massive computing power. Application of neural networks to backcalculation shows some promise.
- Backcalculation programs cannot be verified through theoretical means. Even the most sophisticated theoretical models like dynamic, three-dimensional viscoelastoplastic finite-element programs are based on simplifications, such as not considering the particulate nature of most pavement materials. No theoretical model constitutes the "truth"; they are all simplified models of reality.
- Most pavement materials do not have a modulus. The example of a handful of sand makes that obvious. The apparent modulus depends on the stress condition, which is influenced by moisture or bitumen content, temperature, loading time, and so forth. Trying to verify backcalculation procedures by comparing the moduli derived from the procedure with moduli determined by other means is, therefore, extremely difficult for most pavement materials.
- The most promising method for verification of backcalculation procedures appears to be through comparison of stresses and strains predicted by the procedure to values measured in actual pavements. The measurement of stresses and strains in pavements is very difficult, because the presence of a measuring instrument changes the stress or strain condition. Nevertheless, this appears to be the only solution, and instrumentation of pavements is taking place in many parts of the world.
- It is important to verify the models used for calculating pavement response if pavement engineering is to move away from being a craft to being a science. It should be kept in mind, however, that pavement response is seldom the final answer, but only an intermediate result used to predict the pavement performance. It is equally important that the relationships between response and performance be verified through the use of accelerated full-scale testing (preferably on instrumented pavements) and through long-term pavement performance studies.

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