Delays at Oversaturated Unsignalized Intersections Based on Reserve Capacities

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There is a practical need for estimating average delays at intersections, especially during peak periods, even if a temporary overload must be managed by the intersection. Suitable computation formulas must always be based on approximations. The formulas available are based on the degree of saturation x, where $x > 1$ describes the oversaturated situation. The application of x has proven to be successful in the context of signalized intersections. For unsignalized intersections, the reserve capacity R is a more elegant parameter. Here $R < 0$ describes the oversaturated situation. The coordinate transformation technique applied for R to derive average delays during peak periods is explained. The complexity of many influencing parameters and the algebraic solutions make it impossible to solve the complete problem analytically. However, for two levels of approximation, a set of formulas is derived to estimate the average delay during a peak hour for the vehicles on the minor street of an unsignalized intersection. These formulas can be used for practical application.

A variety of formulas are available for estimating delays at intersections. The fact that so many formulas exist emphasizes that none describes the complete reality. Indeed, each of the formulas represents another approximation that focuses on different special capabilities. Table 1 characterizes the more well-known approaches.

GENERAL QUEUEING MODEL

The basic sophistication of each delay formula is the understanding of traffic operations at an intersection as an analogy to a queueing system. As an example, Figure 1 illustrates this analogy for an unsignalized intersection with only two traffic streams (q_n is the traffic flow on the minor street as the input to the queueing system). Here, the time, s, that a vehicle spends at the first position can be regarded as its service time. The time when a vehicle is waiting in higher positions of the queue than 1 can be regarded as the delay, w, in the sense of queueing theory. Therefore, the total time that a vehicle spends in the whole queueing system ($d = s + w$) is the delay, d, of the vehicle in the sense of traffic engineering.

The problem for traffic engineering is that the type of operation of the service counter cannot be described by one of the classical mathematical solutions from queueing theory, for neither signalized nor unsignalized intersections. Therefore, specific solutions for queueing problems in traffic engineering must be developed.

ANALYTICAL SOLUTIONS

Analytical solutions have been sought for these service systems, which are established by intersections within traffic systems. A realistic chance for the development of such solutions, however, can be expected only for steady-state situations, in which q_n and capacity c are constant over time and $q_n < c$ for unsignalized intersections. For most analytical solutions, it must also be assumed that each of the traffic streams has Poisson properties (i.e., exponentially distributed headways).

For unsignalized intersections under steady-state conditions, several delay formulas have been proposed. The solution with the highest degree of sophistication appears to be the Kremser solution (*1,2*) in Brilon's formulation (*3*) (Equations 12 and 13), which is based on Yeo's formula (*4*):

$$D = \frac{E(W_1)}{v} + \frac{q_n}{2} \cdot \frac{y \cdot E(W_1^2) + z \cdot E(W_2^2)}{v \cdot y} \tag{1}$$

where

$$v = y + z$$
$$y = 1 - q_n \cdot E(W_2)$$
$$z = q_n \cdot E(W_1)$$

$$E(W_1) = \frac{1}{q_p}\left(e^{q_p t_c} - 1 - q_p t_c\right) + t_f$$

$$E(W_1^2) = \frac{2}{q_p}\left(e^{q_p t_c} - 1 - q_p t_c\right) \cdot \left(\frac{e^{q_p t_c}}{q_p} + t_f - t_c\right) + t_f^2 - t_c^2$$

$$E(W_2) = \frac{e^{q_p t_c}}{q_p}\left(1 - e^{-q_p t_f}\right)$$

$$E(W_2^2) = \frac{2 \cdot e^{q_p t_c}}{q_p^2}\left(e^{q_p t_c} - q_p t_c\right)\left(1 - e^{-q_p t_f}\right) - q_p t_f \cdot e^{-q_p t_f}$$

where t_c equals critical gap, in seconds, and t_f equals follow-up time, in seconds.

Here, however, the expressions for $E(W_2)$ and $E(W_2^2)$ are correct only if $t_c = t_f$ (*5*). For the more realistic situation of $t_c > t_f$, the improved expressions by Daganzo (*6*) or, even better, by Poeschl (*7*) could be used.

TIME-DEPENDENT SOLUTIONS BASED ON DEGREE OF SATURATION

From the complexity of the equations mentioned before, it is evident that they are not useful for practical application. Moreover, steady-state situations are not realistic in road traffic operations. Instead, the input flows to street intersections fluctuate over the time of day. For instance, Figure 2 illustrates an example for a typical

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TABLE 1 Characterization of Delay Formulas for Unsignalized Intersections

	steady state	Time dependence with temporary oversaturation	
		$q_0 = 0$; $q_1 = 0$	$q_0 > 0$; $q_1 > 0$
Combination of Kremser, Yeo, Brilon, Poeschl (eq. 1)	•		
HCM 1994 *(13)* cf. (eq. 2)		•	
Kimber, Hollis *(11)*			•

workday. Here the peak hour is particularly important for the layout of the intersection and its control. Therefore, the average delay and its tolerable maximum during the peak hour determine the whole design procedure. Hence, it is most important to describe the quality of traffic operations at intersections for a peak period with sufficient precision. By the definition of a peak period, it is clear that lower input traffic flows exist before and after the peak.

As solutions for this problem at signalized intersections with fixed cycle times and greens, the well-known formula by Akçelik *(8,9)* or the formula by Wu *(10)* can be used. For unsignalized intersections the user can choose between the formula by Kimber and Hollis *(11)* and that by Akçelik and Troutbeck *(12)*, which is also recommended by the new Chapter 10 of the *Highway Capacity Manual* (HCM) *(13)*. The derivation of both formulas is based on x = degree of saturation. For Akçelik and Troutbeck's solution *(12,* Equation 1.2), this is obvious.

The Kimber and Hollis formula *(11)* has proven to be especially useful to estimate delays, and it appears to have a reliable background, particularly for temporarily oversaturated conditions. The formula and its derivation are supposed to be based on the degree of saturation. Unfortunately, the derivation is not published anywhere, which is a remarkable drawback. It also has not been possible to redetermine the formula. From its description it can be obtained that the capacity c_0 and the input flow q_0, before and after the peak period, are taken into account.

The delay formula by Akçelik and Troutbeck *(12)* is given in modified version by the following equation:

$$d = \frac{1}{c} + \frac{T}{4}\left[(x-1) + \sqrt{(x-1)^2 + \frac{8 \cdot x}{c \cdot T}}\right] \qquad (2)$$

where

d = average delay (sec),
T = duration of peak period (sec),
x = degree of saturation = q/c,
$q = q_n$ = minor street traffic flow (veh/sec), and
c = capacity (veh/sec).

The derivation of the formula can be found elsewhere *(14)*.

The capacity, c, can be obtained from any useful capacity formula—for example, the formula according to Siegloch *(15)*, which is also the basis of the HCM procedure *(13)*:

$$\mu = \frac{3{,}600}{t_f} \cdot e^{-(t_c - 0.5 \cdot t_f) \cdot \frac{q_p}{3{,}600}} \qquad (3)$$

where μ is given in vehicles per hour.

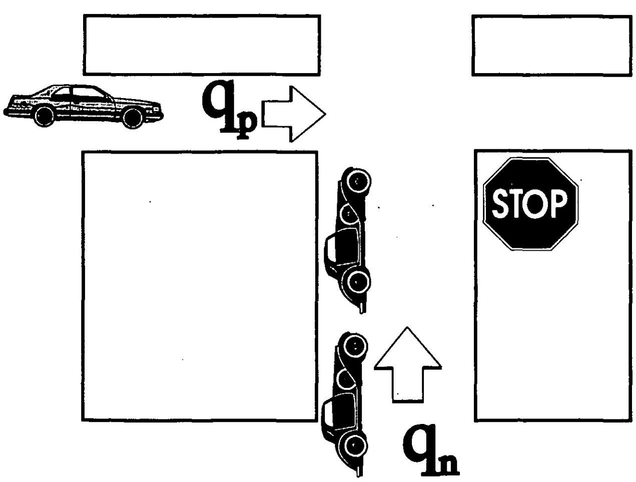

FIGURE 1 General outline of queueing system that represents unsignalized intersection with one minor traffic stream (volume q_n) and one priority stream (volume q_p).

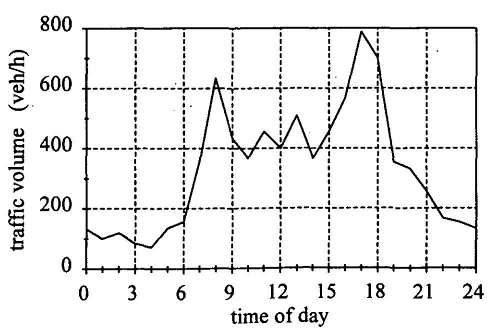

FIGURE 2 Traffic flow pattern over time for typical workday on street with predominating afternoon peak hour.

Equation 2 has one great drawback: it assumes that the traffic flow q_n before and after the peak period of duration T is 0, which is unrealistic. A set of formulas based on the same sophistication—including, however, positive traffic flows before and after the peak period—is described by Troutbeck (*14*).

The results from Equation 2 formulated in dependence on the degree of saturation x can be obtained from Figure 3 as one example. This example has been prepared for T = duration of the peak period = 3,600 sec. From the graph it can be seen that the curves are scattered over a very wide range if the capacity is altered.

As a counterpoint to this, Figure 4 should be compared. Here, the same relations are illustrated, but the reserve capacity $R = c - q_n$ is used as the independent variable. Again, the parameter of the curves is the capacity. All curves are nearly coinciding, and the parameter is not of high importance, especially in the range of delays that is useful in practice ($d < 60$ sec). Thus, the use of the reserve capacity, R, appears to facilitate some of the interrelations within the theory of unsignalized intersections. This fundamental idea originates from Harders' work (*16*). He was the first to find that the reserve capacity R is a strong determinant for the traffic flow quality at unsignalized intersections. This was the reason that R was used as the measure of effectiveness in Chapter 10 of the 1985 HCM (*17*), which was a realization of Harders' concept.

CONCEPT OF RESERVE CAPACITY

The concept introduced in this paper tries to develop formulas for the average delay at an unsignalized intersection, only based on the reserve capacity R:

$$R = c - q \qquad \text{(veh/sec)} \quad (4)$$

where q is the traffic volume of the movement under observation in vehicles per second, and R is given in vehicles per second. Here especially, the approximation of peak periods including temporary oversaturation (i.e., $R < 0$) should be solved.

Simplification of Flow Pattern and D/D/1 System

Consider a queueing system with two traffic streams as in Figure 1. The minor street traffic flow, q_n, is the input flow to the system. For

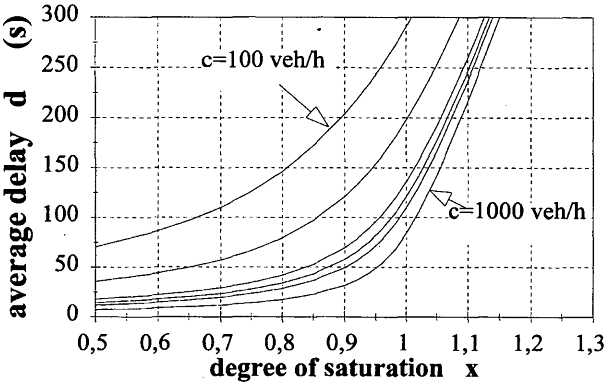

FIGURE 3 Average delay of vehicles in front of stop line calculated from Equation 2 depending on *x*. Different lines apply to different capacities *c* (100, 200, 300, 400, 500, 600, and 1,000 veh/hr); here *T* is given as 1 hr.

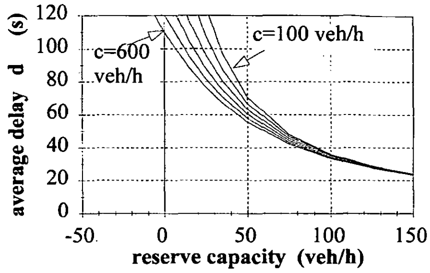

FIGURE 4 Average delay of vehicles in front of stop line calculated from Equation 2 depending on *R*. Different lines apply to different capacities *c* (100, 200, 300, 400, 500, and 600 veh/hr); here *T* is given as 1 hr.

simplicity, it is now called q. Assume that the capacity of the system can be calculated from q_p by any useful capacity equation; for example, the Siegloch formula (Equation 3).

Look at a traffic flow pattern over time as illustrated in Figure 5 (*top*). The variables being used are explained in Table 2, and T equals the duration of peak period (e.g., T = 3,600 sec). Of course, in reality, the headways of vehicles entering the system are distributed randomly. However, for further simplification in the case of oversaturation (i.e., $R < 0$), imagine—as a preliminary approach to the solution—that during the peak period of duration T, the queueing system is operating like a D/D/1 queueing system, in which arrival headways (a) of the minor street vehicles and their departure

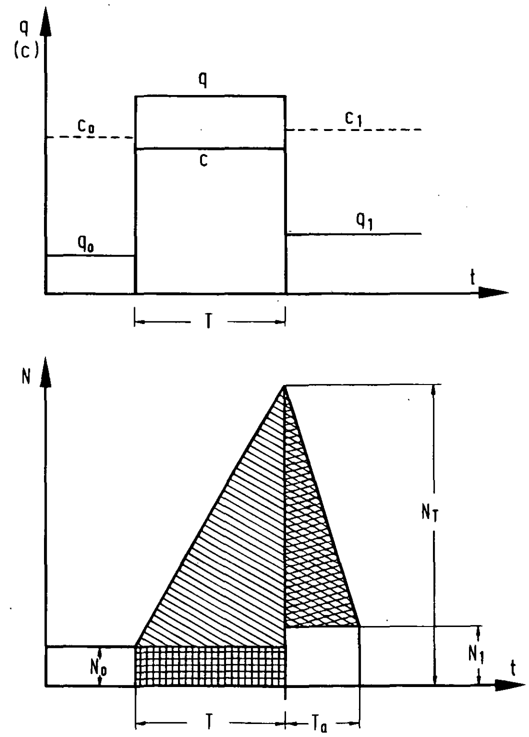

FIGURE 5 Flow patterns over time including peak period: *top,* traffic flow *q* and capacity *c*; *bottom,* queue length *N*.

TABLE 2 Variables for Traffic Flow Pattern

	before	during	after
	the peak period		
traffic demand on the minor street	q_0 $q_0 < q$	q	q_1 $q1 < q$
capacity	c_0	c	c_1
reserve capacity	$R_0 = c_0 - q_0$ $R_0 > 0$	$R = c - q$ $R > 0$ or $R \leq 0$	$R_1 = c_1 - q_1$ $R_1 > 0$
queue length	N_0	N	N_1

All variables for q, c, and R are given in vehicles per second.

headways (b) from the stop line both are constant for all vehicles. For such a system, it can easily be imagined that for $R > 0$ the queue length and the delay both are 0.

However, for the time of temporary oversaturation with $R < 0$, queue length is constantly increasing (Figure 5, *bottom*). At the end of the oversaturated peak period, the queue length is N_T, with

$$N_T = (q - c) \cdot T + N_0$$
$$N_T = N_0 - R \cdot T \tag{5}$$

where $R = c - q$ has a negative value.

The time needed to clear the queue down to N_1 after the peak is

$$T_a = \frac{N_T - N_1}{c_1 - q_1}$$

$$T_a = \frac{N_T - N_1}{R_1} \tag{6}$$

where N_1 is the expectation of the queue length after the peak, on the assumption that no overload was observed within the peak period. Thus, N_1 is only a result of q_1 and c_1 (with $c_1 > q_1$) without regard to the results of the peak period.

In each queueing system, as a general property, the sum of all delays is the area under the function of the queue length. Before this basic idea can be applied, the type of average to be used must be defined. According to most of the authors mentioned earlier, the shaded area in Figure 5 (*bottom*) represents the sum of all delays induced in the system by the vehicles arriving during the peak period. Thus the sum S of all delays is

$$S = N_0 \cdot T + \frac{(N_0 - N_1)^2}{2 \cdot R_1} - R \cdot \left(\frac{N_0 - N_1}{R_1} \cdot T + \frac{T^2}{2}\right)$$
$$+ \frac{T^2}{2 \cdot R_1} \cdot R^2 \tag{7}$$

The average delay, d, per vehicle caused by those vehicles arriving during the peak period, then, is

$$d = \frac{S}{q \cdot T} \tag{8}$$

This delay d already includes the time spent in the service position (first position of the queue on the minor street) of the queue, because the vehicle in this position already has been included in the queue length according to Figure 5 (*bottom*). Therefore, d is a representation of the delay in the sense of traffic engineering.

Equation 8 gives the delay, d, for the D/D/1 system. From alternating the parameters N_0, N_1, and R_1 by a series of sample calculations it is learned that the delay curve is a straight line for $R_1 = c$. For $R_1 < c$, the curve becomes concave (concave side above the curve). For $R_1 > c$, the curve would become convex. However, $R_1 > c$ is not a reasonable case, because it would be unusual that the reserve capacity, R_1, could become even greater after the peak than the peak-hour capacity, c. The function for d is not very sensitive to N_1 as far as N_1 is varied over a reasonable range of values. Therefore, for practical cases, it could be sufficient to assume that $N_0 = N_1$ (see Case S1 later in the paper).

Equation 7, which is also a part of Equation 8, looks rather complicated. Hence, for better understanding, we also look at simplified special cases.

Simplification Case S0

The most simplified case is the one corresponding to the assumptions of the formula of Akçelik and Troutbeck (Equation 2):

$$\begin{aligned} c_0 &= c &= c_1 \\ q_0 &= 0 &= q_1 \\ N_0 &= 0 &= N_1 \\ R_0 &= c &= R_1 \end{aligned}$$

For these conditions, Equation 7 can be written as

$$S = -R \cdot \frac{T^2}{2} \cdot \frac{q}{c} \tag{9}$$

Equation 8 then can be expressed as

$$d = -\frac{R \cdot T}{2 \cdot c} \tag{10}$$

Some results for d as a function of R are shown in Figure 6. Here it is clear that d has a linear relationship to R, where the gradient depends on T and c. The solution of Equation 10 toward R is

$$R = -\frac{2 \cdot c \cdot d}{T} \tag{11}$$

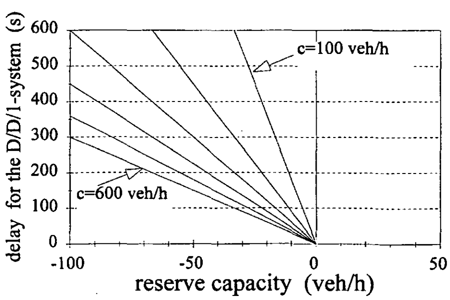

FIGURE 6 Average delay for D/D/1 system as function of reserve capacity, *R*, during peak period for Case S0. *T* has been fixed to 1 hr; *c* is capacity of system.

Simplification Case S1

A more general and realistic simplified case is to assume that the average queue lengths before and after the peak are of the same size:

$$N_0 = N_1$$
$$c_0 = c = c_1$$
$$R_1 < c$$

This case comes very close to reality, because evaluations of Equation 7 showed that N_1 has only a very minor influence on the result for the average delay (see previous discussion).

Under this assumption, Equation 7 becomes

$$S = N_0 \cdot T - R \cdot \frac{T^2}{2} + \frac{T^2}{2 \cdot R_1} \cdot R^2 \tag{12}$$

Equation 8 in this case can be written as

$$d = \frac{1}{c - R}\left[N_0 - \frac{R \cdot T}{2}\left(1 - \frac{R}{R_1}\right)\right] \tag{13}$$

Because the function is quadratic in R, it also has positive d values for a large R. However, only the part for negative R is of interest in this context. A solution of Equation 13 toward R is possible:

$$R = \frac{R_1}{2 \cdot T} \cdot \left[A - \sqrt{A^2 + \frac{8 \cdot T}{R_1} \cdot (d \cdot c - N_0)}\right] \tag{14}$$

where $A = T - 2 \cdot d$.

But this solution cannot be used for further derivations because it leads to equations that cannot be solved (explained later). Therefore, at the moment, this solution is obsolete. Instead, it turns out that a simplified approximation of this equation is needed. To achieve this, approximate the curves for negative R values by straight lines that have the same gradient b for all N_0 values. This is possible with good approximation. More numerical evaluations of Equation 13 showed that this reduction to a uniform gradient of the curves does not cause much bias. This gradient, b, is given by the following equation, which is an application of Equation 13.

$$b = \left\{\frac{1}{c - R_f}\left[N_0 - \frac{R_f \cdot T}{2}\left(1 - \frac{R_f}{R_1}\right)\right] - \frac{N_0}{c}\right\} \cdot \frac{1}{|R_f|} \tag{15}$$

R_f is an arbitrary point along the function of Equation 13 for $R << 0$, where the original function of Equation 13 should be met exactly by the linear approximation. Further derivations show that R_f should be chosen in accordance with the other parameters, mainly the peak-hour duration T and the reserve capacity R_1 after the peak. For application, the following is recommended:

$$R_f = \frac{100 \cdot 3{,}600}{T}$$

Then the approximation for Equation 13 is

$$d = \frac{N_0}{c} - b \cdot R \tag{16}$$

The solution toward R is

$$R = \frac{1}{b} \cdot \left(\frac{N_0}{c} - d\right) \tag{17}$$

Approximation for Steady-State Solution

As pointed out earlier one could indicate an analytical solution for the delay in the steady-state queueing system that is established by an unsignalized intersection (Figure 1). This solution, however, is so complicated that it is not useful for further derivations. It has turned out in many investigations (*11,18*) that the M/M/1 queue is a very close approximation for an unsignalized intersection. In the M/M/1 queue, the total time that a customer spends in the queueing system is

$$d = \frac{1}{R} \tag{18}$$

This approach is used as an approximation of the delay at an unsignalized intersection in the steady state (i.e., $R >> 0$). The curve only can be used for $0 < R \leq c$. Equation 18 can be solved for R as

$$R_s = \frac{1}{d} \tag{19}$$

where the index s stands for steady state.

The M/M/1 queue also can be used, with rough approximation, to estimate the average queue length for steady-state conditions. The expectation for the number of vehicles in the system, then, is

$$N_0 = \frac{q_0}{R_0} = \frac{c_0 - R_0}{R_0}$$
$$N_1 = \frac{q_1}{R_1} = \frac{c_1 - R_1}{R_1} \tag{20}$$

This solution is assumed to apply for the periods before and after the peak when R_0 and R_1 are considerably larger than 0.

Coordinate Transformation

For longer peak periods or small R values (e.g., $R << 0$), the delay in any type of queueing system tends toward the D/D/1 delay. Then the details of the arrival and departure process will be of less importance. The dominating property of the queueing system, then, is the tremendous increase of the queue during the oversaturation period. Therefore, the real delay must be found along a transition curve that connects the steady-state delay curve for $R >> 0$ with the D/D/1 delay for $R << 0$. This transition curve is illustrated in Figure 7. The equation for this curve cannot be derived analytically; again, only an approximation can be derived. A reasonable approach to the derivation of this approximation is to assume that y is equal to z (Figure 7). This is identical to

$$R_s = -R_D + R \tag{21}$$

where

R_s = reserve capacity that causes average delay d in steady-state system (Equation 19),

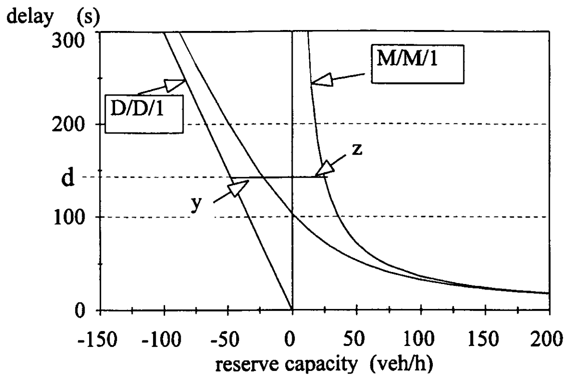

FIGURE 7 Principle of coordinate transformation.

R_D = reserve capacity that causes average delay d in D/D/1 system (solution of Equation 8 toward R and simplified Equations 11 and 14 or 17, respectively), and
R = actual reserve capacity during peak.

The expressions for both R_s and R_D include the delay, d. Therefore, Equation 21 establishes a relation between the delay, d, and the reserve capacity, R. The problem now is to solve Equation 21 toward the solution d = function (R). It is not easy to solve this function by algebra for a more general case. Moreover, it can be shown that for the general case (Equation 7) this solution is not possible because it results in imaginary equations containing expressions for $\sqrt{-1}$. Therefore, one is forced to focus on the simplified cases S0 and S1.

Case S0

Concentrate first on Case S0 to understand the method. Enter Equation 11 for R_D and Equation 19 for R_s into Equation 21, which results in

$$\frac{1}{d} = R + \frac{2 \cdot c \cdot d}{T} \tag{22}$$

The solution toward d gives

$$d = -\frac{1}{4 \cdot c} \cdot \left[R \cdot T - \sqrt{(R \cdot T)^2 + 8 \cdot c \cdot T} \right] \tag{23}$$

An illustration of this formula is given in Figure 7 by the transition line for T = 1 hr and c = 600 veh/hr. To repeat: Equation 23 estimates the average delay at an unsignalized intersection with the same degree of approximation as Equation 2, which is based on a similar derivation using x (degree of saturation) instead of the reserve capacity, R. The capacity, c, can be estimated by any useful type of formula [e.g., the Siegloch formula (Equation 3)]. Capacity formulas based on the empirical regression method (*14*) or any other method can be used. The new Equation 23 is a little bit shorter and thus easier for practical use than Equation 2.

Case S1

To find a solution for Case S1, again the coordinate transformation technique must be used (Equation 21). For R_s, enter Equation 19, and for R_D, use Equation 17. Thus,

$$\frac{1}{d} = R - \frac{N_0}{b \cdot c} + \frac{d}{b} \tag{24}$$

with b according to Equation 15.

The solution of this equation toward d gives

$$d = -B + \sqrt{B^2 + b} \tag{25}$$

where $B = {}^1/_2 \cdot [b \cdot R - (N_0/c)]$. The result of this equation is illustrated in Figure 8. Equation 25 gives an estimation of the average delay during a peak period of duration T when an initial queue of length N_0 exists at the beginning of the peak interval. Moreover, the reserve capacity R_1 after the peak is included via b and Equation 15. The important independent parameter, however, is the reserve capacity, R, during the peak period.

One might argue that on the way to this result, many approximations were made. This, however, also is the case for each of the alternative approaches to solving the peak-hour delay problem. Thus, at the moment, Equation 25 is the most detailed formula for average delay at an unsignalized intersection for times of temporary oversaturation that can be recommended as a result of these derivations.

Of course, it would be desirable to use the more exact solution for R_D (Equation 14) in the coordinate transformation (Equation 21). This, however, turns out to be impossible. The result would be a transcendental equation using imaginary numbers (containing $\sqrt{-1}$) as part of the result.

DISCUSSION OF RESULTS

It would be even more desirable to enter the complete and general solution for the D/D/1 delay given by Equations 7 and 8 into the coordinate transformation technique. This attempt also results in transcendental equations. Thus, this most complete solution also is

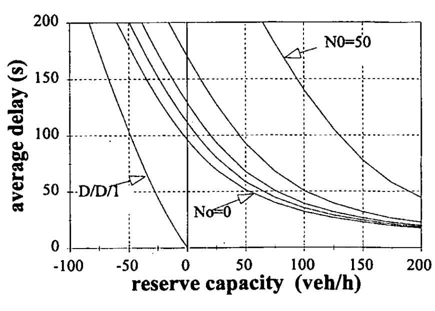

FIGURE 8 Average delay as function of reserve capacity, *R*, during peak period of duration *T* = 1 *hr* calculated from Equation 25 for Case S1 for *T* = 1 hr, R_1 = 300 veh/hr, and *c* = 600 veh/hr. Parameters of curves are N_0 = 0, 5, 10, 20, and 50 veh. Complete D/D/1 solution shown in figure is obtained from Equation 13.

not possible now. To find a solution to that problem, a more general approximation to Equation 7 that does not have two roots for R = function (d) (e.g., an exponential function) must be found. This would be useful only if all the parameters, N_0, N_1, R_1, T, and R, could be included in a realistic way.

Another possible improvement should be considered further: the attempt to allow a time dependency also within the peak period itself (e.g., a parabolic pattern could be used). For signalized intersections, this has been solved by Wu (*10*) on the basis of the standard philosophy of degree of saturation. For the unsignalized intersections, this task might have a chance to be solved together with the approximation mentioned before, as some preliminary numerical sample calculations showed.

Moreover, the same approach that has been developed here for unsignalized intersections could be used for signalized intersections, too. According to Kimber and Hollis (*11*) and subsequent publications by Kimber, one could try to model the signalized intersection delay by an M/D/1 queueing system. The time customers are in the system is

$$d = \frac{1}{2 \cdot c}\left(\frac{c}{R} + 1\right) \tag{26}$$

This equation also could be solved toward R. Nevertheless, compared with Equation 18, the same technique applied to signalized intersections promises to reveal more complications. Therefore, one could say that the degree of saturation, x, is a suitable parameter to describe signalized intersection performance. The reserve capacity, R, is a more suitable parameter for unsignalized intersections.

Finally, the number of possible approximative solutions for the peak-hour delay problem appears to be unlimited. Therefore, for the user it is of greatest importance to understand the sophistication of each of the provided solutions. The numerical results of these theoretically equivalent solutions could make a decisive difference, especially in situations with large overloads of the intersection. Thus, a confirmation of the validity of the solutions by either simulation studies or empirical evaluations would be desirable, a task that will initiated at the author's institute.

CONCLUSIONS

The paper presents another approach for estimating average delays of minor street vehicles at an unsignalized intersection for oversaturated and nearly oversaturated peak periods. The derivations point out that the reserve capacity, R, is useful for application as the independent parameter to describe traffic performance. For the computation of average delays, Equation 23 can be used as a rough approach. It describes the delay problem with the same degree of sophistication as the delay equation of Chapter 10 in the latest edition of the HCM (*13*). However, it describes the average delay with considerable simplifications.

A more realistic solution is given by Equation 25 (including Equation 15 for b). This solution takes into account that before and after the peak period, only limited capacity reserves are available. Thus, the equation can be recommended for practical use. Of course, the whole set of equations is quite lengthy, as are all other solutions presented. However, in computer programs it is not a problem to apply this set of equations. The overall quality of the solution might be comparable to that of the solution of Kimber and Hollis (*11*). However, the derivation also is given here. To improve the applicability of the solution, the whole set of formulas is repeated step by step in the appendix.

APPENDIX
Steps for Applying Equation 25

It is assumed that all values for the variables mentioned in Table 2 are given, including the duration T of the peak period. For practical cases, T should be at least 15 min (900 sec). It is also assumed that the steady-state queue length before and after the peak period are nearly equal with sufficient approximation ($N_1 \approx N_0$). All variables in these equations should be used in units of seconds, number of vehicles (veh), and vehicles per second.

The average queue length during the time before the peak is

$$N_0 = \frac{q_0}{R_o} = \frac{c_0 - R_0}{R_0} \tag{20}$$

Then the sequence of the following equation must be applied:

$$R_f = -\frac{100 \cdot 3{,}600}{T}$$

$$b = \left\{\frac{1}{c - R_f}\left[N_0 - \frac{R_f \cdot T}{2}\left(1 - \frac{R_f}{R_1}\right)\right] - \frac{N_0}{c}\right\} \cdot \frac{1}{|R_f|} \tag{15}$$

$$B = \frac{1}{2} \cdot \left(b \cdot R - \frac{N_0}{c}\right)$$

$$d = -B + \sqrt{B^2 + b} \tag{25}$$

where d is the average delay for vehicles arriving during the peak period.

The maximum of the average queue length must be expected at the end of the peak period (Figure 5, *bottom*). The expectation for this queue length at the end of an oversaturated peak period is

$$N_T = \max \begin{Bmatrix} N_0 - R \cdot T \\ 0 \end{Bmatrix} \tag{5}$$

On average, vehicles arriving at the end of the oversaturated peak period must face the longest delays. The expectation for their delay is

$$T_a = \frac{N_T - N_0}{R_1} \tag{6}$$

N_T and T_a, in addition, are subject to random variation, which is not described in this paper.

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