Unsignalized Intersection Capacity and Level of Service: Revisiting Critical Gap

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Operational performance at a minor street stop-controlled intersection is a function of motorist gap acceptance behavior. Issues in modeling gap acceptance are reexplored using discrete choice methods. Logit models of varying levels of sophistication are used in simulation to generate average delays at the intersection stop bar. Comparison of simulated and empirical delays suggests that deterministic methods for modeling gap acceptance may represent a reasonable trade-off between accuracy and ease of application, but two potential concerns are at issue—namely, delay estimates are very sensitive to the value used for mean critical gap, and the use of a single-valued critical gap necessitates the exclusion of disaggregate factors influencing the gap acceptance decision. Logit models estimated for intersection traffic movements have identified a number of such influential factors. Further research to explore fundamental issues of gap acceptance should be undertaken before adopting a capacity and level-of-service methodology for minor street stop-controlled intersections.

Operating conditions at a two-way stop-controlled intersection are a function of driver choice characteristics. The propensity of motorists traveling on the minor street to use available gaps in the major street traffic streams will dictate operational performance. Efforts to model the gap acceptance behavior of motorists have been the focus of considerable research. There may be value in reexamining these previous efforts in light of newly revised capacity and level-of-service procedures for stop-controlled intersections (*1*). This paper reexplores key issues in modeling gap acceptance.

BACKGROUND

The term "critical gap" is defined as the minimum time gap (exhibited by major street vehicles) allowing one vehicle to enter the intersection from a minor street. The gap acceptance process is probabilistic in nature. Each driver has his or her own perception of a critical gap, and the value of this "minimum acceptable" gap may change with changing conditions at the intersection. Functions have been developed from suggested distributions of critical gap to relate the probability of gap acceptance to the gap length (*2–4*). Miller (*5*) assumed that critical gaps conform to a normal distribution and used probit modeling techniques to estimate the probability of accepting a given gap on the basis of its length. These works addressed the variation in gap acceptance tendencies from one driver to the next.

Daganzo (*6*) extended Miller's work by accounting for variation within drivers as well as across drivers. Daganzo used multinomial probit to estimate the parameters of the distribution of critical gaps. Mahmassani and Sheffi (*7*) modeled the gap acceptance process as a series of independent, sequential choices to either accept or reject each gap in a conflicting traffic stream. By modeling the probability of gap acceptance using a probit function, Mahmassani and Sheffi demonstrated that an *individual* motorist's propensity to accept gaps varies as a function of the time spent waiting at the stop bar (*or* the number of gaps previously rejected). Recent empirical studies by Kittelson and Vandehey (*8*) support this finding. Most recently, Madanat et al. (*9*) used logit modeling to demonstrate that the delay spent in queue (before arriving at the stop bar) also influences drivers' gap acceptance behavior.

These previous findings underscore two important features:

1. Drivers are not homogeneous. Different drivers display different gap acceptance tendencies.
2. Drivers are not consistent. Drivers display time-dependent gap acceptance tendencies (e.g., a driver may ultimately accept a gap smaller than gaps that were previously rejected).

There appears to be little dispute concerning the probabilistic nature of gap acceptance. The literature does, however, document efforts to model gap acceptance decisions using deterministic methods (*1,10*). Single-valued, mean *critical gaps* are estimated from a distribution of gaps. The motorist is assumed to reject all prevailing gaps smaller than the critical gap, and all gaps larger than the critical gap are presumed to be accepted. These deterministic methods for capturing gap acceptance behavior are often assumed to possess adequate predictive strength or the benefits of exploiting deterministic models (which are easy to apply) are often considered to outweigh potential inaccuracies.

The need may exist to examine more carefully the trade-offs between the simplicity of a deterministic methodology and the robustness provided by a probabilistic, properly specified gap acceptance function. And if through careful examination the traffic engineering community eventually elects to adopt a deterministic model, the methodology used for estimating values of critical gap should be based on behaviorally defensible theories. This paper presents evidence concerning variation in gap acceptance behavior from one driver to the next, as well as time-dependent factors influencing the gap acceptance decision. The potential significance of this variability across and within drivers is presented both statistically and through simple example.

The work exploits a very limited empirical data base for model estimation. As such, gap acceptance functions presented herein are not definitive. The purpose of this paper, however, is not to propose the adoption of any particular model but to explore the relative merits of deterministic and probabilistic gap acceptance functions.

EMPIRICAL DATA

Empirical data used to estimate gap acceptance functions were collected from two neighboring T-intersections in Indiana. The geo-

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metric configurations of these suburban intersections are illustrated in Figure 1. The stop-controlled approach at Intersection 1 consists of separate lanes for left- and right-turning traffic, and the stop-controlled approach at Intersection 2 consists of a single shared-turn lane. The major (uncontrolled) street has one lane in both directions. No traffic control devices, other than the minor street stop signs, influence operation at the intersections.

Fifteen minutes of operation were recorded at each intersection using video. Data manually extracted from videotape included

- The lengths of all gaps observed in the major street traffic streams and whether each gap was accepted or rejected by motorists on the minor street,
- The time that minor street motorists waited in queue (before arriving at the stop bar),
- Vehicle move-up times to the stop bar, and
- The amount of time that individual motorists waited at the stop bar before accepting a gap.

HIERARCHY OF GAP ACCEPTANCE FUNCTIONS

The authors first estimated a series of gap acceptance functions ranging from the very simple to the more sophisticated. Each function was estimated through discrete choice techniques (*11*). The application of discrete choice methods produced models estimated from disaggregate observations of individual behavior. Thus, the logit models estimated herein reflect the probabilistic nature of the gap acceptance process (i.e., the variability across and within drivers).

Single-Valued Critical Gap Function

The simplest gap acceptance model recognizes variations in critical gap values across drivers; each driver is assumed to have his or her own critical gap. All gaps confronting a motorist that are smaller than his or her specified critical gap are invariably rejected. Conversely, motorists accept all gaps greater than or equal to their critical gaps. It is further assumed that these critical gap values follow some probability distribution across the population of motorists. The mean and variance of this distribution are the relevant parameters in this model.

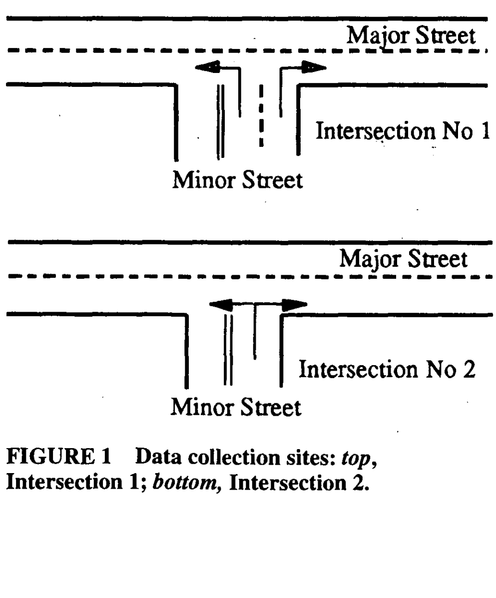

FIGURE 1 Data collection sites: *top*, Intersection 1; *bottom*, Intersection 2.

The mean critical gap cannot be observed directly. To estimate this value, the authors express the probability that a motorist accepts a given gap, Pr(a), as the probability that a gap of length t is greater than or equal to the motorist's critical gap, T_{cr}.

$$\Pr(a) = \Pr(t \geq T_{cr})$$

Denoting by $\overline{T}_{cr}$ the mean critical gap, and by ϵ the random deviation of each motorist's critical gap from the mean, this probability can be rewritten as

$$\Pr(a) = \Pr(t \geq \overline{T}_{cr} + \epsilon) = \Pr(\epsilon \leq t - \overline{T}_{cr})$$

Depending on the distribution of ϵ, this probability function can take different forms; if ϵ is assumed to be distributed logistically with scale parameter μ (which implies that the distribution of critical gaps across drivers is logistic with mean $\overline{T}_{cr}$ and variance $\pi^2/3\mu^2$), then the probability takes the logit form

$$\Pr(a) = \frac{1}{1 + e^{-\mu(t-\overline{T}_{cr})}} = \frac{1}{1 + e^{-\mu t+\mu\overline{T}_{cr}}}$$

setting $\mu\overline{T}_{cr} = \alpha$

$$\Pr(a) = \frac{1}{1 + e^{\alpha-\mu t}}$$

Values of $\hat{\alpha}$ and $\hat{\mu}$ (the estimates of α and μ) are obtained through maximum likelihood estimation (*12*). From the relationship $\hat{\overline{T}}_{cr} = \hat{\alpha}/\hat{\mu}$, the mean value of critical gap is estimated. The resulting model can be used to predict motorist gap acceptance behavior as a homogeneous and consistent process: all drivers can be assumed to accept gaps larger than $\hat{\overline{T}}_{cr}$ and to reject other gaps.

Probabilistic Gap Acceptance Function

The deterministic model just presented fails to exploit the full capabilities of logit models. The logit model used for estimating mean critical gap also provides a *distribution* of critical gaps. As shown, these critical gaps are distributed logistically with a mean of $\overline{T}_{cr}$ and a variance of $\pi^2/3\mu^2$. Because the model identifies only the distribution of T_{cr} and not the actual critical gap for each driver, the model can only generate probabilistic statements. The logit model yields the probability of accepting a specific gap as a function of its length, t. Thus, the function recognizes that drivers are not homogeneous, although drivers are still assumed to behave in a consistent manner.

Probabilistic Function with Disaggregate Factors

Discrete choice methods (e.g., logit models) facilitate the identification of factors influencing gap acceptance as well as the inclusion of these factors in the resulting gap acceptance function. Thus, explanatory variables in addition to gap length can be incorporated into a logit model to further enhance estimation capabilities. The output of this more sophisticated model is the motorist's probability of gap acceptance as a function of relevant prevailing conditions. Through the inclusion of time-dependent factors that further explain gap acceptance decisions, the resulting function recognizes that a motorist's critical gap may change with changing conditions at the intersection. Effectively, the function still assumes that motorists respond to critical gaps, but the value of critical gap varies with each motorist and with each new situation. As such, the logit function captures nonhomogeneous and inconsistent gap acceptance behavior among motorists.

Estimating Gap Acceptance Functions

To demonstrate further the characteristics of each aforementioned gap acceptance function, the authors estimated specific models using empirical observations of right-turning vehicles at Intersection 1. To estimate the mean critical gap for a single-valued function, the logit model was derived incorporating only gap length t as an explanatory variable:

$$\Pr(a) = \frac{1}{1 + e^{5.212 - 0.89934t}}$$

Independent Variable	*Estimated Coefficient*	t-*Statistic*
1 (constant)	−5.21200	−7.65608
t	0.89934	7.01240
Auxiliary Statistics	*At Convergence*	*Initial*
Log likelihood	−67.129	−150.41
Number of observations	217	—
Adjusted rho-square	0.547	—

The factor t is highly significant in explaining gap acceptance propensity at the 95 percent level (i.e., the t-statistic is well above 2). Moreover, the model's overall fit is very satisfactory as evidenced by the adjusted rho-square value of 0.547.

From the relationship $\hat{T}_{cr} = \hat{\alpha}/\hat{\mu}$, the estimated mean critical gap is 5.212/0.899, or 5.8 sec. This single-valued function is illustrated at the top of Figure 2. The characteristics of the logit model are illustrated in the middle of Figure 2.

The logit model incorporating all explanatory variables found to be significant is as follows:

$$\Pr(a) = \frac{1}{1 + e^{8.109 - 1.373\,t - 0.042\,td + 1.720dv}}$$

where

t = gap length confronting the right-turning motorist,
td = total individual delay imparted to motorist up to occurrence of subject gap (where td is sum of queueing and stop bar delays), and
dv = dummy variable distinguishing gaps and lags;
dv = 1 where motorist is confronted with a lag,
dv = 0 otherwise (lag is defined as the elapsed time between the arrival of the minor street vehicle to the stop bar and the arrival of the first conflicting vehicle to the intersection).

Independent Variable	*Estimated Coefficient*	t-*Statistic*
1 (constant)	−8.10861	−6.45629
t	1.37346	6.56972
td (total delay)	4.19540e−002	3.02295
dv	−1.71950	−2.86030
Auxiliary Statistics	*At Convergence*	*Initial*
Log likelihood	−53.828	−150.41
Number of observations	217	—
Adjusted rho-square	0.620	—

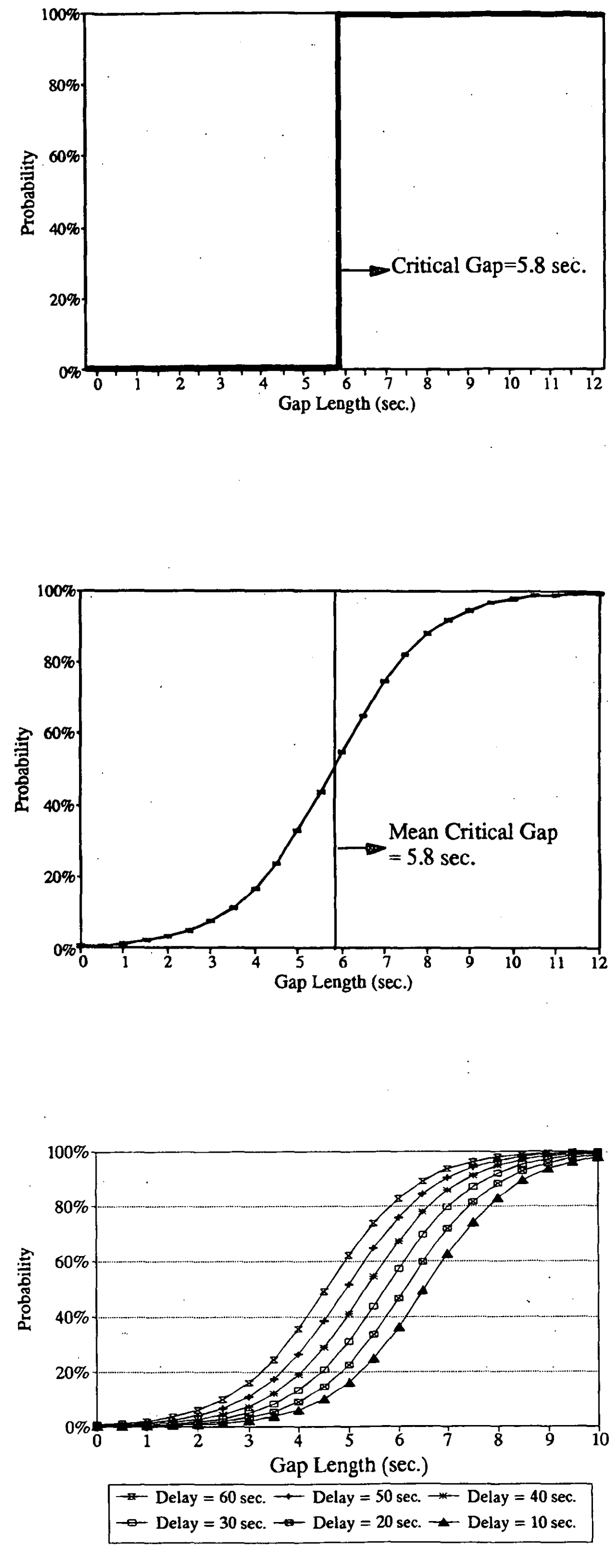

FIGURE 2 Three hierarchal models of gap acceptance.

The right-turn model indicates that a motorist's propensity to accept a gap increases with increasing gap length, consistent with virtually all previous studies of gap acceptance. Likewise, the model indicates that gap acceptance propensity increases with delay incurred on the intersection approach, a finding consistent with previous research evidence (*7–9*). Finally, the model indicates that, all else

TABLE 1 Mean Stop Bar Delays

Measure	Empirical	Disaggregate Logit Model	Simple Logit Model	5.8 sec. Critical Gap
Mean Stop Bar Delay (secs)	7.5	7.0	5.6	6.0
Percent Error		6.7	25.3	20.0

being equal, motorists have a greater tendency to accept gaps than to accept lags, consistent with the findings of Daganzo (*6*).

All independent variables in the model are statistically significant at the 95 percent level. The model's overall fit is very satisfactory, as indicated by the adjusted rho-square of 0.620, a higher value than that of the simpler logit model. The bottom of Figure 2 illustrates the gap acceptance probabilities estimated by the disaggregate logit model for a range of total individual delays. Lag acceptance probabilities are not displayed in this figure.

DELAY ESTIMATION

Using the functions just described in conjunction with simulation, the potential impacts of gap acceptance functions on delay prediction will be explored. Moreover, the sensitivity of predicted delay to single-valued gap acceptance functions is demonstrated. On the basis of this sensitivity, the authors argue the importance of estimating a mean critical gap through behaviorally defensible techniques.

Estimation Method and Simulation Model

The gap acceptance functions were incorporated into a microscopic, stochastic simulation model. Delay estimates generated from each function were evaluated.

Simulated vehicle arrivals on all approaches conformed to a Poisson distribution and were based on the observed mean arrival rates. Vehicle move-up times on the stop-controlled approach conformed to empirically identified distributions.

The initial simulation experiments separately used each of the three gap acceptance functions presented earlier. When a logit function was used in the simulation model, the gap acceptance probability of each right-turning vehicle at the stop bar was computed at the onset of each gap or lag. If the gap acceptance probability exceeded a randomly generated number from the [0,1] uniform distribution, the gap was accepted. When a single-valued function was used in the simulation, the process was purely deterministic (i.e., all gaps less than the specified critical gap were rejected, all gaps greater than or equal to the critical gap were accepted).

Simulation Findings

Each gap acceptance function was evaluated by comparing empirical and simulated stop bar delays (i.e., the delays incurred by motorists waiting at the stop bar for a suitable gap). Table 1 presents average stop bar delays as obtained (*a*) empirically, (*b*) through simulation with the logit function incorporating disaggregate explanatory variables, (*c*) through simulation with the simple logit function accounting only for the influence of gap length, and (*d*) through simulation with a single-valued critical gap of 5.8 sec (as estimated previously).

The logit model incorporating disaggregate factors generated an average delay prediction that most closely matches the empirical value. Statistical tests indicated that discrepancies between simulated and empirical means and variances were not significant at the 95 percent level.

The discrepancies between stop bar delay mean and variance as generated with the single-valued critical gap of 5.8 sec likewise were not statistically significant from the empirical values. Thus, from the example scenario evaluated in this paper, there is no evidence that a single-valued gap acceptance function cannot be used to model driver behavior reliably at a stop sign. As is explained in the following section, however, a deterministic approach to gap acceptance may be reliable only if the specified value of critical gap is an appropriate estimate.

Finally, significant differences did not exist between delay values generated from the simple logit model accounting only for gap length and from the single-valued critical gap function. This was to be expected as the long-run estimates generated from an average value of critical gap will be equivalent to the outcomes generated from a distribution of critical gaps.

Delay Sensitivity to Critical Gap

Where the gap acceptance model is a single-valued function, simulation experiments suggest that predicted delay is very sensitive to the specified value of critical gap. The following table presents the simulated estimates of average stop bar delay for various single-valued critical gaps:

Critical Gap (sec)	*Average Stop Bar Delay (sec)*
5.5	5.1
5.8	6.0
6.0	6.6
6.5	8.8
7.0	13.0

Marginal changes in the specified critical gap value produce relatively large differences in estimates of average stop bar delay, consistent with the tendencies of analytical queueing models.

Further simulation experiments revealed that the specified value of critical gap substantially alters estimates of average approach delay, a common measure of effectiveness. Table 2 presents simulated steady-state values of average approach delay as a function of critical gap. Critical gap values that vary slightly from 5.8 sec yield sizable differences in estimated approach delay.

TABLE 2 Simulated Average Approach Delays Using Critical Gap

	Probabilistic	Deterministic				
Gap Acceptance Function	Disaggregate Logit Function	5.5 sec	5.8 sec	6.0 sec	6.5 sec	7.0 sec
Average Approach Delay (secs)	11.7	8.6	10.9	12.1	17.5	28.2

Estimating Mean Critical Gap

If a single-valued gap acceptance function is to be used for analysis, the apparent sensitivity of predicted delay dictates that critical gap values reflect proper estimates. Thus, any published values of critical gap should be estimated by means consistent with motorist behavior.

The revised *Highway Capacity Manual* procedures (*1*), for example, adopt a method for estimating critical gap previously described by Miller (*5*) and Troutbeck (*13*). With this method, a mean critical gap (for a particular maneuver) is inferred statistically from sequences of observed gaps at a stop-controlled approach using the assumption that the observed largest gap rejected by a motorist is smaller than the driver's critical gap, which, in turn, is smaller than the gap actually accepted by the motorist (i.e., drivers are assumed to be consistent). Estimating mean critical gap using this assumption has several shortcomings:

- All gaps rejected by the motorist except for the largest rejected gap are not included in the estimation of critical gap. This results in a loss of important information.
- Important information is also lost if the observed motorist accepts a *lag*. As no gaps are rejected, data specific to the driver are discarded. The loss of such information may cause bias in the estimated critical gap (i.e., sample selectivity bias) given that previous research (*6*) and findings reported in this paper indicate that motorists respond differently to gaps than to lags.
- A problem occurs whenever drivers reject gaps larger than the one that they eventually accept, a frequent occurrence (*6–9*, also the disaggregate logit model). Data specific to these drivers are either discarded or "*modified*" to be consistent with the assumption of motorist homogeneity and consistency. Discarding or changing observations to match postulates is a concern.

In contrast to the method just described, the application of discrete choice techniques to estimate critical gap is consistent with observable phenomena. By exploiting all observations, the resulting estimates of mean critical gap capture the variability across and within motorists. Given the apparent sensitivity of delay, discrete choice methods should be used for estimating mean critical gap. Such estimates can be derived easily with standard software packages, as demonstrated earlier.

A logit function estimated with sample data in which the fraction of rejected gaps differs significantly from that of the population will be biased in the estimated constant term. If the population's fraction of rejected gaps is known, a correction can be applied (*11*). Bias becomes an issue when estimating functions that are to be generalized. The concern can be avoided by developing gap acceptance models for intersections operating under specified sets of conditions.

FURTHER EVALUATION OF GAP ACCEPTANCE FACTORS

The example scenario does not suggest that exploiting a single-valued gap acceptance function is inappropriate for intersection analysis. Nonetheless, it will be demonstrated that the application of a mean critical gap leads to a potential dilemma: excluding disaggregate factors that influence gap acceptance erodes estimation power. A likelihood ratio test indicated that the predictive strength of the disaggregate logit model is significantly greater at the 95 percent level than that of the simpler logit model. (This finding was inevitable given that all coefficients in the disaggregate function are statistically significant.)

For further exploring the significance of influential factors, gap acceptance functions estimated for the remaining minor street movements at Intersections 1 and 2 are presented.

Intersection 1

The gap acceptance function estimated for left-turn minor street vehicles at Intersection 1 is as follows:

$$\Pr(a) = \frac{1}{1 + e^{7.909 - 1.382\,ming - 0.013\,td + 1.192 dvnf}}$$

Independent Variable	*Estimated Coefficient*	t-*Statistic*
1	−7.90869	−6.81553
ming	1.38196	6.79695
td	1.26032e-002	2.13013
dvnf	−1.19245	−2.10543
Auxiliary Statistics	*At Convergence*	*Initial*
Log likelihood	−53.204	−218.34
Number of observations	315	
Adjusted rho-square	0.740	

Disaggregate total delay, *td*, increases driver propensity to accept smaller gaps. The influence of gap length is complicated in that left-turn maneuvers are executed through two conflicting traffic streams. A specification search indicated that a more powerful model results from the inclusion of a single coefficient, *ming*, representing the smaller of the two gaps in both traffic streams. This suggests that left-turning motorists evaluate opposing gap lengths collectively and react to the smaller of the two gaps.

Finally, the estimated coefficient *dvnf*, a dummy variable, is equal to 1 if the smaller prevailing gap is in the near-side lane (*dvnf* = 0 otherwise). The sign of this coefficient indicates that drivers have a reduced propensity to accept a smaller gap occurring in the near-side lane, contrary to an earlier empirical finding (*8*).

Intersection 2

The estimated gap acceptance functions for right- and left-turn movements at Intersection 2, the T-intersection with a shared turn lane, are as follows:

$$\Pr(a) = \frac{1}{1 + e^{6.126 - 1.11\, t - 1.258\, dv1}}$$

Independent Variable	*Estimated Coefficient*	t-*Statistic*
1	−6.12569	−6.75654
t	1.10980	6.23020
*dv*1	1.25750	2.17838
Auxiliary Statistics	*At Convergence*	*Initial*
Log likelihood	−46.553	−160.12
Number of observations	231	—
Adjusted rho-square	0.69	—

$$\Pr(a) = \frac{1}{1 + e^{7.531 - 0.916\, ming - 0.011\, td - 1.466\, dvnf - 1.209\, dv2}}$$

Independent Variable	*Estimated Coefficient*	t-*Statistic*
1	−7.53077	−7.88504
ming (min. gap)	0.91590	8.62597
td	1.05003e-002	2.18858
dvnf	1.46597	2.82062
*dv*2	1.20854	1.94173
Auxiliary Statistics	*At Convergence*	*Initial*
Log likelihood	−79.989	−302.91
Number of observations	437	—
Adjusted rho-square	0.720	—

For right-turn movements, driver propensity to accept a gap increases with gap length. The dummy variable *dv*1 reflects driver propensity to follow behind a "leading" right-turn vehicle (an influence unique to shared turn lanes)—that is, *dv*1 = 1 where the preceding vehicle executed a right turn. The sign of this coefficient indicates that right-turning drivers are motivated to accept lags remaining from a previous right turn.

The right-turn function in this model does not have a coefficient reflecting the effect of individual delay at the stop-controlled approach. The apparent exclusion of this influence is most likely attributable to a lack of variability in total delay observed in the data set. The actual influence of delay on gap acceptance may not be insignificant.

In the gap acceptance function for left-turn movements at Intersection 2, the influence of individual delay, *td*, and minimum gap, *ming*, are interpreted as in the logit model for left-turn vehicles at Intersection 1. At Intersection 2, however, the sign of the dummy variable *dvnf* indicates that a motorist has a lower propensity to perform left-turn maneuvers where the smaller of the two opposing gaps occurs in the far-side lane, a finding consistent with an earlier study (*8*). Finally, the dummy variable *dv*2 characterizes a left-turn driver's propensity to accept the lag "left behind" by a preceding left-turn vehicle. The sign of this coefficient is compatible with the factor identified for right-turn movements at the intersection.

Pooling Models Across Intersections

Tests of taste variation were conducted to assess gap acceptance behavior across intersections. The assessments indicated that gap acceptance functions for either turning movement should not be combined across intersections, implying that differences in geometrics may create differences in gap acceptance behavior.

Summarizing Model Estimation

The coefficients found to affect gap acceptance decisions include disaggregate measures such as individual approach delay, influences of near- and far-side gaps in the conflicting traffic streams, motorist propensity to follow closely behind leading motorists executing the same maneuver, and a general preference for gaps over lags. Because these factors were all significant at the 95 percent level, the disaggregate gap acceptance functions have much greater predictive power than models including only gap length. Thus, for the scenarios evaluated in this paper, the estimation capabilities of mean critical gaps are inferior to those provided by disaggregate models.

CONCLUSIONS

This paper has highlighted issues relevant to modeling gap acceptance behavior at stop-controlled intersections. Findings from this study do not suggest that deterministic methods for modeling gap acceptance are unacceptable. Because using deterministic functions leads to analysis techniques that are easy to apply, modeling gap acceptance using a single-valued critical gap may be justified.

However, this paper illustrates two concerns. First, if the traffic engineering community adopts a deterministic gap acceptance methodology, values of mean critical gap should be estimated using techniques consistent with motorist behavior. Critical gap values that differ only marginally from proper estimates produce dramatic delay prediction errors.

Second, the use of single-valued functions necessitates the exclusion of disaggregate factors influencing the gap acceptance decision. The limited data exploited in this paper provide some insight into how the exclusion of these factors harms estimation. More conclusive assessments (using larger empirical data bases) are required. Expanded empirical evaluations would probably identify additional factors that affect gap acceptance. Such discrete influences might include socioeconomic driver characteristics, conflicting vehicle speeds and flows, and intersection geometrics.

Before the trade-offs between deterministic and probabilisitic gap acceptance functions can be identified, the estimation capabilities of both function types should be evaluated carefully. The application of discrete choice methods may represent the appropriate means for satisfying research needs in gap acceptance modeling. The relative strengths of deterministic and probabilistic gap acceptance functions may be evaluated through discrete choice. No matter which function type is ultimately adopted, the gap acceptance model can be estimated by logit or probit.

Should probabilistic functions be warranted, incorporating the gap acceptance model into an intersection assessment procedure becomes a consideration. Perhaps the only practical means of applying probabilistic models is through computer simulation. If manual evaluation techniques are desired, nomographs or some other graphical-based method can be constructed from simulation experiments.

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DISCUSSION

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The paper by Cassidy et al. is an interesting one in that it discusses a problem that has relevance today as researchers look at the performance of unsignalized intersections and of permitted right-turn movements at signalized intersections. The authors presented a well-documented case, but omitted an important point: the proportion of gaps accepted is influenced by driver characteristics and by flows. The reason is, drivers whose perceptions of a critical gap is longer than other drivers' will reject a number of gaps. Similarly, as the major stream flow increases, the number of shorter gaps will increase, and the drivers who need longer gaps will reject even more gaps. Hence, as the priority stream flow increases, the gap size with a 50 percent probability of being accepted increases.

An approximate relationship exists between the gap size with a 50 percent probability of being accepted and the priority stream flow, as a function of the mean and variance of the critical gap distribution (*1*). Ashworth's correction was based on the critical gap distribution for drivers whose critical gaps follow a normal distribution and on a probit function for the probability that a gap will be accepted. If the critical gap distribution has a log-normal distribution and the gaps in the priority stream are exponentially distributed, the proportion of gaps of a particular size that are accepted, $\Pr(a)$, does not have a log-normal distribution (*2*).

However, the equation

$$\mu_p \approx \mu_f + q\sigma_f^2 \tag{1}$$

can be applied for most critical gap distributions, including normal, log-normal, and gamma distributions. Here, μ_p is the mean used to quantify the $P(a)$ distribution. If the critical gaps are normally distributed, the gap size with a 50 percent chance of being accepted is μ_p. If the critical gap distribution has a log-normal distribution, the gap size with a 50 percent chance of being accepted, t_{50}, is given by

$$t_{50} = \frac{\mu_p}{\sqrt{1 + \sigma_p^2/\mu_p^2}}$$

where

s_p^2 = variance used to quantify $\Pr(a)$ curve,
μ_f and σ_f^2 = mean and variance of the critical gap distribution, and
q = flow (vehicles/sec).

Miller also indicated that the coefficient of variation of the $\Pr(a)$ distribution is approximately the same as the critical gap distribution. Hence

$$\sigma_p/\mu_p \approx \sigma_f/\mu_f \tag{2}$$

For example, if the driver's critical gap has a log-normal distribution with a mean of 6*s* and a standard deviation of 2*s*, Ashworth's technique indicates that t_{50} would be given by

$$t_{50} = \mu_p/\sqrt{1.1111} = 0.949\ \mu_p$$

In addition, by using Equation 1

$$t_{50} \approx 0.95\ (\mu_f + q\ \sigma_f^2)$$

or

$$t_{50} \approx 5.69 + 3.79q \tag{3}$$

To demonstrate that this equation is reasonable, I took a sample of 500 drivers whose critical gaps followed a log-normal distribution and presented them with simulated gaps with a Cowan M3 distribution of 100 times. For each of these times, a logit analysis was applied to the accepted and rejected gaps, and a t_{50} value was estimated.

There was a small difference between the logit function I used and the one used by the authors in that the logit function was assumed to be a function of logarithm of gap size. That is,

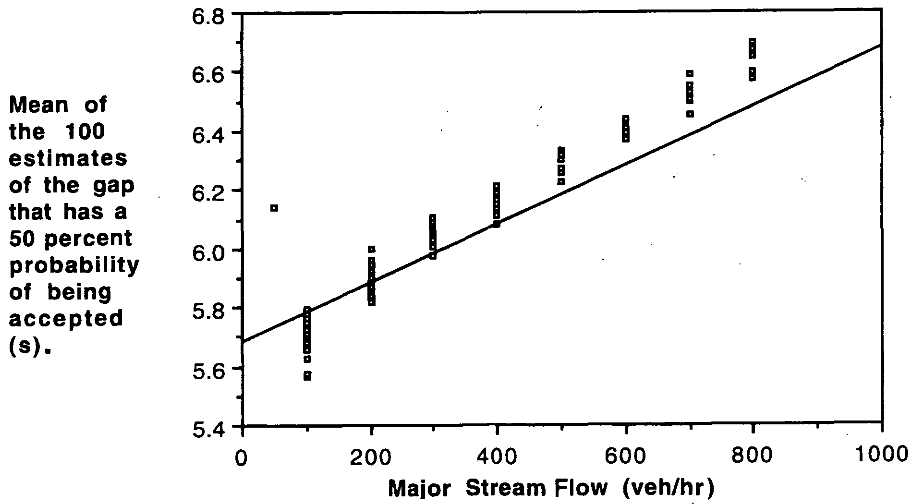

FIGURE 3 Ashworth's line determined by Equation 3.

$$\ln\left\{\frac{\Pr(t)}{1-\Pr(t)}\right\} = a\ln(t) + b \tag{4}$$

hence

$$\Pr(t) = \frac{1}{1 + e^{-a\ln(t)-b}} \tag{5}$$

This ensures that as t approaches zero $\Pr(t)$ approaches zero. The functions in the Cassidy et al. paper indicate that there is a probability that a gap of zero will be accepted.

By using Equation 3 or 4,

$$t_{50} = e^{-b/a} \tag{6}$$

Thus, for each set of minor stream and major stream arrival flows, I obtained 100 estimates of t_{50}. The mean of these estimates, determined by using Equation 3, appear in Figure 1. The drivers were assumed to be consistent with a mean critical gap of 6s and a standard deviation of 2s. If this condition is relaxed so that drivers have a degree of inconsistency, the results show a similar trend.

Figure 3 indicates that Ashworth's equation provides a reasonable fit, and explains that there is a relationship between the proportion of gaps with a 50 percent chance of being accepted and the major stream flow.

The conclusions I reach are, first, that there is a monotonic, increasing relationship between the proportion of accepted gaps and the major stream flow if driver behavior remains constant and flows change. This can be explained using Ashworth's method. Second, logit or probit analyses should account for this trend, but more important, researchers should expect terms such as total delay to be statistically significant in the logit or probit analyses. However, this delay term also is a proxy for the flow term. I caution others (using these simplified logit or probit analyses) not to assume that a delay term has a substantial affect on the critical gap function.

REFERENCES

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AUTHORS' CLOSURE

We thank Troutbeck for his discussion concerning the relationship between flow on major streets and gap acceptance. We do not disagree with his assertion that "as the priority stream flow increases, the gap size with a 50-percent probability of being accepted increases." We do not have data substantiating this claim because our study relied solely on a small data set; that is 15 minutes of observations from each of two intersections. This small data set did not provide a wide range of flows.

We suspect that if we had estimated gap acceptance models by using a larger data base with a range of major street flows, we would have found this factor to be significant. We note the likelihood of this in the conclusions of the manuscript.

We emphasize that the delay term in our models is not a proxy for the flow term, and our finding that delay is a significant predictor of gap acceptance is not attributed to a change in major street flows. Our models' delay term is driver specific. It is the disaggregate delay imparted to a motorist who, while waiting on a minor street, is confronted with a nearly fixed flow on a major street. We found that drivers who experienced longer delays had a propensity to accept shorter gaps and found this difference in gap acceptance behavior to be statistically significant. This observed effect was independent of the major street traffic flow.

It is worth reiterating that two factors—minor street delay and major street flow—influence gap acceptance in opposite ways. The discussant's formulas and accompanying figure point out that the gap size with a 50-percent chance of being accepted increases with major street flow. Conversely, we found that added delay leads to a decrease in the estimated value of critical gap.

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