Methodology for Assessing Dynamics of Freeway Traffic Flow

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A methodology for the detailed evaluation of measured freeway traffic stream features is presented. The method compares cumulative vehicle arrival curves at multiple locations, and empirical data illustrate traffic flow dynamics. However, conclusions with regard to traffic flow features will not be drawn until ongoing research is completed.

The paper presents a methodology for performing a detailed assessment of features of measured freeway traffic stream. Application of the proposed methodology identifies how disturbances propagate in time and space. Empirical data are used to present examples of traffic flow features revealed by the proposed method and to illustrate the methodology's advantages over conventional techniques for evaluating freeway data. The paper is methodological in nature; the authors, therefore, defer drawing conclusions on traffic flow dynamics until the ongoing research is completed.

BACKGROUND

Traffic flow on any freeway system cannot exceed the capacity of its most severe restriction (i.e., bottleneck). Thus, bottlenecks often characterize freeway operating conditions (1, p.288). The measurement and assessment of bottleneck flow has been the subject of much research.

Past studies of bottleneck operation often have relied on observations measured at a single location along the freeway. Such observations might have included measured values of flow, q; speed, v; and density, k (or occupancy) from which q-k or q-v scatterplots were constructed (2-7). More recent work has sought to assess capacity flow by measuring vehicle arrival rates at locations presumed to be downstream of restrictions and comparing these rates before and after the observed onset of queueing (8-11).

The evaluation of operating states measured at a single location is, for lack of a better term, myopic. Restricting assessments of traffic stream behavior to a single location obscures flow dynamics occurring over space and time. A number of studies have constructed q-k or q-v scatterplots for multiple locations along a freeway and compared the relative features of these fundamental relations in an effort to assess the influence of physical location (12-15). Likewise, past work has examined changing traffic patterns in response to time-variant conditions by constructing plots of \(q(t), k(t)\) versus time \(t\) (12, 15-17). Yet these techniques do not identify explicitly the propagation of changing flow states in the traffic stream. As such, bottleneck flow dynamics may have yet to be identified in a definitive manner.

PROPOSED METHOD

The method described herein is based on the work by Newell (18), who used assumptions about wave motion to predict the features of cumulative vehicle arrival curves. Analogously, the authors use the observed features of cumulative arrival curves to identify the motion of changing traffic states.

The cumulative vehicle arrival curve plots cumulative arrival number to time \(t\) (19-22). In Figure 1, the value \(j\) on the vertical axis is the cumulative number of vehicle arrivals to the given location by time \(t_j\). Analogously, \(t_j\) is the time that the \(j\)th vehicle arrives at the location. In constructing cumulative curves, the authors plot smooth, differentiable interpolations through the stepwise function illustrated in Figure 1. The derivative (i.e., slope) of this interpolation is flow.

The cumulative arrival curve is a visual representation of observations collected directly from the highway. The measure flow, on the other hand, requires specification of a time interval, and the interval selected can influence the magnitude of flow. Moreover, cumulative curves do not model relationships, as is often the intent of q-k and q-v scatterplots.

The methodology herein uses cumulative curves constructed in series. The input-output diagram in Figure 2 shows cumulative curves measured at two locations along the highway. Curve \(A(x, t)\), the cumulative vehicle arrivals past upstream location \(x_s\) to time \(t\), is constructed from the same collection of vehicles used for \(A(x, t)\), the cumulative curve at downstream location \(x\). That is, an upstream observer records (and cumulatively graphs) the arrival times of vehicles as they pass \(x_s\). The times at which these same vehicles pass \(x\) are also recorded (and plotted). The vertical distance between curves at some time, say \(t_j\), for example, is the number of vehicles in section \(x-x_j\) at \(t_j\). In the absence of vehicle overtaking maneuvers, the horizontal distance between curves at height \(j\) for example, is \(j\)'s trip time, \(t_j\), from \(x_s\) to \(x\).

The input-output diagram is an effective tool for tracing the motion of disturbances in time and space. As an example, the "thick" portions of the arrival curves in Figure 3 depict a short-term fluctuation in arrival rate. The fluctuation on arrival curve \(A(x, t)\) is passed horizontally to downstream curve \(A(x, t)\), indicating that this fluctuation propagates forward among the same collection of vehicles. The phenomenon of changing flow states moving forward with vehicles has been observed consistently in this study.

The input-output diagram in Figure 3 can be transformed into a queueing diagram by translating upstream curve \(A(x, t)\) horizontally to the right by a distance equal to the average free-flow trip time from \(x_s\) to \(x\). Translated curve \(A(x, t)\) is a "desired" arrival curve mapping what would be vehicle arrival times to downstream location \(x\) in the absence of delay. Where a desired arrival curve is superimposed on the downstream arrival curve, traffic is flowing...
without delay (i.e., desired and actual arrival times to the down­
stream location are equal). In the presence of delay, displacement
will exist between the desired and the downstream arrival curves.
The horizontal distances between these displaced curves at unit
heights define vehicle delays.

A system of moving time coordinates \((18)\) is used to describe
conveniently the process of translating upstream curve \(A(x_0, t)\) by
the appropriate free-flow trip time. In the moving time coordinate
system, time advances forward over space at a pace equal to aver­
age free-flow trip time. “Moving” time, \(t'\), at any downstream loca­
tion, \(x\), lags behind “actual” time, \(t\), by the free-flow trip time from
an upstream reference point, \(x_0\). That is,

\[
t' = t - u_f(x - x_0)
\]

where \(u_f\) is the average free-flow trip time per unit distance.

The use of moving time facilitates the presentation of desired and
downstream arrival curves with a single time axis, as in a queueing
diagram. Free-flow vehicles exhibit zero trip time to downstream
locations (i.e., curves are superimposed), whereas displacements
between curves reveal added trip times (i.e., delays).

Figure 4 presents the motion of a forward-moving wave propa­
gating at a rate slower than prevailing vehicle speed, as described
by Kinematic Wave Theory (23) for moderately heavy, uncongested
flow conditions. As vehicles advance downstream faster than the
wave, the disturbance past location \(x_0\) manifests itself at down­
stream location \(x\) among a collection of vehicles of higher arrival
number. A horizontal translation of upstream curve \(A(x_0, t)\) will not
result in the superposition of both curves in Figure 4.

Figure 5 illustrates the motion of backward-moving waves. At
time \(t_i\), Curve \(A(x, t)\) exhibits a dramatic discontinuity in flow states
created by a sudden flow reduction past point \(x\). This flow reduction

FIGURE 1 Cumulative vehicle arrival curve.

FIGURE 2 Input-output diagram.
might be caused by a downstream incident or a surge in on-ramp flow downstream. The resulting discontinuity in flow, called a shock wave, propagates in the upstream direction. Upon the shock wave's arrival at \( x_0 \), Curve \( A(x_0, t) \) presumably conforms to the shape of \( A(x, t) \). Figure 5 indicates that the shock's trip time from \( x \) to \( x_0 \) is given by \( w \) and that the number of vehicles traveling through the shock during this trip is \( M \).

Later, at time \( t_2 \), a disturbance is created by a rise in flow through the downstream bottleneck (e.g., the incident is partially cleared or downstream on-ramp flow slightly diminishes). The backward motion of the resulting wave in Figure 5 describes, according to Kinematic Wave Theory, how disturbances propagate in congestion. This methodology is used to study the evolution of traffic flow. Upstream curves are translated horizontally, as in a queueing diagram, thereby employing a system of moving time coordinates. Subsequently, the features of traffic disturbances are evaluated by comparing the attributes of the cumulative curves in series.

The arrival curves presented in Figures 3 through 5 idealized as empirical count data seldom reveal changing flow states in a pronounced or obvious manner. Thus, the methodology incorporates a simple but important graphical “trick”: cumulative counts used for arrival curves are reduced uniformly by a “background” flow. A fixed number is cumulatively subtracted from the vehicle counts in each count interval. The reduction is applied sequentially to counts at each observation location starting with intervals that correspond to the same moving time at all locations.

The process used for background flow reduction can be visualized using Figure 6, which already displays a horizontally translated \( A(x_0, t) \). Once in moving time, a fixed reduction is cumulatively applied to both curves simultaneously (e.g., starting from \( t' = 0 \)). Using this technique, the vertical distances between consecutive curves are preserved following the background flow reduction. Similarly, a background flow reduction will not alter the occurrence times of flow changes on the cumulative curve, which is the feature...
of interest in this analysis. Note, however, that where displacements between curves prevail because of delay (as in Figure 6), a background flow reduction enlarges the horizontal distances between consecutive curves. The number “reassigned” to the jth vehicle on upstream curve $A(x, t)$ is different from the number reassigned to vehicle $j$ on downstream curve $A(x, t)$, and horizontal distances no longer equal delays.

A background flow reduction amplifies flow changes on the cumulative curve. Following a background flow reduction of sufficient magnitude, changing flow states can be identified visually on the cumulative curve. The resulting curve may exhibit negative slopes denoting prevailing flows less than the specified background flow.

**FIGURE 5** Backward-moving waves.

**EMPIRICAL DATA**

The methodology is applied using data measured on a section of the Queen Elizabeth Way near Toronto, Canada. Data collected during multiple weekdays in March 1994 were generously provided by personnel at the Ontario Ministry of Transportation.

The study site is illustrated in Figure 7. Mainline and on-ramp demands at the Cawthra Road junction create recurring congestion during the morning commute. Detector stations for measuring traffic stream data are located throughout the system and have been labeled in Figure 7 according to the numbering strategy adopted by the Ontario Ministry of Transportation.

**FIGURE 6** Background flow reduction.
Data measured at Stations 23, 24, and 25 are presented. Detectors at these locations record counts, average speeds, and occupancies over 30-sec intervals. Average free-flow trip time between these stations is, likewise, 30 sec. Data presented herein are aggregated across all travel lanes.

Observations are presented from only Stations 23 through 25 purely in the interest of brevity. The methodology can be applied even in the absence of conservation. Consider, for example detector Stations 25 and 26. These consecutive stations exist upstream and downstream of ramp junctions. Where ramp counts are not available, cumulative curves at Stations 25 and 26 would not be superimposed. One could, however, readily evaluate the motion of disturbances by comparing the relative changes in slope of these consecutive curves.

**FORWARD-MOVING FLUCTUATIONS IN ARRIVAL RATE**

Our initial example presents the motion of forward-moving disturbances as revealed by the proposed method. In this example, the method is applied to a 25-min period on a single observation day (labeled "Day 1" in Figures 8 through 10). Figure 8 illustrates cumulative arrival curves constructed from traffic counts at Stations 23, 24 and 25. These arrival curves illustrate a key issue: the "eye" does not readily identify subtle changes in a function's slope. Examining how disturbances propagate is almost impossible because changing flow states are not apparent from the curves in Figure 8.

Figure 9, on the other hand, presents the cumulative curves previously shown in Figure 8 following (a) the use of moving time by translating curves at Stations 23 and 24 by the respective free-flow trip times to Station 25, and (b) a background flow reduction applied to all three curves. Cumulative curves at Stations 24 and 25 were reduced uniformly by a rate of 4,300 vehicles per hour (vph). A slightly higher background flow reduction of 4,436 vph was applied to the curve at Station 23 as the detectors at this station were found, on this day, to be overcounting vehicles at a rate of 136 vph. Having applied background flow reductions, changing vehicle arrival rates are now displayed prominently as "wiggles" on the curves in Figure 9.

In Figure 9 a sudden flow reduction (manifest as a near-zero average slope) occurs at approximately \( t' = 6:43 \) a.m. and prevails for approximately 10 min. The general superimposition of curves denotes an absence of delay between Stations 23 and 25. Thus, the observed flow reduction initially occurs upstream and the resulting disturbance propagates forward past the observation locations. If the 10-min flow reduction is the consequence of an upstream incident (a plausible explanation), then including data collected during this 10-min interval could corrupt certain experiments, such as maximum flow measurements to estimate capacity.

Conventional wisdom suggests that the 10-min flow reduction in Figure 9 can be identified by constructing time-series plots of flow at each detector station. There are potential shortcomings with this approach, however. Vehicle count is a random variable exhibiting a variance-to-mean ratio comparable with 1. Figure 10 shows the time-series plots of flow computed from 30-sec vehicle counts at

**FIGURE 8** Arrival curves constructed from "raw" data, Day 1.

**FIGURE 9** Arrival curves in moving time with background flow removed, Day 1.
each station during the 25-min period of interest. Given the observed variability, the 10-min drop in flow is not readily apparent in Figure 10.

One can reduce the dispersion in Figure 10 by computing flow on the basis of vehicle counts taken over longer time intervals. The problem here, however, is that resulting flow values are average magnitudes occurring during each specified interval, not the actual rates prevailing at any given time. This approach to "smoothing" flow obscures the details of traffic dynamics.

The advantage of the cumulative curve is its representation of detail. Referring to Figure 9, short-term changes in vehicle arrival rate (i.e., wiggles) replicate across cumulative curves. The ability to superimpose wiggles using a horizontal translation denotes that changing arrival rates are propagating among the same collection of vehicles. That is, disturbances travel forward with vehicles. These disturbances are not waves that propagate across vehicles creating velocity changes in the traffic stream. Instead, the wiggles in Figure 9 are arrival rate fluctuations created by the varying headways chosen by different motorists. The replication of wiggles across cumulative curves reveals that motorists "remember" and maintain their respective headways while traversing the freeway segment. This flow feature is not predicted by any conventional continuum model of freeway flow (23-25).

BACKWARD-MOVING WAVES

The forward-moving fluctuations in vehicle arrival rate described previously are not the only type of disturbance that can occur in the traffic stream. Other disturbances, such as shock waves, will create velocity changes by propagating across vehicles. Using data from a different observation day (Day 2 in Figures 11 and 12), detailed features of shock wave propagation can be demonstrated.

Figure 11 presents cumulative arrival curves in moving time with background flow reductions of 4,300 vph for Stations 24 and 25 and a slightly larger background flow reduction of 4,450 vph for Station 23, as detectors were again overcounting vehicles at this upstream station. Starting at \( t' = 6:26:30 \) a.m., cumulative curves in Figure 11 exhibit their maximum flows (i.e., slopes). At approximately \( t' = 6:30:30 \) a.m., the curve at Station 25 begins to diverge in a pronounced manner from the others, depicting added vehicle delay between Stations 24 and 25. Substantial flow reductions occur at upstream Stations 24 and 23 at \( t' = 6:35:30 \) a.m. and at \( t' = 6:37:30 \) a.m., respectively. These sudden flow reductions, which occur sequentially in time and space, depict the motion of a backward-moving shock wave. These dramatic, short-term "collapses" in flow (manifest as near-zero slopes) appear to reflect initial motorist tendency to overreact to the shock's arrival.

From Kinematic Wave Theory, one would expect that as the shock arrives at each station, the respective cumulative curve would take on the slope of its downstream counterpart. After the shock arrived at Stations 24 and 23, the curves would exhibit fixed displacements denoting delay and the presence of additional vehicles between detector stations.

The occurrence of the flow collapse, however, creates conditions that are different than expected. Namely, a flow collapse "starves" the downstream freeway section as seen in the bulges displayed by the cumulative curves in Figure 11. Note that the additional vehicles accumulated during the shock’s propagation are depleted and the cumulative curves tend to reconverge. This unstable flow behavior might obscure the bottleneck’s location. Figure 12 presents measured speed profiles at Stations 23, 24, and 25 during an extended
period of Day 2. During the shock’s propagation just after 6:30 a.m., all three detector stations exhibit speed reductions associated with congestion. Eventually, however, speeds at downstream Stations 24 and 25 recover to higher levels while speeds at Station 23 remain substantially lower. By observing this prolonged difference between upstream and downstream speeds, one might identify a bottleneck in close proximity to the Cawthra Road on-ramp junction while overlooking the initial bottleneck well downstream, as shown in Figure 11.

CONCLUSIONS

The paper has presented a methodology for the dynamic assessment of freeway traffic flow. The method facilitates identification of the details of flow features. The objective of this paper has not been to draw conclusions or conjectures with regard to freeway traffic stream dynamics. Instead, the authors have described an assessment methodology. For demonstration, the methodology has been applied to assess freeway operation on two days. Some of the flow features identified by the method and presented herein are not yet completely understood. Further research to investigate these dynamics is ongoing. In the future, the authors’ intent is to demonstrate that the proposed method is a valuable tool for assessing bottleneck capacity, speed-flow-density relationships, and highway traffic flow dynamics in general.

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REFERENCES


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