Effect of Wide-Base Tires on Rollover Stability

ANDREW D. ST. JOHN AND WILLIAM D. GLAUZ

One of the most cited advantages of using wide-base tires is to increase the stability relative to rollover of a truck or trailer. The rollover thresholds of a two-axle full trailer when it is equipped with conventional dual-wheel axles and when it is equipped with wide-base tire–axle assemblies are compared, and the stability benefits of design changes to trailers enabled by the wide-base configuration are evaluated. Because it is important in such analyses to account for the locations of the forces exerted by the pavement on the tires, it was necessary to include the effects of tire deflections in the analyses. The results showed that the tire deflection effects reduced the rollover stabilities over those determined by a rigid tire analysis by 5 to 6 percent. The increases in the rollover stabilities of trailers with wide-base tires rather than dual tires were relatively small (up to 6 percent). However, if the user of wide-base tires takes advantage of the ability to spread the suspension springs and lower the center of gravity of the trailer, increases in rollover stability of 18 to 21 percent are possible.

Wide-base tires, also referred to as super singles, are truck tires that are typically between 380 and 460 mm (15 and 18 in.) in nominal width, in contrast to normal truck tires that have a nominal width of 280 mm (11 in.). Certain segments of the trucking industry find a number of safety and economic advantages to substituting a wide-base tire for each pair of dual standard tires on the axles. Perhaps the major safety advantage claimed is a reduced propensity for the truck or trailer to roll over. Improved rollover stability is believed to be enhanced even further if, in addition to simply mounting the wide-base tires in place of the dual tires, one also takes advantage of the resulting wider wheel stance to spread the truck suspension springs, enabling the roll-restoring moment due to the springs to increase. In addition, with the wider stance on a tank trailer, one can lower the tank and its center of gravity and reduce the distance between the center of gravity and the roll center. The purpose of this paper is to investigate these claims analytically. More specifically, the purposes of the paper are to

1. Compare the rollover thresholds of a two-axle full trailer when it is equipped with conventional dual-wheel axles and when it is equipped with wide-base tire–axle assemblies, and
2. Evaluate the stability benefits of design changes enabled by the wide-base configuration.

All analyses presented deal with static or steady-state conditions. Two rollover situations are treated: side slopes and cornering on a pavement with zero superelevation.

Trailer data were taken from Fancher and Mathews (*1*), in which data typical of those for gasoline tankers used in California are provided. The trailer considered is a full trailer, with no vertical loads or moments shared with the tractor. Also, the front and rear axle suspensions both have the same characteristics. The tire data are taken from Fancher et al. (*2*) and from manufacturers. The duals are 11R22.5 tires, and the wide-base tires are 425/65R22.5 tires.

Table 1 lists the data used. The data are for a single axle; the terms are described in more detail later in this paper, along with their associated equations.

The use of wide-base tires instead of dual tires enables potential trailer design improvements. They include a 15 percent increase in the suspension system roll stiffness K_r, a decrease of 150 mm (6 in.) in the sprung weight roll arm $(h_s - h_r)$, and a reduction of 150 mm (6 in.) in the height of the sprung weight h_s. These modifications would apply to the unmodified values in Table 1.

TIRE DEFLECTIONS

In analyzing units with dual wheels it has been conventional to assume that the load on the two wheels can be taken to act midway between the wheels. This is important here because where the forces from the pavement act on the trailer has a major effect on the moments that resist rollover. Furthermore, the only way to decide where these forces act is to include the effects of tire deflections in the analyses. The inclusion of tire deflections is especially important here because the wide-base tires are more compliant; they deflect more than comparable dual tires. This compliancy factor, by itself, is destabilizing.

The work of Fancher et al. (*2*) indicates that the vertical stiffness, or the spring rate, of a truck tire is not always linear. When a truck tire is lightly loaded its deflection is not a linear function of the load. Above certain loads, however, the deflection does become a linear function of the load. If the rated load of a tire is denoted by F_R, the deflection Z of the tire is a linear function of the applied vertical force F, if F is greater than $F_R/3$. F and Z are both measured perpendicular to the pavement surface. At a load of $F_R/3$ the tire deflection is about 0.55 Z_R, where Z_R is the deflection at a load of F_R. This is illustrated in Figure 1. Over the linear range the slope of the curve dZ/dF is $1/S$, where S is defined as the vertical stiffness of the tire. The subsequent analyses can be simplified to the case of rigid tires (no tire deflections) by taking the limit as $S \to \infty$.

From Figure 1 it can be seen that the slope of the linear portion of the curve is

$$1/S = 0.45Z_R/(2/3)F_R \qquad (1)$$

so that the tire vertical stiffness is $S = 1.48\ F_R/Z_R$.

The equation for the linear portion of the curve is

$$Z = \frac{1}{S}\ (0.48F_R + F) \qquad (2)$$

A. D. St. John, 8470 East Amethyst Place, Tucson, Ariz. 85715. W. D. Glauz, Midwest Research Institute, 425 Volker Boulevard, Kansas City, Mo. 64110.

TABLE 1 Data for Each Single Axle of a Two-Axle Trailer

Parameter	Symbol	Units[a]	Dual Tires	Wide Base Tires
Height, unsprung weight	h_u	mm	495	495
Height, sprung weight	h_s	mm	1990	1980
Height, roll center	h_r	mm	740	740
Unsprung weight	W_u	kg	680	600
Sprung weight[b]	W_s	kg	1,020	1,020
Total weight[c]	W	kg	1,700	1,620
Roll stiffness for the sprung weight	K_r	mm-kg per degree	923,000	923,000
Single tire vertical stiffness	S	kg per mm	89	120
Single tire rated load	F_R	kg	2,450	3,090
Track width,[d] outer duals	T_o	mm	2160	—
Track width,[d] inner duals	T_i	mm	1500	—
Track width,[d] wide base singles	T	mm	—	2010

[a]1 mm = 0.03937 in., 1 kg = 2.203 lb.
[b]Empty weight. Weight when fully loaded is 7,950 kg (17,500 lb).
[c]Empty weight. Weight when fully loaded is 8,630 kg (19,000 lb) for duals, 8,550 kg (18,825 lb) for wide base tires.
[d]Center-to-center.
Dash (—) indicates data not applicable.

The nonlinear portion of the deflection/force curve can be fitted with a quadratic function passing through the origin, meeting the linear portion at $F_R/3$, and having a slope of $1/S$ at $F_R/3$. The result is

$$Z = \frac{1}{S}\,(3.89F - 4.33F^2/F_R) \qquad (3)$$

AXLE ROLL ANGLE

The axle roll angle ϕ_A is the angle between the axle and the surface of the pavement. When the loads applied to all of the tires on the axle are equal, it is assumed that ϕ_A equals zero. When the trailer rolls, the load is shifted to the tire(s) on one side of the axle, and the roll angle ϕ_A is determined from the changes in the vertical deflections of the tire. The expressions for the roll angle depend on the axle configuration (i.e., whether there are dual tires or wide-base single tires) and whether the tire deflections are linear or nonlinear, which depends on the load ranges.

Dual Tires on a Side Slope

Figure 2 illustrates the roll angle for an axle with dual tires on a side slope that has an angle of θ. Figure 2 also illustrates the vertical force on one tire (Tire 2) along with its components parallel and perpendicular to the pavement surface.

For the trailer to roll over it is necessary that all of the load be shifted off of Tires 1 and 2 and onto Tires 3 and 4. Attention is first directed to the situation in which F_1 and F_2 have been reduced to zero but in which Tire 2 still remains barely in contact with the pavement. Then, all of the trailer weight is carried by Tires 3 and 4, but not equally. However,

$$F_3 + F_4 = W \qquad (4)$$

where W is the sum of the unsprung and sprung weights W_u and W_s, respectively.

If Z_n is the deflection of the nth tire and Tire 2 is touching the pavement, the axle roll angle is given by either of the equations,

$$\sin\phi_A = (Z_3 - Z_2)/T_i$$

or

$$\sin\phi_A = (Z_4 - Z_2)\Big/\left(\frac{T_o + T_i}{2}\right) \qquad (5)$$

Alternatively, if Tire 2 carries no load,

$$\sin\phi_A = (Z_4 - Z_3)\Big/\left(\frac{T_o - T_i}{2}\right) \qquad (6)$$

It is convenient to consider that when the entire axle load is carried by Tires 3 and 4 (that is, when Tire 2 carries no load), these two tires each carry half of the entire axle load plus or minus a difference or load shift of ΔF. That is, one can write the loading on Tires 3 and 4 as

$$F_3 = \frac{W}{2} - \Delta F \qquad (7)$$

and

$$F_4 = \frac{W}{2} + \Delta F \qquad (8)$$

Now, if it is assumed that $F_3\cos\theta$ and $F_4\cos\theta$, the loads on Tires 3 and 4 normal to the pavement, respectively, are such that the tire

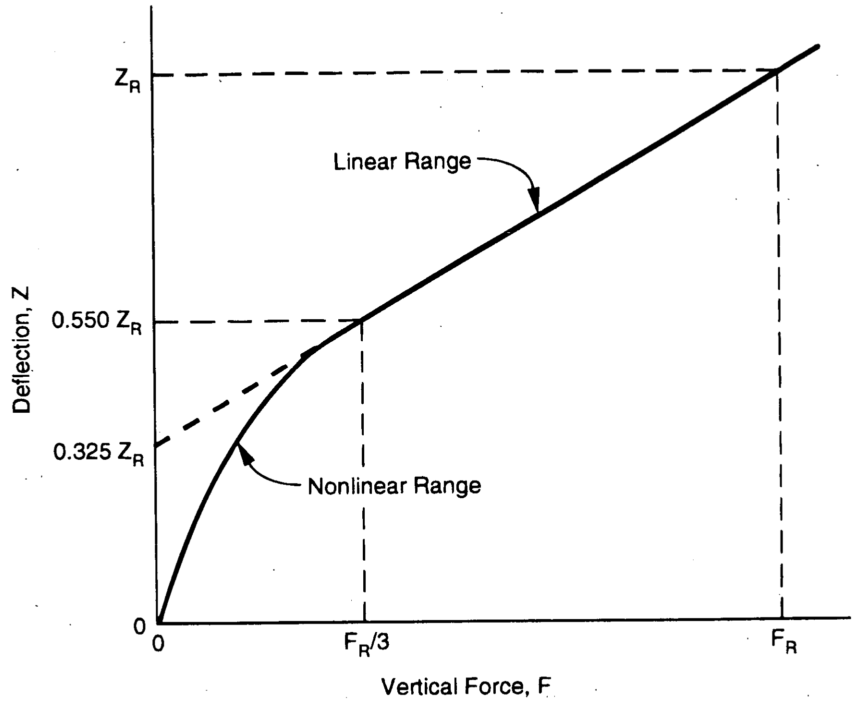

FIGURE 1 Tire deflection versus vertical force curve.

deflections Z_3 and Z_4 are in the linear range, Equation 2 can be used and Equations 7 and 8 can be substituted into the two forms of Equation 5 (with $Z_2 = 0$) to obtain

$$\Delta F = \frac{(T_o - T_i)}{(3T_i + T_o)} \left(\frac{0.48F_R}{\cos\theta} + \frac{W}{2} \right) \tag{9}$$

Equation 9 defines the load shift between Tires 3 and 4 when Tire 2 is just barely touching the pavement, assuming that the deflections of both tires are in the linear range. The assumption that both tires are in the linear deflection range means that the (lighter) load on Tire 3 be at least $F_R/3$. That is,

$$\left(\frac{W}{2} - \Delta F \right) \cos\theta \geq F_R/3 \tag{10}$$

If Equation 10 is not satisfied, Equation 3 must be used instead of Equation 2 for Z_3 and Equation 9 is replaced by

$$A(\Delta F)^2 + B(\Delta F) + C = 0 \tag{11}$$

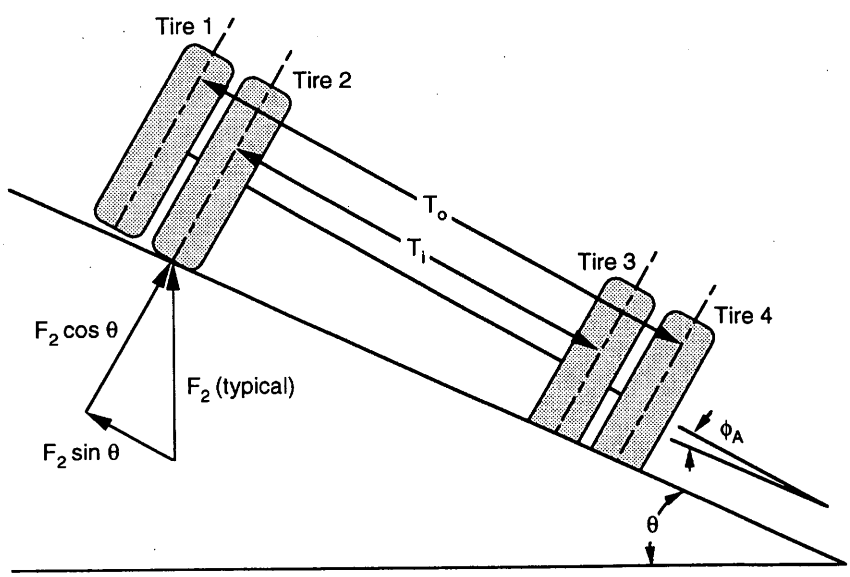

FIGURE 2 Dual tires on a side slope.

where

$$A = \frac{4.33}{F_R}\left(\frac{T_o + T_i}{2}\right)\cos\theta$$

$$B = 3.89\left(\frac{T_o + T_i}{2}\right) - \frac{4.33}{F_R}\left(\frac{T_o + T_i}{2}\right) W\cos\theta + T_i$$

$$C = -3.89\left(\frac{W}{2}\right)\left(\frac{T_o + T_i}{2}\right) + \frac{4.33}{F_R}\left(\frac{W}{2}\right)^2\left(\frac{T_o + T_i}{2}\right)\cos\theta + \frac{0.48T_i}{\cos\theta} + T_i\left(\frac{W}{2}\right) \qquad (12)$$

Equation 11 is to be used when Equation 10 cannot be satisfied. The load shift ΔF can be found by solving Equation 11 for a specified value of θ when Tire 2 is just barely touching the pavement and Tire 3 is in the nonlinear range or by using Equation 9 if Tire 3 is in the linear range. (The case in which both Tires 3 and 4 are in the nonlinear range is not addressed because the weight of the trailer would have to be unrealistically low.) The axle roll angle ϕ_A, is then found by using the value of the load shift ΔF in Equations 7 and 8 to get the resultant tire loads, using these (times $\cos\theta$) in Equation 2 or 3, as appropriate, to get the tire deflections, and then using the deflections in Equation 6.

With the dual tire configuration on a side slope the load shift ΔF can be found corresponding to θ, which provides static equilibrium of the trailer with Tire 2 barely touching the pavement. However, it has not yet been shown whether this state of balance is stable or unstable. If the roll axle angle ϕ_A is increased a slight additional amount, the overturning moment due to gravity will increase and the heights of the centers of gravity will also increase, both of which are destabilizing. However, the load will be shifted from Tire 3 to Tire 4, which will tend to resist rollover. To determine the net effect it is necessary to determine the relationships between the additional load shift and the increase in the axle roll angle for a fixed side slope angle.

Starting with a side slope angle θ, the load shift ΔF, and the axle roll angle θ_A, determined by the method described previously, it is then necessary to determine the additional load shift δF corresponding to an additional amount of axle roll, $\delta\phi_A$.

If Tires 3 and 4 are both deflected in their linear range, substitution of Equation 2 into Equation 6 yields

$$\sin(\phi_A + \delta\phi_A) = \frac{4}{S(T_o - T_i)}(\Delta F + \delta F)\cos\theta \qquad (13)$$

If ϕ_A and $\delta\phi_A$ are small angles (as they would typically be), $\sin(\phi_A + \delta\phi_A) \doteq \sin\phi_A + \sin\delta\phi_A$, Equation 13 becomes

$$\sin(\delta\phi_A) = \frac{4}{S(T_o - T_i)}(\delta F)\cos\theta \qquad (14)$$

However, if Tire 3 is deflected in its nonlinear range, Equation 3 is used in place of Equation 2 and equations parallel to Equation 13 and 14 can be developed. For the small angle assumption, Equation 14 would be replaced with

$$\sin\delta\phi_A = \frac{2\cos\theta}{S(T_o - T_i)}\left\{(\delta F)\left[1 + 3.89 - \frac{8.67}{F_R}\left(\frac{W}{2} - \Delta F\right)\cos\theta\right] + \frac{4.33}{F_R}(\delta F)^2\cos\theta\right\} \qquad (15)$$

Wide-Base Single Tires on a Side Slope

Figure 3 illustrates the geometry for the wide-base tire situation. The tire deflections and the axle roll angle are related by

$$\sin\phi_A = (Z_2 - Z_1)/T \qquad (16)$$

In the following, ΔF again represents the amount of the axle load transferred between tires, but in this case it is the load transferred between Tires 1 and 2. If both tires are in the nonlinear range, then substituting from Equation 3 for both Z valves in Equation 16 gives

$$\sin\phi_A = \frac{2\Delta F}{ST}\left(3.89\cos\theta - \frac{4.33}{F_R}W\cos^2\theta\right) \qquad (17)$$

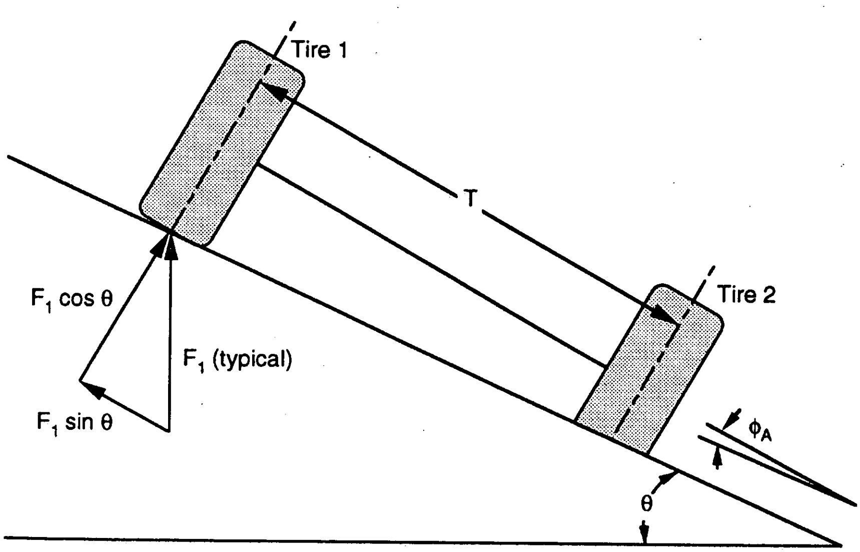

FIGURE 3 Wide-base single tires on a side slope.

whereas if Tire 1 is in the nonlinear range and Tire 2 is in the linear range,

$$\sin\phi_A = \frac{1}{ST}\left[0.48F_R + \left(\frac{W}{2} + \Delta F\right)\cos\theta - 3.89\left(\frac{W}{2} - \Delta F\right)\cos\theta\right] + \frac{1}{ST}\left[\frac{4.33}{F_R}\left(\frac{W}{2} - \Delta F\right)^2\cos^2\theta\right] \quad (18)$$

In the discussion of dual tires on a side slope, it was noted that for the side slope that produces equilibrium when Tire 2 carries no load but barely touches the pavement, it is necessary to determine whether the equilibrium is stable or unstable. With the wide base tires this equilibrium situation is always unstable, because any increase in ϕ_A will increase the overturning moments due to gravity as well as increase the heights of the centers of gravity, but no additional load shift is possible, so there will be no compensating restoring moment.

Cornering Acceleration with Zero Superelevation

All of the earlier discussion about the geometrics of the tire–axle systems is equally valid during a cornering maneuver except if the pavement has no superelevation (the simplest case), in which cos θ is equal to 1.0.

MOMENT EQUILIBRIUM EQUATIONS

If the trailer is to not roll over the overturning and stabilizing moments on the trailer must be in equilibrium (that is, be balanced). The trailer will roll over when the moments caused by the forces acting on the tires from the pavement can no longer balance the moments caused by gravity or lateral acceleration. In this subsection the equilibrium equations are developed for dual and wide-base tires for both side slope and cornering situations.

Moment Equations for Dual Tires on a Side Slope

Here and in the following, discussion the convention that counterclockwise moments are stabilizing (resist rollover) and are positive is adopted. Clockwise moments are negative, and if the net moment is negative, the trailer will roll over. The key angles and dimensions are shown in Figure 4. Figure 4 also shows the origin, 0, about which moments are summed. The selection of an origin is arbitrary, but this location, which is at the pavement surface along a perpendicular to the center of the axle, is convenient because the forces due to friction parallel to the pavement can be ignored because they create no moment.

The following symbols are further defined; typical values for the fixed quantities were given in Table 1:

h_s = the height of the center of gravity of the sprung mass, which is that portion of the trailer supported by the suspension springs;
h_u = the height of the center of gravity of the unsprung mass; the remainder of the trailer is not supported by the suspension, such as the tires and axles;
h_r = the height of the roll center, a point in space about which the sprung mass rotates;
ϕ_s = the roll angle of the sprung mass relative to a perpendicular to the axle; and
K_r = the roll stiffness of the trailer (per axle) created by the springs; as the sprung mass is rotated relative to the axle these springs create a moment resisting the roll.

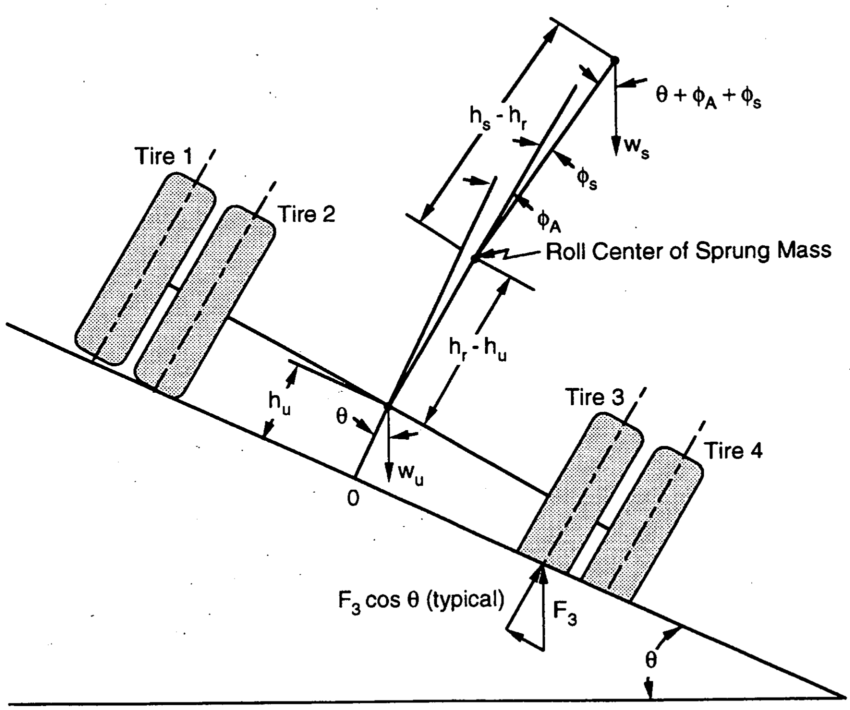

FIGURE 4 Key angles and dimensions for an axle with dual tires on a side slope.

First, it is convenient to consider an approximation in which the tires and the suspension are all rigid. That is, the trailer is treated as a rigid body. Summing moments about the origin for the case in which θ is just great enough to cause all of the trailer weight to be placed on Tire 4 gives

$$\frac{T_o W}{2}\cos\theta - (W_u h_u + W_s h_s)\sin\theta = 0 \tag{19}$$

For any lesser angle some weight would be on the other tires and no rollover will occur. If the angle is any greater, the (rigid) trailer will roll over. The critical side slope is thus

$$\tan\theta = \left(\frac{T_o W}{2}\right)/(W_u h_u + W_s h_s) \tag{20}$$

This approximation will prove useful later as a starting point for the iterative computation of the critical angle θ when the assumption that the trailer is rigid is dropped.

Now, dropping the rigidity assumption, if the trailer is allowed to roll against its suspension, the springs create a counterclockwise restoring moment $K_r\phi_s$, but the rolling displaces the sprung mass center of gravity, which creates a clockwise moment. Considering, first, only the sprung mass and taking moments about the roll center, the net moment due to the roll of the sprung mass is

$$M_r = K_r\phi_s - W_s(h_s - h_r)\sin(\theta + \phi_A + \phi_s) \tag{21}$$

If the sprung mass is in equilibrium, the net moment must be zero. If the angle ϕ_s is small one can approximate the sine term in Equation 21 as

$$\sin(\theta + \phi_A + \phi_s) \doteq [\sin(\theta + \phi_A) + \cos(\theta + \phi_A)\sin\phi_s] \tag{22}$$

in which the approximation $\cos\phi_s$ is taken as 1.0. Furthermore, one can set $\sin\phi_s$ equal to $(2\pi/360)\phi_s$, in which the factor $2\pi/360$ enables the angle to be written in degrees rather than radians. Making this substitution in Equation 22 and setting M_r to zero gives

$$\phi_s = \frac{\left[\left(\frac{W_s}{K_r}\right)(h_s - h_r)\sin(\theta + \phi_A)\right]}{\left[1 - \left(\frac{2\pi}{360}\right)\left(\frac{W_s}{K_r}\right)(h_s - h_r)\cos(\theta + \phi_A)\right]} \tag{23}$$

Next, considering the entire trailer depicted in Figure 4 and taking the situation in which all of the load is on Tires 3 and 4 and Tire 2 is barely in contact with the pavement, the sum of moments about the origin, 0, gives

$$M = \left(F_3\frac{T_i}{2} + F_4\frac{T_o}{2}\right)\cos\theta - W_u h_u\sin\theta - W_s[h_u\sin\theta$$
$$+ (h_r - h_u)\sin(\theta + \phi_A) + (h_s - h_r)\sin(\theta + \phi_A + \phi_s)] \tag{24}$$

Use of Equations 7 and 8 for the tire forces in Equation 24 gives

$$M = \left[W\left(\frac{T_o + T_i}{4}\right) + \Delta F\left(\frac{T_o - T_i}{2}\right)\right]\cos\theta - Wh_u\sin\theta$$
$$- W_s[(h_r - h_u)\sin(\theta + \phi_A) + (h_s - h_r)\sin(\theta + \phi_A + \phi_s)] \tag{25}$$

When this equation is solved for M equal to 0, it gives the value of θ at which the trailer is in static equilibrium, with the trailer's weight totally on Tires 3 and 4 and with Tire 2 barely touching the pavement.

When a solution of Equation 25 for M equal to 0 is found, it indicates a condition of static equilibrium, but the condition may be stable or unstable. If it is in stable equilibrium a small increase in the axle roll ϕ_A will cause the net moment M to become positive (counterclockwise), and the trailer will tilt back to the equilibrium position. If it is unstable the small change will result in a negative net moment and the trailer will roll over.

If the axle roll angle is increased by a small amount, $\delta\phi_A$ (given in the previous subsection as a function of δF), each of the heights, h_u, h_r, and h_s will be increased by the amount

$$\delta h = \left(\frac{T_o + T_i}{4}\right)\sin(\delta\phi_A) \tag{26}$$

Under these circumstances the roll moment on the sprung mass M_r is increased by an amount δM_r. Replacing M_r with $M_r + \delta M_r$ in Equation 21 as well as replacing ϕ_A and ϕ_s with their incremental equivalents, making the usual small angle approximations for the incremental angles, and subtracting Equation 21 from the result yields

$$\delta\phi_s = \frac{\left[\frac{W_s}{K_R}(h_s - h_r)\left(\frac{2\pi}{360}\right)\cos(\theta + \phi_A + \phi_s)\right](\delta\phi_A)}{\left\{1 - \left[\frac{W_s}{K_R}(h_s - h_r)\left(\frac{2\pi}{360}\right)\cos(\theta + \phi_A + \phi_s)\right)\right]\right\}} \tag{27}$$

Thus, for a small increase in δF the axle roll angle is increased by $\delta\phi_A$ and the sprung weight angle is increased by $\delta\phi_s$, which is given in Equation 27. Inserting the total amounts of the forces, heights, and angles in Equation 25 and then subtracting the M equal to 0 terms from the equilibrium solution, the incremental moment after using small angle approximations is expressed as

$$\delta M = \delta F\left(\frac{T_o - T_i}{2}\right)\cos\theta - W(\delta h)\sin\theta$$
$$- W_s[(h_r - h_u)\cos(\theta + \phi_A)]\sin(\delta\phi_A)$$
$$- W_s[(h_s - h_r)\cos(\theta + \phi_A + \phi_s)]\sin(\delta\phi_A + \delta\phi_s) \tag{28}$$

This equation gives the incremental moment that would be produced if, from the static equilibrium position with Tire 2 barely touching the pavement, a small added rotation raised Tire 2 slightly. If δM is positive the trailer will return to the equilibrium position; if it is negative the trailer will roll over.

Moment Equations for Wide-Base Single Tires on Side Slope

Figure 4 is applicable for wide-base single tires on a side slope except that there are only two tires, as shown in Figure 3. Equation 23, for the angle ϕ_s, when the sprung mass is in equilibrium, is applicable here as well as for the dual tire case. For the moment on the overall unit, Equation 25 is replaced by

$$M = (\Delta F)T\cos\theta - Wh_u\sin\theta - W_s[(h_r - h_u)\sin(\theta + \phi_A)$$
$$+ (h_s - h_r)\sin(\theta + \phi_A + \phi_s)] \tag{29}$$

where ΔF is the load transferred from Tire 1 to Tire 2.

Dual Tires Subjected to Cornering Acceleration

Figure 5 illustrates the key dimensions, angles, and forces acting on a trailer, viewed from the rear, when it is turning left on a pavement with zero superelevation. To turn left horizontal forces on the tires (not shown) must be equal to the mass times the lateral acceleration (V^2/R, where V is the trailer speed and R is the radius of the turn) required to turn the trailer. The equal reacting forces tending to overturn the trailer are shown at the centers of gravity of the sprung and unsprung masses. These are $A(W_s/g)$ and $A(W_u/g)$, respectively, where A is the lateral acceleration and g is the acceleration due to gravity.

Proceeding as before, first consider the sprung mass. Writing the equation for the moments about the roll center for the sprung mass, using the small angle assumptions for ϕ_s, setting the net moment equal to zero for equilibrium, and solving for ϕ_s yields

$$\phi_s = C_1(\sin\phi_A + A/g) \quad (30)$$

where

$$C_1 = \frac{W_s(h_s - h_r)}{\left[K_r - W_s(h_s - h_r)\left(\frac{2\pi}{360}\right)\right]} \quad (31)$$

The moment acting on the overall unit for the case in which Tire 2 is just barely touching the pavement but carrying no load is

$$M = \frac{F_3T_i}{2} + \frac{F_4T_o}{2} - (W_uh_u + W_sh_s)\frac{A}{g} - W_s[(h_r - h_u)\sin\phi_A + (h_s - h_r)\sin(\phi_A + \phi_s)] \quad (32)$$

Using Equations 7 and 8 for F_3 and F_4, the small angle assumptions for ϕ_s, and Equation 30 for ϕ_s yields

$$M = \frac{W}{4}(T_o + T_i) + \frac{\Delta F}{2}(T_o - T_i) - W_s(h_s - h_u)\sin\phi_A - \frac{2\pi}{360}W_sC_1(h_s - h_r)\left(\sin\phi_A + \frac{A}{g}\right) - (W_uh_u + W_sh_s)\frac{A}{g} \quad (33)$$

Setting M equal to 0, for equilibrium, and solving Equation 33 for A/g gives

$$\frac{A}{g} = \left\{\frac{W}{4}(T_o + T_i) + \frac{\Delta F}{2}(T_o - T_i) - W_s\sin\phi_A[(h_s - h_u) + \frac{2\pi}{360}(h_s - h_r)C_1]\right\} \Big/ \left[\frac{2\pi}{360}W_s(h_s - h_r)C_1 + (W_uh_u + W_sh_s)\right] \quad (34)$$

Equation 34 gives the lateral acceleration that corresponds to the trailer being in equilibrium when Tire 2 carries no load but is barely in contact with the pavement.

The acceleration given by Equation 34 may or may not cause the trailer to overturn, because the equilibrium condition may be unstable or stable. A small additional amount of acceleration may cause the trailer to overturn, or it may just cause Tire 2 to raise from the pavement, causing a small increase in the angle ϕ_A, and thus transferring additional load from Tire 3 to Tire 4. The calculation procedure is analogous to that for the side slope situation. Equation 34 is modified by replacing ΔF with $\Delta F + \delta F$, ϕ_A with $\phi_A + \delta\phi_A$ from Equation 13 or 15, and h_i with $h_i + \delta F$ from Equation 26. The final solution is found as the maximum possible value of A/g corresponding to a value of δF in the range $0 \leq \delta F \leq F_3$.

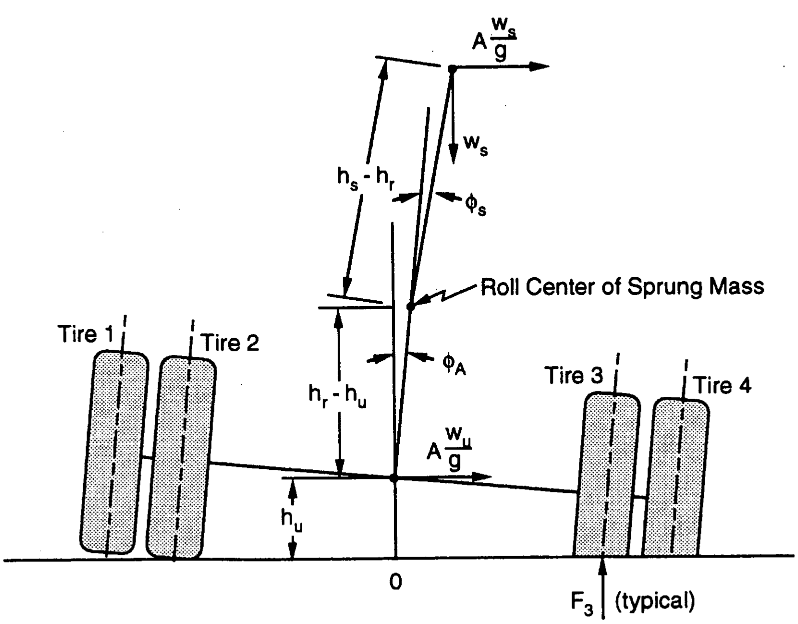

FIGURE 5 Dual wheel axle subject to cornering acceleration.

Wide-Base Tires Subject to Cornering Acceleration

Figure 5 is applicable for wide-base tires subject to cornering acceleration except that there are only two tires, as shown in Figure 3. Equation 30, which gives ϕ_s as a function of A/g for dual tires, is applicable for wide-base singles as well. Denoting the load shift from Tire 1 to Tire 2 as ΔF, the acceleration as a function of the load shift is given by

$$\frac{A}{g} = \frac{T(\Delta F) - W_s \sin\phi_A\left[(h_s - h_u) + \frac{2\pi}{360}(h_s - h_r)C_1\right]}{\frac{2\pi}{360} W_s(h_s - h_r)C_1 + (W_u h_u + W_s h_s)} \tag{35}$$

NUMERICAL RESULTS

Dual Wheels on a Side Slope with an Empty Trailer

The general aim of the calculations is to determine the side slope angle θ for which the trailer is in equilibrium with Tire 2 unloaded but barely touching the pavement. Equilibrium is the condition when the net moment is zero. Having established the equilibrium condition, it is then necessary to determine if that condition is stable. If so, the calculations continue with larger side slope angles until enough load is transferred from Tire 3 to Tire 4 to render the trailer unstable.

The calculations must be conducted iteratively. The general approach is to choose a trial value of θ. Then, assuming that Tire 3 is loaded in the linear range, calculate ΔF by using Equation 9. Next, compare the normal load on Tire 3 with $F_R/3$ by using Equation 10. If the load is insufficient to place Tire 3 in the linear range, recompute ΔF by using Equation 11. Now, calculate ϕ_A from Equation 6, ϕ_s from Equation 23, and M from Equation 24. If M is positive, the trailer is stable and Tire 2 has not yet become unloaded, so a larger value of θ can be tried. If M is negative, Tire 2 has lifted from the pavement, so a smaller θ should be tried. This process is repeated until a result of M equal to 0.0 is found.

To start, recall that a first approximation can be found by assuming that the trailer and tires are rigid by using Equation 20. By using the data in Table 1, the solution is θ equal to 37.78 degrees. This is an upper bound on the side slope that will produce rollover. Suppose the iteration is started at 35 degrees. From Equation 9 it can be found that the trial value of ΔF is 648 kg (1,427 lb). Equation 7 indicates that the normal component of the load on Tire 3 is only 167 kg (367 lb), which is substantially less than $F_R/3$. Thus, the tire is not in the linear range, so ΔF is found to be 155 kg (342 lb) by solving Equation 11.

The loads on Tires 3 and 4 can then be found from Equations 7 and 8, respectively, as 570 and 824 kg (1,256 and 1,816 lb). Thus, Tire 3 is in the nonlinear range and Tire 4 is in the linear range. By using Equations 3 and 2, for Tires 3 and 4, respectively, their deflections are determined to be 18.4 and 22.4 mm (0.724 and 0.883 in.), respectively. The axle roll angle is then calculated from Equation 6 as 0.703 degrees. Next, the sprung mass roll angle ϕ_s is determined from Equation 23 as 0.827 degrees. Finally, the moment is calculated from Equation 25 to be −73 260 mm-kg (−6,353 in.-lb). This is negative, indicating that Tire 2 has lifted and the trailer will roll over on this side slope, so a smaller slope must be tried.

Table 2 shows the results of iterative calculations. At 34 degrees it will also roll over, but at 33 degrees the moment is positive, indicating that Tire 2 will not lift. For small changes in the side slope angle, the moment is nearly linear with the side slope angle. A simple interpolation between 33 and 34 degrees indicates that an angle of 33.549 degrees should be close. Table 2 bears this out by showing that the moment for an angle of 33.549 degrees is approximately zero.

Examination of this equilibrium status by using Equations 15, 26, 27, and 28 indicates that the trailer is in stable equilibrium; it will not roll over on this side slope. Therefore, larger side slope angles are examined by allowing load to be transferred to Tire 4. Carrying out further iterations by using Equations 23 and 25, but replacing h_u with δh in the latter, ultimately leads to the solution that θ is equal to 34.16 degrees. This is the angle at which all load is transferred to Tire 4, the equilibrium becomes unstable, and the trailer will roll over.

Dual Wheels on a Side Slope with a Loaded Trailer

By using the data from Table 1 for a loaded trailer, Equation 20 provides a starting estimate of θ equal to 29.95 degrees. Proceeding as before, a trial value of ΔF of 555 kg (1,223 lb) is found for θ equal to 23 degrees. Equation 10 confirms that Tire 3 is in the linear range (as is Tire 4), so the tire forces given by Equations 7 and 8 can be used with Equation 2 to find the tire deflections. Proceeding as with the unloaded trailer case, the moment M is −813 mm-kg (−9,373 in.-lb) for this side slope. Proceeding iteratively, a solution of 22.67 degrees is found for θ, which produces a negligible moment, and it can be shown to be an unstable equilibrium condition.

Wide-Base Tires on a Side Slope with an Empty Trailer

Examination of Equation 10 shows that for the empty trailer the tires are both in the nonlinear range, even if θ is large enough to transfer the entire weight to Tire 2. Therefore, Equation 17 is used to calculate ϕ_A, where ΔF is taken as $W/2$, corresponding to transferring all of the load from Tire 1 to Tire 2. Then, Equation 23 is used to find ϕ_s and Equation 29 is used to find M.

Proceeding in this fashion, iteration yields a solution of θ equal to 34.34 degrees as the equilibrium solution with M nearly equal to 0. This is a position of unstable equilibrium, as noted earlier.

TABLE 2 Calculations for Empty Trailer with Dual Tires

θ (°)	ΔF (kg)[a]	ϕ_A (°)	ϕ_s (°)	M (mm-kg)[a]
35	155.1	0.7028	0.8271	-551
34	157.2	0.7069	0.8072	-171
33	159.1	0.7109	0.7870	208
33.549	158.1	0.7087	0.7981	0

[a] 1 mm = 0.03937 in., 1 kg = 2.203 lbs.

Wide-Base Tires on a Side Slope with a Loaded Trailer

In the case of wide-base tires on a side slope with a loaded trailer, Tire 2 will be in the linear range but Tire 1 will be in the nonlinear range for the conditions of interest; that is, all or nearly all of the load is shifted to Tire 2. The calculations are conducted just as for the case describe in the previous section, except that Equation 18 instead of Equation 17 is used to calculate ϕ_A. The final solution is that unstable equilibrium will occur at θ equal to 23.95 degrees.

Wide-Base Tires on a Side Slope with a Loaded and Improved Trailer

Several factors combine to give tanker trucks with wide-base tires a potentially greater resistance to rollover. If the truck is fitted with axles with increased track width, the wider stance increases stability. In fact, these analyses already have assumed that this increased track width was used. When using wide-base tires on an axle designed for them it is also possible to widen the spring mount width, which also increases roll stability. Finally, with the wider wheel spacing it is possible to lower the tank on the frame; the lower center of gravity causes an additional improvement in roll stability. The one negative factor is that the wide-base tires have a lower vertical stiffness than a set of dual tires because a wide-base tire has only two side walls, compared with four side walls for a set of duals; this somewhat reduces their resistance to rollover.

Calculations were carried out to incorporate three types of trailer design improvements. The three improvements consist of widening the spring spacing on the frame by 15 percent, lowering the center of gravity of the tank and its contents by 150 mm (6 in.), and decreasing the moment arm from the center of gravity to the trailer's roll center (the point in space about which the trailer rotates) by raising the height of the roll center by 150 mm (6 in.) without changing the center of gravity. The latter difference is typical of what is experienced in the industry; the 15 percent increase in the spring spacing is illustrative, but an even greater increase is possible.

Changing K_r and h_s to reflect these improvements and carrying out the calculations in the same way as was done earlier, a solution of θ equal to 26.72 degree is found for the case of a loaded trailer on a side slope. This represents an angle that is nearly 12 percent greater than that associated with the loaded trailer without these improvements.

Cornering with Dual Tires on an Empty Trailer

For cornering with dual tires on an empty trailer and in other situations described later, the goal is to find the largest cornering acceleration possible that still provides an equilibrium situation. This maximum acceleration corresponds to the equilibrium being unstable, and the trailer will roll over. There is no superelevation, so θ is taken as 0 degrees. The case in which Tire 2 becomes unloaded but remains in contact with the pavement is examined first.

First, Equation 11 is used to determine ΔF, assuming that Tire 3 is in the nonlinear range, which it is. The value of ΔF is found to be 181 kg (398 lb). The value of ϕ_A is found from Equation 6, after calculating the tire deflections, Z_3 and Z_4 from Equations 3 and 2, respectively. Equation 34 is used to find A/g, and Equation 30 can then be used to find ϕ_s, if desired. The value of A/g, the rollover threshold for lateral acceleration, is found to be 0.6638. These results and others are given in the first row of Table 3.

The remainder of Table 3 provides the results of assuming that additional load can be shifted from Tire 3 to Tire 4. [The total amount of the load shift cannot exceed 851 kg (1,875 lb), half of the total weight on the axle.] The added load shift is designated by δF, the added axle roll is calculated with Equation 15, and δh is found from Equation 26. The modified Equation 34 is used for A/g. As shown in Table 3, increasing the load shift from Tire 3 to Tire 4 increases the cornering rollover threshold slightly, but A/g reaches a maximum of 0.6715 with a load shift of about 545 kg (1,200 lb). However, the difference between this acceleration and that at which Tire 2 begins to leave the pavement (0.6638) is not of great practical interest.

TABLE 3 Cornering Calculations for Empty Trailer with Dual Tires

$\Delta F + \delta F$ kg[a]	$\phi_A + \delta\phi_A$	δh mm[a]	A/g
181	0.7757	0	0.6638
454	2.3689	25.4	0.6712
545	3.0122	35.7	0.6715
635	3.7126	46.9	0.6708

[a] 1 mm = 0.03937 in., 1 kg = 2.203 lbs.

Cornering with Dual Tires on a Loaded Trailer

For cornering with dual tires on a loaded trailer Tires 3 and 4 can be shown to be in the linear range when Tire 2 is unloaded but barely touching the pavement. By using the appropriate equations, Equations 9, 2, 6, and 34, the acceleration producing an equilibrium condition is A/g equal to 0.4083. As before the effect of additional load transfer from Tire 3 to Tire 4 can be examined; in this case A/g does not increase, so this equilibrium condition is unstable and the trailer will roll over with this cornering acceleration.

Cornering with Wide-Base Tires on an Empty Trailer

As the entire load on Tire 1 is shifted to Tire 2, Tire 2 is in the linear range but Tire 1 is in the nonlinear range. Equation 18 is used for ϕ_A and Equation 35 is used for A/g. The solution is A/g equal to 0.6806.

Cornering with Wide-Base Tires on a Loaded Trailer

The solution for cornering with wide-base tires on a loaded trailer is found in the same way as the previous case. The solution is A/g equal to 0.4326, with the entire load on Tire 1 shifted to Tire 2.

TABLE 4 Effects of Improvements to Trailer with Wide-Base Tires

Improvement	A/g	$\delta A/g$
None	0.4326	—
K_r	0.4421	0.0095
h_s-h_r	0.4480	0.0154
h_s	0.4661	0.0335
Sum	—	0.0585
All	0.4923	0.0598

Dash (—) indicates data not applicable.

TABLE 5 Summary of Results from Trailer Rollover Analyses

Tire type	Loading	Design improvements	Critical side slope (degree)	Critical cornering accel. (g's)
Dual tires	Empty	None	34.16	0.672
Wide base	Empty	None	34.34	0.681
Dual tires	Loaded	None	22.67	0.408
Wide base	Loaded	None	23.95	0.433
Wide base	Loaded	Increased spring spacing width	—	0.442
Wide base	Loaded	Decreased center of gravity	—	0.466
Wide base	Loaded	Decreased moment arm	—	0.448
Wide base	Loaded	All three improvements	26.72	0.492

Dash (—) indicates data not calculated.

Cornering with Wide-Base Tires on a Loaded and Improved Trailer

A number of calculations were carried out with different improvements made to the trailer. These were

1. Widening the spring mount width to increase the sprung weight roll stiffness by 15 percent, to 1 061 000 mm-kg (92,000 in.-lb) per degree;
2. Raising the roll center h_r by 150 mm (6 in.) with no change in h_s, which reduces the moment arm $h_s - h_r$ by a like amount;
3. Lowering the center of gravity of the sprung mass h_s by 150 mm (6 in.) with no other change;
4. Summing the effects of changes 1, 2, and 3; and
5. Implementing changes 1, 2, and 3 simultaneously.

The results are given in Table 4, along with the case with no changes. Making all of the changes together causes a slightly greater improvement in rollover stability than the sum of the individual improvements, indicating that the improvements slightly aid each other. Overall, the rollover threshold with all of the improvements is improved about 13.8 percent over the case with none of these improvements.

Importance of Including Tire Deflections in the Calculations

As indicated early in this paper the analyses could be simplified by making the assumption that the trailer tires are rigid, that the tires do not deflect. One could make this assumption initially and rederive the various equations. Alternatively, one could repeat the (computerized) calculations with values of S, the tire stiffness, asymptotically approaching infinity. That is the approach used in the present study.

For illustration, in the case of a loaded trailer with rigid dual tires, the critical side slope is 23.93 degrees, a 5.6 percent increase over the more accurate calculation. Similarly, use of the assumption of rigid wide-base tires on a fully loaded trailer will increase the critical side slope by 6.1 percent to 25.40 degrees.

CONCLUSIONS

Improved rollover stability is probably the greatest potential safety advantage to the use of wide-base tires. This paper presented the basic methods of analysis of this stability question from the viewpoints of resistance to rolling over while resting on a side slope and of a trailer's ability to undergo a cornering maneuver without rolling over. The results of a series of calculations are presented in Table 5.

When conventional dual tires and axles are replaced with wide-base tires and axles designed for them and no other changes are made to the trailer, stability is improved, but by relatively small amounts. If the trailer is empty, in particular, the increases in the critical side slope and in the critical cornering acceleration are barely perceptible. If the trailer is loaded, however, the improvement in stability that can be realized by switching from dual tires to wide-base tires is more noticeable (about 6 percent), but still small.

If, in addition to switching from dual tires to wide-base tires, the trailer is modified to take advantage of the wider stance of the tires, further stability increases are possible. If all three trailer improvements considered here are incorporated, the critical cornering acceleration can be increased by an additional 13.6 percent over that achievable by replacing wide-base tires with dual tires but with no other changes. Furthermore, the allowable cornering acceleration that is possible is increased by nearly 21 percent over that with dual tires when the dual tires are replaced with wide-base tires and all three trailer improvements are made. The increase in critical side slope is similar (nearly 18 percent) when these same changes are made.

ACKNOWLEDGMENT

The research reported here is a portion of a Midwest Research Institute study sponsored by the Western States Petroleum Association.

REFERENCES

1. Fancher, P. S., and A. Mathews. *A Vehicle Dynamics Handbook for Single-Unit and Articulated Heavy Trucks.* Report DOT HS 807 185. NHTSA, U.S. Department of Transportation, May 1987.
2. Fancher, P. S., et al. *A Factbook of the Mechanical Properties of the Components for Single-Unit and Articulated Heavy Trucks.* Report DOT HS 807 125. NHTSA, U.S. Department of Transportation, Dec. 1986.

Publication of this paper sponsored by Committee on Motor Vehicle Technology.