OCPLOT: PC Program To Generate Operating Characteristic Curves for Statistical Construction Specifications

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The performance of the nation's highway system is inexorably linked to the quality of design and the quality of construction. To control the quality of construction, transportation agencies have developed elaborate quality-assurance programs, most of which employ end-result specifications that rely on statistical sampling and acceptance procedures to ensure that the work is done in accordance with the plans and specifications. Whether the acceptance procedure leads to a simple pass-or-fail decision or an adjustment in contract price, the proper design of such plans is critical to their performance. Poorly conceived plans may be either totally ineffective or impractically severe, and both extremes have been found in published and proposed national standards. To encourage the proper design of plans that are both effective and fair, an interactive PC program has been developed that enables the user to construct operating characteristic curves to analyze the performance of a wide range of acceptance plans. An example is presented to demonstrate the versatility of the program and the ease with which it can be applied.

One of the nation's most valuable assets is the network of roads and bridges linking the suppliers of goods and services with their customers. State transportation agencies, which bear most of the responsibility for maintaining the highway system in good working order, have responded by developing elaborate programs to ensure that adequate quality is achieved and maintained. Most agencies rely on end-result specifications that use statistical sampling and acceptance procedures to make sure that the work is done in accordance with the plans and specifications. The acceptance tests are performed on random samples taken at either the job site or the supplier's plant. The acceptance procedure may lead to a simple pass-or-fail decision or it may lead to an adjustment of contract price. Whichever method is used, the proper design of such plans is critical to their performance. Poorly conceived plans may be totally ineffective or impractically severe, and both extremes have been found in published national standards (*1*).

OPERATING CHARACTERISTIC CURVES

Although it is not yet used widely in the highway field, there exists a well-established analytical procedure to check that an acceptance procedure will be both fair and effective. The procedure consists of constructing the operating characteristic (OC) curve (*1*, Part 3, Item 6), a graphical representation of the discriminating power of the acceptance procedure.

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Even though the acceptance procedure or pay equation spells out precisely the decision to be made for any level of measured quality, there is always some degree of uncertainty in the quality measurement itself. This uncertainty occurs because only a small fraction of each lot is sampled and tested and the test procedures themselves are not perfectly repeatable. The OC curve, if constructed properly, is capable of accounting for this uncertainty.

A conventional OC curve is shown in Figure 1. The probability of acceptance is indicated on the *y*-axis for the range of quality levels indicated schematically on the *x*-axis. The contractor's risks of having good—acceptable quality level (AQL)—work rejected and the agency's risk of accepting poor—rejectable quality level (RQL)—work are both illustrated in this figure.

Figure 2 presents an OC curve constructed for a statistical specification with an adjusted pay schedule. Quality levels are indicated on the *x*-axis in the usual manner but, instead of probability of acceptance, the *y*-axis gives the expected pay factor.

Although the risks have a slightly different interpretation in Figure 2, essentially the same information is provided. In this example, AQL work receives an expected pay factor of 100 percent, as desired, whereas RQL work receives an expected pay factor of 70 percent. Presumably, the specification is based on a quantitative performance model (*2*) that has enabled the highway agency to estimate the amount of payment to be withheld to cover the anticipated cost of future repairs (*1*, Part 3, Item 10). It can also be seen in this figure that truly superior quality may receive a bonus pay factor up to 102 percent.

The opportunity to earn at least some degree of bonus payment is necessary in order for a statistical acceptance procedure to pay an average of 100 percent when the work is exactly at the AQL. Because of the inherent variability of any sampling and testing process, some samples will underestimate the quality while others will overestimate it. Unless the acceptance procedure is designed to allow bonuses and reductions to balance out in a natural way, the average pay factor will be biased downward at the AQL and acceptable work may be penalized unfairly. The failure to award an average pay factor of 100 percent at the AQL, even by only 1 or 2 percent, can result in many thousands of dollars of unwarranted pay reductions throughout the course of a construction season.

PROBLEMS WITH OVERLOOKING OC CURVES

The following two examples are taken from national standards before their recent correction. They illustrate the two extremes—unduly lenient and unduly severe—that can occur when acceptance procedures are based on faulty premises and are not subsequently checked by constructing the OC curves.

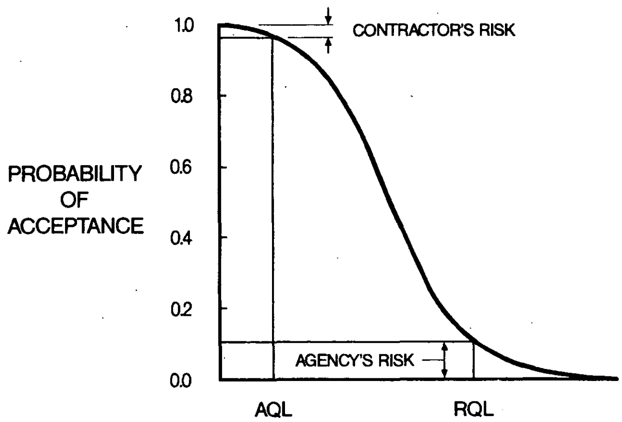

FIGURE 1 Conventional OC curve.

The first example is taken from a generic acceptance procedure that contains a number of desirable features. It uses the statistical measure of percent within limits (PWL) to account for both mean level and variability. [Percent defective (PD) is equally suitable]. The PWL estimate is computed by the standard deviation method which, because it is more statistically efficient, requires smaller sample sizes than plans based on the range (R). The procedure also includes a bonus provision, an essential feature if plans of this type are to operate fairly. And although it does not use a pay equation, which avoids potential disputes over test precision because of the smooth progression of payment as the quality varies, this procedure is nearly as effective because it uses a pay schedule with many small steps.

Despite these advantages, however, this procedure had one major shortcoming: it paid an average of nearly 104 percent for work that just met the AQL. In fact, it was so lenient that it paid an average of 100 percent for quality that was substantially below the AQL. Table 1 presents the OC curve for this plan in tabular form for a typical sample size of $N = 5$.

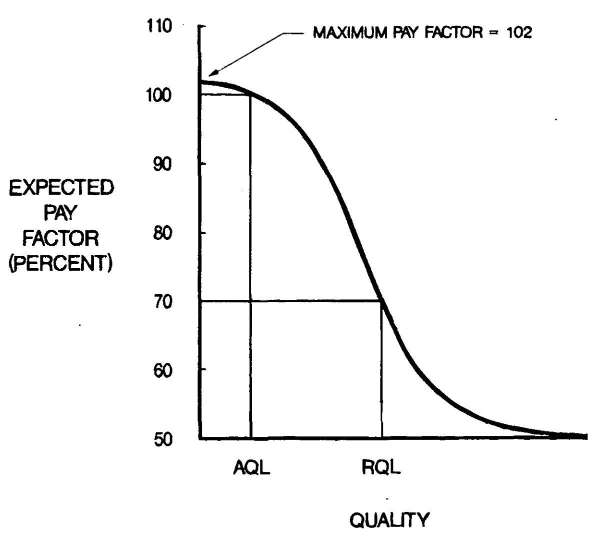

FIGURE 2 Typical OC curve for statistical acceptance procedure with adjusted pay schedule.

TABLE 1 OC Values of Lenient Acceptance Plan

PERCENT WITHIN LIMITS (PWL)	AVERAGE PAY FACTOR (PERCENT)
100	105.0
95 (AQL)	103.7
90	102.2
85	100.5
80	97.8
75	94.3
70	89.9

It is unlikely that any agency would want to use an acceptance plan that provides this degree of overpayment at the AQL. This example illustrates in a dramatic way the value of constructing the OC curve as part of the specification development process so that problems of this nature can be detected and corrected before implementation.

Although it is not known that this was the case, it is possible that this problem was the result of a common misconception about risk analysis as it applies to adjusted payment procedures. For pass-or-fail acceptance plans that produce OC curves of the type shown in Figure 1, the contractor's risk may typically be about 0.01 to 0.05. In other words, there is a 0.01 to 0.05 probability that work that is truly acceptable will be rejected. If the developer of a pay adjustment plan with a bonus provision were to attempt to control the risk of obtaining a pay factor of less than 100 percent at the AQL at a similarly low level, the vast majority of pay factors for acceptable work would exceed 100 percent and the overall average pay factor would be well above 100 percent, as happened in this example. For the acceptance procedure to perform properly, the risk of a pay reduction at the AQL must be approximately 0.50 so that, over an extended period of time, the pay factors for AQL work are split about evenly between bonuses and reductions.

The second problematical example demonstrates how unduly severe an acceptance plan can be if it is not designed properly. This example also includes most of the desirable features in that it uses the standard deviation method to estimate the PWL and a pay equation to compute the lot pay adjustment. However, the maximum pay factor was limited at 100 percent and, because this eliminated the opportunity to receive bonus payments, the procedure was not capable of paying an overall average of 100 percent when the work was precisely at the AQL.

Equation 1 is the basic form of the pay equation that was used. Because the specified AQL for this example is a PWL value of 90, this pay equation can also be expressed in the forms given by Equations 2 and 3, either of which is suitable for analysis with program OCPLOT.

$$\text{PAY REDUCTION} = \text{PWL}_{\text{specified}} - \text{PWL}_{\text{computed}} \quad (1)$$

$$\text{PF} = 10 + \text{PWL} \qquad (\text{maximum} = 100) \quad (2)$$

$$\text{PF} = 110 - \text{PD} \qquad (\text{maximum} = 100) \quad (3)$$

where PF equals the pay factor.

The actual input and output stages with program OCPLOT will be shown later. For now, just the tabular form of the OC curve is presented in Table 2, computed for a typical sample size of $N = 5$.

TABLE 2 OC Values of Severe Acceptance Plan

PERCENT WITHIN LIMITS (PWL)	AVERAGE PAY FACTOR (PERCENT)
100	100.0
90 (AQL)	95.4
80	87.8
70	78.4
60	68.9
50	59.9

Table 2 indicates that a contractor who performs consistently at the AQL will receive an average pay reduction of nearly 5 percent. To emphasize the impact that this would have on the construction industry, this means that a contractor responsible for $10 million worth of pay adjustment work over the course of a construction season would be penalized $500,000 for successfully providing the level of quality that was defined as acceptable in the contract documents. This obviously is misleading and unfair.

Two simple steps will correct this problem. The first is to include a bonus provision as part of the acceptance procedure. In Equations 2 and 3, for example, this would mean removing the limitation that the maximum pay factor cannot exceed 100 percent. The magnitude of the maximum pay factor and the slope of the pay equation should be consistent with established (or estimated) performance relationships and the anticipated economic consequences of any departures—increases or decreases—from the specified AQL. The second step is to construct the OC curve to make sure that the resultant acceptance plan is neither too lenient nor too severe, as was the case for these two examples.

FEATURES AND CAPABILITIES OF OCPLOT

The type of acceptance plan represented by Equations 1–3 is one of literally hundreds that are capable of analysis with OCPLOT. Figure 3 lists some of the options that may be selected and the versatility of the program is apparent from the many ways in which these selections might be combined.

The programming is done in Microsoft QuickBASIC. It is highly structured and modular—consisting of three primary analytical modules, four auxiliary modules, and more than two dozen subroutines—and requires somewhat less than 1 megabyte of disk space. The program and its support modules may be loaded onto the hard drive or run from a diskette from the drive in which the diskette is placed. When the name OCPLOT is entered, preliminary screens identify the program as part of FHWA Demonstration Project 89 on Quality Management (*3*) and provide basic operational information.

```
ACCEPTANCE METHOD
        Pass/Fail ------------------------------------------ TYPE OF PLAN
        Pay Adjustment                                          Attributes
                                                                Variables

QUALITY MEASURE
        Percent Defective (PD)
        Percent Within Limits (PWL)

LIMIT TYPE
        Single-Sided
        Double-Sided

PAY EQUATION TYPE
        Linear/Nonlinear ----------------------------------- Enter Values

MAXIMUM PAY FACTOR
        Yes ------------------------------------------------ Enter Value
        No

ACCEPTABLE QUALITY LEVEL (AQL)
        Enter Value

REJECTABLE QUALITY LEVEL (RQL)
        Enter Value

RQL PROVISION
        Yes ------------------------------------------------ Enter RQL Pay Factor
        No

RETEST PROVISION
        Yes ------------------------------------------------ INITIAL TESTS
        No                                                      Combined
                                                                Discarded

SAMPLE SIZE
        Enter Value(s)
```

FIGURE 3 Available selections in OCPLOT.

Once this is complete, the first menu appears on the screen. Figure 4 shows this menu as it appears after all the selections have been made to analyze the acceptance procedure in the form represented by Equation 2. The various items appear on the menu one at a time in a logical sequence, and later items are dependent on the responses to earlier ones. For example, if a pass-or-fail type of acceptance method had been selected in response to the first query, a different set of subsequent queries would have followed. Besides the many combinations of features that can be accommodated, there is considerable latitude in selecting the values of specific parameters for any particular acceptance plan.

To the extent possible, an attempt has been made to include various checks in the programming to anticipate and avoid a variety of potential problems. For cases in which it is possible to know in advance that certain input values are improper, appropriate parts of the keyboard have been inactivated. In other cases, the program performs many internal checks to guard against the entry of inappropriate values. Depending on the degree of inappropriateness, two different responses may be displayed on the screen: a CAUTION message, color-coded yellow, that allows the user the option of either continuing or reentering a different value, and a WARNING message, color-coded red, that requires the user to enter a different value.

For example, the key with the minus sign is inactivated when the pay equation coefficients are selected and, when the input requires a choice among three menu items, only the keys representing the numerals 1–3 are active (except for <PrintScreen>, <ESC>, and <END>). If the user were to enter an RQL value that is unusually close to the AQL value, a yellow CAUTION message would appear and the user could enter either <ESC> to go back to select a different RQL value or any other key except <END> or <PrintScreen> to continue with the current selection. If the user

```
ENTER THE FOLLOWING INFORMATION

ACCEPTANCE METHOD                   ACCEPTABLE QUALITY LEVEL
   Pay Adjustment                      PWL = 90

QUALITY MEASURE                     REJECTABLE QUALITY LEVEL
   Percent Within Limits               PWL = 50

LIMIT TYPE                          RQL PROVISION
   Single-Sided                        None

PAY EQUATION                        RETEST PROVISION
   PF = 10 + 1 PWL                     None

MAXIMUM PAY FACTOR                  SAMPLE SIZE
   PF = 100                            5

             Press any key to continue

<ESC> = Back                              <END> = Exit
```

```
SELECT LEVEL OF PRECISION

(1)  Low -- Faster Execution

(2)  Intermediate

(3)  High -- Slower Execution

        SELECTION ■

<ESC> = Back        <END> = Exit
```

FIGURE 4 First (*top*) and second (*bottom*) menus for OCPLOT.

were to enter the same value for the AQL and the RQL, a red WARNING message would appear and, when any key other than <END> or <PrintScreen> is pressed, the cursor would move back for another RQL selection. If the pay equation coefficients were chosen such that the pay factor could become negative at any point, the program would run, but a CAUTION message would appear, stating that all negative pay factors will be set equal to 0.

Once the entries in the first menu are complete, the user may elect to use the <PrintScreen> key to obtain a copy of the input selections. Striking almost any other key will cause the second menu in Figure 4 to appear.

Because the program uses computer simulation to analyze whatever acceptance procedure is specified, it is very computationally intensive. Selection 1 provides the fastest execution, which is useful for exploratory work but may not be good enough for a final result. When this level is selected, 200 sample sets of the desired size are generated randomly from a normal population for each of several known levels of quality. This process is far more thorough and many times faster than testing the acceptance procedure with actual field data. Each sample set is evaluated in accordance with the acceptance plan specified in the primary menu in Figure 4, and the results are stored in memory. This function provides the data base with which the acceptance procedure is analyzed.

Selection 2 provides an intermediate level of precision for which 1,000 sample sets are generated at each quality level. This level of precision is usually satisfactory to report as a final result, producing points on the OC curve representing either probability of acceptance or expected pay factor that are typically accurate to within about 1 or 2 percent. If still better precision is required, Selection 3 will cause 5,000 sample sets to be generated at each quality level. This level of precision tends to produce a very smooth line when the OC curve is plotted.

Once the precision level is selected from the second menu, the computational process begins. For either low or intermediate precision, OCPLOT displays detailed information at the two key points at which risk levels are usually expressed—the AQL and RQL—as shown in Figures 5 and 6. This display serves two important purposes. For users less familiar with statistical estimation procedures and acceptance plans, the graphical displays at the AQL and RQL are both informative and educational. It may come as a surprise to some, for example, how widely distributed the quality estimates are, especially for small sample sizes. For users more familiar with statistical acceptance procedures, these displays provide assurance that the simulation process is working properly. The actual displays on a color monitor are color-coded to clearly distinguish acceptable and rejectable test results and the corresponding pay factors.

It can be seen in Figure 5, for example, why the absence of a bonus provision in the pay equation causes the average pay factor to be well below 100 percent at the AQL. The population from which these data were generated is precisely at the AQL of PWL = 90. For the sample size of $N = 5$ and analysis at an intermediate level of precision, the 1,000 estimates of lot quality range from a minimum of about PWL = 48 to the maximum possible value of PWL = 100. It is predicted theoretically, and can be demonstrated empirically, that the average of the PWL estimates in the upper histogram in Figure 5 will be very close to the true value of PWL = 90 because the PD/PWL measurement process is an unbiased statistical estimation procedure. In the lower histogram in this figure, the corresponding pay factors range from a minimum of about 58 percent to the maximum of 100 percent that is permitted with this acceptance plan. As a result, the average pay factor is only 95.4 percent, even though all the samples were drawn from a population that was exactly at the level of quality that was defined as acceptable.

It can be seen in Figure 6 that when the true quality level is at the RQL of PWL = 50, the PWL estimates for a sample size of $N = 5$ cover the complete range from 0 to 100 percent, with the majority falling between about 20 and 80 percent. The average pay factor at the RQL is 59.9 percent, which may be appropriate, depending on the degree of economic loss that the agency believes it incurs when RQL work is accepted.

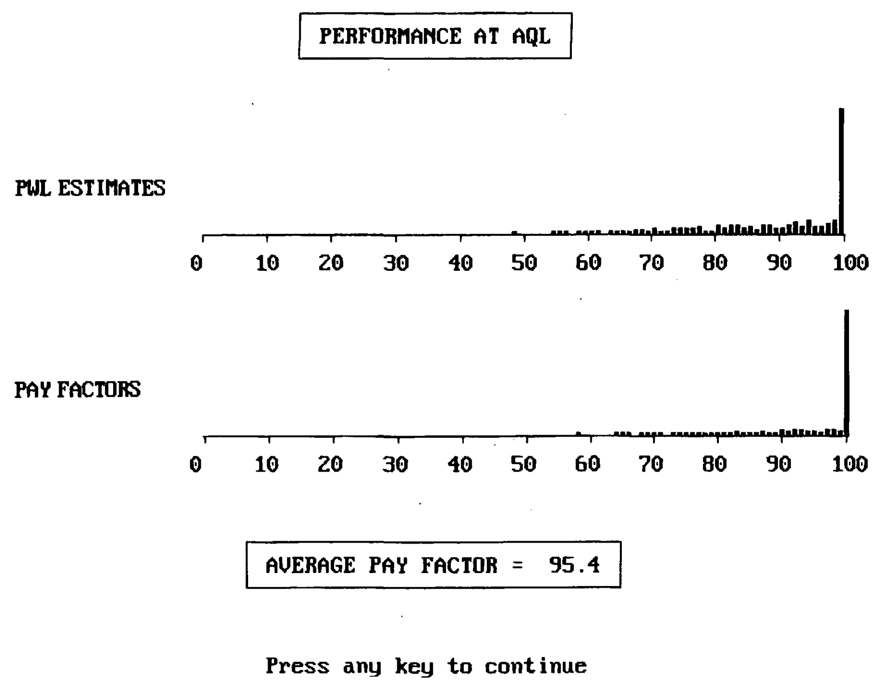

FIGURE 5 Display at AQL resulting from input shown in Figure 4.

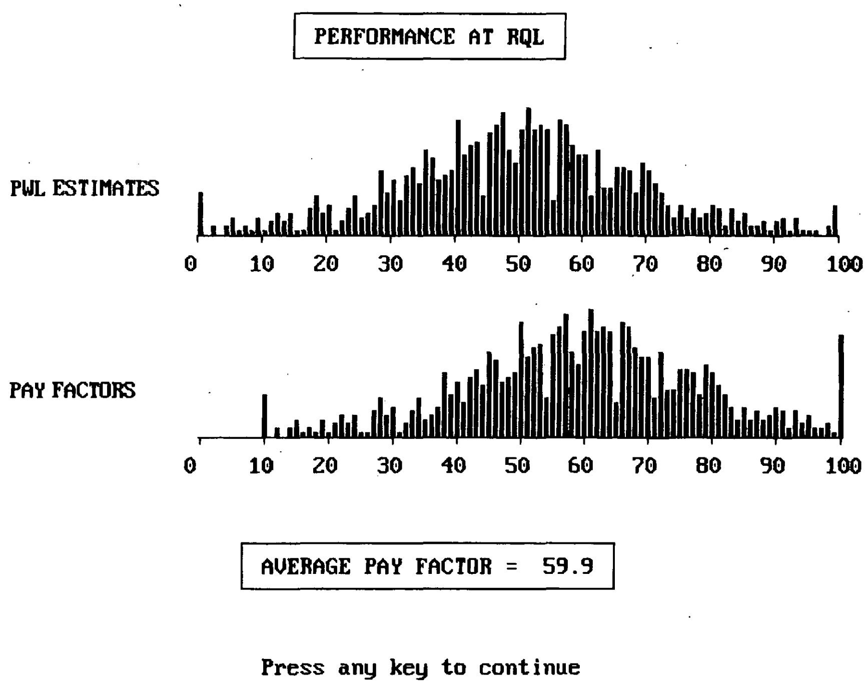

FIGURE 6 Display at RQL resulting from input shown in Figure 4.

Printouts of either Figure 5 or Figure 6 may be obtained provided the user's system has graphics capability, a commonly included feature with recent versions of DOS. A command similar to GRAPHICS [PRINTER TYPE] must be entered before running OCPLOT in order to obtain a printout using the <PrintScreen> key. A DOS manual should be consulted to obtain the appropriate syntax for the particular printer being used.

Although the AQL and the RQL are probably the most important points at which it is desired to know how the acceptance procedure will perform, it usually is useful to have a plot of the entire OC curve that provides a picture of the performance over the complete range of quality that might be encountered. The prompt at the bottom of the screen in Figure 6 instructs the user to strike any key to continue with this step, as shown in Figure 7. The *x-y* axes and the two previously calculated points at the AQL and RQL appear on the screen immediately. The remaining points appear one at a time at a speed determined by the level of precision that has been selected and the speed of the machine on which the program is being run.

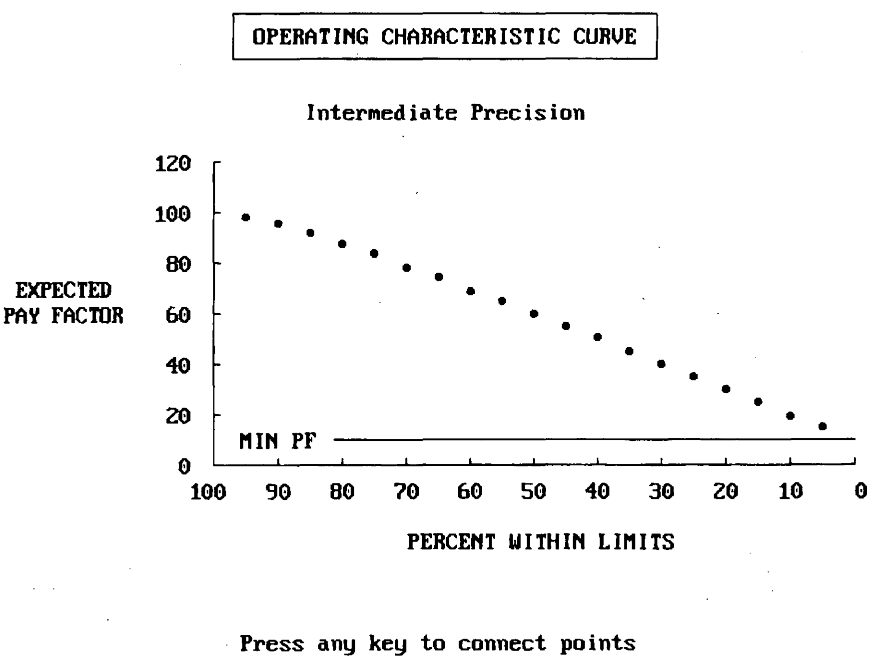

FIGURE 7 Points on OC curve plotted by OCPLOT.

For a 386 machine with a math coprocessor, this may require 1 or 2 min at low precision and 3 or 4 min at intermediate precision. With a 486 or faster machine, there is considerably less delay.

After all the points have been calculated and plotted, the user may strike any key to connect the points with a solid line. The next key stroke will add vertical and horizontal lines highlighting the performance of the acceptance plan at the AQL and RQL, as shown in Figure 8. And, like the histograms in Figures 5 and 6, any of these displays may be printed with the <PrintScreen> key, provided that graphics capability is present.

Following this display, striking a key will produce the menu shown in Figure 9. If the first item in this menu is selected, the output shown in Figure 10 is displayed. This feature permits the user to print out the values of the data points shown in Figure 7 from which the OC curve was constructed. The other selections in this menu make it possible to return to earlier points in the input stage of the program or to exit.

SOLUTION TO FAIRNESS PROBLEM

To demonstrate that the problem of paying less than 100 percent at the AQL can be corrected by allowing the pay equation to award bonus pay factors, another run was made with OCPLOT using the same input shown in Figure 4 except that no restriction was placed on the maximum pay factor.

Ordinarily, the maximum pay factor and the slope of the pay equation should be consistent with established (or estimated) per-

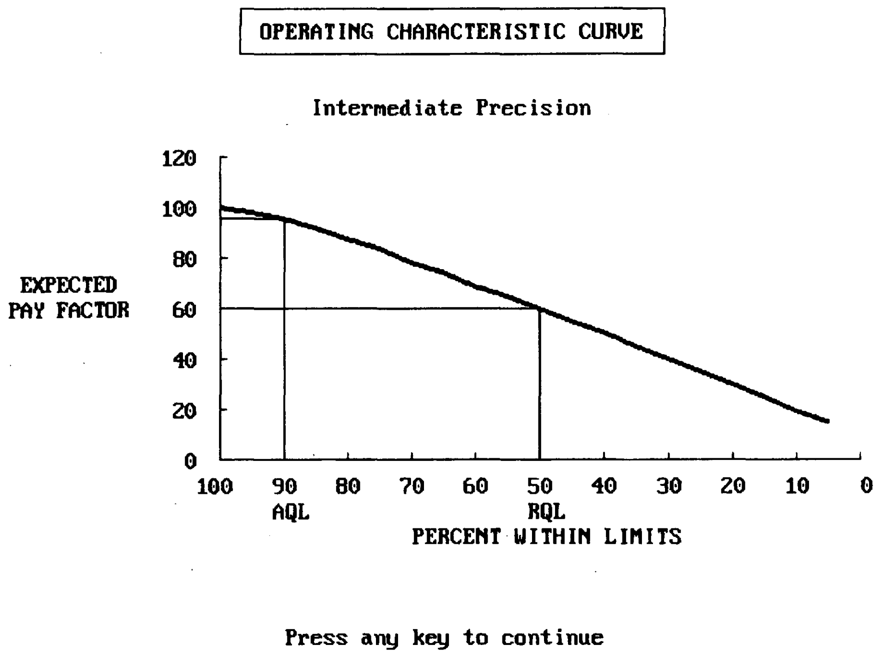

FIGURE 8 Display of OC curve with AQL and RQL performance highlighted.

SELECT DESIRED OPTION

(1) Display operating characteristic table

(2) Run again at different precision level

(3) Change some values and run again

(4) Run again with new input data

(5) Exit program

SELECTION ■

<ESC> = Back <END> = Exit

FIGURE 9 Third menu for OCPLOT.

PERCENT WITHIN LIMITS		EXPECTED PAY FACTOR
100.0		100.0
95.0		98.2
90.0	——— AQL ———	95.4
85.0		92.0
80.0		87.8
75.0		84.0
70.0		78.4
65.0		74.6
60.0		68.9
55.0		65.1
50.0	——— RQL ———	59.9
45.0		55.1
40.0		50.7
35.0		44.9
30.0		40.0
25.0		35.3
20.0		30.1
15.0		24.7
10.0		19.6
5.0		15.3

Press any key to continue

FIGURE 10 Display of numerical values of data points on OC curve.

formance relationships and the anticipated economic consequences of receiving quality levels other than the specified AQL. Because the pay equation used for this example is fairly steep, with a slope of 1.0, pay factors as large as 110 percent will be permitted when the restriction on maximum pay factor is removed. If a shallower slope had been used, a correspondingly lower maximum pay factor would have been appropriate.

The result at the AQL is shown in Figure 11. It can be seen that the PWL estimates are distributed almost exactly as they were in Figure 5 but that the pay factor estimates now range up to a maximum of 110 percent. Because of this, an average pay factor of almost exactly 100 percent has been achieved, as desired.

SUMMARY

An essential step in the writing of a statistical construction specification is the development of the OC curve. This is the only way to determine whether the acceptance procedure will distinguish properly between satisfactory and unsatisfactory work and award appropriate levels of payment. Two examples were presented to show that, in the absence of this step, the resulting acceptance plans could be either totally ineffective or impractically severe.

One reason that the construction of OC curves has not been a standard practice is that one of the most appealing measures of highway quality—PD, or its counterpart, PWL—is also one of the most complex to analyze. OCPLOT makes it possible for anyone with a minimal amount of statistical training to analyze a broad range of acceptance procedures of this type and, as such, provides a capability well beyond that previously available (*4*). For the less experienced user, the program provides additional guidance in the form of CAUTION and WARNING messages whenever a questionable entry is made.

This program, along with other quality-assurance software being distributed as part of FHWA Demonstration Project 89 (*3*), puts an enormous amount of analytical power in the hands of specification writers. It is hoped that the availability of this software will encourage a general upgrading of highway construction specifications, many of which may have shortcomings of the type illustrated in this paper, and that it will create a greater awareness of the need to develop acceptance plans that are both effective and fair.

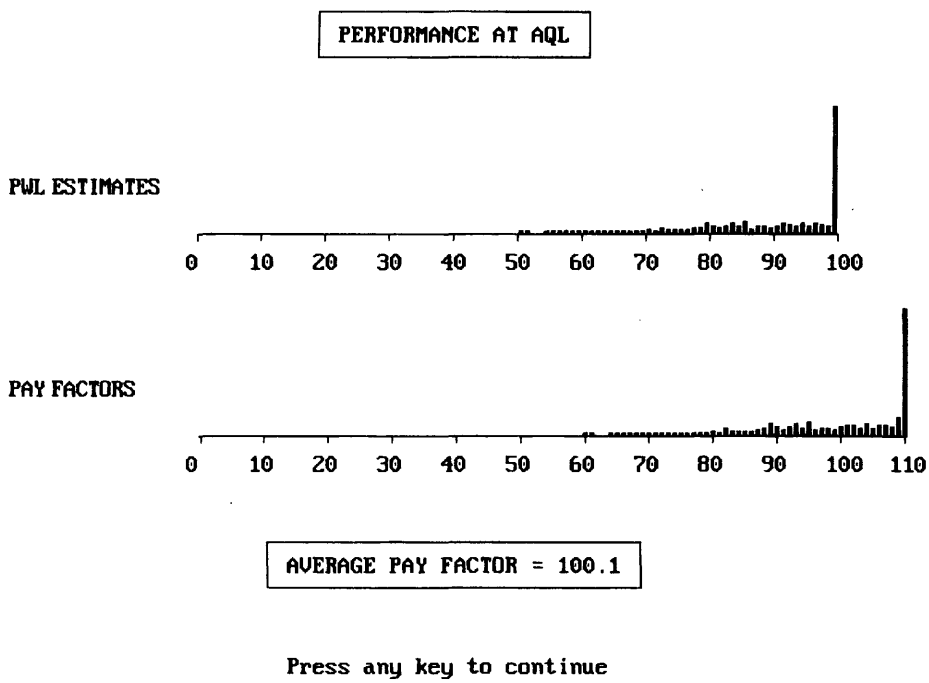

FIGURE 11 Demonstration that bonus provision produces average pay factor of 100 percent at AQL.

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