Travel-Time Uncertainty, Departure Time Choice, and the Cost of Morning Commutes

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Existing models of the commuting time-of-day choice were used to analyze the effect of uncertain travel times. Travel time included a time-varying congestion component and a random element specified by a probability distribution. The results from the uniform and exponential probability distributions were compared and the optimal "head-start" time that the commuter chooses to account for travel time variability, that is, a safety margin that determines the probability of arriving late for work, was derived. The model includes a one-time lateness penalty for arriving late as well as the per-minute penalties for early and late arrival that are included by other investigators. It also generalizes earlier work by accounting for the time variation in the predictable component of congestion, which interacts with uncertainty in interesting ways. A brief numerical analysis of the model reveals that uncertainty can account for a large proportion of the costs of the morning commute.

The choice of home departure time for commuters is an important element in determining how congestion levels will vary during morning peak travel. This choice has been related empirically to the cost of early or late arrival relative to some preferred work arrival time (*1,2*). The planning of on-time arrivals is, however, complicated by the presence of uncertainty in actual travel times.

This paper describes a model in which commuters simultaneously trade off costs of inconvenient schedules, lateness penalties, and the desire to minimize time spent in congested traffic. Like Gaver (*3*) and Polak (*4*), the authors assumed that commuters face a probabilistic distribution of travel times and choose departure time to minimize an expected cost function. In contrast to these authors, the cost function includes a discrete lateness penalty as well as per-minute penalties for both early and late arrival; it also accounts for variation over time in the predictable component of congestion. Furthermore, the optimized expected cost function (i.e., the costs resulting after an optimal departure time is chosen) is derived analytically. This is done for both a uniform and an exponential distribution for uncertain travel time.

The results show how changes in the uncertainty of travel time affect both the departure time decision and the resulting expected costs. For example, as uncertainty increases, commuters shift their departure schedules to earlier hours to compensate for the increased probability of late arrival; in some cases they overcompensate in the sense that the probability of late arrival decreases as uncertainty increases. As for the resulting expected costs, the functional relationship that is derived by relating costs to the underlying parameters of the model is of great interest for empirical studies of traveler behavior under uncertainty (*5–7*). For example, only when lateness penalties are disregarded is that functional relationship linear in the standard deviation of travel time, as is frequently assumed.

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Changes in the level of congestion over the course of the peak period also play an important role in commuter decisions. Rapidly rising congestion shifts the commuter to earlier departure times but also lowers the probability of late arrival. The opposite is true when congestion levels are falling. These types of trade-offs are fully accounted for in the model.

The paper begins with a review of the literature on departure time and route choice, especially previous work dealing with uncertain travel times. The analytical model is then presented and solved. Some numerical examples that provide quantitative information about the possible importance of various components of the model are given. Implications for both research and policy are discussed in the conclusion.

LITERATURE REVIEW

The reliability of arriving at a destination on time is a key component in the decisions made by commuters for their morning trips. Prashker (*8*) attempts to classify some perceived components of reliability into a measurable framework using factor analysis. More recently, researchers have produced direct empirical estimates of how travelers respond to reliability (*5–7*). Much of this work has been aided by the development of stated preference survey techniques.

It is useful to begin with an understanding of how travelers choose departure time choice under certainty. Most research has focused on schedule delay, defined as the difference between the actual time of arrival and some ideal time, usually identified with an official work start time. Typically the commuter is assumed to receive some disutility from schedule delay as well as from travel time (*1,2,9*). In Small's specification (*2*) this disutility is piecewise linear in schedule delay, that is, disutility rises linearly in either the early or late direction. In addition, there is a discrete penalty for being late. In all these studies scheduling disutility is traded off against the possible advantages, caused by variation in congestion over the rush hour, of shifting one's schedule to take advantage of lower congestion. In Cosslett's (*1*) continuous model, this tradeoff appears as a maximization condition involving the slope of the congestion function.

Scheduling models such as these have been incorporated into equilibrium analyses of congestion formation. Basic models for a single link (*10–15*) have been extended to a variety of circumstances including elastic demand (*16,17*); networks (*16,18,19*); heterogeneous commuters, including arbitrary population distributions for desired arrival times (*19,20*); and uncertain capacity or demand (*21,22;* Arnott et al., unpublished data). Small gives a more complete review (*23*). Although most of these analyses use deterministic models of the traveler's choice of departure time, a few (*16,24*) use a discrete-choice model analogous to that of Small (*2*).

Other researchers have incorporated a simpler version of this utility specification into models analyzing uncertain travel times. Gaver (*3*), Polak (*4*), and Bates (*5*) all consider the piecewise linear disutility specification when travel time is uncertain, but none considers congestion that varies over the rush hour. Hence they examine only the trade-offs inherent in trying to minimize the expected disutility from given arrival times given the randomness in travel times.

Mahmassani and associates (*22,25–27*) simulate time-of-day departure choices using hypothetical data collected from actual commuters and fed through a traffic simulation model. These papers focus on day-to-day variations in travel time as commuters gain experience with the system. Although travel times may be uncertain, these simulations emphasize how people learn about the shape of the congestion profile as opposed to uncertainties caused by nonrecurrent events.

Mannering (*28*) and Abdel-Aty et al. (*7*) investigate how likely commuters are to make changes in their departure time or route choices, or both. Mannering finds that those commuters with longer travel times are more likely to make changes and speculates that these trips may have larger variances. His results also indicate that nonrecurrent events may not allow a steady-state equilibrium to evolve, which may have implications for simulating traffic congestion.

Mannering and Hamed (*29*) find empirical evidence that work-to-home departure decisions are influenced by similar factors. Such decisions may not be independent of home-to-work departure decisions: for example, some commuters may delay the morning departure with the intent of staying at work until evening congestion levels have fallen. Neither the model in this paper nor any other one known to the authors attempts to deal with this dependence.

Mahmassani and Herman (*25*) and Mahmassani et al. (*30*) show that commuters tend to adjust departure times more readily than they do routes. In fact, route switches tend to occur when commuters are continually dissatisfied with the outcomes from departure time switches alone (*27,30*). The lower likelihood of route switching adds credibility to models that examine only the choice of departure time, which can have important impacts on the development and timing of peak congestion levels.

ANALYTICAL DERIVATION OF MODEL

A model is described that explains how uncertainty in travel time affects the expected cost of the morning commute. First, the basic components of the cost model, including how changes in congestion levels are accounted for are specified. Then the commuter's scheduling problem is formulated and solved using both a uniform and an exponential probability distribution. The solution is then inserted into the expected cost function to determine how total commuting cost depends on the parameters describing the commuter's travel environment. This cost consists of various components that offer a better understanding of how significant unreliability is as a contribution to travel cost.

Cost Model

The following cost function for the morning commute is assumed:

$$C = \alpha T + \beta\,(\mathrm{SDE}) + \gamma\,(\mathrm{SDL}) + \Theta D_L \qquad (1)$$

where

T = travel time;
SDE, SDL = schedule delay early and late, respectively (defined later),
D_L = 1 when SDL > 0 and 0 otherwise;
α = cost of travel time;
β, γ = costs per minute of arriving early and late, respectively; and
Θ = discrete lateness penalty.

The variables SDE and SDL are defined with respect to the official work start time, t_w, and the home departure time, t_h. Let $\mathrm{SD} \equiv t_h + T - t_w$ be "schedule delay," the difference between actual arrival time and official work start time. Define

$$\mathrm{SDL} = \begin{cases} \mathrm{SD} & \text{if SD} > 0 \\ 0 & \text{otherwise} \end{cases} \qquad (2)$$

$$\mathrm{SDE} = \begin{cases} -\mathrm{SD} & \text{if SD} < 0 \\ 0 & \text{otherwise} \end{cases} \qquad (3)$$

This formulation of costs is that of Small (*2*) Table 2, Model 1. It could result if pay is docked for late arrival, or if in some other way the frequency and magnitude of late arrival are costly to one's career. Many analyses of time-of-day decisions have used the first three terms of Equation 1; others have implicitly added the fourth term with Θ set to infinity by excluding the possibility of late arrivals. A more complex model formulation could also vary the amount of time spent at work and could thus account for evening travel conditions as additional determinants of the morning commute decision.

The total commute time, T, consists of three elements. T_f is the free-flow travel time when there is no congestion. T_x is the extra travel time caused by congestion, which the traveler is sure to encounter; it is a function of t_h, the home departure time. T_r is the extra travel time caused by nonrecurrent congestion and is modeled formally as a random variable. Following the standard classification of congestion delays into recurrent and incident-related delays (*31,32*), T_x is "recurrent delay" and T_r is "incident delay."

For simplicity it is assumed that the probability distribution of T_r is independent of recurrent congestion and of the time of day of travel. This assumption has the advantage that it enables one to isolate the impact of exogenous changes in travel time uncertainty. Although the assumption may appear unrealistic, there is a surprising absence of clear-cut empirical evidence for alternative assumptions. Satterthwaite (*33*), in a review, finds no reported relations between congested traffic and accidents (which are a primary cause of nonrecurrent congestion). Hendrickson et al. (*34*) analyzed data in Pittsburgh and concluded that variance of travel times is independent of departure times. Richardson and Taylor (*35*) posit a relationship between congested traffic and increases in travel time variability but do not derive an explicit relationship.

To simplify the analysis, define the variable T_e to be the amount one would arrive early if there were no incident-related delays:

$$T_e = t_w - t_h - T_f - T_x \qquad (4)$$

As defined by Gaver (*3*), T_e is the "head start" time. Polak's (*4*) "safety margin" is equal to $T_e - E(T_r)$, where $E(T_r)$ denotes the expected incident delay. Note that $T_e > 0$ implies the possibility of early arrival (if recurrent congestion turns out to be nil), whereas $T_e < 0$ implies certain late arrival. Schedule delay can now be written as $SD = T_r - T_e$ and the lateness dummy, D_L, is equal to 1 if $T_r > T_e$ and 0 otherwise.

These definitions enable the cost function to be written as follows:

$$C(T_r) = \alpha[T_f + T_x + T_r] + \beta(1 - D_L)[T_e - T_r] + \gamma D_L[T_r - T_e] + \Theta D_L \quad (5)$$

Two alternative probability distribution functions for T_r are specified. The first uses a uniform distribution, which assumes that the likelihood of a delay is equal for any level in the domain; the second is an exponential distribution, as in Gaver (*3*), which allows lower levels of delay to have a greater likelihood than longer levels of delay. Many authors, including Richardson and Taylor (*35*), have fit log normal curves to travel time variance data; Giuliano (*36*) has found specifically that nonrecurrent congestion follows a log normal distribution. Unfortunately the log normal distribution is found to be intractable in this model, so it is not pursued here.

Changes in Congestion Levels

Before proceeding with the derivation of expected cost functions, it is convenient to describe how congestion levels change with the choice of departure time, t_h. First, it is possible to describe the commuter's choice of departure time by head start time, T_e, instead of departure time, t_h. To do this, one assumes that T_x, the travel time associated with congestion, is a differentiable function of t_h, $T_x(t_h)$. Differentiating the implicit definition $t_h = t_w - T_f - T_x(T_h) - T_e$, one finds that

$$\frac{dt_h}{dT_e} = -\left(\frac{dT_x}{dt_h}\right)\cdot\left(\frac{dt_h}{dT_e}\right) - 1 \quad (6)$$

or, solving

$$\frac{dt_h}{dT_e} = \frac{-1}{(1 + T'_x)} \quad (7)$$

where $T'_x \equiv dT_x/dt_h$. The requirement $T'_x > -1$ is imposed to rule out "overtaking," in which a person can arrive earlier by leaving later (*23,37*). This condition guarantees that Equation 7 is well defined and negative. Using Equation 7, the functional relationship between T_x and T_e, defined by $T_x[t_h(T_e)]$, has total derivative

$$\frac{dT_x}{dT_e} \equiv T'_x \cdot \left(\frac{dt_h}{dT_e}\right) = \frac{-T'_x}{(1 + T'_x)} \equiv -\Delta \quad (8)$$

The quantity Δ is a measure of how steeply congestion increases if departure is delayed; more precisely, Δ is the rate at which congestion increases as the "planned" arrival time, $t_h + T_f + T_x \equiv t_w - T_e$, is made later. It has the same sign as T'_x. If $\Delta > 0$, conditions worsen as planned arrival time is delayed, thus favoring earlier schedules; whereas $\Delta < 0$ favors later schedules. Note that the restriction $T'_x > -1$ implies $\Delta < 1$.

Henceforth T_x is regarded as a function of T_e, with well-defined derivative $-\Delta$. As it turns out, making T_x a function of traffic volume at T_e rather than that at t_h is necessary for consistency in an important equilibrium model of endogenous scheduling choice associated with Henderson (*10,11*); see work by Chu (*37*) for a demonstration. If T_x has a kink so that Δ is undefined, corner solutions in addition to those described below become possible.

It is now possible to solve the model for two alternative probability distributions for T_r. In each case the expected cost given scheduling choice T_e is computed; then the choice of T_e that minimizes the expected cost is computed and this chosen value is inserted into the expected cost equation. The resulting expected cost is then a function solely of those parameters that the commuter faces in choosing the schedule for a morning commute trip.

Uniform Distribution

A uniform probability distribution is defined for the domain $[0, T_m]$. The probability density function is defined as $f(T_r) = 1/T_m$ for $0 \leq T_r \leq T_m$, and 0 otherwise. The mean of T_r is $\frac{1}{2} T_m$, and its standard deviation is $T_m / \sqrt{12}$. The mean and standard deviation for the total travel time are $T_f + T_x + \frac{1}{2} T_m$ and $T_f + T_x + (T_m/\sqrt{12})$, respectively.

The expected cost for the morning commute is

$$\mathrm{EC} = \frac{1}{T_m}\int_0^{T_m} C(T_r)dT_r \quad (9)$$

Substituting Equation 5 into Equation 9, there are three possible cases: (*a*) $0 < T_e < T_m$; (*b*) $T_e \geq T_m$; and (*c*) $T_e \leq 0$. For Case *a*, the chosen departure time can lead to either early or late arrival, depending on the realization of the random variable T_r; Equation 9 becomes

$$\mathrm{EC} = \alpha\left(T_f + T_x + \frac{T_m}{2}\right) + \frac{1}{T_m}\int_0^{T_e} \beta(T_e - T_r)dT_r + \frac{1}{T_m}\int_{T_e}^{T_m}[\gamma(T_r - T_e) + \Theta]\,dT_r \quad (10)$$

$$= \alpha\left[T_f + T_x + \frac{T_m}{2}\right] + \frac{1}{T_m}[\Theta(T_m - T_e)] + \frac{1}{2T_m}[\beta T_e^2 + \gamma(T_m - T_e)^2] \quad (11a)$$

$$= \alpha E(T) + \Theta P_L + \beta E(\mathrm{SDE}) + \gamma E(\mathrm{SDL}) \quad (12)$$

In Equation 11a the first term is merely the expected travel time multiplied by its cost. The second term is the probability of arriving late, P_L, multiplied by the lateness penalty, Θ. The last two terms are the expected cost associated with the amounts of early and late schedule delays.

The other cases result in simple modifications of Equations 10 and 11a. For Case *b*, where $T_e \geq T_m$ (implying the commuter is early with a probability of 1), the limit of integration T_e is replaced by T_m in Equation 10; the result is

$$EC = \alpha\left[T_f + T_x + \frac{T_m}{2}\right] + \beta\left[T_e - \frac{T_m}{2}\right] \quad (11b)$$

For Case *c*, where $T_e \leq 0$ (implying the commuter is late with a probability of 1), then T_e is replaced by 0 as a limit of integration in Equation 10 and the result is as follows:

$$EC = \alpha\left[T_f + T_x + \frac{T_m}{2}\right] + \Theta + \gamma\left[\frac{T_m}{2} - T_e\right] \quad (11c)$$

In Cases *b* and *c* the per-minute scheduling cost is simply that associated with the expected arrival time because there is no uncertainty about whether the commuter will arrive late. Equation 12 continues to apply, with appropriately modified expressions for the probability P_L and for the expectations of SDE and SDL. As will be seen, Cases *b* and *c* can occur when the cost parameters and the rate of change in the level of congestion have specified ranges; for example, if Θ is very large or if congestion is increasing rapidly in departure time, one may choose to always arrive early (Case *b*).

The value of T_e that minimizes the expected cost can now be calculated. For Case *a*, the derivative of Equation 11*a* is set to 0, while regarding T_x as a function of T_e as in Equation 8. Solving for T_e gives the following result:

$$T_e^* = \frac{1}{(\beta + \gamma)} (\Theta + \gamma\, T_m + \alpha\, \Delta^* T_m) \tag{13}$$

where $\Delta^* = -\, dT_x/dT_e$ evaluated at T_e^*. The second-order condition requires that $d\Delta/dT_e < (\beta + \gamma)/(\alpha\, T_m)$, which may be interpreted as requiring that congestion be convex, or at least not too strongly concave, in planned arrival time $(t_w - T_e)$. If T_x is a concave function of $(t_w - T_e)$, then $d^2T_x/dT_e^2 < 0$, that is, $\Delta \equiv -\, dT_x/dT_e$ is increasing in T_e. This solution is valid only if it is consistent with Case *a* as an interior solution, which requires that $0 < T_e^* < T_m$, that is, $-\gamma\, T_m < (\Theta + \alpha\, \Delta^* T_m) < \beta\, T_m$.

To evaluate the expected cost when T_e is chosen optimally, Equation 13 is substituted into Equation 11a, yielding the following:

$$EC^* = \alpha\, E(T^*) + \Theta\, P_L^* + C_S^* \tag{14}$$

where

$$E(T^*) = T_f + T_x[t_h(T_e^*)] + \frac{1}{2}\, T_m \tag{15}$$

$$P_L^* = \frac{T_m - T_e^*}{T_m} = \frac{\left(\beta - \alpha\,\Delta - \dfrac{\Theta}{T_m}\right)}{(\beta + \gamma)} \tag{16}$$

$$C_S^* = \frac{1}{2}\,\delta\, T_m + \frac{(\Theta + \alpha\,\Delta\, T_m)^2}{2(\beta + \gamma)T_m} \tag{17}$$

and

$$\delta = \frac{\beta\gamma}{(\beta + \gamma)} \tag{18}$$

When $\Theta = \Delta = 0$, Equations 14 through 17 are especially easy to interpret. The probability of being late is then chosen independently of travel time variance and is decreasing in γ/β. In addition, the uncertainty of travel time creates a cost $C_S^* = \frac{1}{2}\,\delta\, T_m$ which is proportional to the standard deviation $(T_m/\sqrt{12})$ of travel time and also to the coefficient δ, which is a kind of geometric average of the two schedule delay cost parameters; this cost arises because the commuter is unable to eliminate the likelihood of some schedule delay, either early or late. When $\Theta = \Delta = 0$, the probability of being early is $1 - P_L^* = \gamma / (\beta + \gamma)$ in agreement with Gaver (*3*) Equation 2.3; Polak (*4*) Equation 3.8 (with notation $c_E = \beta$ and $c_L = \gamma$); and Bates (*6*) Equation 17 (with notation $l = \gamma$ and $e - h = \beta$).

The last term in C_S^* may be regarded as the scheduling-cost consequences of shifts in T_e that are made to reduce congestion (if $\Delta \neq 0$) or to reduce the likelihood of a discrete lateness penalty (if $\Theta > 0$). For example, when $\Delta \neq 0$, indicating that some congestion can be avoided by changing the head start, the commuter does so; expected travel time is thereby reduced and C_S^* increased. When $\Theta > 0$, indicating an extra penalty for being late by any amount, T_e^* is increased so as to reduce P_L^*; C_S^* will go up unless a negative Δ was already causing a tendency toward lateness.

Consider now Case *b* of an individual who arrives early with a probability of 1; this occurs if, in Equation 13, $T_e^* > T_m$, that is, if

$$\alpha\Delta^* \geq \beta - \frac{\Theta}{T_m} \tag{19}$$

This case can occur when Θ is high or when congestion is increasing at a rapid rate. In this case, the commuter seeking to minimize cost will choose T_e to minimize Equation 11b. An interior solution occurs when

$$\alpha\Delta = \beta \tag{20}$$

which requires $\Delta > 0$; the second-order condition requires that $d\Delta/dT_e < 0$. Hence the congestion function must have a region where it is a rising convex function of planned arrival time $t_w - T_e$. At Solution 20 the consumer trades off the extra schedule-delay costs of still-earlier arrival (βdT_e) against the saving in travel time cost caused by less congestion ($\alpha\Delta dT_e$); this is the same tradeoff that forms the basis for determination of early-side arrival times in the models of Vickrey (*12*), Cosslett (*1*), Fargier (*13*), Hendrickson and Kocur (*14*), Arnott et al. (*15*), and others. Alternatively, Case *b* may lead to the corner solution $T_e = T_m$. This will occur if Equation 19 is satisfied but Equation 20 cannot be, as could easily happen if Θ/T_m is large. The interpretation here is that the discrete lateness penalty is large enough for the commuter to eliminate entirely the possibility of late arrival, but variation in congestion, Δ, is not large enough to cause a desire for still earlier planned arrivals.

Consider finally Case *c* of an individual who decides to arrive late with a probability of 1, that is, someone who chooses $T_e \leq 0$. This occurs if $T_e^* \leq 0$ in Equation 13, if

$$\alpha\,\Delta^* \leq -\left[\gamma + \frac{\Theta}{T_m}\right] \tag{21}$$

This requires that Δ^* be negative, that is, congestion is decreasing and also that neither γ nor Θ be too large. In such a situation, the commuter chooses to incur the relatively mild lateness penalties to take advantage of lessening congestion. Expected cost (Equation 11c) has a local minimum where

$$\alpha\,\Delta = -\gamma \tag{22}$$

provided again that $d\Delta/dT_e < 0$ (convex congestion function). Again, there could also be a corner solution $T_e = 0$. Note that Equations 21 and 22 are compatible only if Δ changes considerably over the range of possible values of T_e. This could happen if, for example, the interval $[t_w - T_m, t_w]$ occurs near the end of the rush hour, so that Δ^* is strongly negative (representing rapidly falling congestion at T_e^*); the commuter may then choose a later time than T_e^* when both congestion T_x and its rate of change, Δ, are smaller in magnitude, making Equation 22 possible. In fact, if Δ^* is strongly negative there must be a later region where $|\Delta|$ is smaller because T_x cannot fall below 0.

A practical difficulty is to find a reasonable congestion profile that allows one to solve these equations for the optimal head start. A linear congestion profile will work for Equation 13 but not for Equations 20 and 22. Conversely, other functional forms work for Equations 20 and 22 but will not give analytic solutions for Equation 13. An explicit congestion profile is not defined; additional research is examining simulations that endogenously generate congestion profiles (*38*).

Exponential Distribution

The exponential distribution for T_r is defined by the probability density function,

$$f(T_r) = \frac{1}{b} e^{(-T_r/b)} \tag{23}$$

which applies for $0 \geq T_r$. The parameter b is the mean and the standard deviation of the distribution (this differs from the uniform distribution in which the mean is $\sqrt{3}$ times larger than the standard deviation). The exponential distribution more accurately reflects the actual probability of the occurrence of an incident by allowing short delays to have a higher probability of occurrence than longer delays.

Following the same procedures as those described earlier yields an expected cost for the exponential distribution. Assuming that $T_e > 0$, to guarantee an interior solution,

$$\begin{aligned} \text{EC} = {} & \frac{1}{b}\int_0^\infty \alpha\,(T_f + T_x + T_r)e^{-T_r/b}\,dT_r \\ & + \frac{1}{b}\int_0^{T_e} \beta\,(T_e - T_r)\,e^{-T_r/b}\,dT_r \\ & + \frac{1}{b}\int_{T_e}^\infty [\gamma(T_r - T_e) + \Theta\,]\,e^{-T_r/b}\,dT_r \end{aligned} \tag{24}$$

Note that it is now possible to specify an infinite range for the distribution function. Integration yields the following result:

$$\text{EC} = \alpha(T_f + T_x + b) + \beta\,(T_e - b) + e^{-T_e/b}\,(\Theta + b\beta + b\gamma) \tag{25}$$

which can be rewritten as

$$\text{EC} = \alpha(T_f + T_x + b) + \beta\,(T_e - P_E b) + P_L(\Theta + b\gamma) \tag{26}$$

where $P_L = e^{-T_e/b}$ is defined as the probability of arriving late, and $P_E = 1 - P_L$ is the probability of arriving early, given $T_e > 0$. This can again be put in the form of Equation 12, where in this case $E(T) = T_f + T_x + b$, $E(\text{SDE}) = T_e - P_E b$, and $E(\text{SDL}) = bP_L$. These expectations can be verified by direct calculations from Equations 2 through 4.

The value of T_e that minimizes expected cost can now be calculated. Taking the derivative of Equation 25 with respect to T_e, setting it equal to 0, and solving for T_e^* gives the following result:

$$T_e^* = b\cdot\ln\left[\frac{\Theta + b(\beta + \gamma)}{b(\beta - \alpha\Delta)}\right] \tag{27}$$

where ln denotes the natural logarithm. When Θ and $\Delta = 0$, implying no late penalty and no change in congestion levels, this formula corresponds to that of Gaver (*3*), Equation 2.5. The second-order condition requires that $d\Delta/dT_e < -1/\alpha b^2 \cdot \exp(-T_e/b) \cdot [\Theta + b(\beta + \gamma)]$, which can simplify to $d\Delta/dT_e < (\alpha\Delta - \beta)/\alpha b$. The probability of being late, $P_L^* = e^{-T_e^*/b}$, can be rewritten as

$$P_L^* = \frac{b(\beta - \alpha\Delta)}{(\Theta + b\beta + b\gamma)} \tag{28}$$

Lateness is favored by small values of θ and γ and by a large negative slope to the congestion function. Equation 27 will have no solution where $\alpha\Delta \geq \beta$, but this is not a problem because if Δ is large enough for this to occur at some head start, then the commuter will seek larger head starts and must eventually find a region where Δ is small. Such a region must exist because T_x cannot be negative.

The interior solution of Equation 27 is valid only when it is compatible with $T_e \geq 0$, the range under which it was derived. That condition is violated if the term in square brackets is ≤ 1, i.e., if

$$\alpha\Delta^* \leq -\left(\gamma + \frac{\Theta}{b}\right) \tag{29}$$

This condition is the same as that in Equation 21, except that T_m is replaced by b (recall that the standard deviation in the uniform distribution is $T_m/\sqrt{12}$, whereas for the exponential distribution it is equal to b). If it holds, the commuter chooses to always be late; expected cost is found by replacing T_e by 0 in the limits of integration in Equation 24, resulting in the following

$$\text{EC} = \alpha(T_f + T_x + b) + \Theta + \gamma(b - T_e) \tag{30}$$

which is equivalent to Equation 11c for the uniform distribution. Head start, T_e, would be chosen either at the corner solution, $T_e = 0$, or at a point where $\alpha\Delta = -\gamma$, just as in Equation 22. This is analogous to Case *c* of the uniform distribution; there is nothing analogous to Case b because the exponential distribution has no upper limit and therefore there is no way to set T_e so that one always arrives early.

Returning to the interior solution (Equation 27), the optimized value of expected cost can be calculated by substituting Equation 27 into Equation 25:

$$\text{EC}^* = \alpha(T_f + T_x + b) - b\alpha\Delta + b\beta\cdot\ln\left[\frac{\Theta + b(\beta + \gamma)}{b(\beta - \alpha\Delta)}\right] \tag{31}$$

The first term is the expected cost of travel time. This can be rewritten to compare with Equation 14:

$$\text{EC}^* = \alpha(T_f + T_x + b) + \Theta P_L^* + C_s^* \tag{32}$$

where P_L^* is given by Equation 28 and

$$C_s^* = b\left\{\beta\cdot\ln\left[\frac{\Theta + b(\beta + \gamma)}{b(\beta - \alpha\Delta)}\right] - \frac{\Theta(\beta - \alpha\Delta)}{\Theta + b(\beta + \gamma)} - \alpha\Delta\right\} \tag{33}$$

The equations derived above describe the expected cost functions associated with uncertainty in travel times. These can be used to evaluate the relative proportion of expected cost associated with travel time uncertainty. The analyses in the next section provide some useful examples showing the relative importance of travel time variance for the cost of commuting.

NUMERICAL EXAMPLES

To analyze the head start times and expected costs associated with travel variance, estimates of the cost coefficients in the models are

TABLE 1 Head Start Times by Standard Deviation and Change in Congestion

	Uniform Distribution: T_e^* (in minutes)		
$T_m / \sqrt{12}$ = Std. Dev.	Δ = -0.1	Δ = 0	Δ = 0.1
5	15.03	15.61	16.19
10	28.23	29.39	30.55
15	41.44	43.18	44.92
20	54.64	56.96	59.28
30	81.05	84.54	88.02
	Exponential Distribution: T_e^* (in minutes)		
b = Std. Dev.	Δ = -0.1	Δ = 0	Δ = 0.1
5	8.74	9.50	10.40
10	16.05	17.57	19.36
15	23.28	25.56	28.25
20	30.49	33.53	37.11
30	44.89	49.44	54.82

needed. Empirical estimates by Small (*2*) of the ratios β/α and γ/α are used in combination with a value of time of \$6.40/hr. These values are also used by Arnott et al. (*15*). The result, using α = 6.40/hr, is β = 3.90/hr and γ = 15.21/hr (rescaled to minutes for these calculations). The authors also use $\Theta = 0.58$ from Small (*2*).

Table 1 shows the values of T_e^* for standard deviations of travel time between 5 and 30 min and for the congestion slopes, Δ, between -0.1 and 0.1. The optimal head start time is always larger with the uniform distribution than with the exponential distribution; this is because of its higher probability weighting for large delays. The head start is larger (earlier departure) when the standard deviation is larger and when congestion is increasing. Table 2 shows the corresponding optimal values of P_L^*, the probability of arriving late, which is smaller when congestion is increasing.

If a hypothetical commuter has scheduling flexibility, then it is possible to assume that $\beta = \gamma$, that is, the commuter is indifferent between schedule delay early and schedule delay late. In addition, this hypothetical commuter would have no lateness penalty, Θ. This can be considered a form of flextime. A commuter with flextime may still have some preferred arrival time, perhaps determined by constraints on the work departure time or personal preferences, such that β and γ are not 0. Table 3 indicates the head start times chosen by such a commuter (with $\beta = \gamma = 3.90$). In all cases the commuter still desires a head start time to avoid congestion, although these values are significantly less than those in Table 1. Note that the head start times increase linearly with respect to the standard deviation because $\Theta = 0$. In the case with no change in congestion levels, $T_e = \sqrt{3} \cdot b$ with the uniform distribution, and $T_e = b \cdot \ln(2)$ with the exponential distribution.

Our analytical derivations have separated the costs associated with travel time, $E(T^*)$, from those associated with the uncertainty of travel time, $C_s^* + \Theta P_L^*$. How important are the relative contributions made by these terms toward the total expected cost of travel, EC*? Table 4 provides a breakdown for each distribution for different levels of travel time uncertainty and Table 5 provides a breakdown for different levels of Δ, excluding the cost of certain travel time, $\alpha (T_f + T_x)$. The total EC* does not differ much between the two distributions, the largest difference being about \$0.73 (when SD = 30). However, C_s^*, the expected cost of schedule delay, is much larger under the exponential distribution than the uniform distribution. For large standard deviations of travel time, C_s^* from the uniform distribution becomes virtually insignificant regardless of

TABLE 2 Optimal Probability of Being Late by Standard Deviation and Change in Congestion

	Uniform Distribution: P_L^*		
$T_m / \sqrt{12}$ = Std. Dev.	Δ = -0.1	Δ = 0	Δ = 0.1
5	13.24%	9.89%	6.55%
10	18.50%	15.15%	11.80%
15	20.25%	16.90%	13.55%
20	21.13%	17.78%	14.43%
30	22.00%	18.66%	15.31%
	Exponential Distribution: P_L^*		
b = Std. Dev.	Δ = -0.1	Δ = 0	Δ = 0.1
5	17.41%	14.96%	12.50%
10	20.10%	17.26%	14.43%
15	21.19%	18.20%	15.21%
20	21.77%	18.71%	15.64%
30	22.40%	19.24%	16.08%

TABLE 3 Head Start Times by Standard Deviation and Change in Congestion with Flex Time

	Uniform Distribution: T_e^* (in minutes)		
$T_m / \sqrt{12}$ = Std. Dev.	$\Delta = -0.1$	$\Delta = 0$	$\Delta = 0.1$
5	7.239	8.660	10.081
10	14.478	17.321	20.163
15	21.717	25.981	30.244
20	28.956	34.641	40.326
30	43.435	51.962	60.489
	Exponential Distribution: T_e^* (in minutes)		
b = Std. Dev.	$\Delta = -0.1$	$\Delta = 0$	$\Delta = 0.1$
5	2.706	3.466	4.362
10	5.412	6.931	8.724
15	8.118	10.397	13.086
20	10.824	13.863	17.448
30	16.236	20.794	26.172

the level or direction of changes in congestion. However, under the exponential distribution, the proportion of expected costs attributable to C_s^* remains relatively stable at about 46 to 48 percent of the total expected costs for each level of standard deviation. This is about the same contribution made by the expected value of uncertain travel time, αb or $\frac{1}{2} \alpha T_m$, which in the case of the uniform distribution accounts for virtually all of the expected costs of commuting. In both distributions the proportion of expected cost associated with the probability of arriving late, ΘP_L^*, decreases as the standard deviation increases; apparently the shifts in head start time shown in Table 1 more than compensate for the increases in standard deviation.

CONCLUSIONS

This research has analyzed the costs associated with uncertain travel times. The work of Gaver (*3*) and Polak (*4*) has been followed but with some new contributions, focusing primarily on scheduling considerations. The effects of congestion that the commuter encounters every day have been explicitly separated from the nonrecurrent congestion that accounts for day-to-day variability in travel times. In addition, a discrete lateness penalty, which was originally detected empirically by Small (*2*) has also been introduced.

Using one set of empirically estimated parameters, the expected cost of schedule delay is found to be a relatively minor component of costs when the uniform distribution is used but quite large when the exponential distribution is assumed. In both cases the residual probability of being late is set by the commuter at a small enough value that the expected discrete lateness penalty is only a small fraction of the total costs.

The model described in this paper enables the analyst to predict the expected cost of schedule delay, including penalties for lateness, taking into account how the traveler adjusts the trip schedule in response to the travel environment. Our numerical example suggests costs of several dollars per trip, arising from standard devi-

TABLE 4 Expected Costs of Scheduling and Incident Delay with Uncertain Travel Time (Δ = 0)

	Uniform Distribution				
$T_m / \sqrt{12}$ = Std. Dev.	EC*	C_s^*	%	θP_L^*	%
5	1.0375	0.0564	5.43%	0.0574	5.53%
10	1.9765	0.0411	2.08%	0.0879	4.45%
15	2.9054	0.0360	1.24%	0.0980	3.37%
20	3.8317	0.0335	0.87%	0.1031	2.69%
30	5.6817	0.0309	0.54%	0.1082	1.90%
	Exponential Distribution				
b = Std. Dev.	EC*	C_s^*	%	θP_L^*	%
5	1.1508	0.5307	46.11%	0.0868	7.54%
10	2.2084	1.0416	47.17%	0.1001	4.53%
15	3.2612	1.5557	47.70%	0.1056	3.24%
20	4.3126	2.0708	48.02%	0.1085	2.52%
30	6.4139	3.1023	48.37%	0.1116	1.74%

Note: Costs in dollars per morning commute.

TABLE 5 Expected Costs of Scheduling and Incident Delay with Uncertain Travel Time (SD = 10)

	Uniform Distribution				
Δ	EC*	C_S^*	%	θP_L^*	%
-0.1	1.9827	0.0279	1.41%	0.1073	5.41%
0	1.9765	0.0411	2.08%	0.0879	4.45%
0.1	1.9827	0.0667	3.37%	0.0685	3.45%
	Exponential Distribution				
Δ	EC*	C_S^*	%	θP_L^*	%
-0.1	2.2163	1.0331	46.61%	0.1166	5.26%
0	2.2084	1.0416	47.17%	0.1001	4.53%
0.1	2.2183	1.0679	48.14%	0.0837	3.77%

Note: Costs in dollars per morning commute.

ations of travel time varying from 10 to 30 min. Furthermore, if the exponential distribution applies, about half this cost is due purely to the variance of travel times (the other half being the expected value of the incident delay).

If the expected cost of schedule delay is indeed a major cost of unreliable commute trips, as this suggests, then policies that reduce travel time variance may be preferable in many cases to policies that reduce travel times, especially when the latter are costly. Policies that decrease the response time needed to clear incidents, for example, may be much cheaper than and provide cost savings comparable to capacity expansion.

The information the commuter has about congestion will influence the departure time decision and ultimately the expected cost of commuting. Future work will analyze the impact of providing commuters with accurate information about changes in congestion levels and travel time variance. For example, how will changing information affect head start times when combined with a supply-side congestion model? What are the impacts on congestion when commuters have varying degrees of information about both congestion and reliability? The model presented here offers a starting point for addressing such questions, which are central to the evaluation of intelligent transportation systems.

ACKNOWLEDGMENTS

This research was funded by California Partners for Advanced Transit and Highways and the California Department of Transportation, whose support the authors gratefully acknowledge.

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All responsibility for the contents of this paper remains with the authors.

Publication of this paper sponsored by Committee on Traveler Behavior and Values.