# Analysis of Temporal and Spatial Variability of Free Speed Along a Freeway Segment

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To determine the speed-flow relationship for a highway section, a number of parameters must be estimated. These include free speed, speed at capacity, capacity, and jam density. Because of fluctuations in demand, variations in driver behavior, and geometric and environmental conditions, these parameter values may vary both spatially for different stations and temporally for different days. To use these speed-flow relationships to estimate link travel times or diversion capacities, or for incident detection algorithms, these spatial and temporal variations in the speed-flow relationships need to be quantified so that accurate estimates of the relevant traffic parameters can be made. This work presents a statistical analysis of the variability of free speed estimates for 24 stations along a section of I-4 in Orlando, Florida during a 4-month period. This analysis is a first step in performing similar analyses of capacity, speed at capacity, and jam density. In the analysis presented in this work, it was found that free speed estimates along I-4 had a standard deviation of 4.7 km/hr and were most dependent on the location at which they were observed. This location factor explained 60 percent of the sum of squared errors. Minor variations in free speed from one day to another were overshadowed by these spatial differences and accounted for approximately 6 percent of the sum of squared errors. These two findings suggest that on this freeway section there is little loss in accuracy if many days of data are aggregated for a specific location, but a great loss in accuracy if many locations are averaged for the same day. There is also little to be gained by estimating day-of-theweek specific free speeds.

The objective of the research reported in this work was twofold. The primary purpose was to ascertain whether there were statistically significant differences in free speed estimates from one location to the next, or from one day to the next. In the absence of such variations, it would be sufficient to calibrate a speed-flow relationship for an entire highway section based on either 1 day's worth of data at a single station or a composite single data set of all stations and days combined. The second objective was to determine whether significant temporal or spatial differences existed in the estimated free speeds. For example, it is often perceived that midweek (Tuesday through Thursday) traffic behavior is different from driver behavior on Friday or Monday. If this perception is substantiated, then it would be necessary to establish different free speeds for the same section of highway to model these different types of days.

The characteristics of the study network and the data collection time frame are presented, followed by an overview of the study procedure. The details of the Analysis of Variance (ANOVA) tests are then described, followed by the conclusions of this study.

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## STUDY DESCRIPTION

#### **Network Configuration**

A 16-km (10-mi) portion of the I-4 freeway in Orlando, Florida was considered in this study, modeled as part of an Intelligent Vehicle Highway System—Institute of Transportation Studies benefit assessment. I-4 is a major route that extends across the center of Florida from the southwest (Tampa) to the northeast (Daytona), passing by the Disney World complex to the west of the study area. The detectorized portion of the I-4 freeway is located near downtown Orlando, extending from 33rd Street to the southwest, and ending downstream of Maitland Boulevard to the northeast, as illustrated in Figure 1.

A total of 24 loop-detector stations were located along I-4, numbered from 1 to 25, with no data provided for Station 10. The spacing of the detector stations ranged from approximately 0.40 to 0.90 km (0.25–0.54 mi). There were no major terrain variations along the detectorized section of the I-4 freeway, as Orlando is rather flat. However, at many interchanges with arterials, the freeway was elevated. The entire detectorized section of I-4 was composed of three lanes in each direction.

## **Data Collection Time Frame**

The analysis period included traffic data for portions of a 4-month period during the winter of 1992–1993. The data included 11 days in November 1992, 29 days in January 1993, 26 days in February 1993, and 11 days in March 1993. This data set amounted to a total of 75 days of 30-sec data, with approximately 10 different days of data available for each day of the week.

The Freeway Management Center (FMC) dual loop detectors measured and logged the flow, occupancy, and space mean speed for each of the three lanes at 30-sec intervals. These data were aggregated for this analysis into 5-min data summaries to reduce the amount of data to be handled, while still capturing most of the variability in the traffic conditions. An average lane flow, occupancy, and mean speed estimate for each station were generated from the individual loop detector measurements. In estimating the average speed at a specific station, the loop speeds were weighted by the volume on each set of loops.

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FIGURE 1 Location of FMC detector stations along I-4 freeway

## **OVERVIEW OF STUDY PROCEDURES**

## **Typical Traffic Conditions Along I-4**

Based on the FMC data for all of the available days within the 4month period, it was possible to generate surfaces that represented the average speed, average flow, or average occupancy at a particular station and at a particular time of day. Figure 2(a) represents the resulting average flow surface in the eastbound direction along the I-4 section. The x-axis represents the time period from 0 at midnight at the start of the day, to 24 at midnight at the end of the day, and the y-axis represents the station numbers that are traversed. The eastbound flow proceeds in the upward direction from Station 1 to Station 25. For each cell combination of time of day and station, the z-axis represents the average hourly lane flow rate measured.

Figure 2(a) shows that the flow gradually increased at 6:00 a.m. along all eastbound stations until it reached a flow of approximately 2,000 vehicles per hour (vph) per lane at 8:00 a.m. along detector





(b)

FIGURE 2 (a) Temporal and spatial variation in 30-min. eastbound average lane flow (vph/lane). (b) Temporal and spatial variation in 30-min. eastbound average lane speed (km/hr).

Stations 5 through 18. The flow increased again during the p.m. peak from approximately 3:00 p.m. until 6:30 p.m. at Stations 12 through 22. Figure 2(a) shows that the flow from 5:00 to 7:00 p.m. at Stations 7 through 12 was lower than 2,000 vph/lane. However, after examining Figure 2(b), it appears that during this period the speeds were also low (20 to 40 km/hr). Thus, the lower flow measurements were most likely caused by the presence of congestion, and not by a reduction in demand.

Figure 2(b) illustrates that only Stations 5 through 20 experienced speeds near or in the congested portion of the speed-flow relationship (speeds less than 60 km/hr) during the p.m. peak in the eastbound direction. As subsequent analyses of the variability of speed at capacity, capacity, and jam density required the use of congested data, only stations with congested data were subsequently considered for the fitting of a complete speed-flow relationship.

### **Structure of Speed-Flow Relationships**

The selection of a particular shape for a speed-flow relationship has been a topic discussed by traffic engineers for more than 50 years. May (1) provides an excellent discussion and comparison of the various single- and multiregime models, and describes their respective strengths and limitations in the context of producing reasonable free speed, speed at capacity, capacity, and jam density estimates. In response to these limitations, a new single regime speed-flow relationship was developed [see Figures 3(a) and 3(b)]. This relationship is described in detail by Van Aerde (2). The main features are the highly linear and almost horizontal behavior in the uncongested region, the speed at capacity in excess of one-half of the free speed, and the jam density value, which is higher than two times the density at capacity, yet still finite. Of particular interest to this study is the fact that the free speed, which theoretically occurs when the volume is 0, can be extrapolated reliably from the near-linear uncongested portion of the curve. An average of the speeds observed when flows are below a given maximum flow threshold (e.g., V/C < 0.5) would always represent an underestimate of the free speed in view of the small negative slope of the curve in this region.

#### **Estimation of Speed-Flow Parameters**

Figures 3(a) and 3(b) illustrate sample fits of Equation 1 and its density counterpart to data collected over an entire day at Detector Station 13. The discrete points represent the 5-min loop detector measurements, and the continuous curve represents the fit estimated by the curve-fitting model.

To generate the free speed estimates at each station, a heuristic curve-fitting program was developed that selects the speed-flow relationship parameters that produce the minimum normalized square error in a three-dimensional flow-speed-density data space (3). This curve-fitting model estimates four parameters, namely free speed  $(u_f)$ , speed at capacity  $(u_c)$ , capacity  $(q_c)$ , and jam density  $(d_j)$ . The structure of the speed-flow relationship is represented in Equation 1. Equations 2 through 5 are used to calculate the three model parameters,  $c_1$ ,  $c_2$ , and  $c_3$ , in addition to the intermediate parameter k.

It appears from Figure 3(a) that the macroscopic relationship captures most of the deterministic variation in speed-flow while achieving a reasonable compromise estimate when stochastic variability exists. The four parameters for Station 13 selected by the model were:  $u_f = 87.2$  km/hr,  $u_c = 70.6$  km/hr,  $q_c = 1,925$  vph, and  $d_j = 92.2$  vehicles/km. In Figure 3(*a*), free speed is identified as the higher y-axis intercept, speed at capacity is the y-axis value that corresponds to the maximum flow point (nose of curve), and capacity is the maximum x-value. Jam density is the inverse of the slope of the fitted curve as it emerges at the origin to the axes in Figure 3(*a*), but it is more easily identified as the x-value at 0 speed in Figure 3(*b*).

$$q = \frac{u}{c_1 + \frac{c_2}{u_f - u} + c_3 u}$$
(1)

$$k = \frac{c_1}{c_2} = \frac{(2u_c - u_f)}{(u_f - u_c)^2}$$
(2)

$$c_2 = \frac{1}{d_l \left(k + \frac{1}{u_l}\right)} \tag{3}$$

$$c_1 = kc_2 \tag{4}$$

$$c_{3} = \frac{-c_{1} + \frac{u_{c}}{q_{c}} - \frac{c_{2}}{(u_{f} - u_{c})}}{u_{c}}$$
(5)

where

- $c_1$  = fixed distance headway constant (km),
- $c_2 =$  first variable distance headway constant (km<sup>2</sup>/hr),
- $u_f = \text{free speed (km/hr)},$
- $u_c$  = speed at capacity (km/hr),
- u = prevailing speed associated with headway h (km/hr),
- q = flow rate of traffic traveling at speed u (km/hr) (vph),
- $q_c =$  flow at capacity (vph),
- $d_i$  = jam density (vehicle/km), and
- k = dimensionless constant to set the speed at capacity relative to the free speed.

It should be noted in Figures 3(a) and 3(b) that to generate a satisfactory fit of a typical speed-flow relationship's jam density and speed at capacity at a specific location, sufficient data points in both the uncongested and the congested portion of the curve are required. This is the basis for the fact that the curve-fitting model was set to not estimate the desired four parameters if no points existed in the congested region (speeds less than 60 km/hr) of the curve.

#### **Typical Spatial and Temporal Variation in Free Speed**

Figure 4 demonstrates the temporal and spatial variation in the free speed estimates at Stations 9 to 22 over a sample 10-day period. Because of a lack of points in the congested portion of the speed-flow relationship at Stations 1 to 8 and Stations 23 to 25, the curve fits and therefore the free speed estimation was performed only for the stations located in the downtown area (Stations 4 to 22). The surface plot shows that the free speed ranged from 80 to 110 km/hr. It appears that the free speeds were relatively constant during the 10-day period, as indicated by the minor variations in the *y*-axis direction. However, the speeds varied to a greater extent for the different locations along the *x*-axis. The variation in free speed was in the range of approximately  $\tilde{n}15$  percent of the average free speed



FIGURE 3 (a)A typical speed-flow fit to I-4 data ( $u_f = 87.2 \text{ km/hr}$ ,  $u_c = 70.6 \text{ km/hr}$ ,  $q_c = 1,925 \text{ vph}$ ,  $d_j = 92.2 \text{ veh/km}$ ).

(continued on next page)



FIGURE 3 (b) A typical speed-density fit to I-4 data ( $u_f = 87.2$  km/hr,  $u_c = 70.6$  km/hr,  $q_c = 1,925$  vph,  $d_j = 92.2$  veh/km.



FIGURE 4 Temporal and spatial variation in free speed along I-4 (km/hr).

and had a standard deviation (SD) of 4.7 km/hr. A more detailed analysis of the free speed variation follows.

## INTRODUCTION TO ANALYSIS OF VARIANCE

An examination of the speed contours in Figure 4 suggests that the free speed is much more spatially dependent than temporally dependent. This qualitative assessment prompted a statistical ANOVA of the free speed data to ascertain whether different days of the week or different station locations, or both, affected the value of the free speed in a statistically significant fashion. To complete this analysis, a data set of free speeds as a function of the day of the measurement (75 different days) and the location (24 station locations) was produced for subsequent analysis using SYSTAT (4).

## **Screening of Data**

For this data set, data for the eastbound direction at Stations 1, 2, 3, 10, 23, and 24, and for the westbound direction at Stations 1, 2, 3, 4, 5, 10, 13, and 21 were removed because of a lack of congested data. Data from most Saturdays and Sundays were also removed. After removing several other days for the same reason, approximately 31 days of acceptable data remained for the westbound direction and 33 days of acceptable data remained for the eastbound direction.

#### **Analysis Scenarios**

After the data set had been conditioned, a series of ANOVAs was carried out (5, 6). A brief introduction to the procedure follows.

The data set was split first into two main sets, one for the eastbound and one for the westbound direction. These data sets were treated separately for the rest of the analyses. Three different ANOVA models were fit for the aggregated data set. A one-way ANOVA was performed on calendar date (Analysis 1*a*) and the location factor was analyzed (Analysis 1*b*). In the third analysis, a two-way ANOVA was conducted because the date factor and location factor could be significant (Analysis 1*c*). Because the data set contained free speeds for each location for several weeks, it was possible to group the data by the day of the week rather than by the calendar date. This grouping permitted a one-way ANOVA with replication of measurement (Analysis 2).

To explore the effect of location and date within a single week of data, the eastbound direction for Monday, January 25, through Friday, January 29, 1993, was analyzed. For the westbound direction, screening of the data set made it impossible to find a continuous Monday through Friday period. Hence, the period from Friday, January 22, to Thursday, January 28, (excluding the weekend) was analyzed. As with the entire data set, three ANOVAs were fit, along with two one-way ANOVAs (grouped either by day or location) and one two-way ANOVA (Analyses 3a, 3b, and 3c, respectively).

An additional set of ANOVAs was fit to explore the premise that traffic behavior during the core midweek period (Tuesday, Wednesday, and Thursday) is different from Monday or Friday. For this reason, a two-way ANOVA with replication, similar to Analysis 2, was performed on a data set of Tuesday, Wednesday, and Thursday data (Analysis 4).

A total of 13 different analyses of variance models was fit for each direction. Typical data sets used in these analyses are shown in Tables 1 and 2. Tables 3 and 4 summarize the results of the most important ANOVAs performed. The following sections discuss each series of ANOVAs.

Table 2 shows that the mean free speed changed significantly along the route in the eastbound direction. The westbound direction experienced a similar change in free speed; however, because of space limitations, the results are not presented in this work. The large drop in free speed at Station 9 in the eastbound direction and

	02-Nov-92	09-Nov-92	25-Jan-93	01-Feb-93	22-Feb-93	29-Mar-93		
Station	MON	MON	MON	MON	MON	MON	MEAN	STD
4	92.2	90.9	90.0	93.1	97.5	92.5	92.70	2.38
5	87.5	85.3	82.5	87.2	89.1	86.9	86.42	2.07
6	87.5	89.4	84.7	88.1	84.4	84.1	86.37	2.05
7	87.5	90.0	85.3	86.3	89.4	86.9	87.57	1.66
8	90.0	90.9	83.1	87.5	86.9	87.2	87.60	2.50
9	85.0	82.5	78.1	80.0	79.7	77.8	80.52	2.52
11	91.3	91.3	88.8	90.0	89.4	88.8	89.93	1.05
12	95.6	95.0	92.5	92.5	90.6	91.3	92.92	1.82
13	91.3	92.5	88.8	91.3	96.6	90.3	91.80	2.42
14	88.4	87.5	83.8	86.6	89.4	86.6	87.05	1.76
15	90.6	89.4	85.0	86.9	88.4	90.0	88.38	1.92
16	88.8	95.6	87.2	88.4	92.5	87.5	90.00	3.05
17	86.6	91.6	89.1	88.8	86.3	88.8	88.53	1.76
18	89.4	93.8	90.3	91.6	90.0	90.6	90.95	1.44
19	88.8	90.9	88.1	90.0	93.8	89.4	90.17	1.85
20	91.3	92.2	90.6	92.5	100.6	92.5	93.28	3.34
21	97.5	100.0	103.1	110.0	102.5	105.9	103.17	4.02
22	91.3	93.1	90.0	91.3	95.6	91.3	92.10	1.81
MEAN	90.03	91.22	87.83	90.12	91.26	89.36	89.97	4.41
STD	2.99	3.78	5.10	5.70	5,56	5.24	1.17	

 TABLE 2
 Data Set for Eastbound Free-Flow Speed km/hr (January 25-29, 1993)

	25-Jan-93	26-Jan-93	27-Jan-93	28-Jan-93	29-Jan-93			
Station	MON	TUE	WED	THUR	FRI	MEAN	STD	
4	90	91.9	92.2	95	91.3	92.08	1.64	
5	82.5	82.5	85.3	86.9	88.1	85.06	2.27	
6	84.7	86.3	90	88.8	89.1	87.78	1.97	
7	85.3	84.4 ·	86.9	87.5	86.9	86.20	1.16	
8	83.1	83.1	85.6	85.9	85	84.54	1.21	
9	78.1	77.5	80	79.4	80	79.00	1.02	
11	88.8	85.3	90	91.3	90.3	89.10	2.08	
12	92.5	89.4	91.6	93.8	93.8	92.22	1.64	
13	88.8	87.5	89.7	90.9	90.6	89.50	1.24	
14	83.8	82.5	85.3	85.3	85.6	84.50	1.18	
15	85	83.8	86.3	89.4	90.3	86.96	2.50	
16	87.2	85	87.5	88.8	88.8	87.46	1.39	
17	89.1	87.5	88.1	88.1	87.5	88.06	0.59	
18	90.3	87.5	91.9	92.5	90.6	90.56	1.73	
19	88.1	86.3	89.4	88.8	89.4	88.40	1.15	
20	90.6	90	90.6	91.9	90.6	90.74	0.62	
21	103.1	93.8	100	99.4	97.5	98.76	3.07	
22	90	88.8	91.3	91.3	91.3	90.54	1.01	
MEAN	87.83	86.28	88.98	89.72	89.26	88.42	4.01	
STD	4.96	3.63	3.92	4.08	3.49	1.24		

#### TABLE 3 Summary of ANOVA for Eastbound Free-Flow Speed Along I-4 Freeway in Orlando, Florida

ANALYSIS TYPE	SAMPLE	MEAN SUM OF SQUARES						F		Ferti	
	SIZE	Station	(%)	Date	(%)	Error	(%)	Station	Date	Station	Date
Analysis 1 (without replication)	594	622.74	96%	23.43	4%	3.08	0%	202.36	7.61	1.64	1.47
Analysis 2 (with replication)	450	467.77	97%	9.45	2%	6.58	1%	94.04	1.90	1.65	2.40
Analysis 3 (25 Jan-29 Jan 93)	<del>9</del> 0	85.25	70%	34.34	28%	1.60	1%	53.31	21.47	1.78	2.51
Analysis 4 (midweek with repl.)	330	294.57	97%	3.46	1%	5.38	2%	65.00	0.47	1.67	3.04

TABLE 4 Summary of ANOVA for Westbound Free-Flow Speed Along I-4 Freeway in Orlando, Florida

ANALYSIS TYPE	SAMPLE	MEAN SUM OF SQUARES						F		Ferti	
	SIZE	Station	(%)	Date	(%)	Error	(%)	Station	Date	Station	Date
Analysis 1 (without replication)	496	310.09	92%	21.86	6%	6.04	2%	51.32	3.62	1.69	1.48
Analysis 2 (with replication)	400	244.03	86%	29.32	10%	9.51	3%	48.87	5.87	1.70	2.40
Analysis 3 (25 Jan-29 Jan 93)	80	48.98	72%	16.44	24%	2.16	3%	22.67	7.61	1.84	2.53
Analysis 4 (midweek with repl.)	240	128.93	70%	45.64	25%	10.85	6%	22.99	8.13	1.72	3.04

a corresponding increase in free speed in the westbound direction are most likely caused by an uphill grade at Station 9 in the eastbound direction. The large increase in free-flow speed at Station 20, however (in the east- and westbound directions), is most likely due to a change in free speed limit from 88 km/hr (55 mph) to 104 km/hr (65 mph).

## Analysis 1: ANOVA of Entire Data Set (Monday–Friday)

The data were first grouped by calendar date to test for the significance of the calendar date factor on the free-flow speed for both the east- and westbound directions (Analysis 1*a*). The one-way ANOVA results indicated that the free-flow speed was not significantly different, at the 95 percent confidence level, from one day to the next. When these data were grouped by the location (Analysis 1*b*), the one-way ANOVA revealed that the free speed varied significantly from one location to the next. Finally, when both variables were included in a two-way ANOVA without replication (Analysis 1*c*), the results indicated that both the calendar date factor and location factor were statistically significant.

The summary results of the latter two-way ANOVA analysis are given in Tables 3 and 4, which show that the largest amount of variation (as indicated by the mean sum of squares is accounted for by the station factor. In the eastbound direction 96 percent, and in the westbound direction 92 percent of the variation in the data was due to the location factor. Four and 6 percent of the error in the respective directions was explained by the factor that accounts for the calendar day on which the data were collected. These percentages are based on mean square ratios.

For the total sum of squared errors, the error explained by the station factor was approximately 60 percent. Consequently, when specifying speed-flow relationships for this highway it is more important that a different relationship be developed for each location along the route than for each separate day. The observed minor differences from day to day prompted the analyses to determine whether the differences were systematic or random.

## Analysis 2: Two-Way ANOVA with Replication (Monday–Friday)

Analysis 1c indicated that the free speed at a specific location did vary to some extent with the day on which the data were measured; therefore, an ANOVA was carried out to determine whether a dayof-the-week factor was a systematic source of these differences. In other words, the analysis was done to learn whether traffic behavior varies in a consistent fashion from Monday to Tuesday or Thursday to Friday. The results of these analyses are referred to as Analysis 2 and are indicated in Line 2 of Tables 3 and 4. The mean sum of squares shows that for the eastbound direction very little, if any, differences occurred between the different days of the week, as the F statistic indicated that the day-of-the-week factor is not significant at the 95 percent level of confidence (1.90 < 2.40). However, in the westbound direction the location factor is still the most important source of variation (48.87 > 1.70); there is a statistically significant difference between each day of the week (5.87 > 2.40). At this stage of the research, the reason the east- and westbound directions produce different results remains unclear.

#### Analysis 3: One Week of Data

The next series of analyses, referred to as Analysis 3, examined a continuous period of 5 weekdays. The purpose of this analysis was to determine whether a week of data would be sufficient to determine an average free-flow speed at a specific location. The results of this analysis are given in Line 3 of Tables 3 and 4. As with the entire data set for Analysis 1, three different ANOVAs were performed for each direction, two one-way analyses of the calendar

date (Analysis 3a) and location (Analysis 3b), and one two-way analysis without replication (Analysis 3c) using both factors. In both directions it was found that although the location factor was still the most significant factor, the day-of-the-week factor was also statistically significant. In the eastbound direction 25 percent and in the westbound direction 24 percent of the variation in free speed was due to the day-of-the-week factor. This would suggest that it is not possible to obtain a representative estimate of the free speed at a specific location by gathering data on only one day of the week.

## Analysis 4: Midweek Only

Analysis 3 indicates that there were differences between the free speed obtained from one day to the next. It is often hypothesized that midweek period behavior is different from Friday or Monday behavior. As such, it might be possible to calculate two different estimates of free speed, one for each portion of the week. An analysis of the midweek data was performed to ascertain whether these temporal differences in Analyses 1 to 3 could be adequately explained by simply having midweek and Monday through Friday data grouped together. The results of this analysis, referred to as Analysis 4, are given on Line 4 of Tables 3 and 4. The values for the Mean Sum of Squares and the F statistic indicate that in the eastbound direction there is no statistically significant difference in the free speed from one day to the next during the midweek period. Virtually all of the variation in the data is explained by the location factor. Therefore, it is possible to obtain a location-specific measure of free speed for the midweek period. However, this is not the case in the westbound direction. In fact, 24 percent of the variation is due to the day on which the data were measured. This finding is consistent with the results obtained during the analysis of the entire data set using replication (Analysis 2).

## CONCLUSIONS AND RECOMMENDATIONS

Several conclusions can be drawn from the analyses presented in this work. Although these conclusions are based on the specific I-4 data, the authors believe that the trends in the I-4 freeway behavior are representative of many typical freeways in North America and that the analysis used is applicable elsewhere.

First, free speeds along I-4 depend most strongly on the location where they are observed. Changes in geometry, ramp location or configuration, and speed limit may all be responsible for the observed significant differences in free speed as a function of the location factor. Second, minor variations in free speed from one day to another are due to differences between midweek versus weekend characteristics.

It is therefore recommended that when analyzing freeways such as I-4, location-specific free speeds be estimated first. Subsequently, day-of-the-week specific adjustments may be made, but these will have a less significant effect. However, even when these factors have been accounted for, some residual day-to-day variations will remain.

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