# Incremental Bus Allocation with Competing Mass Transit Services 

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#### Abstract

An efficient allocation of fixed equipment among routes in a bus network can be primarily achieved by choosing appropriate headways or frequencies on each route. An interesting exercise arises when the allocation of additional equipment is likely to result in significant ridership attraction from competing services. In this case, models that take into account the operating environment and the nature and extent of competing services on different routes are needed to estimate the demand response to various headways and to allocate additional equipment efficiently. In this paper, a model that allocates buses among various lines based on a realistic correlation between bus headways and demand for bus service is proposed and applied to the case of the bus transit operation in the city of Beirut, Lebanon. The transit authority in Beirut runs a limited bus network with a small fleet that will be expanded on over the course of the coming 2 years. Competing mass transit services in Beirut include private jitneys operating over an extensive network as well as limited private bus fleets. In this paper, the allocation of additional buses in the bus transit operation in Beirut is addressed. A variable demand representation was applied in the bus allocation model and necessitated a data collection effort to identify the potential for ridership attraction from competing services. The model was then applied based on an incremental analysis and with cost-recovery considerations in order to allocate efficiently the additional buses to the existing network.


Given a transit operation with established routes, areas being served, and hours of service, a common exercise for the bus transit planner involves deciding on the levels of service or frequencies that should be provided on each route. This procedure of setting headways or frequencies on bus routes is commonly referred to as the equipment allocation, or fleet allocation, problem. The fre-quency-setting decision is a complex component in the planning process that involves determination of the distribution of the operating resources over an existing network. In its most general sense, the aim is for an efficient, system-wide allocation of services among routes as well as across time of day (1).
An interesting exercise arises when additional equipment is to be allocated and where such allocation is likely to result in significant ridership attraction from competing mass transit services. In this case, models that take into account the operating environment and the nature and extent of competing services on different routes are needed to estimate the demand response to various headways and to allocate additional equipment efficiently. In this paper, a model that allocates buses among various lines based on a realistic corre-
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lation between bus headways and demand for bus service is proposed and applied to the case of the bus transit operation in the city of Beirut, Lebanon. The transit authority in Beirut runs a limited bus network with a small fleet that will be expanded over the course of the coming two years. As such, an analysis involving an allocation of additional equipment to the existing network based on potential ridership attraction from competing services and cost-recovery considerations was implemented.

## EXISTING APPROACHES TO FLEET ALLOCATION WITH VARIABLE DEMAND

Early approaches to the optimum headway problem assumed demands that did not vary with the headway, although they might vary with time. The objective function commonly related to the minimization of a weighted sum of operators' monetary costs and passengers' waiting time $(2,3)$ or the minimization of passengers' waiting time subject to budget or fleet size constraints (4-6).

Variable demand formulations of the fleet allocation problem have appeared in a number of studies. For instance, the allocation of buses in networks with overlapping routes based on a minimization of a function of passenger wait time and bus crowding is discussed by Han and Wilson (7). The allocation is constrained by the number of available buses and the provision of enough capacity on each route to carry all passengers selecting that route. Although the demand for bus service, expressed by the set of origin-destination flows, is given and assumed fixed in this approach, the number of passengers eventually using each route is variable and depends on the bus allocation, since passenger flows are split between competing routes serving some of the origin-destination pairs. Moreover, Kocur and Hendrickson (8) analyze the design of local bus service and determine the optimal route spacing, headway, and fare for three objective functions. The analysis is based on an equilibrium framework whereby transit ridership is sensitive to the level of service provided by the bus system. Finally, Furth and Wilson (9) propose a model to allocate available buses between time periods and on fixed routes so as to maximize the net social benefit subject to constraints on total subsidy, fleet size, and levels of vehicle loading. The model formulates the problem of setting frequencies on bus routes as a constrained resource allocation problem with the objective function consisting of maximizing the summation of two distinct components: consumer surplus and transit ridership. Of particular interest here is the fact that in this (nonlinear) formulation, headway is used as the basic decision variable and ridership is expressed explicitly as a function of headway.

## PROPOSED APPROACH TO INCREMENTAL BUS ALLOCATION IN A COMPETITIVE ENVIRONMENT

With fixed bus routes, fares, and speeds, the only operating factor capable of attracting riders from competing services is a change in headways, which can be achieved through fleet allocation. The focus of this paper is on proposing an approach for bus allocation over several routes in a network in the case where the bus service is competing for ridership with other mass transit services. To estimate the demand response to various headways, models that take into account the operating environment and the nature and extent of competing services on different routes are needed. Given the need for a variable demand formulation in the competitive environment being considered, an appropriate form of the demand function that relates ridership on a bus route to that route's headway must be determined.

## Problem Context

In the analysis that follows, it is considered that public transit buses will be competing for riders with two other types of mass transit services, namely (a) buses of similar operating characteristics (for instance, private bus operations), and (b) an alternate mass transit service of distinct operating characteristics (for instance, jitney operations). The potential reaction and shift of auto passengers is not considered because the limited improvement in bus level of service is not expected to be sufficient to incur any significant switching from the auto mode.

The analysis will focus on the assignment of additional buses to routes in an existing network to improve on a minimal level of bus service. The assignment will be based on the potential attraction of riders from each of the two competitors.

## Possible Models of Ridership Attraction

To estimate the potential attraction of riders from competing services based on the allocation of additional buses, three variable demand models were considered: the trinomial logit model, the nested logit model, and a third model that considers competition with each of the two other modes separately. First, it was concluded that the application of a trinomial logit model (public bus, private bus, or other distinct service) would not have been feasible because the operating characteristics of public and private buses are quite similar, and therefore such a model would violate the independence of irrelevant alternatives (IIA) assumption of the logit model (10). Second, it was noted that the application of the nested logit model in the competition context being considered is possible in principle. In such a case, the upper-level choices would include "bus service" and "other distinct service," and the lower-level choices (under bus service) would incorporate both the public and private bus operations. A detailed description of the nested logit model can be found in the work of Ben-Akiva and Lerman (10).

A third model, which is being proposed here, considers competition between the public bus operation and each of the two other mass transit services separately. This model stresses the differences in competition mechanisms between public buses and each of the two competing modes, and is useful in cases where including the three modes in the same model is likely to be inappropriate. As such, two different approaches are adopted for computing ridership attracted from each of the two competing services. The proposed model is described in detail next.

## Competition with Services of Distinct Operating Characteristics

## Logit Model

Demand for bus service is modeled as a choice process in which each individual traveler has the possibility of choosing either bus transit or an alternative, distinct mass transit mode (such as jitneys). The demand function to be used here is the logit model, the most commonly used disaggregate mode choice model and one that allows the analyst to predict the modal choice probabilities for individual trip-makers. The logit model takes the form:
$P_{i}(1)=\frac{e^{V_{i}(1)}}{e^{V_{i}(1)}+e^{V_{i}(2)}}$
where
$P_{\mathrm{i}}(1)=$ probability of a mass transit rider served by route $i$ choosing bus (mode 1),
$V_{i}(1)=$ utility of bus transit for population served by route $i$, and
$V_{i}(2)=$ utility of alternative mass transit mode for population served by route $i$ (mode 2 ).

For the population of mass transit riders whose origins and destinations are served by route $i$, or $P O P_{i}$, the number of riders who will choose bus transit is given by
$r_{i}=P O P_{i} * P_{i}(1)=P O P_{i} * \frac{e^{v_{i}(1)}}{e^{v_{i}(1)}+e^{v_{i}(2)}}$
where $r_{i}$ equals the number of mass transit riders served by route $i$ choosing to use buses and $P O P_{i}$ is the total mass transit-riding population served by route $i$.

## Utility Function

The utility of each of the two competing modes is a function of variables describing that mode (travel time, travel cost, comfort and convenience, etc.) and the individual making the modal choice decision (income, automobile availability, etc.). Assuming random passenger arrivals and constant bus headways, the average passenger wait time at the bus stop is equal to half the headway. As such, the impact of the passenger wait time on demand for transit service can be directly related to bus headway. This suggests the inclusion of a variable relating to bus headway in the utility function. In particular, the utility of bus transit may be represented as:

$$
V_{i}(1)=\theta * h_{i}+\text { other terms }
$$

where $h_{i}$ is the headway on route $i$ and $\theta$ is the headway coefficient in the utility function.

## Pivot-Point Form

If the base probability of choosing bus transit is known, and headway on route i is changed by an amount $\Delta h_{i}$ from its base value of $h_{i}^{0}$ to $h_{i}$, the new probability of choosing transit can be predicted
from the pivot-point form of the binary logit model, which can be derived from Equation 1. This form is (10):
$P_{\mathrm{i}}^{\prime}=\left[1+\frac{1-P_{i}^{0}}{P_{i}^{0}} * e^{-\theta \Delta h_{\mathrm{i}}}\right]^{-1}$
In this formulation, $P_{i}^{0}$ is the base probability of choosing bus transit for the population of mass transit riders served by route $i$; that is, it corresponds to the observed proportion of total mass transit travelers served by route $i$ choosing bus transit. $P_{i}^{0}$ can also be referred to as the base bus transit share. $\Delta h_{i}$ is the change in the headway of route i from its base value ( $h_{i}-h_{i}^{0}$ ). Note that $\mathrm{h}_{i}^{o}$ is the observed, base headway for bus transit on route i whereas $h_{i}$ is the new headway on route $i . \mathrm{P}_{i}^{\prime}$ is the probability of choosing bus transit given the new headway $h_{i}$. The primary feature of the pivot-point logit model that makes it a suitable demand function is that it reproduces the observed ridership, so that demand variation is considered only from the actual point of observation. This at least ensures that the present steady-state conditions are reproduced in the model.

## Attracted Ridership

In the model considered here, the base headway, ridership, and transit share correspond to a minimal level of transit service. The allocation of additional buses will strictly improve the headway on all routes, and as such ridership will always be attracted from the competing, distinct mode. The final equation for attracted ridership on route $i$ from this first class of competing services, or the difference between the ridership because of the improved headway and the base ridership, is:

$$
\begin{align*}
\Delta r_{1}\left(h_{\mathrm{i}}\right) & =P O P_{i} * P_{i}^{\prime}-r_{i}^{0}=\frac{r_{i}^{0}}{P_{i}^{0}} * P_{i}^{\prime}-r_{i}^{0} \\
& =\frac{r_{i}^{0}}{P_{i}^{0}} *\left[1+\frac{1-P_{i}^{0}}{P_{i}^{0}} * e^{-\theta \Delta\left(h_{i}-h_{i}^{0}\right)}\right]^{-1}-r_{i}^{0} \tag{2}
\end{align*}
$$

where $r_{i}^{0}$ is the base ridership on route $i$ or the observed number of riders.

## Competition with Private Bus Services

Private buses represent the second class of competitors being considered for the public bus system. In this case, "bus riders" have the choice of riding on either private or public buses that serve their intended trip origin and destination. The two services are assumed to have similar service quality and similar operating characteristics with respect to fares, in-vehicle travel times, and comfort on the different routes they serve. Within this context, the frequency of operation is the major, if not only, factor determining the split in ridership between the private and public systems. As in the first case of competition outlined above, the allocation of additional buses to the public bus system will be considered starting from a minimal basic level of service. Therefore, only the attraction of additional riders from the private buses as the frequency of public buses is increased will be considered. The number of additional riders who are attracted from private buses is:
$\Delta r_{2}\left(h_{i}\right)=\frac{f_{b i}}{f_{b i}+f_{v i}} * r_{T i}-r_{i}^{0}$
where
$f_{b i}=$ frequency of public buses on route $i$,
$f_{v i}=$ frequency of private buses on route $i$, and
$r_{T i}=$ total current bus (public and private) ridership.

## Total Attracted Ridership

The total number of riders attracted to an improved public bus system will be computed as the sum of the two terms appearing in Equations 2 and 3. The final public bus ridership will be equal to:
$r_{i}\left(h_{i}\right)=r_{\mathrm{i}}^{0}+\Delta r_{1}\left(h_{i}\right)+\Delta r_{2}\left(h_{i}\right)$
In other words, the final public bus ridership is the sum of three components, namely: (a) the base ridership, (b) ridership attracted from the competing of distinct operating characteristics (as in Equation 2), and (c) riders attracted from private buses (as in Equation 3).

## Data Requirements

The use of the above modeling approach requires data on the ridership served on each route by the different mass transit services for the base case of current public bus service. This data can be used to determine the base public bus ridership $\left(r_{i}^{0}\right)$, the current modal split between public buses and the first class of competing services ( $p_{i}^{0}$ ), as well as the total current bus ridership $\left(r_{T i}\right)$. Current frequencies of the two types of bus services on each route ( $f_{b i}$ and $f_{v i}$ ) are also required. Finally, the headway coefficient $\theta$, to be used in Equation 2 , is also required.

## CASE STUDY: BEIRUT CONTEXT

The proposed methodology for incremental bus allocation in a competitive environment is illustrated in the case of the city of Beirut, Lebanon. The Beirut context is described first by discussing mass transit operations in existence, modal usage trends, and the potential role of the public bus system. The study objectives and scope are also described at the end of the section.

## Mass Transit Operations

The public bus system in Beirut has been limited in its operations for a number of years, providing a service that is too infrequent to be reliable and highly subsidized. The public bus authority owned and operated 150 buses in the greater Beirut area in 1965. All the buses were destroyed during the first years of the war. In 1978, the authority ordered 220 buses, which were shipped in stages until the late 1980s. However, many of these buses were eventually destroyed or stolen, and as a result only. 60 buses remained in operable condition as of the end of 1992. However, because of a lack of manpower and inefficient management practices, only a fraction of these buses are actually operating on the different routes ( 42 in 1991 and 22 since the latter part of 1992).

In contrast, mass transport in Beirut is characterized by a significant supply of privately operated transit services, which are mostly unstructured and unregulated. Jitneys, locally known as "service"
and which rely on sedan cars, form the backbone of these services. Moreover, limited private bus fleets have begun to grow in recent years to fill the void created by the weakness of the public bus system, and now carry significantly more passengers than public buses. However, the private buses are operating without authorization, and the rather old equipment being used is not properly maintained and lacks certain safety requirements. The private bus operations within Beirut are certain to be scaled down (or phased out altogether) by the government when the public bus system gets revitalized.

## Modal Usage Trends

The limited service offered by the public bus system within Beirut, the absence of public bus services outside Beirut, and the expanding ownership and use of the private automobile (about 100,000 cars were imported into the country in 1991) have resulted in the predominance of the auto as the most important transportation mode. Traffic counts conducted in 1984 (11) indicated that buses carried less than 5 percent of midday trips in Beirut (peak period ridership levels were unavailable) and jitneys carried about 23 percent, with the remaining 72 percent of trips being made by the private auto. Comparable pre-war figures for 1970 (12) were 11, 44, and 44 percent, respectively, indicating a trend toward less reliance on public transit.

## Potential Role of Public Bus System

Based on the factors outlined above, and given the probable financial and physical constraints on major new infrastructure investments in Beirut, public transit has the potential to play a significant role in reducing congestion based on its ability to provide higher capacities and vehicle occupancies than the private auto. The last major government planning study for the Beirut Metropolitan Region (BMR), the Schema Directeur (13), suggests that a greater reliance on public transit to transport passengers from the suburbs to Beirut and within Beirut itself should be a major component of an overall plan to ease traffic congestion in the BMR.

During the coming 2 years, the transit authority in Beirut is planning to expand its bus fleet to close to 120 buses through the rehabilitation of older buses and the acquisition of a limited number of new ones. Based on this modest public investment, it becomes possible for the public bus system to improve travel conditions in Beirut in at least two respects: (a) by providing a viable, cheap alternative to jitneys, and consequently inducing a shift in ridership away from this less-efficient travel mode; and, (b) by reassuming its role as the prime bus mode in Beirut, and consequently helping to phase out the illegal private bus operations by reducing reliance on them.

## Study Objectives and Scope

Since the transit authority in Beirut will, for the near future, be constrained with respect to major equipment acquisition to support bus network expansion, the existing network structure, which has been relatively stable since 1991, is likely to be maintained until further notice. In such a context, the major lever in improving service is in setting the route headways on the existing network. Constrained by a limited number of operating buses and a limited budget, it is primarily in choosing the headways that these limited resources can be
allocated more efficiently. It is therefore imperative that the allocation be performed in a way that maximizes the social benefit and ensures efficiency.

In view of the likelihood that the transit authority will be in charge of running a larger fleet within the next 1 to 2 years, the next section describes a proposed approach for allocating buses among lines based on a realistic correlation between bus headways and demand for bus service. This need for a variable demand representation in the bus allocation model necessitated a data collection effort to identify the potential for ridership attraction. The approach was then applied, in an incremental mode, to efficiently allocate the potential additional buses on the existing network.

## INCREMENTAL BUS ALLOCATION IN COMPETITIVE ENVIRONMENT

During the latter part of 1992, the bus fleet in Beirut comprised only 22 buses, which were being operated on a network consisting of nine lines. The operating fleet has remained at about that size since then. However, during the coming 2 years, the transit agency is likely to be able to expand the bus fleet to nearly 120 buses, raising the question of how these buses should be distributed among the different lines in the existing network.

The expected fleet expansion will introduce modifications to the current system conditions, including a potential for attracting riders from competing modes of mass transit. Therefore, a need exists for variable demand models for setting headways. In the following, the proposed approach to incremental bus allocation (described above and represented by Equations 2 to 4) is adopted instead of the nested logit model since, in the context of the expansion of the public bus system in Beirut, all demand shifts will be in the direction of this revitalized system. Moreover, and with the likely restrictions on and phasing out of the private bus operations, this mode cannot be considered as a viable, equivalent bus alternative in the lower-level choices, as the nested logit model would imply.

In the analysis that follows, it is considered that public buses will be competing with jitneys and private buses for riders. Because the extent of the public bus service will remain rather limited, the potential reaction and shift of auto passengers is not considered. As such, the binary choice represented by Equation 2 is whether to ride the public bus or to use the jitneys. Equation 2 models demand variation that results from bus headway changes only, and hence wait time changes.

## Data Requirements for Model Application

As outlined above, the use of the suggested model requires data on the ridership served on each route by the different mass transit services, frequencies of bus services on each route, and an estimate of the headway coefficient $\theta$. Because the required data were not readily available, a limited data collection effort was undertaken.

## Mass Transit Ridership

The data collection procedure consisted of ride checks to determine public bus ridership levels and traffic counts at the maximum load points to determine ridership on competing mass transit modes. Being limited by a small research budget, each of the counts could be conducted only once. This was thought to be sufficient for the
purpose of this study, which was to illustrate a methodology for allocating additional buses and to obtain a rough first allocation instead of to reach an exact and final conclusion.

Ride checks were conducted for both directions on each route and for peak period trips (7:00 to 9:00 a.m.). Once the peak load point on each route was determined, a traffic count was used to count person trips by all mass transit modes in the peak direction. The counts were conducted on weekdays during November 1992 from 7:00 to 10:00 in the peak a.m. period. The data collected at each of the nine counting stations included (a) traffic volumes classified according to public buses, private buses, and jitneys; and (b) the passenger occupancy of each vehicle.

The model proposed for predicting the potential attraction of riders to public buses from competing services considers a target choice population of mass transit riders whose trips are served by bus routes and who are willing to switch to an improved bus system. However, the number of riders on the competing mass transit services that observed during the traffic counts includes individuals whose origins and destinations may not be served by the public bus system as well as individuals who may be unwilling to switch to the public bus system. To remedy this difficulty, the choice population was obtained from the observed ridership by introducing the following two factors.

O-D Factor The origins and/or destinations of riders on other modes of collective transport may not correspond to bus routes. The boundaries of served origins and destinations are uncertain, and an accurate trip table is not available. The fraction of the observed ridership whose origins and destinations are served by a particular route was estimated for each route based on judgment. For jitney riders, this fraction ranged between 0.45 and 0.75 based on route characteristics such as geometry of road network along the route, type of area served (whether commercial or residential), type of route (whether radial or cross-town), length of the route, and so forth. For private bus riders, the choice fraction was taken as 1.0 (except for Line 9), because private buses follow practically the same routes as public bus lines.

Switching Factor Some riders are unwilling to switch to a bus even if the bus route coincides with their O-Ds. The fraction of jitney riders willing to switch to an improved public bus system was obtained from a survey conducted by students at the American University of Beirut (14), in which 92 percent of jitney riders indicated a willingness to switch to an improved public bus service. This fraction was assumed the same for all lines. The fraction for private bus riders is taken to be 100 percent as its service characteristics are quite similar to those of the public bus system.

The choice jitney population associated with each bus route is inferred by multiplying the O-D factor and the switching factor by the observed jitney ridership (obtained from traffic counts) on each of the routes. A similar procedure was followed to obtain the choice private bus population.

## Estimation of Base Modal Split Ratio, $\mathrm{p}_{\mathrm{i}}^{0}$

Based on the determination of the choice jitney population associated with each of the public bus routes, the base modal split ratio to be used in Equation 2 can be computed as follows:
$p_{i}^{0}=r_{i}^{0} /\left(r_{i}^{0}+\right.$ choice jitney riders $)$

## Determination of Headway Coefficient, $\theta$

The data set that was used to obtain the headway coefficient relates to work trip mode choice in Beirut and is extracted from surveys conducted by students at the American University of Beirut (14). Only two travel modes are taken into consideration, namely, public bus and jitney. The data set that was used consisted of answers to detailed questionnaires and provided 108 observations of socioeconomic and mode characteristics associated with a group of trip makers and the mode choices they made ( 24 used buses and 84 used jitneys). Various utility functions associated with the logit model were specified and their parameters estimated. The model that was chosen based on statistical validity tests revealed a value for the wait time coefficient $\theta$ of approximately -0.05 .

## Demand Elasticities with Respect to Headways

For the logit model, the point elasticity of demand for bus service with respect to headway on route $i$ is
$e_{i}=\theta * h_{i}^{0}\left(1-P_{i}^{0}\right)$
Elasticity values for the different lines are presented below. As one might expect for a bus system that currently provides very lowfrequency service, the demand is quite elastic with respect to headway. Note that the demand on Line 1 is inelastic with respect to headway because the base bus share was relatively high for this line.

## Model Application

Having determined all principal factors of the model, its application based on Equations 2 to 4 becomes straightforward. It is worth mentioning that the analysis was performed based on ridership figures for the 3-hr A.M. peak period (7:00 to 10:00 A.M.). The base public bus ridership on the eight lines being analyzed was 1,345 for a service that was provided by 22 buses. The computed choice populations over the $3-\mathrm{hr}$ period for jitneys and private buses were 5,700 and 3,530 , respectively, for a total of 10,575 and a base public bus share of 12.7 percent.

## Incremental Analysis

Additional buses to be rehabilitated and acquired by the bus transit authority in Beirut during the coming 1 to 2 years will be put in service in stages. Using the proposed model, an incremental assignment of additional buses as they become available is proposed here.

With the adoption of the objective of providing public service to as many travelers as possible, the allocation of each additional bus is made to the route having the highest marginal benefit with respect to passenger attraction. With this technique, a route "priority list" is generated. Starting with a baseline of the existing 22 buses, and terminating at the upper bound of 120 buses, the list serves as a guide each time an additional bus is to be allocated. The starting point in the above analysis was the 22 available buses distributed so as to provide minimum regular service, and according to the service standard of policy headways (15). A summary of the results (allocated buses and corresponding headways) is reported in Table 1. One interesting observation is that, with the 120 buses being allo-

TABLE 1 Summary of Incremental Analysis Results

| Total No. of Buses | 21 | 40 | 50 | 75 | 100 | 120 |
| :--- | ---: | :--- | :--- | ---: | ---: | ---: |
| Line \# | Bus Allocation |  |  |  |  |  |
| 1 | 4 | 5 | 7 | 9 | 10 | 12 |
| 2 | 2 | 5 | 6 | 9 | 12 | 15 |
| 3 | 3 | 7 | 9 | 14 | 19 | 22 |
| 4 | 1 | 2 | 2 | 3 | 4 | 4 |
| 5 | 2 | 3 | 3 | 4 | 5 | 6 |
| 6 | 3 | 6 | 8 | 12 | 16 | 19 |
| 7 | 3 | 7 | 9 | 14 | 20 | 25 |
| 9 | 3 | 5 | 6 | 10 | 14 | 17 |
| Line \# | Headway (min.) |  |  |  |  |  |
| 1 | 30 | 24 | 17 | 13 | 12 | 10 |
| 2 | 45 | 18 | 15 | 10 | 7 | 6 |
| 3 | 40 | 17 | 13 | 8 | 6 | 5 |
| 4 | 90 | 45 | 45 | 30 | 23 | 23 |
| 5 | 45 | 30 | 30 | 22 | 18 | 15 |
| 6 | 30 | 15 | 11 | 7 | 5 | 5 |
| 7 | 40 | 17 | 13 | 8 | 6 | 5 |
| 9 | 30 | 18 | 15 | 9 | 7 | 5 |

cated, five of the lines would have headways of 5 to 6 min , representing peak-period frequencies that are necessary to build up a solid ridership base. This indicates that the 120 buses are needed to provide strong basic service on existing routes, and that additional bus routes have to be considered if the fleet is expanded any further.

Figure 1 indicates the increment in ridership with additional buses allocated according to the priority list. The trend indicates a continuously increasing ridership as the number of operating buses increases. On the other hand, the incremental rate of attraction is decreasing: the effect of adding a bus at latter stages is much lower than at initial stages. For instance, during the 3-hr a.m. peak period, the 22 nd bus attracts 160 additional trip makers, the 50th attracts 48 more, and the 120th attracts 13 more. This relates to the fact that the model is only interested in the limited mass transit market ( 10,575 total travelers during the peak 3 hr along routes served by the existing public bus routes). An extension of the analysis presented in Figure 1 indicates that the increment in ridership with each additional bus becomes almost insignificant starting at 200 buses, at which stage the public bus share of choice mass transit riders would have reached 62.6 percent. As public transit service improves beyond a certain level, a shift is likely to occur from the auto-driver and auto-passenger markets.


FIGURE 1 Ridership with additional buses.

## Bus Allocation With Cost-Recovery Considerations

Based on the incremental analysis results, a number of questions arise: Given the decreasing marginal return of additional buses with respect to ridership attraction, what is the ceiling at which procurement of buses on the existing network should stop? How should a trade-off be made between cost considerations and social benefits (additional ridership)? To answer these questions, cost-recovery ratios, relevant to subsidy levels, are calculated.

To compute cost-recovery figures, the 1991 operating costs per vehicle kilometer were used (after accounting for inflation since that time). For any number of allocated buses on a bus route: $(a)$ the vehicle kilometers and the associated costs are calculated, $(b)$ the expected ridership and the revenues (based on the flat fare currently in effect) are predicted, and (c) the cost-recovery ratio is obtained. This procedure is followed on each route independently. Sample results for Line 9 are reported in Figure 2. This figure indicates that a maximum cost recovery of around 30.5 percent can be achieved for this line based on the allocation of four to six buses to it. The maximum cost recovery that could be achieved for each line and the associated number of buses summarized in Table 2. Figure 3 indicates that cost recovery is optimal ( 29 percent) for a fleet ranging between 35 and 50 buses. Between 100 and 150, cost recovery is within the range of 17 to 21 percent. However, for a much bigger fleet, the cost-recovery ratio becomes unacceptable. With almost the same cost-recovery ratio, it would be recommended to use 47 buses instead of 35 since social service (ridership) is improved by 23.2 percent.

## Fleet Size and Subsidy Considerations

The final question relates to the recommended fleet size needed to operate on the existing network, given no switching from the auto mode. Two main factors will be considered: potential ridership attraction and maximum permissible subsidy. In the latter case, the decision on fleet allocation could be financially dominated. If the transit authority is awarded the same subsidy as in 1991 (inflated), it can operate almost 147 buses with the improved allocation made possible by the proposed approach. If, on the other hand, the authority tries to minimize costs while providing a fairly good service, the 120 buses soon to be available provide a good combination: a 56.2 percent public bus share ( 4.4 times the 1991 share), a 20 percent cost-recovery ratio, and a subsidy level about 80 percent that of the year 1991 (inflated).


FIGURE 2 Cost recovery for Line 9.

TABLE 2 Bus Allocation with Cost-Recovery Considerations

| Line \# | Maximum Cost <br> Recovery | Number <br> of Buses | Headway <br> (min.) | Ridership |
| :--- | :--- | :--- | :--- | :--- |
| 1 | $28.1 \%$ | $6-7$ | $18-19$ | $440-498$ |
| 2 | $69.2 \%$ | $5-6$ | $14-16$ | $618-689$ |
| 3 | $26.8 \%$ | $6-8$ | $17-18$ | $523-655$ |
| 4 | $12.3 \%$ | $2-4$ | $60-90$ | $104-160$ |
| 5 | $17.4 \%$ | $3-4$ | $23-27$ | $212-258$ |
| 6 | $30.8 \%$ | $4-6$ | $17-19$ | $489-660$ |
| 7 | $30.7 \%$ | $5-6$ | $20-22$ | $568-661$ |
| 9 | $30.5 \%$ | $4-6$ | $17-18$ | $391-530$ |
| Total | $29.0 \%-29.2 \%$ | $35-47$ |  | $3345-4121$ |

## CONCLUSIONS

In this paper, an approach for assigning additional buses to routes of an existing network in the context of competition with other mass transit services has been presented. The proposed approach allocates additional buses based on maximum potential ridership attraction from competing services. The bus system in Beirut, Lebanon was considered as a case study for incremental bus allocation using the proposed approach. With jitneys and private buses representing competing modes of mass transit in Beirut, a priority list was generated for the assignment of additional buses, which are expected to become available soon. In addition to ridership attraction, costrecovery implications were considered. The adopted approach clearly provided an improved allocation of buses. For instance, it was found that 147 buses could be assigned to the network and operated at the same effective level of subsidy, as was the case in 1991 when only 42 buses were operational. However, the analysis indicated that with a larger bus fleet, the introduction of new lines to the existing network needs to be considered. Moreover, with a significant improvement in the bus level of service, the potential attraction


FIGURE 3 Systemwide cost recovery.
of riders from the auto mode would become possible, a situation that needs to be addressed. These two items, as well as the consideration of possible response from jitney and private bus operators to public bus fleet expansion, represent interesting avenues for further research.

## ACKNOWLEDGMENTS

Funding for the research described in this paper was provided by the AUB/MIT Collaborative Program on Science, Technology, and Development and by the University Research Board at the American University of Beirut.

## REFERENCES

1. Banasiak, M. Computer Aided Decision Making: Service Allocation by Transit Planners. M.S. thesis. Massachusetts Institute of Technology, Cambridge, 1984.
2. Newell, G. F. Some Issues Relating to the Optimal Design of Bus Routes. Transportation Science, Vol. 13, 1979, pp. 21-35.
3. Wirasinghe, S. C. Nearly Optimal Parameters for a Rail/Feeder-Bus System on a Rectangular Grid. Transportation Research, Vol. 14A, 1980, pp. 33-40.
4. Salzborn, F. Optimum Bus Scheduling. Transportation Science, Vol. 6, 1972, pp. 137-148.
5. Hurdle, V. F. Minimum Cost Schedules for a Public Transportation Route. Transportation Science, Vol. 7, 1973, pp. 109-157.
6. Last, A., and S. E. Leek. Transept: A Bus Model. Traffic Engineering and Control, Vol. 17, 1976.
7. Han, A., and N. Wilson. The Allocation of Buses in Heavily Utilized Networks with Overlapping Routes. Transportation Research, Vol. 16B, No. 3, 1982, pp. 221-232.
8. Kocur, G., and C. Hendrickson. Design of Local Bus Service with Demand Equilibration. Transportation Science, Vol. 16, 1982, pp. 149-170.
9. Furth, P., and N. Wilson. Setting Frequencies on Bus Routes: Theory and Practice. In Transportation Research Record 818, TRB, National Research Council, Washington, D.C., 1981, pp. 1-7.
10. Ben-Akiva, M., and S. Lerman. Discrete Choice Analysis: Theory and Applications to Travel Demand. MIT Press, Cambridge, Mass., 1985.
11. Team International. Comptage de Circulation de la Region Metropolitaine de Beyrouth. Republique Libanaise, 1985.
12. Nakkash, T., and N. Jouzy. Beirut Travel Characteristics-A Comparative Study. Transportation, Vol. 2, 1973, pp. 411-430.
13. Mission Franco-Libanaise Etude et Amenagement. Schema Directeur de la Region Metropolitaine de Beyrouth. Republique Libanaise, June 1986.
14. Bassil, G. Bus Public Transit in Beirut: A Study of System Characteristics and Potential. M.S. thesis. American University of Beirut, Lebanon, 1993.
15. Metropolitan Transit Authority of Harris County (METRO). Bus Service Evaluation Methods: A Review. UMTA, U.S. Department of Transportation, Nov. 1984.

Publication of this paper sponsored by Committee on Transit Management and Performance.

