Selection of Highway Design Parameters in the Presence of Uncertainty

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AASHTO guidelines on highway design have drawn criticism for their inability to deal with the uncertainty of traffic operations, costs, and physical constraints. Some analysts believe that in light of changing economic and socio-environmental values, new procedures are needed to better address uncertainties and to justify engineering decisions. An analytical model that could be used to determine the optimal design curvature (D_d^*) for horizontal curves on two-lane highways is presented. The optimal curvature results in the minimum total cost, defined as the sum of construction, maintenance, and expected user costs. The expected user cost is the sum of expected accident, travel time, and vehicle operating costs. It is shown that: (*a*) D_d^* is highly sensitive to the skewness of the probability distribution of the required curvature; (*b*) when the mean operating speed is high, D_d^* does not change significantly with the changes of standard deviation of speeds compared with the low mean operating speed; and (*c*) when the mean operating speed is low, the polynomial model best represents the relationship between D_d^* and the standard deviation of the operating speeds. When the mean operating speed is high, the linear model best represents the relationship between D_d^* and the standard deviation of the operating speeds. The application of the model and sensitivity of the optimum to model parameters are illustrated using numerical examples.

The current policy on highway geometry, published by the American Association of State Highway and Transportation Officials (AASHTO) (*1*), seeks to promote safety through the use of the highest design standards. This traditional approach to highway design has drawn criticism recently (*2–5*). Critics argue that the higher design standards are not always justifiable when working under budget constraints and do not necessarily guarantee better safety due to numerous uncertainties. The most significant of these uncertainties are the characteristics of drivers and vehicles. These concerns, and the consensus that adding safety factors is not the most cost-effective and prudent way to treat uncertainty, have heightened the need for new approaches to roadway design. As in other disciplines of science and engineering, these approaches should strive for a balance between costs and benefits of a particular design when many factors are uncertain.

The development of an analytical model for determining the optimal degree of design curvature (D_d^*) of a horizontal curve on a two-lane highway is discussed. In many practical cases, the optimum is not always attainable. The decisions are affected by one or more constraints. Road geometry is a classic example of a constrained case, in which physical and environmental factors features limit design options. Thus, the optimization model is developed under two scenarios, constrained and unconstrained, and its application is illustrated by a numerical example.

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CURRENT PRACTICE

The degree of curvature required to permit the vehicle to negotiate a simple horizontal curve at a particular speed can be determined by the following fundamental relationship:

$$D = \frac{85{,}660\,(e + f)}{V^2} \tag{1}$$

where

D = curvature required by the individual vehicle speed (degrees per 100-ft curve length),
V = speed (mph),
e = superelevation rate in feet per feet, and
f = side friction factor at speed V.

If the curve is designed with a curvature of D_d, because V is a random quantity due to differences in vehicle and driver characteristics, the curvature required by a given vehicle may be less than, equal to, or greater than the design curvature. This phenomenon, and several other assumptions underlying the current practice of horizontal curve design, must be addressed by the new approach.

The two key aspects in need of attention stem from the following:

1. Currently, a high percentile speed is chosen as design speed (V_d) irrespective of the shape or form of the distribution of D.
2. Within a given functional class of roadway, variations in traffic volume and mix are disregarded.

Two other concerns have also emerged. The first is that although changes in operating speed due to inconsistencies in horizontal alignment have been found to be a leading cause of accidents (*6–8*), no formal mechanism exists to ensure consistency when selecting design speed. The second concern is related to cost-effectiveness. Although smaller D_ds mean higher construction costs, they also mean lower accident rates (*2,9,10,11*), operating costs, and travel times. But the trade-off between costs and savings is neither clear nor explicit in AASHTO (*1*), which makes assessing cost-effectiveness difficult, if not impossible.

In the next two sections, the components of the optimization model are presented, and the sensitivity of the optimal curvature to the various cost parameters is discussed.

OBJECTIVE FUNCTION

After the cost components are defined, the design degree of curvature (D_d) that minimizes the total cost can be sought in several ways.

The objective function is defined as follows:

$$TC_{\min} = C_a + C_{tt} + C_{op} + C_c + C_m \quad (2)$$

where

$TC_{\min}$ = minimum total cost,
C_a = expected accident cost,
C_{tt} = expected travel time cost,
C_{op} = expected vehicle operating cost,
C_c = expected construction cost,
C_m = expected maintenance cost.

The next section discusses the optimum solution when there are no constraining circumstances, and then identifies some common constraints and their impact on the solution.

USER COSTS

Expected Cost of Accidents

Although some researchers have expressed different views on the relationship between accident rate and D_d, the consensus is that (*a*) accident rates are higher on horizontal curves than on tangents, (*b*) rates increase as D_d increases, and (*c*) D_d is the most significant geometric feature contributing to accidents (*3–5*).

Considering all the variables cited in TRB (*12*) and Zegeer et al. (*13*), a generalized accident prediction model can be expressed as

$$A_m = g(D) = a\left[\frac{0.0189\, bI}{D_d} + cD_d - dS\right] \quad (3)$$

where

a, b, c, and d are calibration constants,
I = external angle in degrees,
A_m = number of accidents in a curve, and
S = superelevation in feet per feet.

If, as stated earlier, the safest design curvature ($D_{\min}$) is that which corresponds to the maximum operating speed, anything larger increases the geometry-related accident rate in proportion to the difference between D_d and $D_{\min}$. This demand-supply concept is explained in detail by the authors in previous articles (*14,15*), and by Newman and Glennon (*16*) in the case of stopping sight distances. Hirsch et al. (*5*) have also used the same reasoning that accidents occur when the design radius of curvature is smaller than the radius required by a vehicle traveling at a specific speed. However, if the selected design value is equal to or greater than the required value, they assumed the accidents were unrelated to radius.

Following the demand-supply concept, two regimes are defined for D in the present case; one when $D \geq D_d$, and the other when $D < D_d$. In the first regime, accidents are assumed to be unrelated to curvature; that is, the nonhuman-error and environment-related accidents that may occur at the curve are not directly influenced by the curvature. Therefore, the number of accidents in this regime is considered to be zero. In the second regime, in which demand exceeds supply, accidents are proportional to the deviation of D from D_d. The number of accidents can, therefore, be expressed as

$$A = \begin{pmatrix} N\,h(D)\,(D_d - D) & D < D_d \\ 0 & D \geq D_d \end{pmatrix} \quad (4)$$

where N is the annual traffic volume in millions of vehicles, and $h(D)$ is the rate change in accident per unit change of D per year per million vehicles.

Considering a generalized form of the probability density of D [i.e., $\phi(D)$], the exceedance probability [$P(D < D_d)$] or likelihood of the deficiency may be written as

$$P(D < D_d) = \int_0^{D_d} \phi\,(D)\,dD \quad (5)$$

Therefore, the expected costs of accidents when design curvature at a given site is D_d can be expressed as

$$C_a = \gamma_a N \int_0^{D_d} h(D)\,(D_d - D)\,\phi\,(D)\,dD \quad (6)$$

where γ_a is the weighed average cost per accident.

The cost parameter γ_a depends on the type of accident and the average cost of each type of accident. The first step is to define the type of accident and its proportion, and the second step is to estimate the average cost of each type of accident. Both types of accident and its costs vary from state to state and can be obtained from the state accident data base. Although the accident type can be a function of curvature, in the present case it is assumed to be a constant. Therefore, the weighted average accident cost (γ_a) can be estimated by using the formula:

$$\gamma_a \sum_{k=1}^{n} [C_k\,P_k] \quad (7)$$

where

C_k = average cost per type k accident,
P_k = proportion of type k accidents, and
k = accident type according to severity, 1, 2, 3, . . . , n.

Expected Vehicle Operating Cost

Drivers reduce their speeds when they approach a curve and accelerate after they enter or pass the curve (*17*). This speed-change cycle consumes fuel and engine oil, wears tires, and increases maintenance costs, which are all listed as significant in the AASHTO guidelines (*18*). When vehicle speeds are distributed over a wide range, the additional operating cost of each vehicle due to a particular curvature is a function of the difference between the operating costs at D_d and the operating cost at D. In other words, when $D \geq D_d$, excess vehicle operating cost on a curve is zero. Otherwise, it is assumed to be proportional to the difference in the operating costs at the two speeds. When D is a random quantity with a known density function, the expected operating cost (C_{op}) is expressed as

$$C_{op} = N\gamma_\beta \int_0^{D_d} (D_d - D)^m \phi(D)\,dD \quad (8)$$

where

C_{op} = expected vehicle operating cost,
γ_β = rate of change of operating cost per unit change of D_d per million vehicles, and
m = exponent.

Calculation of γ_β

AASHTO (*18*) provides tables of operating cost in dollars per 1,000 veh/mi above cost of tangent with respect to D_d and speed. It is also known from Islam and Seneviratne (*17*) that: $V_{85} = 62.4 - 1.46\,D_d + 0.018\,D_d^2$

Expressions for γ_β depend on the central angle. For example, when $I = 50°$, the regression equation in 1975 dollars can be formulated as

$$C_{75} = 5229 + 87.5\,D_d\ (R^2 = 0.87)$$

The preceding equation may be converted to 1992 dollars assuming a 7 percent discount rate as

$$C_{92} = 16524 + 276.5\,D_d$$

that is $\gamma_\beta = 276.5$

Expected Travel Time Cost

According to the TRB Special Report 214 (*12*), the cost of travel time should be a principal determinant of geometric elements, although Lin (*20*) has ignored it in his work. However, the value of travel time and the amount of travel time saved are the two key aspects of travel time. Using design speed to estimate travel time has no justification because all vehicles do not operate at design speed. Furthermore, if the operating speed is less than the design speed, the travel time saved is zero. Therefore, only those vehicles whose operating speeds are greater than design speed are considered. This delay can be expressed as

$$\text{Delay} = \left[\frac{L_d}{V_d} - \frac{L}{V}\right] \quad (9)$$

where

L = required length of the curve at speed V in miles,
V = operating speed on tangent in mph,
L_d = required length of curve at V_d in miles, and
V_d = design speed in mph.

Thus, the expected cost of delay after expressing all parameters in terms of D can be written as

$$C_u = \alpha\gamma_t N \int_0^{D_d} [\omega(D) - \omega(D_d)]\phi(D)\,dD \quad (10)$$

where

γ_t = value of travel time (\$/unit time),
$\alpha = 0.0189\,I$,
$\omega(D) = \dfrac{1}{DV}$,
$\omega(D_d) = \dfrac{1}{D_d V_d}$, and
$L = \dfrac{0.0189\,I}{D}$.

CONSTRUCTION AND MAINTENANCE COSTS

Construction Cost

Construction and maintenance costs depend on a variety of factors, including site conditions, labor and materials costs, design practice, and project scale. Pavement, shoulder, and side slope design standards vary from state to state. For example, New York State usually paves shoulders, whereas Virginia constructs gravel or turf shoulders. Labor costs in San Francisco are nearly double those in Jackson, Mississippi (*21*). Unit price for construction also depends on the size of the project.

The relationship between annual construction cost (C_c) and D_d for a specific site in a particular region may be linear, quadratic, or inverse, and the unit costs may vary from state to state or even from place to place. Three forms of the generalized expression for construction cost $P(D_d)$ are proposed in the present case. They are:

$$C_c = P(D_d) \quad (11)$$

where

$$P(D_d) = a - \gamma_b D_d \text{ or,} \quad (11a)$$
$$P(D_d) = c - \gamma_d D_d + \gamma_e D_d^2 \text{ or,} \quad (11b)$$
$$P(D_d) = \gamma_f / D_d. \quad (11c)$$

Maintenance Cost

The maintenance cost for projects is a function of the length of the roadway. The annual maintenance cost of a curve (C_m) can therefore be expressed in terms of its length as follows:

$$C_m = \gamma_k [L + L_c/2640] \quad (12)$$

where

γ_k = annualized maintenance cost per mile,
L = length of circular curve in miles, and
L_c = length of spiral in miles.

When the length of the curves is expressed in terms of the degree of the curvature, the cost of maintenance also becomes a function of D_d. For developing a model for obtaining D_d^*, it is necessary to know the cost of construction and maintenance per unit change of D_d. First derivative of Equations 11 and 12 with respect to D_d will give the construction and maintenance costs per degree change in D_d, and the tangent length will not be a factor.

OPTIMAL CURVATURE

Unconstrained Case

According to the definition of total cost, the objective function is the sum of the cost given by Equations 6, 8, 10, 11, and 12, which are expressed in terms of D_d. Therefore, the optimal value of D_d could be derived by taking the first derivative of the objective function expressed as Equation 2, and equating it to zero, as follows:

$$\frac{\partial(TC)}{\partial(D_d)} = 0 = f'(D_d) \quad (13)$$

When $m = 1$ and (C_c) is linear, there is no closed form solution to $f'(D_d)$. Thus, Equation 13 can be rearranged as follows and solved graphically or numerically to obtain the value of D_d leading to the lowest TC:

$$\int_0^{D_D} \phi(D)\, dDh(D_d) + \gamma_\beta + \gamma_t \alpha \omega'(D_d) = \frac{\gamma_b + \gamma_k [0.0189\, I]}{N} \quad (14)$$

Additionally, the sensitivity of D_d^* to the various parameters can be tested by changing one while the others are held constant.

Numerical Examples

Assuming that speeds are normally distributed with a mean of 50 mph and a standard deviation of 7 mph, random speeds were generated using the Monte Carlo method. These speed deviates were then substituted in Equation 1 to obtain the distribution of D for that speed distribution. A chi-square goodness-of-fit test performed on the distribution of D, [ϕ(D)] indicated that it was normally distributed with a mean of 9.12° and a standard deviation of 3.21°. For the cost parameters and the road alignment, the following values were used: I = 50°, γ_t = \$12, γ_b = \$1,175, γ_d = \$1,175, γ_e = \$30, γ_f = \$3,175, γ_k = \$116, and $h(D)$ = 0.0336. Most of these values were obtained from the TRB Special Report 214 (*12*).

The graphical solutions to Equation 14 (when m = 1, 2, and 3, and the construction cost function is linear, quadratic, and inverse) are illustrated in Figure 1. It shows that when m = 1, the construction cost is linear, the minimum cost occurs at D_d^* = 10°. The sensitivity of optimum solution to the construction cost can also be seen in Figure 1. When C_c is given by Equations 11(*b*) and 11(*c*), the optimum solutions are D_d^* = 8.75° and 5.5°, respectively.

The optimum solution is also sensitive to the accident cost function, but not to the same extent as construction cost. For example, as shown in Figure 2, when the accident cost function is linear or the rate change of accident rate with respect to the degree of curvature is constant (and construction cost is a linear function), the optimum occurs at D_d = 10°. When the accident cost function is nonlinear (the form given by Equation 3), the optimum occurs at D_d^* = 9.7°.

A comparison of the optimum values and the AASHTO (*1*) recommended values at different design speeds is shown in Table 1. For example, when m = 1, and C_c is linear, the optimum value from the model is 10°, but the AASHTO value at the 85th percentile speed of the same distribution used in the model is 5.4°. However, when m = 1 and C_c is inverse, the modeled value is closer to the AASHTO value when the 80th percentile is used as the design speed. A similar comparison is shown in Table 2 for the case when the accident prediction model is nonlinear.

Constrained Case

In most real-world engineering problems, the objective functions are subjected to several constraints. It is relatively easy to solve a simple optimization problem that is unconstrained, but if constraints are imposed on the problem, few efficient solution techniques are available. The mathematical technique of Lagrange multipliers (*22*) is one of those techniques, but it can be used only when constraints are strict equalities. However, Kuhn-Tucker (*22*) has taken the concept of Lagrange multipliers from mathematical models with active constraints and extended them to mathematical models with active and inactive constraints. In the present case, the follow-

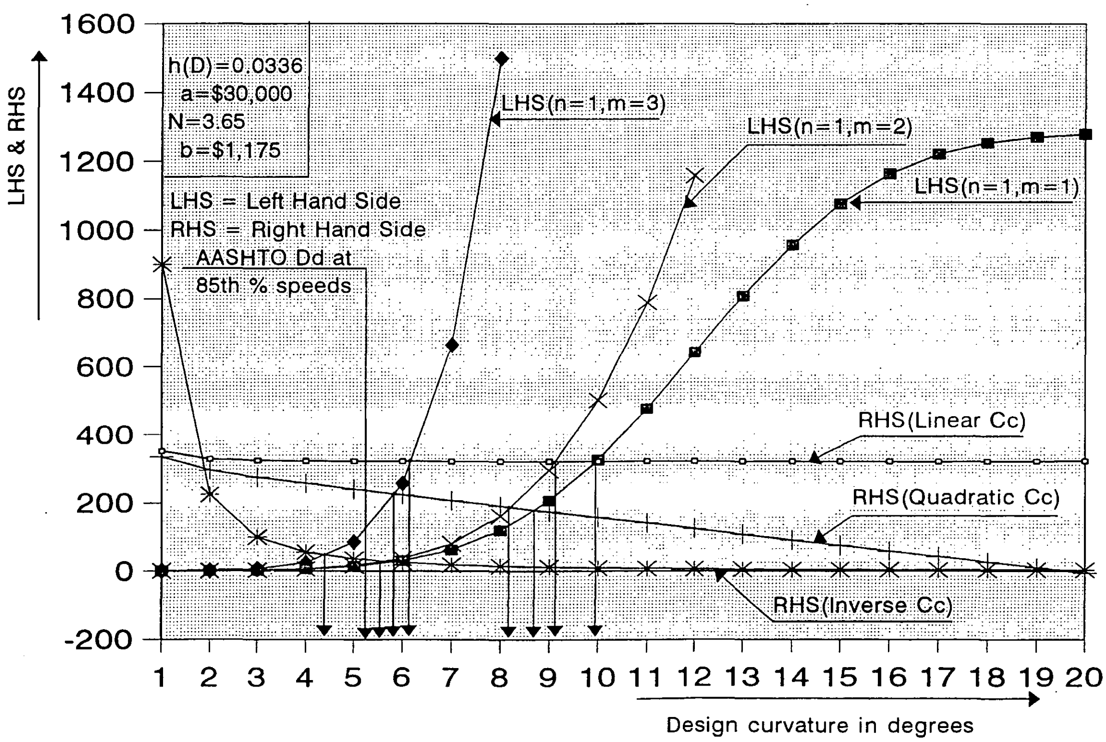

FIGURE 1 Optimum curvature under different curve density and construction cost functions.

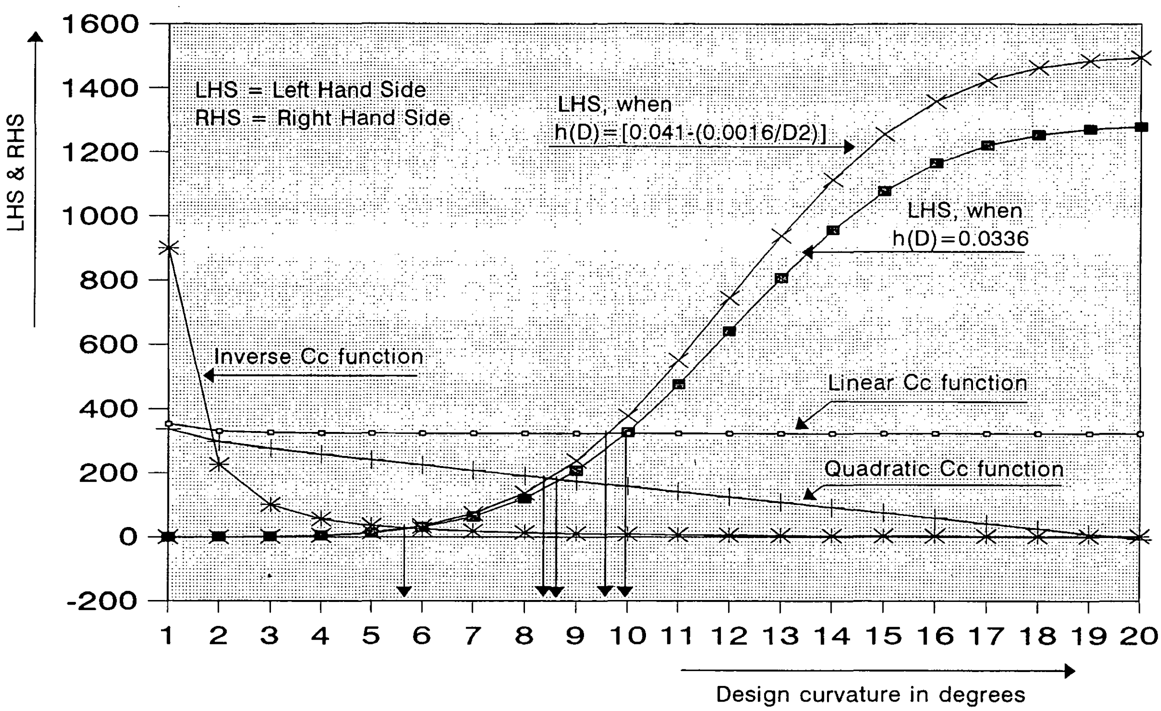

FIGURE 2 Optimum curvature under different accident prediction models.

ing two inequality constraints are considered in developing the model, and the Kuhn-Tucker technique is applied to obtain the optimum value.

Consistency Constraint

Abrupt changes in operating speeds lead to accidents on rural roads, and speed inconsistencies may be largely attributed to abrupt changes in horizontal alignment (i.e., changes in D_d). Lamm et al. (*8*), and Leisch and Leisch (*6*) studied these inconsistencies and suggested maximum allowable speed differentials between two curves and between a tangent and a curve. In the present optimization model, this condition may be expressed in terms of curvature as

$$(D_d - D_a) \leq 0 \tag{15}$$

where D_a is the maximum allowable design curvature from a consistency point of view.

This form of constraint ensures that sharp speed drops are avoided during the optimization process, and that the consistency requirements become an integral part of the analysis.

Environmental and Archaeological Constraint

Environmental and archaeological constraints are key determinants of curvature. However, as environmental and social awareness grow, roadway alignments and dimensions have to be selected in response to those needs. Therefore, a condition was included in the present model to ensure that the sight distance requirements are met under the constraints. This condition may be written in terms of the degree of curvature as follows (*19*):

$$D_s \geq D_d$$
$$\text{or } (D_s - D_d) \geq 0 \tag{16}$$

where D_s is the maximum allowable curvature when middle ordinate is fixed.

The Kuhn-Tucker function and the pertinent constraints can now be written as

$$\begin{aligned} L(D_d, \lambda_1, \mu_1) = {} & \gamma_a N\, h(D_d) \int_0^{D_d} [D_d - D]\, \phi(D)\, dD \\ & + \gamma_\beta N \int_0^{D_d} [[D_d - D]^m \phi(D)\, dD] \\ & + \gamma_t \alpha N \int_0^{D_d} [\omega(D) - \omega(D_d)]\, \phi(D)\, dD \\ & + C_{cm} + \lambda_1 [D_a - D_d] + \mu_1 [D_s - D_d] \end{aligned} \tag{17}$$

subject to:

$$\begin{aligned} & \lambda_1 (D_a - D_d) = 0, \\ & \mu_1 [D_s - D_d] = 0, \\ & D_d \leq D_a, \\ & D_d \geq D_s, \\ & \lambda_1 \leq 0, \\ & D_d \geq 0, \\ & \mu_1 \geq 0, \end{aligned}$$

where λ_1 and μ_1 are control variables associated with less-than-or-equal-to or greater-than-or-equal-to constraints.

The Kuhn-Tucker conditions give a different insight into the nature of the optimum values. For a minimization problem, the Lagrange function must be a minimum. Because it is a sum of terms, each term must be a minimum. Accordingly, in the present case, the

TABLE 1 Optimum D_d and AASHTO Recommended D_d for Different n and m Values, Construction Cost Functions, and $h(D)$ = 0.0336

m	Optimum Design Curvature			AASHTO Recommended Dd			
value	Linear Cc	Quadratic Cc	Inverse Cc	@ 80th % speed	@ 85th % speed	@ 90th % speed	@ 95th % speed
1	10	8.7	5.9	5.8	5.4	4.8	4.4
2	9.3	8.2	5.8	5.8	5.4	4.8	4.4
3	6.3	5.8	4.4	5.8	5.4	4.8	4.4

term $\lambda_1(D_a - D_d)$ will be minimized when $\lambda_1 = 0$ and $(D_a - D_d) \geq 0$, or when $\lambda_1 \leq 0$ and $(D_a - D_d) = 0$. When the constraint is inactive, the Lagrange multiplier will be equal to zero. If λ_1 is equal to zero, the constrained equation will not influence the problem or its minimum value. On the other hand, when the constraint is active, $(D_a - D_d) = 0$. A similar argument can be given to justify the environmental constraint.

Now, D_d, which minimizes the Lagrange function, can be obtained by taking the first derivative of Equation 17 with respect to D_d and equating to zero.

Numerical Example

The impact of the constraints in the optimal solution can be best illustrated using a numerical example. Assuming the following cost parameters are used in Equation 17, the first derivative of it to D_d, when equated to zero, takes the form

$$4675 \int_0^{D_d} \phi(D)\, dD - 1175 - \frac{110}{D_d^2} - \lambda_1 - \mu_1 = 0 \qquad (18)$$

Suppose also that D_a is set equal to 15°, following Leisch and Leisch (*6*), who have suggested that the speed change between a curve and a tangent should be less than or equal to 10 mph. As for the environmental constraints, assume that the middle ordinate cannot exceed 200 ft, and the external angle I is either 50° or 20°. These two curve parameters place a lower bound of 3.2° (when I = 50°) or 9.3° (when I = 20°) on D_s.

For the Kuhn-Tucker conditions to be satisfied, either $\lambda_1 = \mu_1 = 0$; or $\lambda_1 \leq 0$ and $\mu_1 \geq 0$. If $\lambda_1 = \mu_1 = 0$, then D_d^* is not influenced by the constraints and can be derived graphically or numerically as illustrated in the previous case. Otherwise, it should lie between D_s and D_d. For example, it can be seen in Figure 3, where D_d^*s is shown under different speed distribution parameters (V_{mean} and V_{std}), that at V_{mean} = 47 mph and V_{std} = 10 mph, D_d^* is 8.0°. Therefore, it satisfies both constraints. However, if V_{mean} = 45 mph, then the environmental constraint becomes active in that D_d^* should be equal to D_s. Similarly, the active and inactive constraints when V_{std} varies can be seen in Figure 4.

The sensitivity of D_d^* to the cost parameters was tested by changing one parameter at a time while the others were held constant. Subsequently, regression analyses were performed to determine the extent of the sensitivity. In the case of both γ_a and γ_b, nonlinear relationships were observed:

$$D_d^* = 18.3 + 0.00177\, \gamma_a - 0.0796\, \gamma_a^{0.5} \qquad (19)$$
$$(R^2 = 0.98)$$

$$D_d^* = 5.76 + 0.005\, \gamma_b - 0.00001\, \gamma_b^2 \qquad (20)$$
$$(R^2 = 0.99)$$

The relationships are shown in Figures 5 and 6, respectively.

CONCLUSIONS

The optimization model allows uncertainties in traffic operations to be incorporated into the decision-making process by seeking the optimal design curvature under different speed distributions. Because speed distributions depend on traffic mix, terrain, and driver characteristics, if the expected distribution at a selected site can be accurately described, uncertainty stemming from the stochasticity of traffic operation can be treated effectively. In examining the sensitivity of optimal curvature to cost functions, it became evident that the form of the construction function (whether linear or nonlinear) has more of an influence than the form of the accident cost function. Likewise, the unit cost of accidents and construction have different impacts in that the rate of change in optimal curvature with respect to the costs are linear in the case of accidents and nonlinear in the case of the construction. That difference can be seen in Figures 5 and 6.

The authors believe that the most important feature of this model is its ability to make the process of selecting the design curvature a formal and integral one. The constraints and costs can be considered simultaneously instead of at different stages of the design process.

TABLE 2 Optimum Dd and AASHTO Recommended Dd for Different n and m Values, Construction Cost Functions, and $h(D)$ = [0.0412-(0.0016/D²)]

m	Optimum Design Curvature			AASHTO Recommended Dd			
value	Linear Cc	Quadratic Cc	Inverse Cc	@ 80th % speed	@ 85th % speed	@ 90th % speed	@ 95th % speed
1	9.6	8.6	5.5	5.8	5.4	4.8	4.4
2	8.9	8.1	5.4	5.8	5.4	4.8	4.4
3	6.3	5.8	4.4	5.8	5.4	4.8	4.4

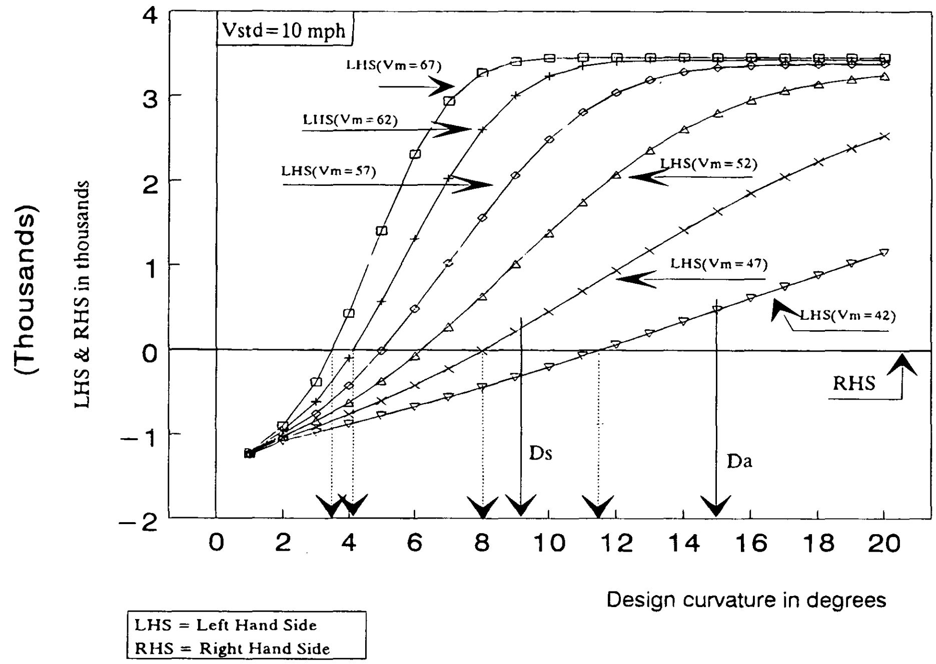

FIGURE 3. Sensitivity of optimum curvature to speed distribution parameters.

The sensitivity can be tested instantly. Moreover, the model provides the engineer with a systematic and rational basis for justifying designs under uncertainty. The ability to incorporate experience and subjective judgment into the decision-making process through the definition of exceedance probabilities and cost parameters gives the designer added flexibility and a sense of personal involvement.

The authors acknowledge that some designers may be apprehensive with this approach, particularly regarding the validity of the underlying accident prediction models and construction cost models. The authors believe, however, that this problem will be resolved as better prediction models become available. Finally, the present model is not intended to provide precise answers or to gen-

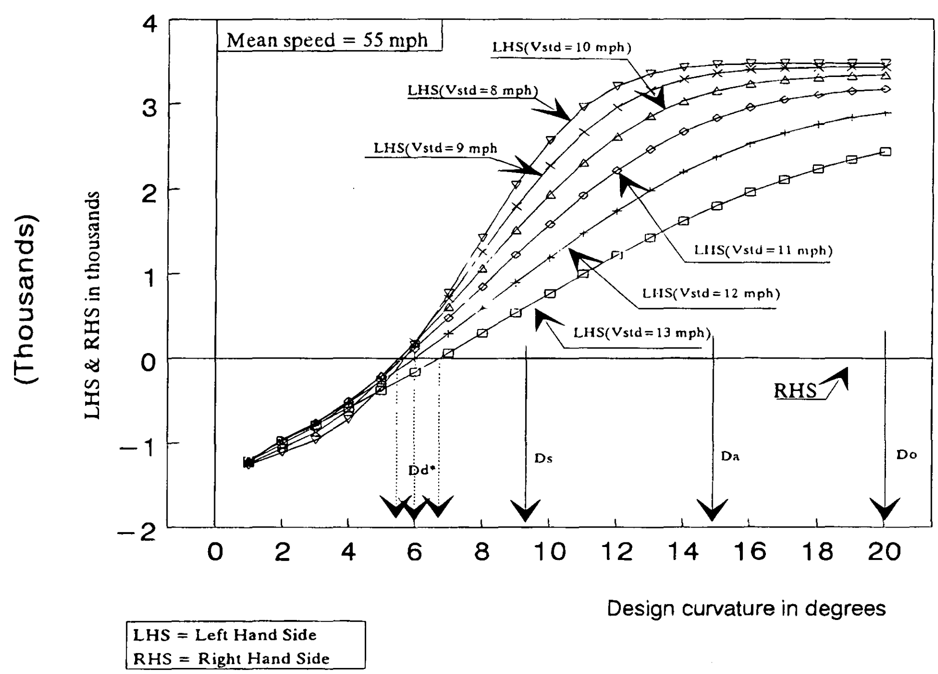

FIGURE 4 Sensitivity of optimum curvature to speed distribution parameters.

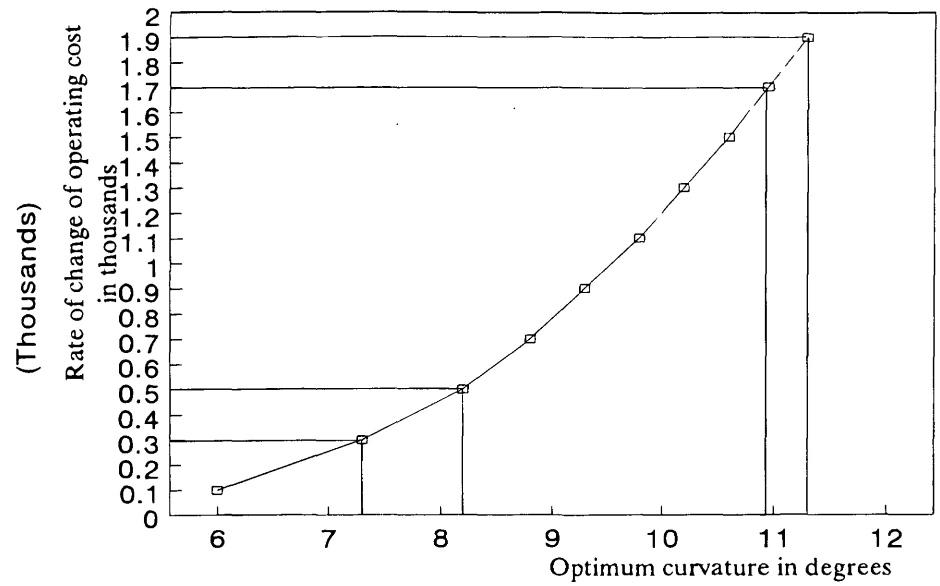

FIGURE 5 The rate of change of operating cost as a function of optimum curvature when construction cost is linear.

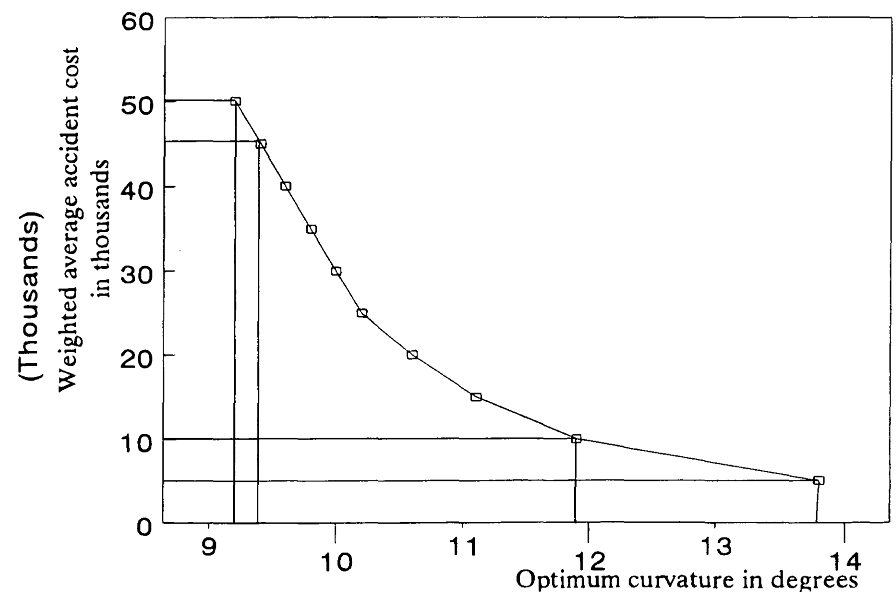

FIGURE 6 The rate of change of weighted average accident cost as a function of optimum curvature when construction cost is linear.

erate exact design values, but is meant as a tool that can be used to compare and perhaps evaluate design values in AASHTO (*1*) or similar manuals.

REFERENCES

1. *A Policy on Geometric Design of Highways and Streets.* American Association of State Highway and Transportation Officials, Washington, D.C., 1990.
2. Newman, T. R. Design Risk Analysis. Address to 71st Annual Meeting of the Transportation Research Board, National Research Council, Washington, D.C., 1992.
3. Navin, F. P. D. Safety Factors for Road Design: Can They Be Estimated? In *Transportation Research Record 1280,* TRB, National Research Council, Washington, D.C., 1990, pp. 181–189.
4. Ben-Akiva, M. Probabilistic and Economic Factors in Highway Geometric Design. *Transportation Science,* Vol. 19, No. 1, Feb. 1985, pp. 38–57.
5. Hirsch, M., J. N. Prashker, and M. Ben-Akiva. New Approach to Geometric Design of Highways. In *Transportation Research Record 1100,* TRB, National Research Council, Washington, D.C., 1986, pp. 50–57.
6. Leisch, J. E., and J. P. Leisch. New Concepts in Design-Speed Application. In *Transportation Research Record 631,* TRB, National Research Council, Washington, D.C., 1977, pp. 4–14.
7. Lamm, R., E. M. Choueiri, J. C. Haywood, and A. Paluri. Possible Design Procedure to Promote Design Consistency in Highway Geometric Design on Two-Lane Rural Roads. In *Transportation Research*

Record 1195, TRB, National Research Council, Washington, D.C., 1988, pp. 111–122.

8. Lamm, R., and E. M. Choueiri. Recommendations for Evaluating Horizontal Design Consistency Based on Investigation in the State of New York. In *Transportation Research Record 1122,* TRB, National Research Council, Washington, D.C., 1987, pp. 68–78.
9. Dart, O. K., Jr., and L. Mann, Jr. Relationship of Rural Highway Geometry to Accident Rates in Louisiana. In *Highway Research Record 312,* HRB, Washington, D.C., 1970, pp. 1–16.
10. Jorgensen, R. *Cost and Safety Effectiveness of Highway Design Elements.* Roy Jorgensen & Associates, National Cooperative Highway Research Program Report 197, TRB, National Research Council, Washington, D.C., 1978.
11. Glennon, J. C., T. R. Newman, and J. C. Jack. *Safety and Operational Considerations for Design of Rural Highway Curves.* Report FHWA/RD-86/035. FHWA, U.S. Department of Transportation, 1985.
12. Transportation Research Board. *Designing Safer Roads.* Special Report 214. TRB, National Research Council, Washington, D.C., 1987.
13. Zegeer, C. R. Stewart, E. Council, P. Reinfurt, and E. Hamilton. Safety Effects of Geometric Improvements on Horizontal Curves. Preprint No. 920893, Transportation Research Board's 71st Annual Meeting, National Research Council, Washington, D.C., 1992.
14. Seneviratne, P. N., and M. N. Islam. Treating Uncertainty in the Design of Horizontal Curves. *Proc., ASCE Conference on Infrastructure Planning and Management,* Denver, Colo., 1993, pp. 479–488.
15. Seneviratne, P. N., and M. N. Islam. Optimum Curvature for Simple Horizontal Curves. *Journal of Transportation Engineering,* ASCE, Vol. 120, No. 5, Sept./Oct. 1994, pp. 773–786.
16. Newman, T. R., and J. C. Glennon. Cost-Effectiveness of Improvements to Stopping-Sight Distance Safety Problems. In *Transportation Research Record 923,* TRB, National Research Council, Washington, D.C., 1983, pp. 26–34.
17. Islam, M. N., and P. N. Seneviratne. *Evaluation of Design Consistency of Two-Lane Rural Highways.* Institute of Transportation Engineers, Washington, D.C., Feb. 1994, pp. 28–31.
18. *A Manual on User Benefit Analysis of Highway and Bus Transit Improvements.* American Association of State Highway and Transportation Officials, Washington, D.C., 1977.
19. Islam, M. N. *Determination of Optimum Horizontal Curvature.* Ph.D. dissertation. Department of Civil and Environmental Engineering, Utah State University, Logan, Utah, 1994.
20. Lin, F. B. Flattening of Horizontal Curves on Rural Two-Lane Highways. *Journal of Transportation Engineering,* ASCE, Vol. 116, No. 2, March/April, 1990, pp. 181–196.
21. *Dodge Guide to Public Works and Heavy Construction Costs.* McGraw-Hill, New York, 1986.
22. Ravindran, A., D. Phillips, and J. Folberg. *Operations Research Principles and Practice,* 2nd ed. John Wiley and Sons, New York, 1987.

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