AM/PM Congestion Pricing with a Single Toll Plaza

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In an effort to reduce queuing delays at toll booths, many toll facilities now collect the toll only in one direction. In fact, many older facilities have removed existing toll plazas/barriers, and many newer facilities are constructing only a single plaza/barrier. Unfortunately, this makes it difficult to charge time-varying tolls in both directions even with electronic toll collection since it is unlikely that all vehicles will be equipped with this technology. This study explores how this difficulty might be overcome.

When toll facilities were first constructed and for many years thereafter it was common to collect tolls from vehicles traveling in both directions. Indeed, this approach is quite natural since in many cases it is not necessary to use the same facility in both directions. Unfortunately, as the amount of traffic on these facilities increased, so did the amount of time spent in queues waiting to pay the toll. In an effort to reduce the amount of time wasted in queues (and reduce the cost of collecting the tolls) many facilities began collecting tolls in one direction only, charging the round-trip toll in that direction. This policy has worked so well that many facilities removed the second (unnecessary) toll plaza/barrier (e.g., the tunnels and bridges connecting New York and New Jersey, the Sumner/Callahan Tunnels in Boston). In addition, many newer facilities are being constructed with a single toll plaza/barrier (i.e., in one direction only).

Unfortunately, while this practice does seem to have worked well in the past, it has been argued that it makes it very difficult to implement some kinds of pricing policies. Recall that toll policies can be used in different ways to influence the decision to travel, destination choice, mode choice, route choice, and departure time choice (*1*). When tolls can be collected only in one direction it becomes impossible to use time-varying tolls to influence the departure time choices of people traveling in both directions.

At first glance, it would seem that this problem could easily be overcome using electronic toll collection (ETC) (*2*). However, since it is virtually impossible (at this point in time anyway) to require that all vehicles make use of ETC, it is not immediately clear that this technological fix is workable.

In this report we will discuss how ETC may make it possible to implement a.m. and p.m. congestion pricing even when there is a toll plaza/barrier in only one direction and all vehicles are not required to make use of ETC. In addition, we will discuss how this approach may correct some of the adverse distributional impacts of congestion pricing, eliminate the need to redistribute the toll revenues, and allay the fears [see, for example, Higgins (*3*)] that congestion pricing is unfair, discriminatory, regressive, coercive and anti-business. The approach we suggest for achieving these goals makes use of both time-varying tolls and time-varying subsidies, as discussed by Bernstein (*4*).

To illustrate the potential benefits of this approach we extend the traditional one-directional model (*5–9*) so that it can be used to study a.m./p.m. commuting. As it turns out, this is not equivalent to simply "considering the a.m. peak twice" for several reasons. First, as discussed by Fargier (*10*), the commuting schedule in the evening is different from that in the morning (e.g., there is no desired arrival time for the p.m. trip). Second, work-to-home trips often involve secondary trips (e.g., shopping, dinner) making the origin/destination, route and departure time choices more irregular. Third, a.m. and p.m. decisions are not independent (i.e., the decision you make in the a.m. affects the one you make in the p.m.).

This study begins with a description of the model itself. It then considers a.m./p.m. tolling with one plaza when there is only one relevant route. Next, it considers the implications of a.m./p.m. tolling on multiple routes. Finally, it considers a variety of implementation details and concludes with a discussion of future research.

THE MODEL

In order to get some insight into commuters' route and departure time decisions, we will work with a model with N homogenous commuters traveling between home and work. The decisions for a commuter are to choose both their a.m. and p.m. departure times and routes in order to minimize their round-trip travel cost. The travel cost is composed of the total travel time and the schedule delay (plus tolls if any).

$$C = \alpha[T(t_a) + T(t_p)] + (\Phi_a + \Phi_p) + (\tau_a + \tau_p) \quad (1)$$

where C is the travel cost; $T(t_a)$ and $T(t_p)$ are the travel times for the a.m. and p.m. trips; Φ_a and Φ_p are the a.m. and p.m. schedule delays; τ_a and τ_p are the a.m. and p.m. tolls (or subsidies); and α is the dollar value of travel time. There is a desired work schedule starting from t_a^* and ending at t_p^*. Whenever a person does not arrive on time or leave on time, a positive schedule delay is incurred.

Of course, the a.m. and p.m. schedule delays may or may not be correlated. For example, suppose people must work exactly 8 hr every day. This implies that the departure time choices for the morning and evening are perfectly correlated (i.e., a person that arrived 20 min late in the morning must leave 20 min late in evening). However, this situation rarely occurs. In most cases, the 8-hr work day can be viewed only as a loose constraint. That is, the departure time decisions for the a.m. and p.m. are not always perfectly dependent. In fact, in some cases they are independent. For example, some people have fixed start and end times for their work day. Hence, even

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if they arrive after 9:00 a.m. they do not get compensated for working after 5:00 p.m. For simplicity, here we assume that the schedule delays are completely independent. The a.m. schedule delay depends only on when the commuter arrives at work in the morning, and the p.m. schedule delay depends only on when he/she leaves from work in the evening. Therefore the schedule delays are given by:

$$\Phi_a = \begin{cases} \beta[t_a^* - (t_a + T(t_a))] & \text{if } [t_a + T(t_a)] < t_a^* \\ \gamma[(t_a + T(t_a)) - t_a^*] & \text{if } [t_a + T(t_a)] \geq t_a^* \end{cases} \quad (2)$$

and

$$\Phi_p = \begin{cases} \delta(t_p^* - t_p) & \text{if } t_p < t_p^* \\ \theta(t_p - t_p^*) & \text{if } t_p \geq t_p^* \end{cases} \quad (3)$$

where β and γ denote the dollar penalties for early and late arrivals to work, and δ and θ are the dollar penalties for early and late leaves from work. In addition, following notations for some important time points are introduced: t_j^0 = beginning of peak ($j = a, p$), t_j^e = ending of peak ($j = a, p$) and $\tilde{t}_a$ = a.m. departure time to arrive at work on time [i.e., $\tilde{t}_a + T(\tilde{t}_a) = t_a^*$].

We also assume that the time needed to travel in each direction can be modeled as a deterministic queuing process in which:

$$T(t_j) = D(t_j)/s, \; j = a, p \quad (4)$$

where s is the service rate (road capacity) and $D(t_j)$ is the queue length at time t_j. This approach is believed to represent actual travel time functions fairly well. Finally, we assume that in equilibrium no individual has any incentive to change his/her departure time or route choice. The equilibrium departure rates that arise from such a model are given by:

$$r_a(t_a) = \begin{cases} s\left(1 + \dfrac{\beta}{\alpha - \beta}\right) & \text{for } t_a \in [t_a^0, \tilde{t}_a] \\ s\left(1 - \dfrac{\beta}{\alpha + \gamma}\right) & \text{for } t_a \in [\tilde{t}_a, t_a^e] \end{cases} \quad (5)$$

and

$$r_p(t_p) = \begin{cases} s\left(1 + \dfrac{\delta}{\alpha}\right) & \text{for } t_p \in [t_p^0, t_p^*] \\ s\left(1 - \dfrac{\theta}{\alpha}\right) & \text{for } t_p \in [t_p^*, t_p^e] \end{cases} \quad (6)$$

OPTIMAL PRICING ON A SINGLE ROUTE

We first assume that there is only one route between work and home, and that there is only one toll plaza (in the a.m. inbound direction). As shown by equations (*5*) and (*6*), both the a.m. and p.m. departure rates are greater than the service rate before the desired departure time and smaller than the service rate after that time. Thus, the queues in both directions reach their maximums at the desired departure times. With these results, we can now consider how to construct pricing schemes that eliminate congestion in both directions. As it turns out, there are at least three optimal pricing schemes, each of which is discussed in detail below. The analytical expressions for these pricing schemes are given in Table 1.

Scheme 1: a.m. Toll and p.m. Toll

The first scheme is a traditional one in which commuters are charged positive tolls for both their a.m. and p.m. trips. The optimal toll structure is shown in Figure 1. Here $\lambda = \beta\gamma/(\beta + \gamma)$ and $\mu = \delta\theta/(\delta + \theta)$. The a.m. peak starts at t_a^0 and ends at t_a^e and the p.m. peak starts at t_p^0 and ends at t_p^e.

Under this scheme, the tolls are zero at the beginnings and the ends of a.m. and p.m. rush hours, and they reach the peaks at the scheduled arrival time t_a^*, and the scheduled departure time t_p^*. The merit of this pricing scheme is that it only charges commuters (i.e., people who travel during the rush hours); noncommuters (i.e., people who travel outside of the rush hours) can continue to enjoy their trips free of charge in both directions.

Though in theory the above pricing scheme can eliminate traffic congestion and has some nice properties, in practice it is not the preferable approach for two reasons. First, this type of scheme is subject to the criticisms that it is unfair, discriminatory, regressive, coercive, and anti-business (*3*). The average individual's share for this "tax" in above scheme is clearly greater than zero and it increases as the peak duration becomes longer. It is not clear how this toll revenue is redistributed to the society. Second, this scheme requires two toll plazas, one in each direction, to collect the tolls. Note that this problem cannot be overcome by ETC unless all vehicles are equipped since there would be no way to charge unequipped vehicles without the toll plaza. Hence, there would be no way to influence their behavior. In addition, equipped vehicles would pay higher tolls than unequipped vehicles and hence would be encouraged to stop using ETC. It is clear that this pricing scheme cannot be implemented without two enforcement barriers no matter whether there exists an ETC or not.

Scheme 2: a.m. Toll/Subsidy and p.m. Subsidy

The second scheme is designed to consider the two practical requirements: that there is no barrier for the p.m. outbound and that the toll revenue must be zero. By imposing these two constraints, the drawbacks of the previous scheme can be eliminated. The method for incorporating these constraints is to impose negative tolls (subsidies) on the p.m. outbound direction in which there is no toll plaza. Such a pricing scheme with tolls and subsidies, which is still optimal, is drawn in Figure 2.

In Figure 2, it is interesting to note that the a.m. toll can be positive or negative depending on the parameters. If $\lambda > \mu$, then all commuters arrive before $t_a^* - [(\lambda + \mu)/\beta](N/2s)$ and after $t_a^* + [(\lambda + \mu)/\beta](N/2s)$ will receive a subsidy and all others have to pay a toll. However, if $\lambda \leq \mu$, then all commuters must pay positive tolls in the morning.

It is also interesting to note that the toll revenue collected in the morning is redistributed to the commuters in the evening. It can be seen from Figure 2 that the total toll revenue is zero. Therefore there is no reason for people to view this type of congestion pricing as a tax. More importantly, the p.m. subsidy can be distributed without a toll plaza. Vehicles equipped with ETC will be able to receive the subsidy and unequipped vehicles will not. Thus, this will encourage people to participate in the ETC systems.

TABLE 1 Pricing Schemes for One Route

Pricing Schemes	AM Peak			PM Peak		
	time	toll/subsidy	sign	time	toll/subsidy	sign
Scheme 1	$t_a^0 \sim t_a^*$	$\lambda \frac{N}{s} - \beta(t_a^* - t_a)$	+	$t_p^0 \sim t_p^*$	$\mu \frac{N}{s} - \delta(t_p^* - t_p)$	+
	$t_a^* \sim t_a^e$	$\lambda \frac{N}{s} - \gamma(t_a - t_a^*)$	+	$t_p^* \sim t_p^e$	$\mu \frac{N}{s} - \theta(t_p - t_p^*)$	+
Scheme 2	$t_a^0 \sim t_a^*$	$(\lambda + \mu)\frac{N}{2s} - \beta(t_a^* - t_a)$	+ if $\lambda<\mu$	$t_p^0 \sim t_p^*$	$-\delta(t_p^* - t_p)$	–
	$t_a^* \sim t_a^e$	$(\lambda + \mu)\frac{N}{2s} - \gamma(t_a - t_a^*)$	+ if $\lambda<\mu$	$t_p^* \sim t_p^e$	$-\theta(t_p - t_p^*)$	–
Scheme 3	$t_a^0 \sim t_a^*$	$\lambda \frac{N}{s} - \beta(t_a^* - t_a)$	+	$t_p^0 \sim t_p^*$	$(\mu - \lambda)\frac{N}{2s} - \delta(t_p^* - t_p)$	– if $\lambda>\mu$
	$t_a^* \sim t_a^e$	$\lambda \frac{N}{s} - \gamma(t_a - t_a^*)$	+	$t_p^* \sim t_p^e$	$(\mu - \lambda)\frac{N}{2s} - \theta(t_p - t_p^*)$	– if $\lambda>\mu$

Scheme 3: a.m. Toll and p.m. Subsidy/Toll

An alternative scheme that may also meet the requirements of zero toll revenue and one barrier is illustrated in Figure 3. In this scheme, a.m. toll is positive during the entire morning rush hours (a pure toll) and this would simplify the toll collection for the a.m. trips. However the p.m. toll can either be negative during the whole rush hour (a pure subsidy) or be negative in some periods and positive in the others (a mixed toll and subsidy), depending on the relationship among the parameters. If $\lambda \geq \mu$, then p.m. toll is a pure subsidy structure which is desirable. If, however, $\lambda < \mu$, then during any period between $t_p^* - (\lambda - \mu)/(N/2s)$ and $t_p^* + (\lambda - \mu)/(N/2s)$ a positive toll is charged. Nevertheless, as can be seen in Figure 3, the total toll revenue is always zero regardless of the parameters.

Comparing Different Schemes

Though in theory all three of the above pricing schemes are socially optimal, Scheme 1 is the most difficult one to implement. However, it is worth considering the pros and cons of Schemes 2 and 3 in somewhat more detail. Depending upon the parameters, either Scheme 2 or Scheme 3 must have a mixed toll/subsidy structure for the same direction trips. In Scheme 3, there might exist some periods during which the p.m. tolls are positive but clearly such positive tolls cannot be collected without a toll plaza. On the other hand, though Scheme 2 could be operated from a purely technological standpoint, it introduces a nontechnological problem. Observe that, under the condition of $\lambda > \mu$, there are some periods in the a.m. and all the periods in the p.m. during which commuters are actually

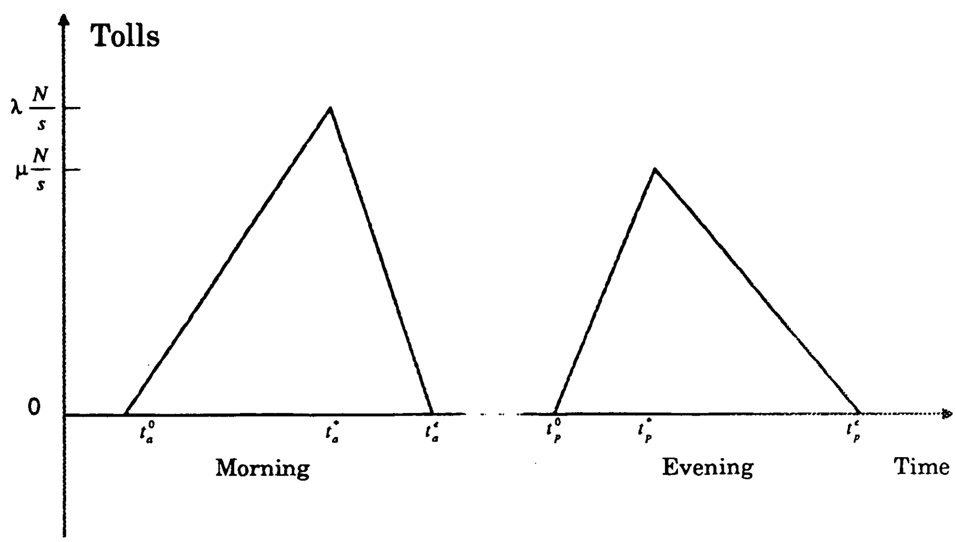

FIGURE 1 Scheme 1—a.m. and p.m. tolls.

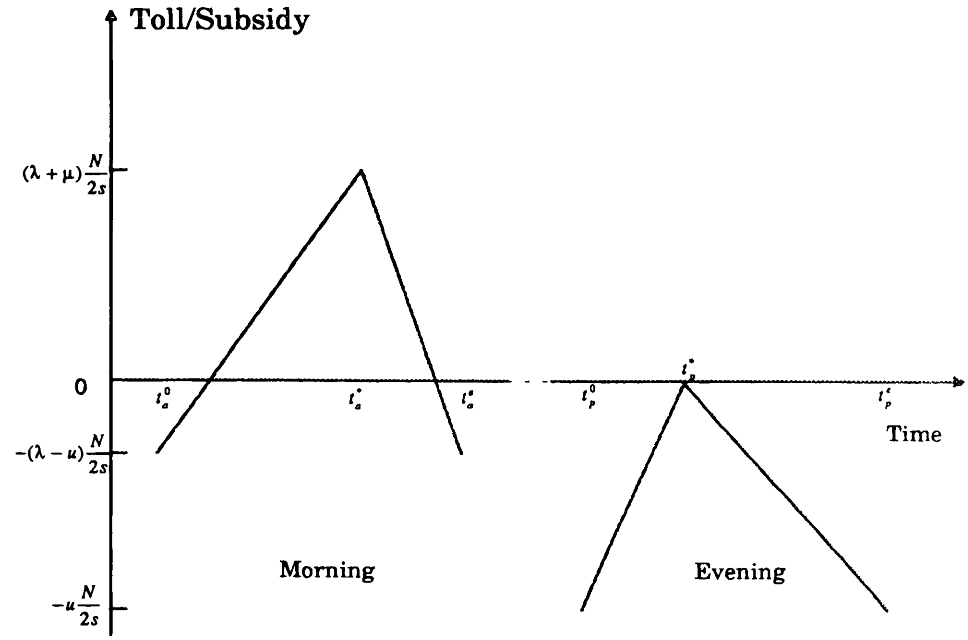

FIGURE 2 Scheme 2—a.m. toll/subsidy and p.m. subsidy.

being paid to use the road. Therefore it is possible that a person can make money simply by traveling back and forth during the subsidy periods. This means that Scheme 2 can encourage spurious trips (i.e., trips simply aimed at receiving subsidies). Though, as discussed later, there may be some ways to discourage such spurious trips using existing technologies, the incentive for such spurious trips should be kept as low as possible. Observe that a pricing program with a pure toll for the a.m. trips and a pure subsidy for the p.m. trips is implementable in our context. Such a pure toll/subsidy program may also discourage spurious trips because the a.m. toll may outweigh the p.m. subsidy.

It follows that if $\lambda < \mu$, then Scheme 2 should be chosen; if, however, $\lambda > \mu$, then Scheme 3 should be chosen. By this selection criterion, we can get a program with a pure toll for the a.m. and a

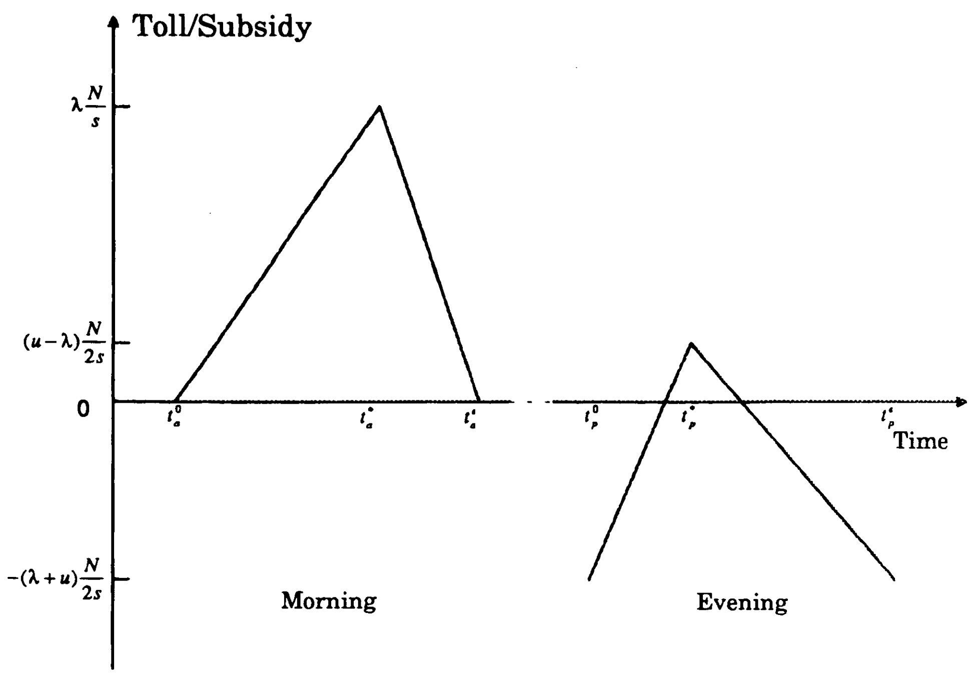

FIGURE 3 Scheme 3—a.m. toll and p.m. toll/subsidy.

pure subsidy for the p.m. When $\lambda = \mu$, there is no difference between Scheme 2 and 3. Observe that this selection criterion does not depend on the roadway condition (i.e., capacity). It is only determined by how people value the schedule delay and thus it is applicable anywhere.

A Numerical Example

In order to illustrate these ideas we consider a numerical example. We assume that $t_a^* = $ 9:00 a.m., $t_p^* = $ 5:00 p.m., $s = 6000$ vehicle-hr and $N = 10{,}000$ vehicles. For the shadow value parameters we use the oft-cited values for the a.m. trips (*11,12*) $\alpha = \$6.40$, $\beta = \$3.90$, $\gamma = \$15.21$. For the p.m. trips we arbitrarily use $\delta = \gamma = \$15.21$, and $\theta = \$3.00$. Here θ is assumed to be smaller than β because people can participate in secondary activities after work and before heading for home, such as shopping, dining and other social activities. This possibility decreases the shadow value of departing late (*13*).

In this case, the rush hour lasts from 7:40 a.m. to 9:20 a.m. in the morning and from 4:44 p.m. to 6:24 p.m. in the evening. Since $\lambda > \mu$ in this example, Scheme 3 is selected. As shown in Figure 4, the a.m. toll first increases smoothly from zero at 7:40 a.m., reaches the peak of \$5.17 at 9:00 a.m. and then falls to zero at 9:20 a.m. The p.m. subsidy begins with a maximum of \$4.17 at 4:44 p.m., falls to a minimum \$1.00 at the 5:00 p.m., and then increases and reaches the maximum again at 6:24 p.m.

We should compare our two-way work trip model with the one-way morning trip model. Table 2 shows the average costs per commuter under four scenarios: no toll, one-direction toll, two-directional tolls and two-directional toll/subsidy. From this table, it is clear that one-directional tolling is not efficient and can be improved (i.e., social savings increase from 27.7 to 50%).

OPTIMAL PRICING ON TWO ROUTES

We now consider a network in which there are two parallel routes. Let the capacity at Route 1 be s_1 and the capacity at Route 2 be s_2. When there is no congestion pricing, the equilibrium departure rates can be shown as follows:

$$r_{ia}(t_{ia}) = \begin{cases} s_i\left(1 + \dfrac{\beta}{\alpha - \beta}\right) & \text{for } t_{ia} \in [t_{ia}^0, \tilde{t}_{ia}] \\ s_i\left(1 - \dfrac{\beta}{\alpha + \gamma}\right) & \text{for } t_{ia} \in [\tilde{t}_{ia}, t_{ia}^e] \end{cases} \qquad i = 1, 2 \tag{7}$$

and

$$r_{ip}(t_{ip}) = \begin{cases} s_i\left(1 + \dfrac{\delta}{\alpha}\right) & \text{for } t_{ip} \in [t_{ip}^0, t_{ip}^*] \\ s_i\left(1 - \dfrac{\theta}{\alpha}\right) & \text{for } t_{ip} \in [t_{ip}^*, t_{ip}^e] \end{cases} \qquad i = 1, 2 \tag{8}$$

where $i = 1, 2$ is the route index. This result is similar to the single route case. It can also be shown that the beginning and ending times of peaks for two routes are the same:

$$t_{1j}^0 = t_{2j}^0, \text{ and } t_{1j}^e = t_{2j}^e \qquad j = a, p \tag{9}$$

The route split between two routes is proportional to the ratio of the capacities (s_1/s_2). This equilibrium split coincides with the system optimum. The intuition behind is clear: the larger the road, the more people are on that road.

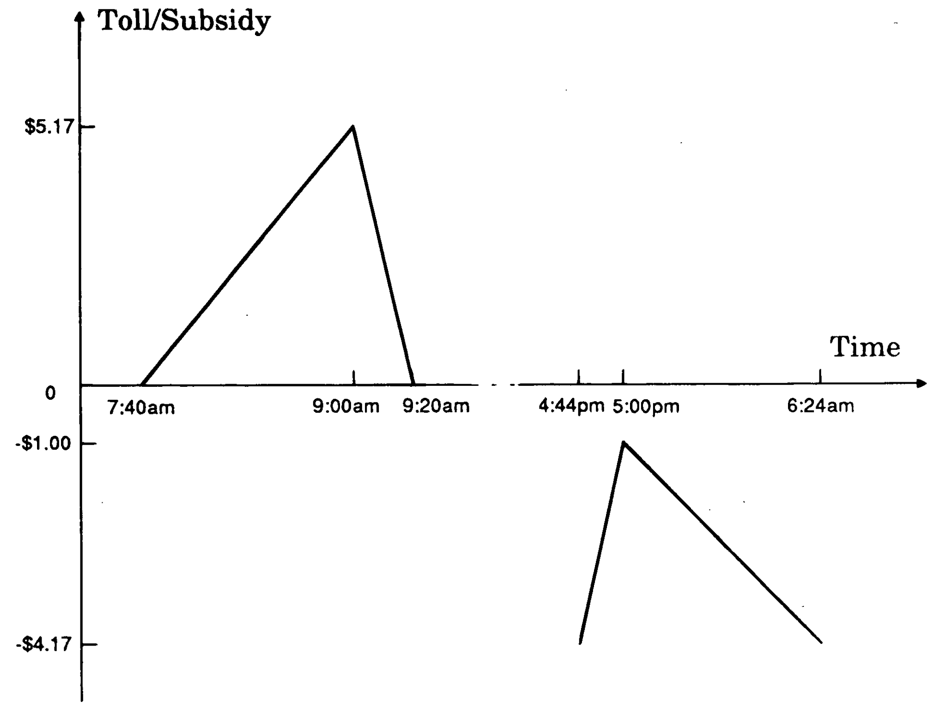

FIGURE 4 a.m. toll and p.m. subsidy.

TABLE 2 Average Costs Under Different Schemes

		Pricing Schemes			
		No Tolls	AM Toll only	AM & PM Toll	AM Toll & PM Subsidy
Schedule Delay	AM	\$2.59	\$2.59	\$2.59	\$2.59
	PM	\$2.09	\$2.09	\$2.09	\$2.09
Travel Time Cost	AM	\$2.59	0	0	0
	PM	\$2.09	\$2.09	0	0
Social Cost (exclude toll)		\$9.36	\$6.77	\$4.68	\$4.68
Commuter Cost (include toll)		\$9.36	\$9.36	\$9.36	\$4.68
Social Savings (%)		none	27.7%	50.0%	50.0%
Commuter Savings (%)		none	none	none	50.0%

This result seems to suggest that an optimal pricing scheme should not alter the users' route choices since they are already optimal. However, this observation may not be true in some cases when not all of the routes are priced. Once some routes cannot be priced, the best (i.e., system optimal) pricing scheme may be not achievable. Instead, the second-best should be used. We now consider a.m./p.m. pricing for two cases: when both roads can be tolled and when one must be left untolled.

Case I: Two Tolled Roads

In the first case we assume that both roads can be tolled. Specifically we assume that there are two toll plazas in the a.m. inbound direction, one in each road and there is no toll plaza in the p.m. outbound direction. Since there is no toll plaza in the p.m. outbound, subsidies are used to price the evening traffic.

Analogous to the one route case, there are two alternative optimal schemes combining tolls and subsidies, as given in Table 3. These two schemes are completely analogous to Scheme 2 and Scheme 3 in the single route case except that the service rate has been replaced by the summation of the two routes' service rates. Again, in order to get a pure toll/subsidy scheme, the selection between these two schemes depends on the parameters λ and μ. It is also interesting to see that the tolls or subsidies at two routes are always equal. Therefore the route split is unaffected since the equilibrium split is optimal.

In this case, we can extend above results to multiple routes in parallel. That is, they can be treated as one single route in which the capacity is the summation of all routes. The road usage is propor-

TABLE 3 Pricing Schemes for Two Tolled Routes

Pricing Schemes	AM Peak			PM Peak		
	time	toll/subsidy (route 1=route 2)	sign	time	toll/subsidy (route 1=route 2)	sign
Scheme 1	$t_a^0 \sim t_a^*$	$(\lambda+\mu)\frac{N}{2(s_1+s_2)} - \beta(t_a^* - t_a)$	+ if $\lambda<\mu$	$t_p^0 \sim t_p^*$	$-\delta(t_p^* - t_p)$	–
	$t_a^* \sim t_a^e$	$(\lambda+\mu)\frac{N}{2(s_1+s_2)} - \gamma(t_a - t_a^*)$	+ if $\lambda<\mu$	$t_p^* \sim t_p^e$	$-\theta(t_p - t_p^*)$	–
Scheme 2	$t_a^0 \sim t_a^*$	$\lambda\frac{N}{(s_1+s_2)} - \beta(t_a^* - t_a)$	+	$t_p^0 \sim t_p^*$	$(\mu-\lambda)\frac{N}{2(s_1+s_2)} - \delta(t_p^* - t_p)$	– if $\lambda>\mu$
	$t_a^* \sim t_a^e$	$\lambda\frac{N}{(s_1+s_2)} - \gamma(t_a - t_a^*)$	+	$t_p^* \sim t_p^e$	$(\mu-\lambda)\frac{N}{2(s_1+s_2)} - \theta(t_p - t_p^*)$	– if $\lambda>\mu$

tional to the capacity regardless of pricing or not. The starting times and the durations of the congestion are the same for all routes. Finally, the optimal time-varying tolls are also the same for all roads during any time of the day.

Case II: One Tolled Road and One Untolled Road

The more common situation in the real world is the existence of a mixture of tolled and untolled facilities. Therefore it is more important to study the case in which one route can be tolled and the other cannot be tolled. This may result because it is either physically impossible or publicly unacceptable to collect tolls on all facilities. More interestingly, this situation can also represent the single route case in which there are two types of toll booths: manual and ETC-only. The non-ETC lane and the ETC lane can be modeled as two routes and the decisions on equipping ETC tags or not can be seen as the choices between two different routes.

We assume, without loss of generality, that Route 1 can be tolled and Route 2 cannot. As shown in Figure 5, there is one toll station located on Route 1 a.m. inbound and one ETC reader on the Route 1 p.m. outbound. Since there is no enforcement mechanism at the ETC reader, the only practical pricing scheme must be an a.m. toll and p.m. subsidy scheme on Route 1.

There are four possible paths for a round trip as follows:

- Path 1: taking both Route 1 in the a.m. and p.m. ($1a \to 1p$);
- Path 2: taking both Route 2 in the a.m. and p.m. ($2a \to 2p$);
- Path 3: taking Route 2 in the a.m. and Route 1 in the p.m. ($2a \to 1p$);
- Path 4: taking Route 1 in the a.m. and Route 2 in the p.m. ($1a \to 2p$).

Once a toll/subsidy pricing program is implemented, the costs for these four paths are described as follows. On Path 1, a commuter must pay a toll in the a.m., receive a subsidy in the p.m. and incur no travel time cost on either trip. On Paths 2 and 3, there is no toll or subsidy but the travel time costs are nonzero. Though link $1p$ is used in the third path of ($2a$, $1p$), there is no subsidy received. As will be explained in the next section, only those persons who have paid the tolls in the a.m. can receive subsidies. On Path 4, a traveler must pay a toll in the a.m. and receive no subsidy and spend some waiting time in the p.m. Thus this path will never be used because its cost is always greater than the cost for Path 1.

We treat the above route choices as if they are made hierarchically. The first path's travelers are viewed as ETC users and the second and third paths' travelers as non-ETC users. The commuters first have to decide to use the ETC system or not. If they use ETC, then there is only one path. If not, they then have to choose between the second path and the third path. This structure is helpful because the congestion pricing scheme can only affect the first level decision—ETC or non-ETC. The route split between the second path and the third path for non-ETC users cannot be influenced since they are not controllable.

Let numbers of people using these three paths be N_1, N_2, and N_3. The equilibrium road usage can be derived as:

$$N_1 = (1 - \mathrm{A})\mathrm{N} \tag{10}$$

$$N_2 = \frac{s_2}{s_1 + s_2}\mathrm{AN} \tag{11}$$

$$N_3 = \frac{s_1}{s_1 + s_2}\mathrm{AN} \tag{12}$$

where $A = [(s_1 + s_2)\lambda + s_2\mu]/[(5s_1 + s_2)\lambda + (s_1 + s_2)\mu]$. The split between Paths 2 and 3 is based on the road capacities while the split between ETC and non ETC users depends on both the schedule delay parameters and the road capacities. This equilibrium route split for the non-ETC users is not optimal for most cases. This is because non-ETC users generate some unbalanced social costs on two paths while users only pay the private costs, which are equal on two paths. As a result, Path 2 is overused if the capacity of Route 1 is greater than the capacity of Route 2 and Path 3 is overused if the capacity of Route 1 is less than the capacity of Route 2. When the two routes have the same capacity, the equilibrium route split will be optimal.

The average toll for a commuter using Path 1 is

$$\bar{\tau} = \frac{\lambda AN}{s_2} - \frac{\lambda + \mu}{2}\,\frac{(1 - A)N}{s_1} \tag{13}$$

This toll revenue $\bar{\tau}$ can be positive, zero, or negative, depending on the relative capacities, (s_1/s_2), and the relative value of the schedule delay parameters, (μ/λ). For example, when the capacity of Route 1 is less than that of Route 2 (i.e., $s_1 < s_2$), the toll revenue is always negative regardless of the parameters of λ and μ. This also suggests that if we can only toll one road, we should toll the bigger road because tolling on the smaller road will result in a deficit. The optimal toll is given by:

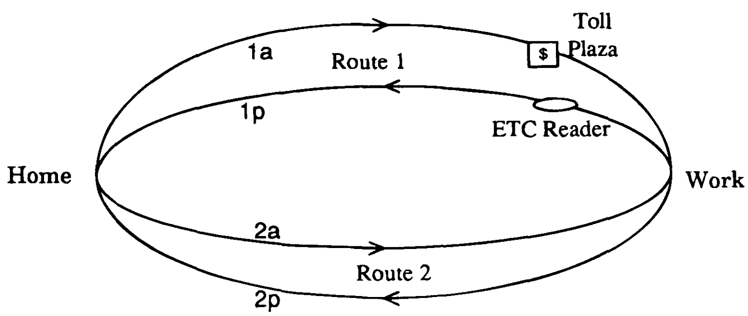

FIGURE 5 Two routes with one toll plaza.

$$\tau_a(t_a) = \begin{cases} \tau_a^0 - \beta(t_a^* - t_a) & \text{if } t_a \in [t_{1a}^0, t_a^*] \\ \tau_a^0 - \gamma(t_a - t_a^*) & \text{if } t_a \in [t_a^*, t_{1a}^e] \end{cases} \tag{14}$$

and

$$\tau_p(t_p) = \begin{cases} \tau_p^0 - \delta(t_p^* - t_p) & \text{if } t_p \in [t_{1p}^0, t_p^*] \\ \tau_p^0 - \theta(t_p - t_p^*) & \text{if } t_p \in [t_p^*, t_{1p}^e] \end{cases} \tag{15}$$

where $\tau_a^0 \geq \lambda(N_1/s_1)$ and $\tau_p^0 \leq 0$ are time-invariant uniform tolls and they must satisfy:

$$\tau_a^0 + \tau_p^0 = \left(\frac{\lambda}{s_2} + \frac{\mu}{s_1 + s_2}\right)AN \tag{16}$$

The pricing scheme for this case is suboptimal, in the sense that both the route split and the departure rate for the non-ETC users are not optimal. Because of the constraint of the optimal split between ETC and non-ETC, the toll revenue can no longer be set to zero.

IMPLEMENTING A.M./P.M. CONGESTION PRICING

There is one issue that needs to be addressed before a congestion pricing program with both tolls and subsidies is implemented. Observe that in the p.m. peak periods drivers are actually being paid to use the road. Hence, such a program, if implemented incorrectly, could generate spurious trips in which people drive simply to receive subsidies. Fortunately, there may exist some ways to prevent such trips [see Bernstein (*4*) for details] in general.

In the specific setting considered here, the most interesting method to discourage spurious trips is to give a subsidy only to those people who were tolled in the other direction. In such a system, if a driver would like to receive a subsidy in the evening peak, he/she would have to take an inbound trip in the morning and pay a toll first. Therefore it is important to have the information of the time and the route of the a.m. inbound trip for each vehicle traveled in the subsidized roads. Such a task is easy to implement using existing technologies. In fact, almost all ETC systems could be modified to record a.m. trip information and charge p.m. tolls (negative) based on the a.m. activities. However, it may be advantageous to use an ETC system with read-write capabilities rather than a read-only system. This is because with a read-write system the information can be recorded in the vehicles themselves rather than in a central computer. Thus there is no worry about "tracking" individual vehicles and invading anyone's privacy. In such a system, whenever a vehicle arrives at the subsidized outbound road in the p.m., the reader/writer on the roadside first checks the information stored in the in-vehicle unit. If it has been tolled in the inbound direction then a credit is refunded to the user's account. Any untagged or not qualified vehicle cannot receive a subsidy.

Of course, it is still possible to receive a pure subsidy even if a toll has been charged in the a.m. This occurs when the subsidy outweighs the toll. However, the time and money costs (e.g., the price of gasoline) would probably outweigh the net subsidy and, therefore, eliminate spurious trips. In addition, if there is a preexisting toll for the purpose of covering construction and maintenance costs, then the a.m. toll may be high enough during any periods to offset the p.m. subsidy.

CONCLUSION

This study explored two-directional congestion pricing for work trips. It extended a previous one-way home-to-work model to a two-way home⇔work model. It showed that by carefully designing a scheme combining tolls and subsidies, a two-directional pricing program could be implemented with one barrier only. Such programs might also assuage some of the opponents of congestion pricing. However, a great deal of further research on dynamic travel behavior is needed before any final conclusion can be drawn.

First, the model needs to incorporate the elastic demand. The travel cost will go down after implementing a toll/subsidy program and this cost reduction may attract more people. For example, noncommuting trips may switch from off-peak periods to peak periods and this can extend the duration of the peak substantially. In addition, it is also expected that some commuters switch from public transportation and this may offset the social savings in implementing such pricing programs.

Second, the schedule-delay function must be extended. Though separable schedule-delay functions greatly simplify the algebra and do yield some insights, it remains unclear how many of the results obtained here rely on this special piecewise linear function.

Third, we need to consider other toll structures besides our continuously time-varying toll/subsidy scheme. In particular, we must consider the step toll/subsidy in which the toll/subsidy is constant for some time intervals because such schemes are likely to be better understood by travelers.

Fourth, it has been assumed that commuters have the same characteristics, such as their work schedule time, their value of travel time, and their value of arriving late. This is clearly not the case in the real world. The extension to treat commuter heterogeneity is very important because it can help us understand how commuters respond to the pricing. The essential insight is that we need to model individuals' decisions instead of an average user's behavior so that the equilibrium can be sustained.

Finally, we need to extend this work to general networks. Simultaneous route and departure time choice equilibrium models (SRD equilibrium models) are now being developed (*14*). Further research needs to be done to apply these models to the study of congestion pricing.

REFERENCES

1. El Sanhouri, I., and D. Bernstein. Integrating Driver Information with Facility-Based Congestion Pricing. Presented at 73rd Annual Meeting of the Transportation Research Board, Washington, D.C., 1994.
2. Bernstein, D., and A. Kanaan. Automatic Vehicle Identification: Technologies and Functionalities. *IVHS Journal*, Vol. 1, No. 2, 1993.
3. Higgins, T. Road-Pricing Attempts in the United States. *Transportation Research*, Vol. 20A, 1986, pp. 145–150.
4. Bernstein, D. Congestion Pricing with Tolls and Subsidies. *Proc., Pacific Rim TransTech Conference*, Vol. 1, 1993, pp. 145–151.
5. Vickrey, W. Pricing in Urban and Suburban Transport. *American Economic Review*, Vol. 53, 1963, pp. 452–465.
6. Vickrey, W. Congestion Theory and Transportation Investment. *American Economic Review*, Vol. 59, 1969, pp. 251–261.
7. Hendrickson, C., and G. Kocur. Schedule Delay and Departure Time Decisions in a Deterministic Model. *Transportation Science*, Vol. 15, 1981, pp. 62–77.

8. Arnott, R., A. de Palma, and R. Lindsey. Economics of a Bottleneck. *Journal of Urban Economics*, Vol. 27, 1990, pp. 111–130.
9. Arnott, R., A. de Palma, and R. Lindsey. Departure Time and Route Choice for the Morning Commute. *Transportation Research*, Vol. 24B, 1990, pp. 209–228.
10. Fargier, P.-H. Effects of the Choice of Departure Time on Road Traffic Conditions. *Proc., 8th International Symposium on Transportation and Traffic Theory*, 1981, pp. 223–263.
11. Small, K. The Scheduling of Consumer Activities: Work Trips. *American Economic Review*, Vol. 72, 1982, pp. 467–479.
12. Small, K. Trip Scheduling in Urban Transportation Analysis. *American Economic Review*, Vol. 82, 1992, pp. 482–486.
13. Mannering, F., and M. Hamed. Occurrence, Frequency, and Duration of Commuters' Work-to-Home Departure Delay. *Transportation Research*, Vol. 24B, No. 2, 1990, pp. 99–109.
14. Bernstein, D., T. L. Friesz, R. L. Tobin, and B.-W. Wie. A Variational Control Formulation of the Simultaneous Route and Departure-Time Choice Equilibrium Problem. *Proc., 12th International Symposium on the Theory of Traffic Flow and Transportation*, 1993, pp. 107–126.

Publication of this paper sponsored by Committee on Taxation and Finance.