# Sight Distance on Horizontal Alignments with Continuous Lateral Obstructions 

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#### Abstract

For safe and efficient highway operation, sight distance has been of great interest to researchers in the field of highway geometric design. Several formulas have been developed to relate the available sight distance to the horizontal and vertical alignment of the highway and the existing obstructions. Among these formulas is the one presented by (AASHTO) to determine the available sight distance on a simple horizontal curve with a length greater than the sight distance. For shorter curves in which the sight distance is greater than the curve length, other methods have been developed. However, none of these methods considered the case of continuous lateral obstructions or complicated horizontal alignments. Consequently, it has been recommended that the available sight distance be checked graphically or in the field. For this study, general analytical procedures were developed to check the available sight distance on horizontal alignments with single and continuous obstructions. A horizontal alignment may consist of any combination of horizontal components, such as straight segments, circular curves, and spiral curves. Based on the analytical procedure, a computer software program was developed to establish the no-passing zones on twolane highways, according to the specifications used by the Ministry of Transportation of Ontario, Canada. The developed procedures and computer software proved to be very accurate. Using them would save time and effort and would avoid possible human errors when the sight distance is checked using current graphical or field techniques. The computer software can also be used to develop design tables and charts for the available sight distance on different horizontal alignments.


Sight distance is vital for safe and efficient highway operation. The driver must be able to see ahead a distance sufficient for stopping to avoid hitting an unexpected object on the roadway. Moreover, it is recommended that the design of two-lane rural highways should provide drivers with a sight distance sufficient to pass slower vehicles. In many cases, however, the sight distance is restricted to a certain length. On horizontal curves, the driver's sight line may be limited by lateral objects such as trees, buildings, and cut slopes. On crest vertical curves, the sight line may be limited by the vertical curve itself. Also, sight distance on sag vertical curves may be limited to the farthest point covered by the vehicle's headlight beam. Therefore, the designer must check the available sight distance against the required stopping site distance (SSD) or decision site distance (DSD) on any highway, and the passing site distance (PSD) on two-lane rural highways.

For horizontal curves, many models have been developed to relate the available sight distance to the lateral clearance. Among these models is the one presented by the AASHTO for the case of $S \leq L$, where $S$ is the sight distance on the curve and $L$ is the curve length ( 1 ). Although this formula is easy and direct, it is "of limited practical value except on long curves" (1). Therefore, AASHTO recommends that "the designer must use graphical methods to check sight distance on horizontal curves" ( $I$ ).

[^0]The other case in which the sight distance is greater than the curve length has been studied by many researchers. Neuman and Glennon (2), Waissi and Cleveland (3), Berg et al. (4), and Easa (5) developed different methods to check the required lateral clearance on simple horizontal curves. Easa $(6,7)$ also studied the case of a single lateral obstruction on compound and reverse curves and developed other formulas to relate the lateral clearance to the available sight distance. None of these methods has considered the case of continuous obstructions. According to the standards of the Manual of Uniform Traffic Control Devices (MUTCD), which is used by the Ministry of Transportation of Ontario (MTO), Canada, for establishing the no-passing zones (8), the available sight distance for drivers in the inside lane is limited by a continuous obstruction represented by a theoretical 3 m wide shoulder. Continuous obstructions also may be encountered because of cut slopes.

For this study, general analytical methods were developed to evaluate sight distance on horizontal alignments for both cases of continuous and single obstructions. The terms "horizontal alignment" and "horizontal curve" refer to any combination of horizontal highway components, such as straight segments, circular curves, or clothoid spiral curves. A computer software program was developed to determine the available sight distance and, in turn, the nopassing zones on two-lane rural highways according to the standards of MUTCD used by MTO.

## THEORETICAL DEVELOPMENT

This study examines available sight distance (which may be SSD, DSD, or PSD) on general horizontal curves consisting of any combination of straight segments, circular curves, and spiral curves. Figure 1 shows some of the cases of horizontal alignments that are covered in this report. More complicated alignments that can be encountered on actual highways are also covered in this paper.

## Continuous Obstruction: General Procedures

Assuming a constant lane width and lateral clearance, the continuous obstruction will be parallel to and have the same geometry of the highway centerline; the sight distance will be restricted by having the sight line tangent to the obstruction. The point of tangency may be located on a circular curve, spiral curve, or the point of intersection of two successive straight segments without curves. The following sections present general procedures that can be used to determine the available sight distance, regardless of the components of the horizontal curve. Relationships are then presented for the special alignments of intersecting long tangents without a curve and for the simple circular curve.


FIGURE 1 Examples of possible horizontal alignments.

In the following sections, the azimuth of the lines, defined as the angle from the north direction to the line, measured clockwise, is referred to as $\Phi$. Also, the east and north coordinates of the points are referred to as $(x, y)$, respectively. However, because it is the relative positioning of the points to each other (not the absolute positions) that determines the available sight distance, the coordinates and azimuths can be taken relative to any reference point and direction. In all of the following derivations, the lateral clearance between the obstruction and the center of the lane is assumed to be constant and is referred to as $m$.

## Sight Line Tangent to a Circular Curve

In this case, the obstruction restricting the sight line is a circular curve, as shown in Figure 2. In general, the beginning and the end of the sight line may be positioned on any horizontal highway segment (a straight or circular segment or a spiral curve). The general procedure developed for this study is iterative, the sight distance is initially assumed as $S$. Then $S$ is checked and decreased or increased until the sight line becomes tangent to the obstruction. The procedure involves the following steps:


FIGURE 2 General procedure: Sight line tangent to circular curve (continuous obstruction).

1. Determine the coordinates of the beginning of the sight line and the center of the curve ( $x_{1}, y_{1}$ ) and ( $x_{\mathrm{c}}, y_{\mathrm{c}}$ ), respectively.
2. Calculate the length $l_{1}$ as
$l_{\mathrm{t}}=\left[\left(x_{1}-x_{\mathrm{c}}\right)^{2}+\left(y_{1}-y_{\mathrm{c}}\right)^{2}\right]^{1 / 2}$
3. Determine the coordinates of the end of the sight line $\left(x_{2}, y_{2}\right)$.
4. Calculate $l_{2}$ and $l_{3}$ similar to $l_{1}$.
5. From the basics of trigonometry, $l_{1}, l_{2}, l_{3}$, and $\theta$ can be related by the following equation:
$l_{2}^{2}=l_{1}^{2}+l_{3}^{2}-2 l_{1} l_{3} \cos \theta$
or
$\theta=\cos ^{-1}\left(\frac{l_{1}^{2}+l_{3}^{2}-l_{2}^{2}}{2 l_{1} l_{3}}\right)$
6. Calculate the length $l_{4}$ as
$l_{4}=l_{1} \sin \theta$
7. If $l_{4}<R-m, S$ is greater than the actual sight distance. Decrease $S$ and repeat Steps 3 to 6 .
8. If $l_{4}>R-m, S$ is less than the actual sight distance. Increase $S$ and repeat Steps 3 to 6 .
9. If $l_{4}=R-m, S$ is equal to the actual sight distance. End iterations.

Although the lengths $l_{1}, l_{2}$, and $l_{3}$ can be calculated without using the coordinates, a unique sequence of calculations is required for each possible combination of horizontal segments.

On the other hand, using the coordinates of the points makes the procedure applicable regardless of the positions of the beginning
and end of the sight line and also makes it very easy for computer programming.

## Sight Line Tangent to Spiral Curve

Generally, a spiral curve is a curve with varying radius, beginning with a straight segment $(R \rightarrow \infty)$; as the curve's length increases, the corresponding radius decreases. Many mathematical formulas can be used to represent spiral curves and may be found in mathematics textbooks, such as in Survey of Applicable Mathematics (9). Among these formulas, Euler's spiral, known as the clothoid spiral, is the most commonly used in road design $(1,10)$. Defining $l$ as segment length of a spiral curve beginning with a straight segment $R$ as the corresponding radius and $\delta$ as the deflection angle of this segment in radian, Euler's spiral is formulated as follows:
$A^{2}=l * R=\frac{l^{2}}{2 \delta}=R^{2} * 2 \delta$
where $A$ is the spiral parameter.
As illustrated in Figures 3 and 4, this case is similar to the previous case, but the obstruction restricting the sight line is a spiral curve. The beginning and the end of the sight line may be positioned on any horizontal highway segment (straight or circular segment or spiral curve). In this case, three iterative procedures were developed, any of which can be used to determine the available sight distance (11). Only the first procedure is presented because, as will be shown later, the same technique can be used for the case of a single obstruction. The procedure involves the following steps:

1. Determine the coordinates of the beginning of the sight line $\left(x_{1}, y_{1}\right)$.


FIGURE 3 Coordinate determination involving spiral curves.


FIGURE 4 General procedure: Sight line tangent to spiral curve (continuous obstruction).
2. Determine the coordinates of the end of the sight line $\left(x_{2}, y_{2}\right)$.
3. Calculate the azimuth of the sight line, $\Phi_{1}$, as
$\Phi_{1}=\tan ^{-1}\left(\frac{x_{2}-x_{1}}{y_{2}-y_{1}}\right)$
4. Knowing the azimuth of the tangent to the spiral, $\Phi_{0}$, determine the coordinates of the point of tangency of a line having an azimuth $\Phi_{1}$ and the given obstruction, $\left(x_{3}, y_{3}\right)$. For right-turn spirals
beginning with a straight segment, $\left(x_{3}, y_{3}\right)$ can be determined by considering another point, $\left(x_{5}, y_{5}\right)$, defined as the point of tangency of a line having an azimuth $\Phi_{1}$ and the highway's centerline. As shown in Figure 3, the coordinates $\left(x_{5}, y_{5}\right)$ can be determined as follows
$\delta=\left(\Phi_{1}-\Phi_{0}\right) \times \frac{\pi}{180^{\circ}}$

Using the spiral formula presented in Equation 5,
$l_{\delta}=A \times \sqrt{2 \delta}$
From the general characteristics of Euler's spiral (9),
$T_{x}=A \sqrt{2 \delta}\left(1-\frac{\delta^{2}}{5 * 2!}+\frac{\delta^{4}}{9 * 4!}-\frac{\delta^{6}}{13 * 6!}+\ldots\right)$

$$
\begin{equation*}
\approx l_{\delta}-\frac{l_{\delta} \delta^{2}}{10} \tag{9}
\end{equation*}
$$

$T_{y}=A \sqrt{2 \delta}\left(\frac{\delta}{3}-\frac{\delta^{3}}{7 * 3!}+\frac{\delta^{5}}{11 * 5!}-\ldots\right) \approx \frac{l_{\delta} \delta}{3}$
Defining $\left(x_{4}, y_{4}\right)$ as the coordinates of the tangent-spiral (TS) point on the centerline of the highway, then
$x_{5}=x_{4}+T_{x} \sin \Phi_{0}+T_{y} \cos \Phi_{0}$
$y_{5}=y_{4}+T_{x} \cos \Phi_{0}-T_{y} \sin \Phi_{0}$
Defining $w$ as the lane width, the line from $\left(x_{5}, y_{5}\right)$ to $\left(x_{3}, y_{3}\right)$ will have an azimuth of $\Phi_{1}+90^{\circ}$ and a length of $m+w / 2$. Therefore, $\left(x_{3}, y_{3}\right)$ can be calculated as follows:
$x_{3}=x_{5}+(m+w / 2) \sin \left(\Phi_{1}+90^{\circ}\right)$
$y_{3}=y_{5}+(m+w / 2) \cos \left(\Phi_{1}+90^{\circ}\right)$
5. Calculate the azimuth of the line between Points 1 and $3, \Phi_{2}$.
6. For right-turn curves, if $\Phi_{1}>\Phi_{2}$ (if $\Phi_{1}<\Phi_{2}$, for left-turn curves), $S$ is greater than the actual sight distance (see Figure 4 for illustration). Decrease $S$ and repeat Steps 2 to 5.
7. For right-turn curves, if $\Phi_{1}<\Phi_{2}$ (if $\Phi_{1}>\Phi_{2}$, for left-turn curves), $S$ is less than the actual sight distance (see Figure 4 for illustration). Increase $S$ and repeat Steps 2 to 5.
8. If $\Phi_{1}=\Phi_{2}, S$ is equal to the actual sight distance (see Figure 4 for illustration). End iterations.

Because the azimuth is always less than 360 degrees, some caution is required in the last check if for right-turn curves, $\Phi_{1}$ is slightly greater than zero and $\Phi_{2}$ is slightly less than 360 degrees (or if $\Phi_{2}$ is slightly greater than zero and $\Phi_{1}$ is slightly less than 360 degrees for left-turn curves). For example, for right-turn curves, if $\Phi_{2}$ is slightly less than 360 degrees, and $S$ is greater than the available sight distance, $\Phi_{1}$ may be slightly greater than zero. In this case, $S$ should be decreased and another iteration is required.

## Continuous Obstruction: Special Cases

The following procedures present closed-form solutions derived for the special alignments of intersecting two long straight segments without a curve and for the simple circular curve.

## Intersecting Straight Segments Without a Curve

In this case, only two long straight segments are intersecting with a small deflection angle, as shown in Figure 1. Although no circular or spiral curves are involved in this case, it represents a possible horizontal alignment and therefore was considered in this paper. In general, the beginning and the end of the sight line may be located on any horizontal highway segment. However, the straight segments in this case are usually long enough for the beginning and the end of the sight line to be located on the two intersecting straight segments. Therefore, only this case is presented; other cases can be found elsewhere (11).

Figure 5 presents two straight segments with a deflection angle $\Delta$. The driver is at a distance $l_{1}$ from the point of intersection (PI). As shown the sight line will touch the continuous obstruction at its PI. From Figure 5,
$l_{3}=m / \tan \frac{180^{\circ}-\Delta}{2}$
$\alpha=\tan ^{-1}\left[m /\left(l_{1}-l_{3}\right)\right]$
$\beta=\Delta-\alpha$
$l_{2}=l_{1} \sin \alpha / \sin \beta$
Then, the available sight distance is
$S=l_{1}+l_{2}$

## Simple Circular Curve

In this case, the horizontal curve consists of a simple circular curve, having a radius, $R$, with two straight segments (tangents) at the two ends. Obviously, the sight line can only be restricted by touching a circular curved obstruction. Therefore, the general procedure pre-


FIGURE 5 Special case: Intersecting straight segments without curves (continuous obstruction).
sented previously can be applied. However, other closed-form relationships were developed for this special case to determine the available sight distance more easily and accurately. These formulas can be considered extensions to the work of Easa (5), which considered only a single (or multiple) lateral obstruction.

For the case of a simple curve, there are four possibilities for the beginning and the end of the sight line touching the obstruction:

1. Sight line begins on first tangent and ends on curve.
2. Sight line begins on first tangent and ends on second tangent.
3. Sight line begins and ends on curve.
4. Sight line begins on curve and ends on second tangent.

Case 1: Sight Line Begins on First Tangent and Ends on Curve As presented in Figure 6
$l_{2}=\left(R^{2}+l_{1}^{2}\right)^{1 / 2}$
where $l_{1}$ is the distance between the river and the point of curve (PC).
$\delta_{1}=\tan ^{-1}\left(l_{1} / R\right)$
$\delta_{2}=\cos ^{-1}\left(\frac{R-m}{l_{2}}\right)$
$\delta_{3}=\cos ^{-1}\left(1-\frac{m}{R}\right)$
$\Delta_{1}=\delta_{2}+\delta_{3}-\delta_{1}$
Then, the available sight distance is
$S=l_{1}+R \Delta_{1} \times \frac{\pi}{180^{\circ}}$

Case (2): Sight Line Begins on First Tangent and Ends on Second Tangent As shown in Figure 6, $\delta_{1}$ and $\delta_{2}$ can be determined as in Case (1). Defining $\Delta$ as the total deflection angle of the curve, then
$\delta_{3}=\Delta+\delta_{1}-\delta_{2}$
$m_{2}=\left[R^{2}+(R-m)^{2}-2 R(R-m) \cos \delta_{3}\right]^{1 / 2}$
$\theta_{1}=\sin ^{-1}\left(R \sin \delta_{3} / m_{2}\right)$
$\theta_{2}=\sin ^{-1}\left[(R-m) \sin \delta_{3} / m_{2}\right]$
As shown in Figure 6, the angle $\alpha$ is in a triangle whose other two angles are $\left(\theta_{1}-90^{\circ}\right)$ and $\left(\theta_{2}+90^{\circ}\right)$. Therefore,
$\alpha=180^{\circ}-\theta_{1}-\theta_{2}$
$l_{4}=m_{2} \sin \left(\theta_{1}-90^{\circ}\right) / \sin \alpha$
Then, the available sight distance is
$S=l_{1}+R \Delta \times \frac{\pi}{180^{\circ}}+l_{4}$


Sight Line Begins on Tangent and Ends on Curve.


Sight Line Begins and Ends on Tangents.
FIGURE 6 Special case: Simple circular curve (continuous obstruction).

Case (3): Sight Line Begins and Ends on Curve In this case, the formula presented by AASHTO (1) can be applied as follows:
$S=2 R \times \cos ^{-1}(1-m / R) \times \frac{\pi}{180^{\circ}}$

Case 4: Sight Line Begins on Curve and Ends on Second Tangent The relationships involved in this case (11) are backward derivations for Case (1) and, therefore will not be presented.

## Single Obstruction

As previously mentioned, the case of a single obstruction has been extensively studied. Formulas relating the available sight distance to
the lateral clearance on simple horizontal curves already exist. For this study a general iterative procedure was developed to check the available sight distance regardless of the components of the horizontal curve or the positions of the beginning and end of the sight line. The procedure uses the azimuths of the lines in a way similar to the procedure of the continuous obstruction, with the sight line tangent to a spiral curve. However, in this case, the coordinates of the obstruction are directly used instead of using the point of tangency $\left(x_{3}, y_{3}\right)$.

## COMPUTER SOFTWARE PROGRAM FOR NO-PASSING ZONES

The theoretical procedures previously presented were translated to a computer software program written in Microsoft Quick Basic, which
determines the available passing sight distance and no-passing zones on two-lane highways. The software uses the MUTCD standards for no-passing zones, which are used by the MTO (8). In these standards, lateral obstructions on the two lanes are considered differently. On the inside lane, defined as the lane nearer to the center(s) of the curve(s), the driver's sight line is always limited by the edge of a theoretical shoulder 3 m wide. On the other hand, the sight line of drivers on the outside lane can cross the right-of-way and is limited only by any existing lateral obstruction. For both lanes, the sight distance is measured along the centerline of the lane. The sight line of a driver on the outside lane obviously does not cross the lane on which the driver travels, and the driver sees the entire opposing lane. For this reason, the sight line would be limited by existing obstructions. For a driver on the inside lane, however, the sight line crosses the lane on which the driver travels, and part of the opposing lane may be hidden from the driver's view. In this case, the edge of a 3meter theoretical shoulder is considered an obstruction to confine the sight line near the traveled way and to improve the visual conditions for the passing driver.

For the inside lane, the program uses the previous special relationships for continuous obstructions, if applicable, to calculate the available sight distance. If, because of the geometry of the curves, these relationships are not applicable, the program uses the general procedures for continuous obstructions to determine the available sight distance. However, the program was designed to interpret the alignment data specified by the user in the form of the station and the radius at each point between two horizontal segments ( $R=0$ is used to specify straight segments). The program determines whether the special or the general procedures will be used, generates the parameters required in either case, and determines the available sight distance. The width of the theoretical shoulder is a variable specified by the user.

For the outside lane, lateral obstructions are defined by the user as an input to the program; they may be continuous or single obstructions. Continuous obstructions are treated exactly as in the case of the inside lane, for single obstructions, the program uses only the general procedures to determine the available sight distance. In the case of more than one obstruction, the program will check the available sight distance against each obstruction and determine the minimum available sight distance.

For both lanes, the user specifies the minimum sight distance, $S_{m}$, required to be checked. If the available sight distance is less than $S_{m}$, the program determines the available sight distance, $S_{a v}$, every user-specified incremental step, STEP, and for a user-specified accuracy, ACC. The values of $S_{m}$, STEP, and ACC are defined by the user in the input file, which also contains the alignment and obstructions data. If $S_{a v}$ is greater than or equal to $S_{m}$, the current station will be skipped, and the available sight distance at the next station will be checked. It should be noted that $S_{m}$ need not be taken at exactly the minimum required PSD for the highway of interest. It is better to specify a greater value for $S_{m}$ so the user has a better idea about the change of $S_{a v}$ on the highway. The user can then establish the marking of the no-passing zones manually according to the actual value of the minimum required PSD.

## Program Verification

The developed methods and program were verified by comparing the results obtained by the software with those obtained graphically for numerical examples having alignments similar to those shown
in Figure 1. The parameters specified for the program were as follows: $S_{m}=250 \mathrm{~m}, \mathrm{STEP}=20 \mathrm{~m}$, and $\mathrm{ACC}=0.1 \mathrm{~m}$. The actual available sight distances were determined by drawing the same curves using Auto-Computer Assisted Design (AutoCAD). Then, the available sight distances on the inside lane were determined by drawing sight lines tangent to the theoretical shoulder. On the outside lane, the sight lines were drawn passing through certain obstructions input in to the program. The results obtained by the program for all cases were in very good agreement with those obtained graphically (11).

## APPLICATION EXAMPLE

The developed computer program was applied to a two-lane stretch on Highway 17 in Ontario to check the available sight distance and to establish no-passing zones. The lane width was 3.75 m and the speed limit was 90 kph . As shown in Figure 7 and Table 1, the stretch contained two curves; each was a circular curve with two spirals. The alignments before the first curve and after the second curve were assumed to be straight segments.

According to the MUTCD specifications used by MTO, the minimum required PSD is 300 m for a $90-\mathrm{kph}$ speed ( 8 ). This required sight distance is much less than the corresponding distances required by AASHTO ( 1 ) and the Roads and Transportation Association of Canada (RTAC) (10). However, based on a model of the kinematic relationships among the passing, passed, and opposing vehicles, Harwood and Glennon (12) agreed with the MUTCD criteria for PSD for a passenger car passing a passenger car. In passing maneuvers involving trucks, the required PSD was greater than that recommended by MUTCD but less than that recommended by AASHTO. Based on this discussion and because the highway is supervised by the MTO, the $300-\mathrm{m}$ minimum PSD was adopted for establishing the no-passing zones. Also, because of the lack of information about single obstructions for the outside lane, the nopassing zones were established for the continuous obstruction represented by the $3-\mathrm{m}$ theoretical shoulder only.

The parameters for the program were: $S_{m}=300 \mathrm{~m}, \mathrm{STEP}=5 \mathrm{~m}$, and $\mathrm{ACC}=0.1 \mathrm{~m}$. Table 2 shows a sample of the program output, which gives the sight distance less than $S_{m}$. Because of the large number of stations having $S_{a v}$ less than $S_{m}$, the results presented in Table 2 are given for both lanes every 50 m . In Figure 8, the available PSD on both lanes was also plotted against the highway stations to show the variation of the available PSD caused by the existing horizontal alignment. The appropriate marking for passing is also given in Figure 8.

## DESIGN VALUES FOR SPECIAL ALIGNMENTS

For design purposes, the computer program was used to determine the available PSD on the inside lane of two-lane highways because of the horizontal alignment, according to MTO specifications. Three types of horizontal alignments were considered: simple curves, simple curves with spirals, and compound curves. Assuming a lane width of 3.5 m and therefore a lateral clearance $(\mathrm{m})$ of 4.75 m , the available sight distance for the three alignments was determined every 5 m and to an accuracy of 0.1 m . Then, the minimum $S_{a v}$ on each alignment was given and was rounded to the nearest lower integer (see Tables 3-5 for different values of the curve radius $R$ and the deflection angle $\Delta$ ).


FIGURE 7 Horizontal alignment of the stretch example (Highway 17).

In the first alignment, a simple circular curve, the only variables are $R$ and $\Delta$. Table 3 shows the range used for each variable. In the second alignment, a circular curve with spirals, the spiral parameter, $A$, is used. The value of $A$ usually depends on the value of $R$, the design speed, and the maximum superelevation rate. An example of the design values of $A$ is the specifications given by RTAC (10). However, because of the differences in the specifications used by different jurisdictions, ideally, values of $S_{a v}$ for various $R$ should be given for different values of $A$. However, when comparing the results in Tables 4 and 5 , the change of $A$ from zero (which corresponds to the simple curve) to 200 m resulted in a change in $S_{a}$ ranging from less than 0.1 to about 5 percent. This would suggest that the spiral parameter has a minor effect on the available sight distance. In this study, however, the value of $A$ was set at 200 m to present a sample of the possible design tables that can be produced using the developed computer program. Finally, the parameters associated with the third alignment, a compound curve, are the radii
of the two circular curves, $R_{1}$ and $R_{2}$, and the deflection angle of each curve, $\Delta_{1}$ and $\Delta_{2}$. The ratio of $R_{2} / R_{1}$ was set as 1.5 , which is the value suggested by AASHTO (1) for open highways, and the values of $\Delta_{1}$ and $\Delta_{2}$ have the range given in Table 5.
Tables 3 to 5 represent a sample of the design tables that can be developed to determine the PSD on horizontal alignments. For example, if the alignment is a curve with spirals having $R=1000$ $\mathrm{m}, A=200 \mathrm{~m}$, and $\Delta=6^{\circ}$, the available PSD on the inside lane is 236 m , compared with 195 m given by the AASHTO formula.
Tables 3 to 5 also show that, for the same radius, as the angle of deflection increases the curve length increases and the available sight distance approaches the value given in Equation 33. Each curve has a deflection angle beyond which the curve is long enough for Equation 33 to be applicable, and $S_{a v}$ will no longer depend on $\Delta$. Applying the AASHTO formula of Equation 33 will considerably underestimate the available sight distance for short curves.

TABLE 1 Horizontal Alignment of Example Highway Stretch

| Station $^{a}$ | Coordinates $^{b}$ |  | Point <br> Identification |
| :---: | :---: | :---: | :---: |
|  | North | East |  |
| $13+000.000$ | 4000.000 | 4000.000 | Begin |
| $13+857.769$ | 4606.534 | 4606.534 | TS |
| $13+918.729$ | 4649.068 | 4650.197 | SC |
| $14+329.870$ | 4837.172 | 5010.389 | CS |
| $14+390.830$ | 4848.699 | 5070.245 | ST |
| $14+588.673$ | 4883.564 | 5264.992 | TS |
| $14+649.633$ | 4895.351 | 5324.794 | SC |
| $15+015.986$ | 5081.903 | 5633.081 | CS |
| $15+076.946$ | 5129.412 | 5671.268 | ST |
| $16+000.000$ | 5858.850 | 6236.907 | End |

a Station $13+000$ means a point at a distance of $13,000 \mathrm{~m}$ from the point of zero chainage, and so on.
$b$ the coordinates of the first point and the azimuth of the first segment were assumed as $(4000,4000) \mathrm{m}$ and $45^{\circ}$, respectively.
c TS = point of Tangent to Spiral.
SC = point of Spiral to Curve.
CS = point of Curve to Spiral.
TS $=$ point of Spiral to Tangent.

## CONCLUSIONS

General analytical procedures to determine the available sight distance on horizontal alignments with single or continuous obstructions were presented. These procedures, along with the developed computer program, can replace the current graphical technique recommended by AASHTO or the field technique used by the MTO to
determine the available sight distance, when sight distance is restricted only by the horizontal alignment.

As an application to the no-passing zones according to the specifications of MUTCD, it was shown that the AASHTO formula extremely underestimates the available sight distance when sight distance is greater than the curve length. Therefore, a sample of design tables was developed using the developed procedures and

TABLE 2 Sample of Computer Software Program Output for Example Highway Stretch

| Right Lane $^{a}$ |  | Left Lane $^{a}$ |  |
| :---: | :---: | :---: | :---: |
| Station | $S_{a V}$ | Station | $S_{a v}$ |
| $13+700$ | 293.6 | $14+750$ | 154.6 |
| $13+750$ | 249.7 | $14+800$ | 148.6 |
| $13+800$ | 210.4 | $14+850$ | 148.6 |
| $13+850$ | 181.7 | $14+900$ | 148.6 |
| $13+900$ | 172.2 | $14+950$ | 148.6 |
| $13+950$ | 171.6 | $15+000$ | 148.6 |
| $14+000$ | 171.6 | $15+050$ | 150.4 |
| $14+050$ | 171.6 | $15+100$ | 167.8 |
| $14+100$ | 171.6 | $15+150$ | 203.2 |
| $14+150$ | 171.6 | $15+200$ | 246.0 |
| $14+200$ | 174.8 | $15+250$ | 291.6 |
| $14+250$ | 255.9 |  |  |

[^1]

FIGURE 8 Available PSD and corresponding marking for the stretch example.
computer program. The tables can be easily used to check the available PSD on some horizontal alignments, such as simple curves, curves with spirals, and compound curves. For complicated alignments, the program can be used directly to determine the available passing sight distance faster, more conveniently, and more accurately than the techniques now in use.

## ACKNOWLEDGMENTS

The authors thank Dennis Siczkar of the MTO, Thunder Bay District, for his support and thoughtful comments during this project. The financial support of the Natural Sciences and Engineering Research Council of Canada is also gratefully acknowledged.

TABLE 3 Available Sight Distance on Simple Horizontal Curves
(Continuous Obstruction, $m=4.75 \mathrm{~m}$ )

|  | $S_{a v}(\mathrm{~m})$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |
|  | $2^{\circ}$ | $4^{\circ}$ | $6^{\circ}$ | $8^{\circ}$ | $10^{\circ}$ | AASHTO <br> Formula |
| 200 | 547 | 279 | 191 | 150 | 126 | 87 |
| 400 | 551 | 286 | 202 | 164 | 143 | 123 |
| 600 | 554 | 293 | 212 | 178 | 161 | 151 |
| 800 | 558 | 300 | 223 | 191 | 178 | 174 |
| 1000 | 561 | 307 | 233 | 205 | 196 | 195 |
| 1200 | 565 | 314 | 244 | 219 | 213 | 213 |
| 1400 | 568 | 321 | 254 | 233 | 230 | 230 |
| 1600 | 572 | 328 | 265 | 247 | 246 | 246 |
| 1800 | 575 | 335 | 275 | 261 | 261 | 261 |
| 2000 | 579 | 342 | 286 | 275 | 275 | 275 |

[^2]TABLE 4 Available Sight Distance on Horizontal Curves with Spirals (Continuous Obstruction, $A=200 \mathrm{~m}, m=4.75 \mathrm{~m}$ )

| $R(\mathrm{~m})$ | $S_{a v}(\mathrm{~m})$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\Delta^{a}$ |  |  |  |  | AASHTO |
|  | $2^{\circ}$ | $4^{\circ}$ | $6^{\circ}$ | $8^{\circ}$ | $10^{\circ}$ |  |
| 600 | N/A | N/A | N/A | 187 | 168 | 151 |
| 800 | N/A | 307 | 228 | 195 | 181 | 174 |
| 1000 | N/A | 310 | 236 | 208 | 197 | 195 |
| 1200 | 569 | 316 | 245 | 221 | 214 | 213 |
| 1400 | 571 | 322 | 255 | 234 | 230 | 230 |
| 1600 | 574 | 329 | 266 | 248 | 246 | 246 |
| 1800 | 577 | 335 | 276 | 262 | 261 | 261 |
| 2000 | 580 | 342 | 286 | 275 | 275 | 275 |

a $\Delta=$ total deflection angle of the circular curve and the two spirals.
$\mathrm{N} / \mathrm{A}=$ Not Applicable (the required length of the circular curve $\leq$ zero).
b See Equation 33.

TABLE 5 Available Sight Distance on Compound Horizontal
Curves (Continuous Obstruction, $R_{2} / R_{1}=1.5, m=4.75 \mathrm{~m}$ )

| $\Delta_{1}$ | $\Delta_{2}$ | $S_{\text {av }}(\mathrm{m})$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $R_{1}(\mathrm{~m})$ |  |  |  |  |
|  |  | 400 | 800 | 1200 | 1600 | 2000 |
| $2^{\circ}$ | $2^{\circ}$ | 289 | 307 | 324 | 341 | 359 |
|  | $4^{\circ}$ | 210 | 239 | 268 | 297 | 326 |
|  | $6^{\circ}$ | 176 | 216 | 256 | 294 | 326 |
|  | $8^{\circ}$ | 160 | 211 | 256 | 294 | 326 |
| $4^{\circ}$ | $2^{\circ}$ | 204 | 228 | 251 | 274 | 297 |
|  | $4^{\circ}$ | 171 | 205 | 240 | 270 | 297 |
|  | $6^{\circ}$ | 155 | 202 | 240 | 270 | 297 |
|  | $8^{\circ}$ | 149 | 202 | 240 | 270 | 297 |
| $6^{\circ}$ | $2^{\circ}$ | 165 | 195 | 225 | 254 | 280 |
|  | $4^{\circ}$ | 149 | 189 | 224 | 254 | 280 |
|  | $6^{\circ}$ | 143 | 189 | 224 | 254 | 280 |
|  | $8^{\circ}$ | 142 | 189 | 224 | 254 | 280 |
| $8^{\circ}$ | $2^{\circ}$ | 145 | 181 | 216 | 247 | 275 |
|  | $4^{\circ}$ | 137 | 181 | 216 | 247 | 275 |
|  | $6^{\circ}$ | 135 | 181 | 216 | 247 | 275 |
|  | $8^{\circ}$ | 135 | 181 | 216 | 247 | 275 |
| AASHTO Formula ${ }^{a}$ |  | 123 | 174 | 213 | 246 | 275 |

a See Equation 33.

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Publication of this paper sponsored by Committee on Geometric Design.


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[^1]:    ${ }^{a}$ The right and left lanes are relative to a driver travelling in the direction of increasing stations.

[^2]:    a $\quad \Delta=$ deflection angle of the curve.
    See Equation 33.

