Roughness Model Describing Heavy Vehicle–Pavement Interaction

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The pavement roughness characteristics that affect interaction between pavement and heavy vehicles are addressed. A roughness model describing the pavement roughness attributes affecting heavy vehicles is presented. Dynamic vehicle response data from two sources were analyzed, namely, experimental data obtained with the instrumented vehicle developed by the National Research Council of Canada and simulated data obtained with a quarter-vehicle simulation. It was found that the vehicle response parameter of interest in this interaction is the sprung mass vehicle acceleration because it relates to both pavement and vehicle damage as well as to ride quality and cargo damage. This was demonstrated by analyzing the transfer functions of both the dynamic axle load and the vertical sprung mass acceleration over a range of pavement roughnesses and vehicle speeds. The sprung mass vertical acceleration transfer function showed sensitivity to a pavement roughness excitation frequency of 3.5 Hz. A pavement roughness statistic was proposed that is calculated as follows: (*a*) calculate the spectral density of the pavement roughness profile, (*b*) multiply this spectral density by the square of a transfer function to obtain the spectral density of the vertical sprung mass acceleration of the reference quarter vehicle selected, and (*c*) calculate the integral of the spectral density of the vertical sprung mass acceleration over the full frequency spectrum and take the square root. The resulting statistic has units of energy per unit mass per unit length of pavement traveled and represents the energy input from the road to the vehicle and vice versa.

Historically, pavement serviceability has been defined in terms of the ride quality perceived by the traveling public (*1*). Ride quality has been considered to be a function of longitudinal pavement roughness. In the past 30 years, considerable effort has been spent in measuring pavement roughness as part of the data collected for pavement management purposes.

In 1982 the World Bank initiated an extensive study of the various pavement roughness measuring systems in order to develop a universal roughness index for describing the ride quality of passenger vehicles (*2*). This study distinguished two main categories of devices for measuring roughness, namely, response type and profilometer type. The roughness index proposed, referred to as the international roughness index (IRI), relates the passenger car ride quality to the accumulated displacement between the axle (i.e., unsprung mass) and the body (i.e., sprung mass) of a passenger vehicle (i.e., units of millimeters per kilometer or inches per mile). To maintain universality, a computer model of a quarter car was developed as the reference vehicle for calculating IRI.

Clearly, the IRI is intended to reflect the pavement roughness attributes that affect the ride quality of passenger vehicles. There was some debate in the literature about whether the accumulated relative vertical axle displacement is the best indicator of passenger car ride quality (*3,4*). Despite this criticism, the IRI is accepted widely as the index of choice for reporting pavement roughness, and it has been used extensively by North American transportation agencies.

From this discussion, it is apparent that the IRI was not intended to describe the pavement roughness characteristics that affect heavy trucks. Indeed, the interaction between heavy trucks and pavements generates dynamic vehicle excitation that results in

- Dynamic axle loads affecting pavement damage and vehicle damage, and
- Vertical vehicle accelerations affecting truck ride quality and cargo damage.

The extent of axle load variation due to vehicle dynamics can be substantial, as demonstrated by a number of studies and summarized by an NCHRP report (*5*). In general, the standard deviation of dynamic load increases with increasing vehicle speed and level of pavement roughness. Various suspensions exhibit different dynamic characteristics affecting the extent and frequency content of the axle loads generated. Extensive work has been done in evaluating the relative damaging effects of these dynamic loads on pavement deterioration (*6–8*). These studies concluded that the higher the variation in dynamic axle loads, the higher the pavement damage. The effect of dynamic axle load excitation on the performance and service life of trucks has also been evaluated (*9,10*). Little work is available on the effect of roughness on truck ride quality and cargo damage.

Clearly, the attributes of pavement roughness that affect pavement–heavy vehicle interaction are of interest to roadway authorities as well as the trucking industry. This aspect of pavement roughness needs to be studied and a pavement roughness model or statistic developed that reflects this interaction. This paper addresses these needs.

OBJECTIVES

This paper offers a summary of a study (*11*) dealing with the characteristics of pavement roughness affecting heavy vehicles. It focuses on

- Review of the most important literature in this area,
- Analysis of experimental data on pavement roughness profile and dynamic axle loads obtained during an earlier experiment involving the instrumented vehicle developed by the National Research Council of Canada (NRCC) (*8*), and
- Development of a quarter-vehicle model, similar to the quarter car developed by the World Bank (*2*), to describe this interaction.

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LITERATURE REVIEW

Gillespie et al. conducted a study on the calibration of response-type roughness-measuring devices (*12*). A number of calibration reference alternatives were considered, namely, a shaker device, "standard" pavement sections, and actual pavement elevation profile measurements. The profile measurements were chosen as a more direct approach in conjunction with a quarter-car simulation. The differential equations governing the motion of this quarter car are shown here:

$$\ddot{Z}_s M_s + C_s (\dot{Z}_s - \dot{Z}_u) + K_s (Z_s - Z_u) = 0 \quad (1)$$

$$-\ddot{Z}_s M_s + M_u \ddot{Z}_u + K_t (Z_u - Z) = 0 \quad (2)$$

The configuration of the quarter car and its mechanical constants are defined in Figure 1.

The contribution of the pavement roughness profile and the quarter-car mechanical properties to vehicle response were studied through the use of transfer functions (Figure 2).

Clearly, the quarter-car simulation is sensitive to excitation frequencies of 1 and 10 Hz. The study proposed the use of the average rectified velocity (*ARV*) as the calibration reference:

$$ARV = \frac{1}{T}\int_0^T |\dot{Z}_s - \dot{Z}_u|\, dt \quad (3)$$

where T is the time required to traverse a section of road, and Z_s and Z_u are the displacements of the sprung and unsprung masses of the quarter-car simulation. *ARV* is in essence the accumulated vertical displacement between the sprung and unsprung masses, which makes it compatible with the readings of conventional response-type measuring devices.

Vehicle Parameters	K_t/M_s	K_s/M_s	M_u/M_s	C_s/M_s
HSRI	667	62.3	0.150	6.0
BPR	667	133.3	0.167	5.0

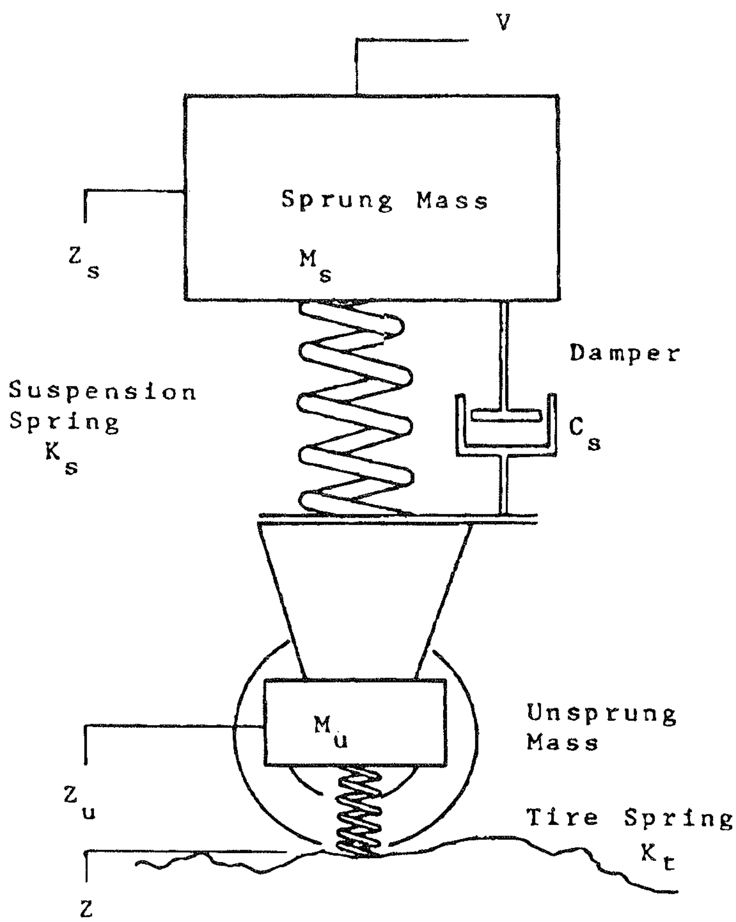

FIGURE 1 Quarter-car simulation (*12*).

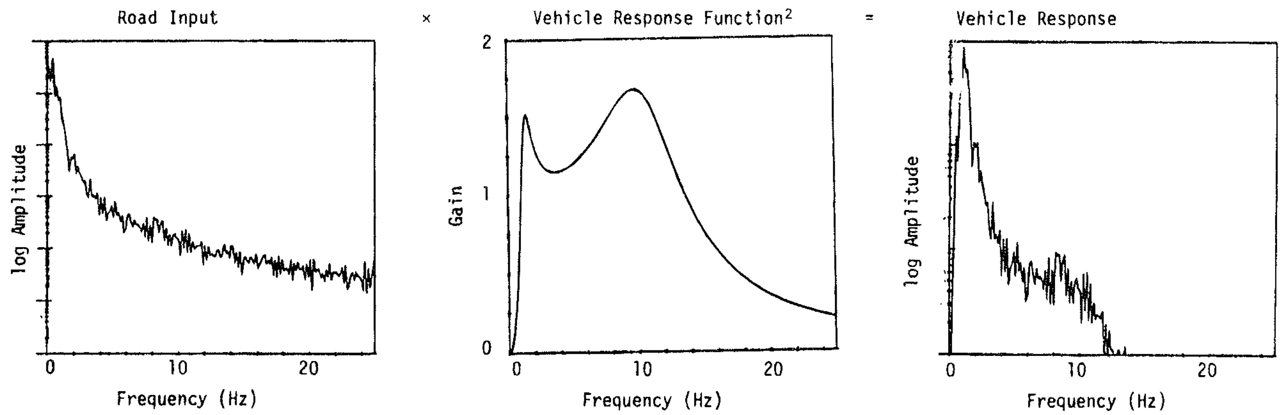

FIGURE 2 Contribution of vehicle and road input to spectral density of relative axle-body displacement (*12*).

Hudson et al. proposed an alternative pavement roughness statistic, referred to as the root mean vertical acceleration (*VA*) (*13*). It is a function of the pavement profile geometry only, defined as

$$VA_b = c\left[\sum_{i=k+1}^{n-k} \frac{(S_b)_i^2}{n-2k}\right]^{1/2} \tag{4}$$

where

VA_b = root mean square vertical acceleration corresponding to base length b;
$(S_b)_i$ = second derivative of pavement elevation Y at point i (Equation 5);
s = horizontal distance between adjacent points, called sample interval;
k = arbitrary integer used to define base length b as a multiple of s,
n = total number of elevation points; and
c = constant required for unit conversion from spatial to time domains.

$$(S_b)_i = \frac{Y_{i+k} + Y_{i-k} - 2Y_i}{ks^2} \tag{5}$$

The study compared the *VA* with the *ARV* statistic and a variation of this statistic, called the average rectified slope (*ARS*), defined as

$$ARS = \frac{1}{L}\int_0^T |\dot{Z}_s - \dot{Z}_u|\, dt \tag{6}$$

where L is the length of a section of road over which the statistic is calculated. Regression equations were developed between the Mays ride meter output *MO* (*inches per mile*) and the *VA* statistics (*feet* per second squared):

$$MO = -20 + 23VA_4 + 58VA_{16} \quad (R^2 = .96) \tag{7}$$

A sensitivity analysis of the *VA*, *ARV*, and *ARS* statistics was undertaken by varying the pavement profile sampling interval. The study also examined the sensitivity of the *ARV* and *ARS* statistics to the frequency of the pavement roughness input. Subjective data on vertical acceleration tolerance were used to establish "isocomfort" curves. Pavement roughness amplitudes were established corresponding to these isocomfort curves using the quarter-car simulation (*12*). It was suggested that the minimum amplitude exhibited at about 10 Hz was a shortcoming of the quarter-car-related statistics because, "for a given amplitude, the roughness statistic should vary in direct proportion to the frequency of the wave forms." It was therefore concluded that the *VA* is a preferable statistic to either the *ARV* or the *ARS*.

Wambold offered an overview of the fundamentals of pavement-vehicle interaction using simple vehicle models (*14*). Quarter-car, half-car, and quarter-truck models were analyzed. Transfer functions were developed, indicating the relationship between the desired vehicle output (e.g., accumulated sprung-unsprung mass displacement) and the input variable (i.e., roughness elevation profile) as a function of input variable frequency. Examples of transfer functions are shown in Figures 3 and 4 for the relative displacement between the sprung and unsprung masses and the tire load of a quarter truck, respectively. Note that the relative displacement between the sprung and unsprung masses of a quarter car is comparable to the *ARV* statistic described earlier (Equation 3). It was concluded

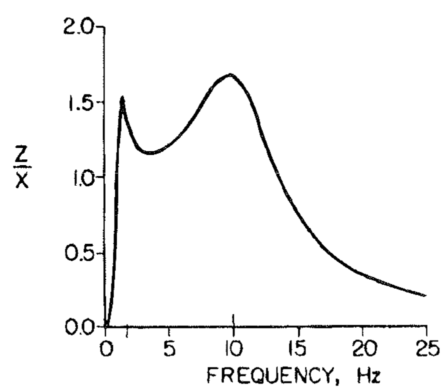

FIGURE 3 Transfer function of sprung mass minus unsprung mass displacements versus roughness amplitude (*14*).

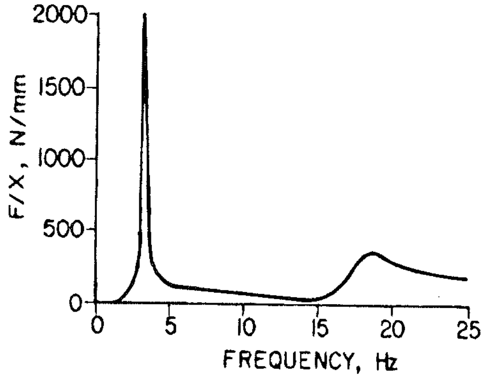

FIGURE 4 Transfer function of tire load versus roughness amplitude for quarter truck (*14*).

that the dominant pavement roughness excitation frequencies affecting the quarter car are different than the frequencies affecting dynamic tire loads.

The World Bank conducted an experimental evaluation and correlation of the variety of systems for measuring pavement roughness (*2*). The intention was to arrive at a universal pavement roughness index that would be stable in time and transferable between jurisdictions. The reference ARS of the quarter car initially proposed by Gillespie et al. (*12*) was recommended as the universal index. This is in essence the *ARS* defined earlier (Equation 6) and was defined as the IRI. The IRI was calculated through a computer simulation of a quarter car such as the one shown in Figure 1 traveling at 80 km/hr. The simulation used a state transition matrix approach for solving the four simultaneous linear differential equations defining the motion of the quarter car. The roughness profile was smoothed through a moving average algorithm to account for tire enveloping.

Todd et al. developed dynamic simulation models for three truck configurations: a quarter-truck model, a half-single-unit two-axle truck model, and a half five-axle semitrailer truck model (*15*). The objectives of the study were to predict ride quality and pavement loading and to arrive at proper mechanical characteristics for these vehicle models. The quarter-truck model developed is shown in Figure 5. The differential equations describing its motion are identical to Equations 1 and 2 described earlier; their expanded form is given by Equations 8–11:

$$\dot{q}_1 = q_3 \quad (8)$$

$$\dot{q}_2 = q_4 \quad (9)$$

$$\dot{q}_3 = [(-q_1 K + q_2 K - q_3 C + q_4 C)]/M_s \quad (10)$$

$$\dot{q}_4 = [(q_1 K - q_2 (K_t + K) + q_3 C - q_4 C + uK_t)]/M_u \quad (11)$$

The mechanical constant values used in the quarter-truck simulation (*15*) are given here:

Symbol	*Value Selected*
M_s	22.9036 lb-sec²/in.
M_u	2.976 lb-sec²/in.
K	6,500 lb/in.
C	15 lb-sec/in.
K_t	5,000 lb/in.

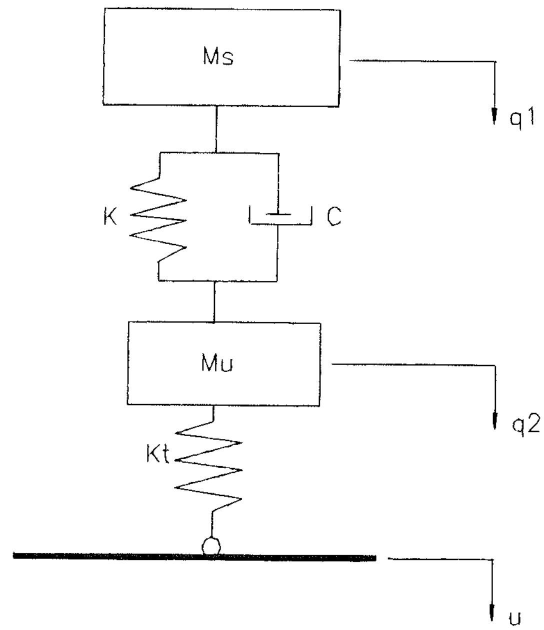

FIGURE 5 Quarter-truck model (*16*).

In a similar fashion, the formulation of the other two truck models treated suspension systems as combinations of linear springs and dash-pots, whereas tires were modeled as linear springs. The formulation for these models was more complex, having 2 and 4 degrees of freedom, respectively. For all three models, a fourth-order Runge-Kutta algorithm was used for solving the simultaneous differential equations involved. These models were then tested using two types of road profiles: (*a*) a simulated sinusoidal profile to determine frequency responses, and (*b*) several actual road profiles to calculate summary statistics of vehicle responses.

For the latter, two statistics were proposed, namely, the root mean square (*RMS*) of the vertical acceleration of the sprung mass and the dynamic impact factors (*DIF*) of the dynamic axle loads, (i.e., Equations 12 and 13, respectively).

$$RMS = \left[\frac{1}{N} \left(\sum_{i=1}^{N} a_i^2 \right) \right]^{1/2} \quad (12)$$

where a_i equals acceleration of the sprung mass at the ith time step and N is the number of observations.

$$DIF = \left\{ \frac{\left[\sum_{i=1}^{N} (F_i - F)^2 \right]}{(N-1) * F^2} \right\}^{1/2} \quad (13)$$

where F_i is the tire force at the ith time step and F is the mean tire force.

These two statistics were related to ride comfort and pavement damage, respectively. Transfer functions were presented for the dynamic axle load of the two-axle truck model versus the amplitude of the sinusoidal profile. The study concluded that the quarter-truck model yielded higher sprung mass *RMS* values and dynamic load *DIF* factors than the two-axle and the five-axle truck models.

ANALYSIS FRAMEWORK

Two vehicle response parameters of interest in studying heavy vehicle–pavement interaction have been identified:

- Dynamic axle loads generated at the tire-pavement interface, which affect pavement damage and vehicle damage, and
- Vertical sprung mass acceleration of the vehicle, which affects the ride quality and the damage of the cargo.

Hence, the proposed pavement roughness model and summary statistic must reflect these two heavy vehicle response parameters. The following sections explore the relationship between pavement roughness profile and these two vehicle response parameters using both experimental data and simulated data obtained with a quarter-vehicle model.

Analysis of Experimental Data

The experimental data were obtained with the instrumented vehicle developed by the NRCC (*8*). The instrumentation included strain gauges and accelerometers located on the axles, yielding dynamic axle load, as well as accelerometers located on the body of the vehicle. Data were obtained on five pavement sections of increasing roughness, (i.e., Site 1, Site 2, and so on) at three vehicle speeds, (i.e., 40, 60, and 80 km/hr). The vehicle was equipped with an air suspension in the drive axles and a rubber suspension in the trailer axles. Pavement roughness was measured at intervals of 0.15 *m*, (i.e., 6 in.) using a Surface Dynamics Profilometer.

The data were analyzed by means of transfer functions between the vehicle response parameters of interest and the pavement roughness excitation input. Figures 6 and 7 show transfer functions for dynamic axle load for Sites 1, 3, and 5 for the rubber suspension and the air suspension, respectively, at a speed of 80 km/hr. Figure 6 shows clearly that regardless of roughness level, the rubber suspension is sensitive to excitation frequencies of about 3.5 and 12.5 Hz. This trend is repeated, in a less obvious fashion, for the air suspension. The "noise" that appears on Site 1 is due to the tire eccentricity [i.e., at 22.2 m/sec (80 km/hr) a tire with a radius of 0.57 m has a circumference of 3.6 m, which results in a load excitation frequency of about 6 Hz]. At low roughness levels, this source of excitation can contribute substantially to the dynamic load variation observed.

Figure 8 shows the transfer function of the vertical acceleration at the rear of the trailer versus the pavement roughness excitation input. Clearly, the vertical sprung mass acceleration is sensitive to the same 3.5 Hz excitation frequency identified earlier. Note that for a vehicle speed of 22.2 m/sec, the 3.5 Hz excitation frequency corresponds to a pavement roughness wavelength of about 6.3 m (i.e., 21 ft). This sensitivity to the 3.5 Hz pavement roughness excitation frequency remains relatively unchanged as the vehicle speed increases (Figure 9). It is evident that the acceleration of the sprung mass contributes significantly to the dynamic axle load variation observed.

Hence, this is the single most important vehicle response parameter related to both aspects of heavy vehicle–pavement interaction, namely, pavement and vehicle damage and ride quality and cargo damage. Clearly, the pavement roughness model needed should reflect the vertical sprung mass acceleration response of heavy vehicles. Furthermore, it should exhibit a dynamic behavior

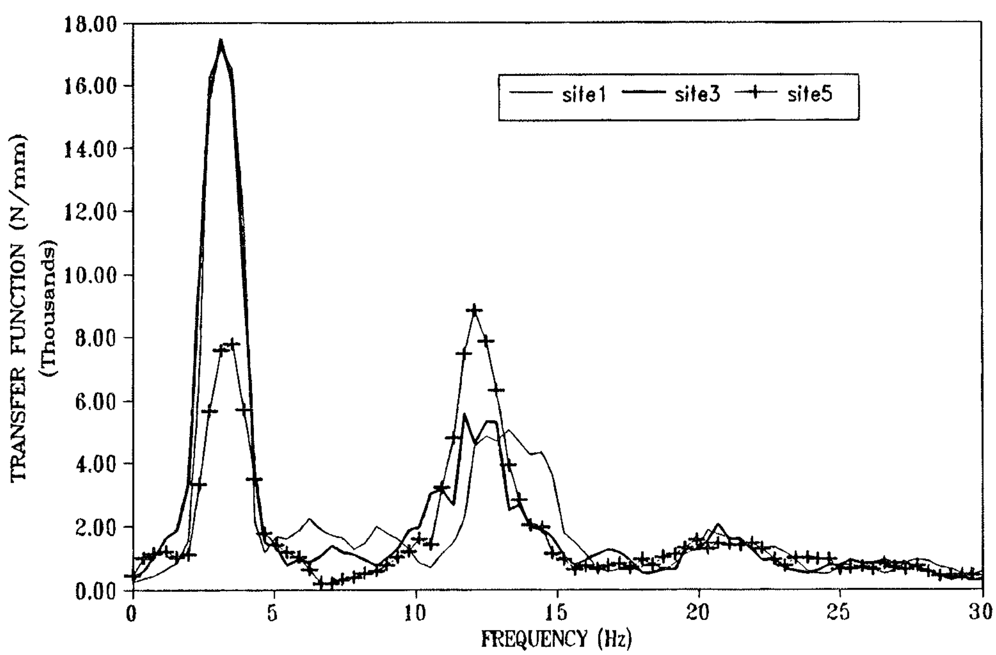

FIGURE 6 Transfer function of dynamic load versus profile elevation difference, rubber suspension at 80 km/hr.

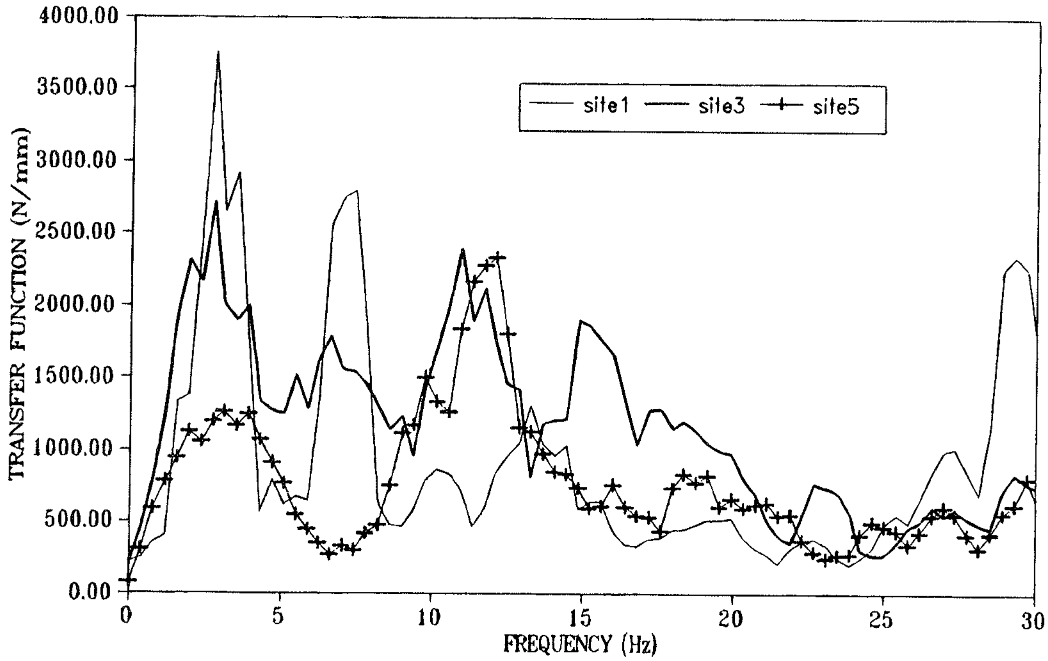

FIGURE 7 Transfer function of dynamic load versus profile elevation difference, air suspension at 80 km/hr.

similar to the one observed experimentally. As explained next, a simple quarter-vehicle model was studied for this purpose.

Analysis of Quarter-Vehicle Simulation Data

A simple quarter-vehicle model was developed by solving numerically the four differential equations described earlier (Equations 8–11). The equations were solved through an Adams-Moulton/Gear algorithm using the IMSL software library (*16*) in the PC environment. The model provided for smoothing of the pavement elevation profile through a moving average technique to account for tire enveloping.

The first quarter-vehicle model tested had mechanical constants identical to the quarter car used as the reference vehicle for calculating the IRI (*2*). The transfer functions for the *ARS* statistic and the

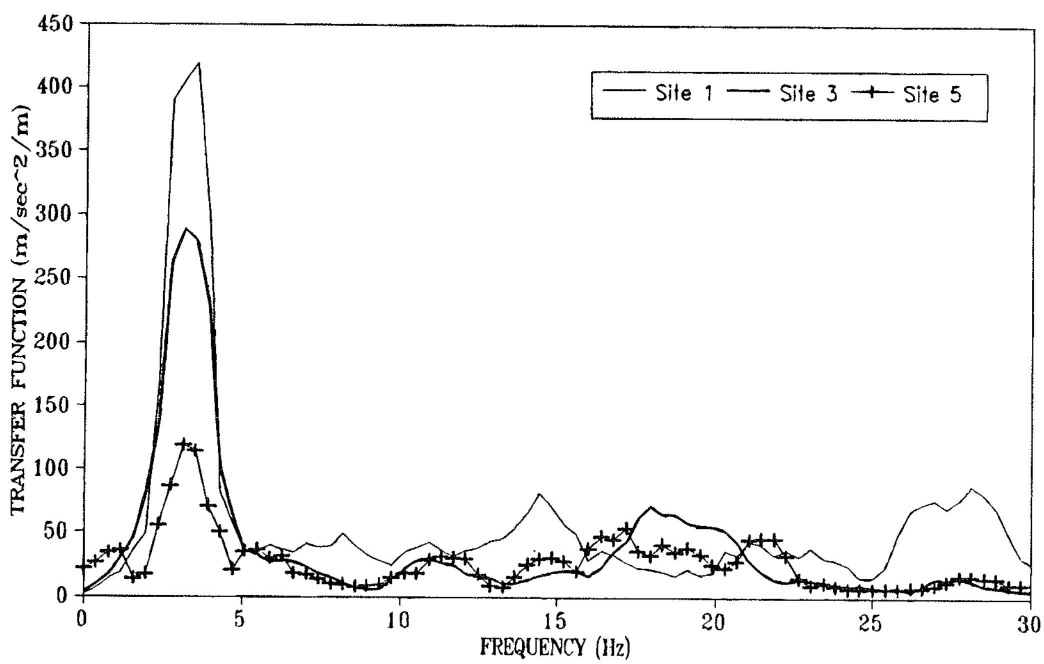

FIGURE 8 Transfer function of vertical sprung mass acceleration versus profile elevation difference at 80 km/hr.

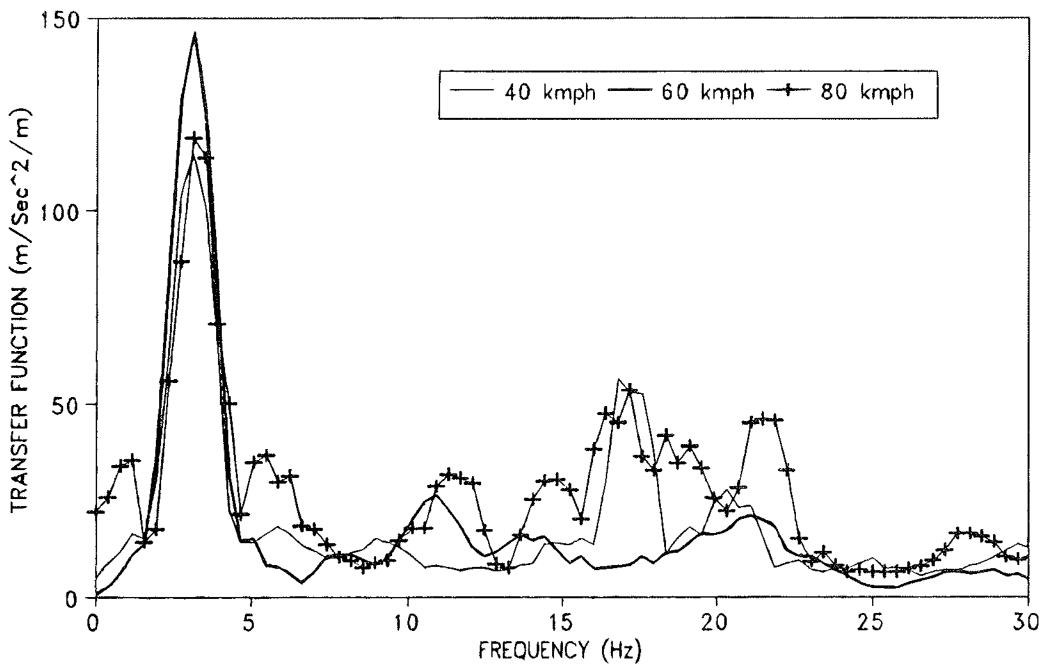

FIGURE 9 Transfer function of vertical sprung mass acceleration versus profile elevation difference, Site 5.

vertical sprung mass acceleration versus the pavement roughness excitation input are shown in Figures 10 and 11, respectively. Figure 10 shows clearly that the *ARS* is sensitive mainly to excitation frequencies of 1.5 Hz, which suggests that the IRI is not suitable as a roughness index reflecting the dynamic observed behavior of heavy trucks as described earlier. Furthermore, Figure 11 suggests that the sprung mass acceleration of the quarter car is sensitive to excitation frequencies of 1.5 and 11 Hz, which is inconsistent with observed behavior. Hence, the quarter-car simulation appears to be unsuitable for modeling the roughness attributes, which affect heavy vehicle–pavement interaction.

The second quarter vehicle tested had the mechanical properties of the quarter truck described by Todd et al. (*15*). These are summarized here:

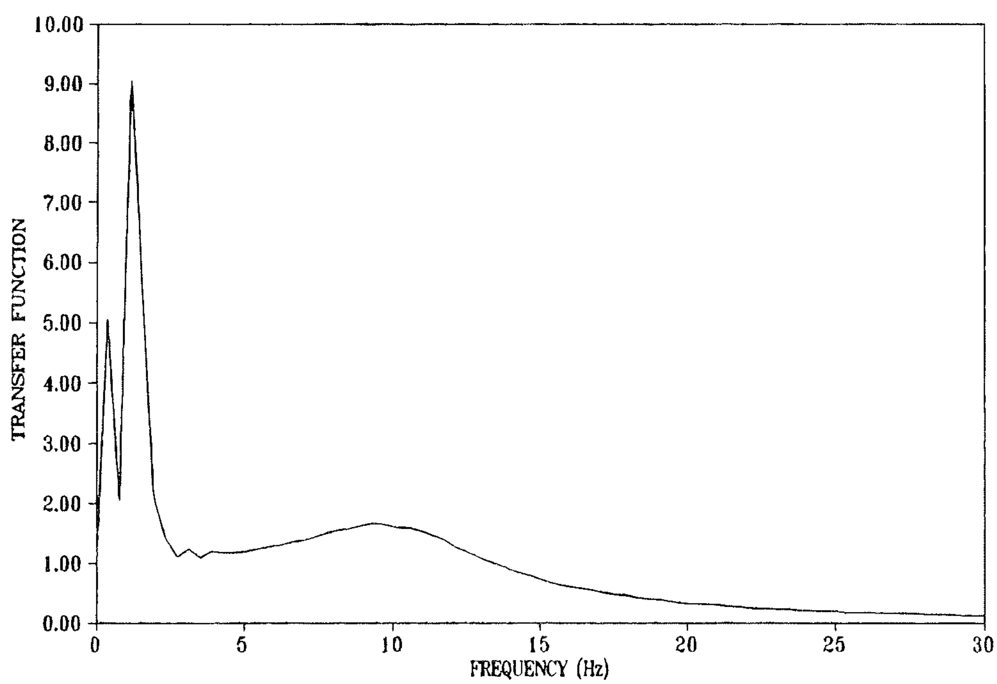

FIGURE 10 Transfer function of ARS versus profile elevation difference, quarter-car model at 80 km/hr, Site 3.

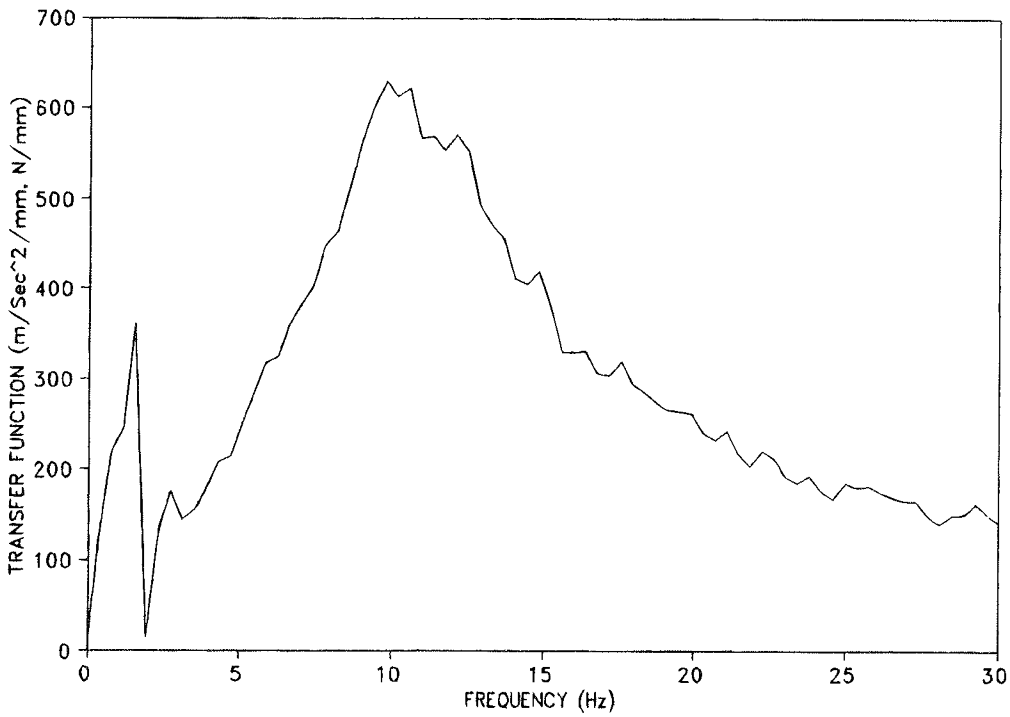

FIGURE 11 Transfer function of vertical sprung mass acceleration versus profile elevation difference, quarter-car model at 80 km/hr, Site 3.

Variable	*Constant*
K/M_s	118 sec^{-2}
K_t/M_s	755 sec^{-2}
M_u/M_s	0.146
C/M_s	4.7 sec^{-1}

Figure 12 shows the transfer function of the sprung mass acceleration of this quarter truck versus the pavement roughness excitation input. The effect of the mechanical constants of this quarter vehicle on the sprung mass acceleration transfer function was explored, as shown in Figure 13. It can be seen that the constants resulting in a transfer function similar to the one observed with the instrumented NRCC vehicle are similar to the ones used by Todd and Kulakowski (*15*) with the exception of the elastic tire constant, which should be considerably lower (i.e., K_t/M_s = 200 sec^{-2}). The selected quarter-vehicle constants are given here:

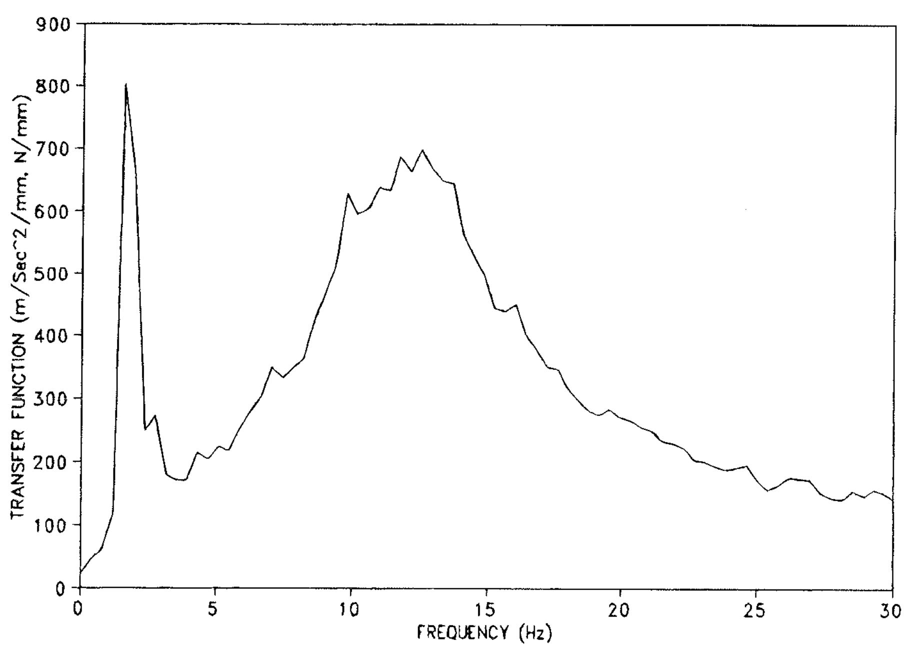

FIGURE 12 Transfer function of sprung mass acceleration versus profile elevation, quarter-truck model at 80 km/hr, Site 3.

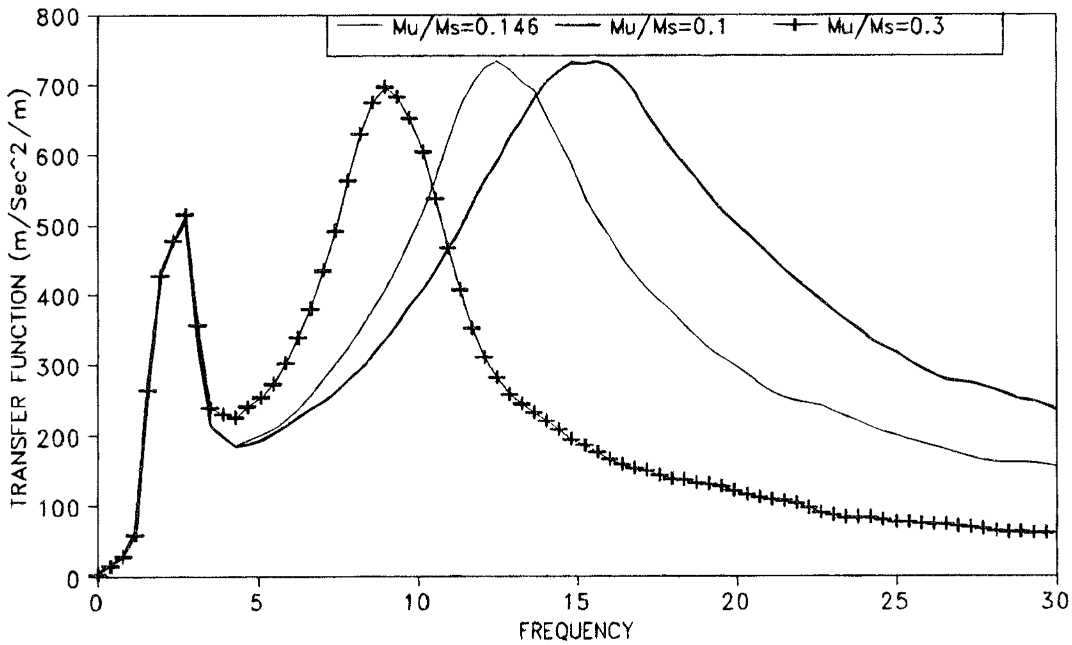

FIGURE 13 Example of effect of quarter-vehicle constants on dynamic response of quarter vehicle, Site 3.

Variable	*Constant*
K/M_s	118 sec^{-2}
K_t/M_s	200 sec^{-2}
M_u/M_s	0.146
C/M_s	4.7 sec^{-1}

Developing a Pavement Roughness Summary Statistic

The earlier discussion established that the vertical acceleration of the sprung mass of a vehicle is the response parameter of interest in describing heavy vehicle–pavement interaction. Furthermore, it demonstrated that a quarter-vehicle model with suitable mechanical constants can exhibit a dynamic response similar to the one observed experimentally.

This response is best described by the transfer function of the vertical sprung mass acceleration with respect to the pavement roughness profile excitation. It was also known that the spectral density of the sprung mass acceleration of a vehicle driving over a known pavement roughness profile can be calculated as the product of the sprung mass acceleration transfer function multiplied by the square of the spectral density of the profile (Figure 2).

Furthermore, integration of the resulting spectral density over all the frequencies of excitation results in a statistic, which is indicative of the mean square of the accumulated vertical sprung mass acceleration over the entire length of the section (*17*, p. 52). This statistic has units of energy per unit sprung mass per unit length of pavement traveled and represents the energy input from the road to the vehicle and vice versa. For a particular pavement section the roughness statistic depends only on the transfer function selected as reference; hence, it is universal and stable.

In summary, the following steps are required for calculating the proposed summary roughness statistic:

1. Calculate the spectral density of the pavement roughness profile.
2. Multiply this spectral density by the square of the transfer function selected to obtain the spectral density of the vertical sprung mass acceleration of the reference quarter vehicle excited by the given roughness profile.
3. Calculate the integral of the spectral density of the vertical sprung mass acceleration over the full frequency spectrum and take the square root.

The procedure described in these steps was implemented into a PC-based computer software program named TRRI (Truck Response to Roughness Index).

CONCLUSIONS

This paper has presented a roughness model describing the pavement roughness attributes affecting heavy vehicle–pavement interaction. Dynamic vehicle response data from two sources were analyzed, namely, experimental data obtained with the instrumented vehicle developed by the NRCC and simulated data obtained with a quarter-vehicle simulation. It was found that the vehicle response parameter of interest in this interaction is the sprung mass vehicle acceleration because it relates to pavement and vehicle damage as well as to ride quality and cargo damage. This was demonstrated by analyzing the transfer functions of both the dynamic axle load and the vertical sprung mass acceleration over a range of pavement roughnesses and vehicle speeds. The sprung mass vertical acceleration transfer function showed a distinct sensitivity to pavement roughness excitation frequencies of 3.5 Hz. A pavement roughness statistic called TRRI was described, summarizing the frequency content of the vertical sprung acceleration of a reference quarter car described by its transfer function.

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