Modeling Skid Resistance for Flexible Pavements: A Comparison Between Regression and Neural Network Models

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The phenomenon of pavement surface friction or skid resistance involves the complex interaction of pavement, vehicle, and environmental factors. Results of skid resistance measurements form the basis of many pavement management safety decisions including the identification of areas of excessive slipperiness, planning of maintenance or rehabilitation activities, and evaluation of material types and new construction practices. Various model forms of skid resistance have been developed using classical statistical methods such as regression. Computational models designed to resemble the human brain and characterized as neural networks have been used successfully in the past in other fields such as economics, medicine, and stock market research to analyze problems involving very complex interrelationships; they are found to perform better than classical statistical methods. The use of neural network models as an alternative to regression models for predicting skid resistance on flexible pavements for assessing future rehabilitation needs is examined. Using data from in-service flexible pavements, separate skid resistance models are developed with both regression and neural network methods. The models are tested and compared, and the results indicate that neural networks can model their environment more convincingly than regression models for the flexible pavements studied.

Significant advances in equipment and test methods, coupled with increasing road user concern about pavement safety, recently have made the routine measurement of skid resistance a popular task among federal and state highway departments. In pavement management, skid resistance is considered an indicator of pavement serviceability. It provides a basis for decisions about pavement management safety, including the identification of areas of excessive slipperiness, planning of maintenance or rehabilitation activities, and evaluation of material types and cost of new construction practices related to improving pavement surface friction.

In an effort to develop a comprehensive pavement management system for its network of roads, the Connecticut Department of Transportation (ConnDOT) initiated a long-term (10-year) pavement performance study in 1984 to gather pertinent information on in-service pavements to use in evaluating pavement performance in three areas—roughness, condition, and safety. The concept of pavement safety generally has been related to its surface friction, hence a skid test program involving the routine measurement of skid friction was included in the overall program. The in-service pavements considered for long-term monitoring consisted of 57 flexible, 11 rigid, and 14 composite pavements. The flexible pavements were broken further into three categories: 33 Type B pavements, which contain bituminous concrete overlays on liquid treatments or chip seals; 14 Type F pavements, which contain bituminous concrete overlays on original bituminous concrete pavements; and 10 Type E pavements, which consist of original bituminous concrete pavements with no overlays. This paper examines the variation in skid resistance for Type E pavements only, using data gathered by ConnDOT between 1984 and 1989. The purpose of the analysis is to develop and compare neural network and regression models to predict skid resistance for assessing future rehabilitation needs on this class of pavement.

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CONCEPTS OF LOSS OF PAVEMENT SKID RESISTANCE

The concept of skid resistance loss has been discussed by various researchers (*1–3*). A combination of weather, traffic, pavement surface, and vehicle characteristics influences the deterioration of pavement skid resistance over time. Under the rolling and braking action of vehicular traffic, gritty surfaces of exposed aggregates in asphalt surfaces become smooth, rounded, and susceptible to polishing and wear. This results in a reduction in skid resistance between the tire and pavement surface, especially in wet weather. Other factors—including variations in temperature, intensity of rainfall, accumulation of contaminants (e.g., oil spillage, chemicals), structural deficiencies such as rutting due to compaction, and lateral distortion or wear—can also lead to a reduction in skid resistance and may be hazardous depending on their magnitudes. Under high temperatures, for example, poorly designed asphalt mixes tend to bleed and cover the gritty surface of the aggregate and reduce skid friction.

With time or traffic the serviceability profile of flexible pavements in terms of skid resistance may be represented as shown in Figure 1. The skid resistance attains a maximum value immediately after construction when all asphalt film on the surface wears off and the fresh gritty surface of the pavement becomes exposed. However, with time the resistance reduces under the action of traffic and the environment. A reduction below the minimum acceptable level prevents the pavement from serving its desired purpose; hence, knowing when this stage will be reached is essential to an overall life-cycle analysis of pavement performance.

GENERAL DESCRIPTION OF DATA BASE

The data base used for this analysis includes a time series of skid numbers (measure of skid resistance) acquired by ConnDOT for Type E flexible pavements between 1984 and 1989 in accordance

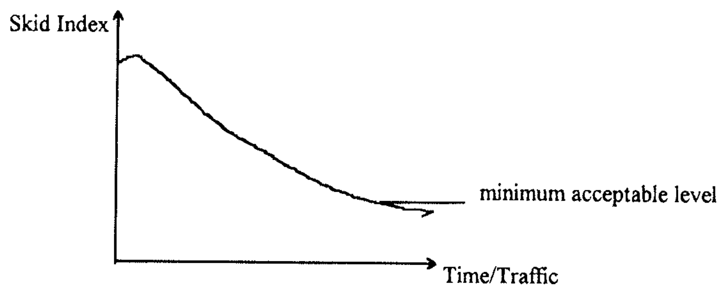


FIGURE 1 Concept of variation of skid resistance with time and traffic.

with ASTM E274-79 (*4*). The skid number is usually corrected to and reported for a standard speed of 40 mph, according to the following equation:

$$SN_{40} = SN_F + (V_F - 40)/2$$

where SN_{40} is the skid number corrected to 40 mph, and SN_F is the skid number recorded in the field at speed V_F. This equation is valid for V_F between 30 and 45 mph (D. Laursen, unpublished work), and SN_{40} is considered an indicator of the potential for skidding in wet weather. Skid resistance measurements were taken around the same time (May–July) for the period under consideration.

The data base also included information on pavement age, posted speed limit, traffic volume, and pavement regional location. The pavement regional locations included the Shore-line area close to the Atlantic Ocean (Long Island Sound), the Connecticut River Valley (CRV), and the Hills Region, which tends to have a much cooler temperature and more frequent snowfall than the other two locations. Detailed information on pavement surface characteristics that influence skid resistance, such as aggregate type, shape, gradation, texture, mineral composition, and hardness, was not available in the data base. However, a close examination of the surface course mix information indicated that all Type E pavement surface courses consisted of ConnDOT-designated Class 1, dense-graded, plant-mixed bituminous concrete material with medium to fine surface texture (PMB). The characteristics of a Class 1 PMB mixture are as follows:

Sieve Size	*Percentage Passing*
#200	3–8
#50	6–26
#8	28–50
#4	40–65
3/8 in.	60–82
1/2 in.	70–100
3/4 in.	90–100
1 in.	100

- Percentage bitumen: 5–8
- Aggregate temperature: 280–350°F
- Bitumen temperature: 325°F max
- Mix temperature: 265–325°F
- Percentage voids: 3–6

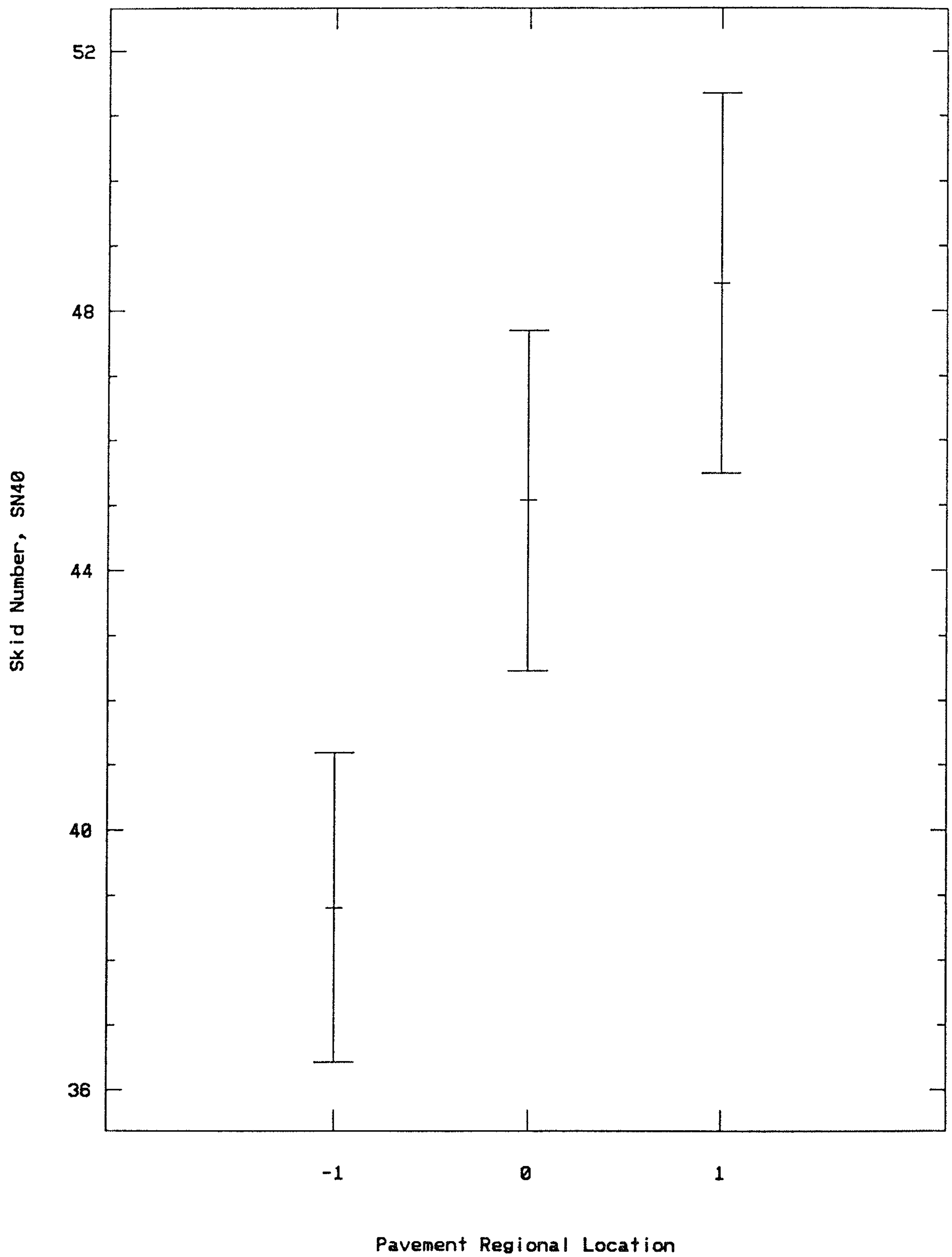
DEVELOPMENT OF SKID RESISTANCE MODEL USING REGRESSION METHODS

The relationship between pavement skid resistance and accidents has been analyzed widely by various researchers, (*5–7*) with moderate success. Others have attempted to develop statistical models relating skid resistance to texture parameters (*8–10*) and have often compared measurements obtained using bald versus ribbed or treaded tires. No significant effort, however, has been made to model skid resistance for assessing or projecting future rehabilitation needs. An evaluation of skid resistance for the purpose of assessing future rehabilitation needs should consider changes on the basis of time, traffic, and climatic effect. Such considerations require periodic measurements (*1*). Hence, the potential variables from the ConnDOT data base considered to influence pavement skid resistance include traffic, in the form of average annual daily traffic (AADT); pavement age; posted speed limit; and climate represented by pavement regional location.

Pavement surface material characteristics such as aggregate type, shape, gradation, hardness, mineral composition, and quality of mix design are significant contributors to skid resistance changes. Such variables were, however, not available from the ConnDOT data base. A close examination of the Type E pavements does indicate that all the pavement surface courses consist of ConnDOT-designated Class 1 PMB, so it is assumed that variations in skid resistance for this class of pavements are due to external factors rather than characteristics of surface course material.

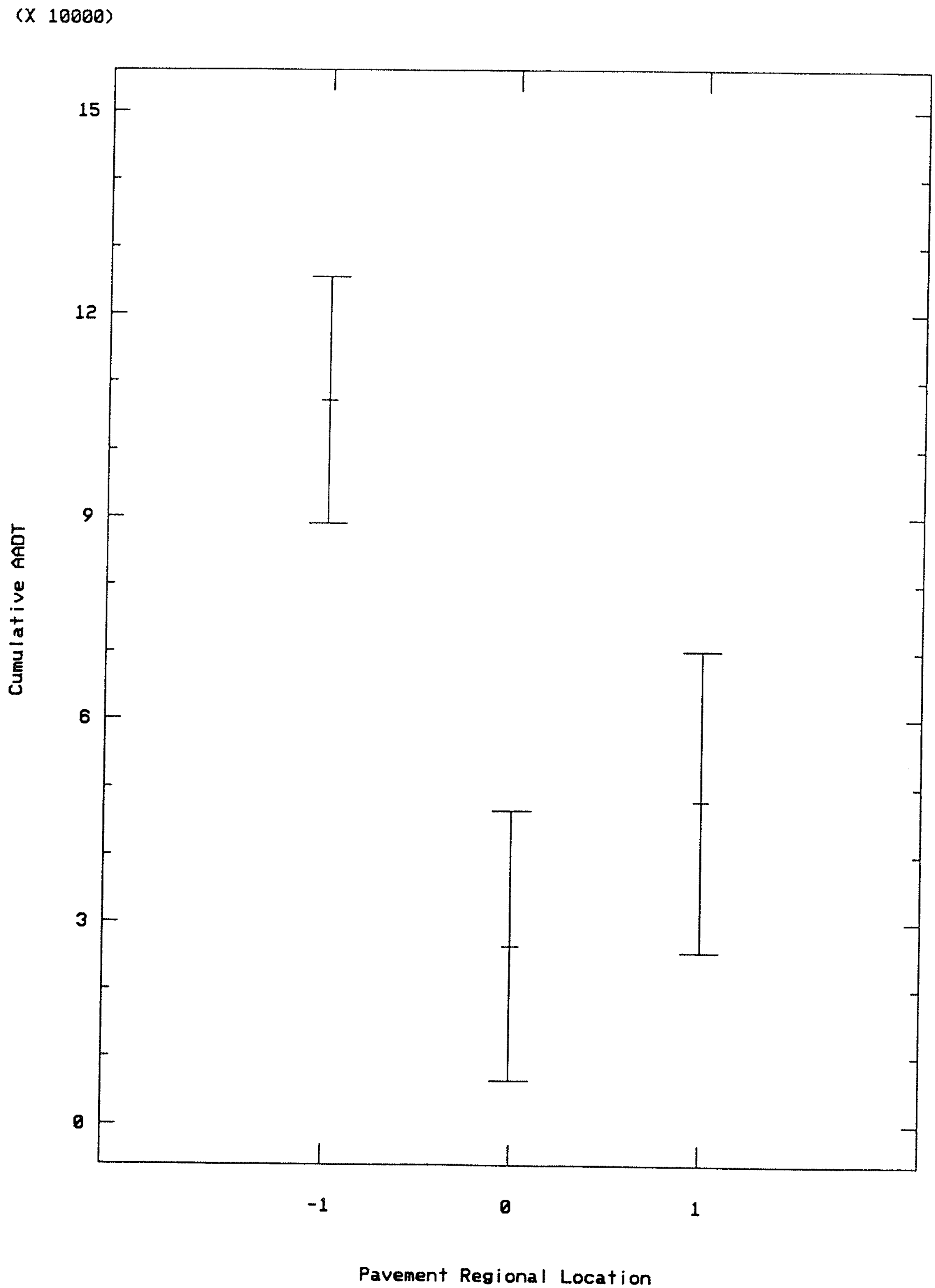
The data analysis consisted of two phases: a preliminary phase and a model building phase. The preliminary phase used analysis of variance (ANOVA) to examine whether regional location has any influence on the level of skid resistance. A correlation matrix was set up to investigate the strength of association between skid resistance and the potential independent variables and to examine problems of collinearity among the independent variables. Figure 2 shows results of the ANOVA tests; it indicates that for a 95 percent confidence level, differences exist in the means of skid numbers found in the three regional locations.

In addition, there is no significant difference between skid number means for the Shore-line and Hills Region compared with that obtained for the CRV, which is significantly lower. The lower skid numbers in the CRV may be due to the very high traffic level found in the region; this conclusion is drawn from analysis of Figure 3. In Figure 3 the effect of pavement regional location on accumulated traffic volume (AADT) is presented for 95 percent confidence intervals for means of accumulated AADT. Figure 3 further indicates that the traffic level for the Shore-line area is higher than that of the Hills Region. It generally is expected that higher skid numbers are associated with regions with low traffic levels such as the Hills, but the converse is true in this case (see Figure 2). This anomaly may be due in part to the more frequent snowfall in the Hills, which result in a correspondingly high plow frequency. The abrasive effect of snow plow equipment causes the pavement to lose its microtexture and become more susceptible to wear and polishing under the action of traffic.



-1 =CRV, 0 = HILLS, 1=SHORELINE REGION

FIGURE 2 Effect of pavement regional location on skid number.



-1 =CRV, 0 = HILLS, 1=SHORELINE REGION

FIGURE 3 Effect of pavement regional location on cumulative AADT.

The model building phase used results from the preliminary phase to select key variables that could explain the variation in skid resistance. Results of the preliminary analysis suggest that the main variables that affect skid resistance variations on the flexible pavements include the environmental factor denoted by the pavement regional location, accumulated traffic volume (AADT), and pavement surface age. Using these variables, a multiple linear regression model (Equation 1) was developed. The regression equation was evaluated using the following criteria at a 95 percent confidence level: (*a*) the *F*-statistic was used to assess the overall significance of fitting the regression equation, and (*b*) the *t*-statistic was used to test the importance of any one term in the model after all other terms have been included.

$$\begin{aligned} SN_{E(t)} &= 46.12 + 2.43PLC - 9.44 \times 10^{-11}\,MADT^{2.6} \\ &\quad\ (66.75) \quad (2.86) \qquad\qquad (-5.46) \\ &\quad <0.000> \ <0.002> \qquad <0.000> \\ \text{overall } F &= 52.64,\ \text{adjusted } R^2 = .63, \text{SE} = 3.92,\ n = 45 \end{aligned} \qquad (1)$$

where

$SN_{E(t)}$ = skid number corresponding to pavement surface at age t;
PLC = pavement regional location = −1 for CRV, 0 for Hills, and 1 for Shore-line;
n = number of observations;
$MADT$ = mean accumulated AADT corresponding to pavement surface age t (veh/day/year)
$= \left(\sum_{i=1}^{t} \text{AADT}\right)/t$
() = computed t-value statistics for independent variables; and
<> = significance, $\propto$, of coefficients.

The speed variable was not statistically significant, hence it did not appear in the final model. Equation 1 indicates that for flexible pavements with Class 1 PMB surface course mix, as much as 63 percent of the variation in skid number is explained by the mean accumulated AADT and the pavement regional location. The unexplained portion could be due to factors such as variability in the mix and in its placement, which could differ in some aspects for the same written specifications especially when different contractors are used for the construction of the different pavements.

NEURAL NETWORKS

Neural networks consist of computational models in the form of interconnected nonlinear mesh-like processing elements (PEs) or nodes capable of mimicking neurons, the basic PEs of the human brain. PEs typically are arranged in layers in a neural network with the main inputs to the network forming the first, or input, layer. Each PE (with the exception of the ones forming the main inputs to the network) generally receives values from all of its abutting input connections. These inputs to a PE are typically defined as a weighted sum of the outputs from all other PEs to which it is connected. The weights are learned by training examples supplied to the network. Each PE, then, performs a predefined mathematical function and produces a single output value that is transferred to other PEs until an overall output is generated by the network.

Building a neural network for solving a particular problem involves addressing a number of issues. These include the type of mathematical function to be used by PEs in processing information, specification of weights for connections between input and output PEs, the learning paradigm suitable for the problem under consideration, and the architecture of the network.

The number of PE functions possible for a neural network are infinite, but five are used regularly by most neural networks: the linear, step, ramp, sigmoid, and Gaussian functions. All of these functions, except the linear type, introduce a nonlinearity in the network dynamics by bounding the output values within a fixed range (*11*). The linear PE function produces a linearly modulated output from an input X as described by the function

$$f(X) = \alpha X$$

where X ranges over the real numbers and α is a positive scalar. If $\alpha = 1$, it is equivalent to removing the the PE function completely. The step PE function produces only two values, β and δ. If the input to the PE function X equals or exceeds a predefined value θ, then the step PE function produces the value β; otherwise it produces the value $-\delta$, where β and δ are positive scalars. Mathematically, this function is described as

$$\begin{aligned} f(X) &= \beta \qquad \text{if } X \geq \theta \\ &= -\delta \qquad \text{if } X < \theta \end{aligned}$$

Typically, the step PE function produces a binary value in response to the sign of the input, emitting +1 if $X \geq 0$ and 0 if $X < 0$. The ramp PE function is a combination of the linear and step PE functions. It places upper and lower bounds on the values that the PE function produces and allows a linear response between the bounds. These saturation points are symmetric around the origin and are discontinuous at the points of saturation. Mathematically, the ramp function is defined as

$$\begin{aligned} f(X) &= \gamma \qquad \text{if } X \geq \gamma \\ &= X \qquad \text{if } |X| < \gamma \\ &= -\gamma \quad \text{if } X \leq -\gamma \end{aligned}$$

where γ is the saturation value for the function and the points $X = \gamma$ and $X = -\gamma$ are where discontinuities in $f(X)$ exist. The sigmoid PE function is a continuous version of the ramp PE function. The sigmoid (S-shaped) function is a bounded, monotonic, nondecreasing function that provides a graded, nonlinear response within a prescribed range. The most common sigmoid function is the logistic function

$$f(X) = 1/(1 + e^{-\alpha X})$$

where $\alpha > 0$ (usually $\alpha = 1$), which provides an output value from 0 to 1.

The Gaussian PE function, on the other hand, is a radial function (symmetric about the origin) that requires a variance value $\nu > 0$ to shape the Gaussian function. The PE function takes the form

$$f(X) = e^{(-x2/\nu)}$$

where X is the mean and ν is a predefined variance.

Connections in a neural network form the links or paths along which information from one PE is passed to another, that is, they define the information flow through the network. Each connection typically carries a weight that modulates the amount of information passed between PEs. The value of the connection weights is often determined by a neural network learning procedure, but it can also be predefined to the network randomly within a particular range before the start of training a network. It is through the adjustment of the connection weights that the neural network is able to learn, update operations for each PE, and recall information.

The arrangement of the PEs, connections, and patterns into a neural network is known as the network architecture. The network typically is organized into layers of PEs: an input layer, output layer, and intermediate or "hidden" layers which provide a link between the input and output layers. The number of layers is determined as part of the network design process; in theory any number of layers can be used, but in practice, three layers are used most often (*12*).

Within a layer, PEs are similar in two respects: first, the connections that feed the layer of PEs are from the same source (i.e., they receive inputs from the same layer). Second, the PEs in each layer use the same type of update dynamics; for example, all the PEs use the same type of connections and the same type of PE function. The number of PEs in the input layer is equal to the number of independent variables. The number of PEs in the output layer is the number of items that is being predicted (i.e., the dependent variables). The number of hidden PEs to use is more difficult to determine. Often the network must be trained several times, each with a different number of hidden PEs, to find the optimal number of hidden PEs for a given problem. Generally, reducing the number of hidden PEs tends to improve the ability of the network to attain a generalized solution and hence be able to respond correctly to input patterns to which it has never been exposed before.

NEURAL NETWORK BUILDING WITH AUTONET

Neural networks have gained considerable attention in recent years because of their ability to learn and deal with problems involving complex interactions. Several researchers have reported exceptional results with the use of neural networks in various fields (*13–16*) using a variety of neural network software packages. The software package employed in this paper is Autonet, which uses an algorithm based on adaptive modeling procedures. The basic approach in Autonet is that each PE in the network receives input from exactly two other PEs, with the exception of the PEs representing the inputs to the network. For any two inputs p and q, an output O is obtained using a complete quadratic PE function of the form

$$O = a + bp + cq + dp^2 + eq^2 + fqp \qquad (2)$$

The coefficients $a, b, c, \ldots f$ are estimated by comparing the observed output with the desired output based on training examples presented to the network. The network learns the correct weights by example and adjusts them in the direction that minimizes the error between current network outputs and the target values. The input variables represent the first layer. Each of these are combined two at a time using the quadratic PE function described in Equation 2 to create a second layer of the network and other subsequent layers. The second-layer PEs are quadratic functions of the input variables, the third layer involves fourth-degree polynomials, the fourth reaches eighth degree, and so on. In this manner extremely complicated output patterns can be recognized by the network even with only a few layers. Yet each node is a quadratic function, arguably the simplest mathematical function with the ability to combine to form more complicated functions.

The number of PEs generated by this approach clearly will be unmanageable as the number of layers increases. Hence, Autonet limits the number of PEs in each layer as well as the number of layers once the performance of the network begins to deteriorate. The stopping rule is based on a squared error criterion, and data used to estimate the coefficients (weights) are kept separate from those used in the stopping rule. Hence, networks developed with Autonet are validated on data that have not been used to estimate the various coefficients of the quadratic functions (*17*).

DEVELOPMENT OF SKID RESISTANCE MODELS USING NEURAL NETWORKS

Model building for a neural network consists of two phases: learning and testing. During the learning process, the network is presented with data describing the problem to be solved—basically the values of the dependent and independent variables. The number of observations for training is typically specified; The network uses these observations to learn the relationships between the dependent and independent variables. During the learning process, layers of PEs are created as long as the best PE in each new layer represents an improvement over the best PE in the previous layer. The training process is terminated once the performance of the network begins to deteriorate. After training, the remaining observations are used on the network to determine how the network has generalized the problem.

The data for Type E pavements consisted of 60 observations: 45 (75 percent) were selected randomly and used to train the network, and the remaining 15 (25 percent) were used to test the validity of the network. Each observation consisted of skid number, pavement surface age, pavement regional location represented by a dummy variable, accumulated AADT, and posted speed limit.

Figure 4 shows the overall architecture of the neural network model generated for the pavement using Autonet. The architecture consists of 10 PEs arranged in four layers. The first layer is the input layer with four PEs, which brought into the network the values of the independent variables; the output layer has one PE, through which the network delivered its estimate of the values of the dependent variable (skid number). In between the output and the input layers are two hidden layers containing five PEs, which performed most of the work. They are called "hidden" because their PEs make contact only with PEs in the input and output layers; they are "hidden" from the outside world.

COMPARISON OF NEURAL NETWORK AND MULTIPLE LINEAR REGRESSION MODELS

Parametric estimators such as linear regression are high-bias estimators in that they assume an a priori model (e.g., a linear relationship). Neural networks, however, are analogous to nonparametric regression methods in that they make no a priori assumptions about the problem; the data are allowed to speak for themselves. The fundamental objective of each of the two methods presented was to fit an accurate model for the universe of the skid number data. The

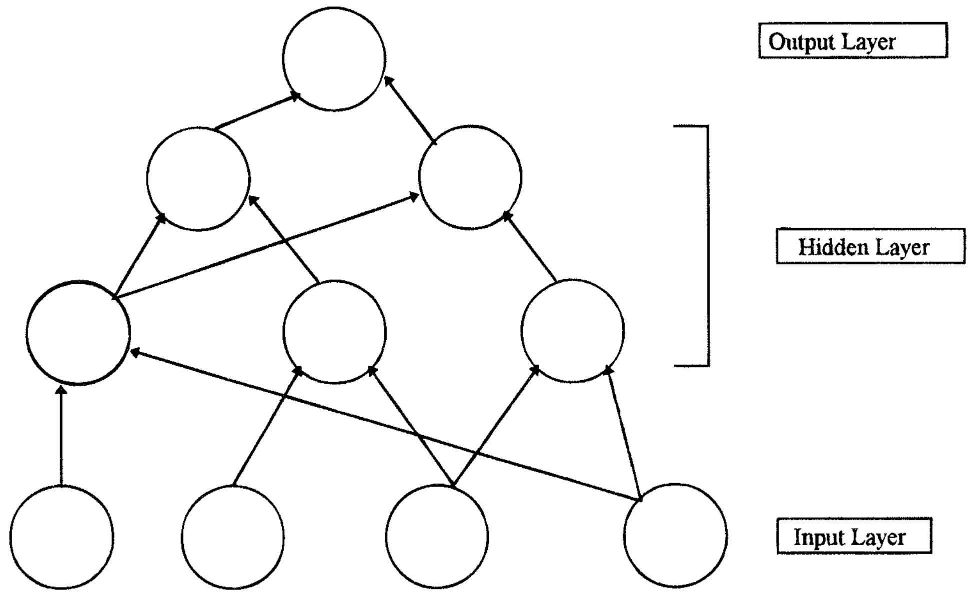


FIGURE 4 Neural network model architecture for flexible pavement.

accuracy of fitness typically is measured by the mean absolute error or the standard error of the estimate (SE) or the coefficient of determination (R^2) of predicted versus actual values. The criterion determining whether the neural network (NN) model performs better than multiple linear regression (MLR) is if it converges to smaller SE and has higher R^2 for actual versus predicted data when compared with the multiple linear regression model.

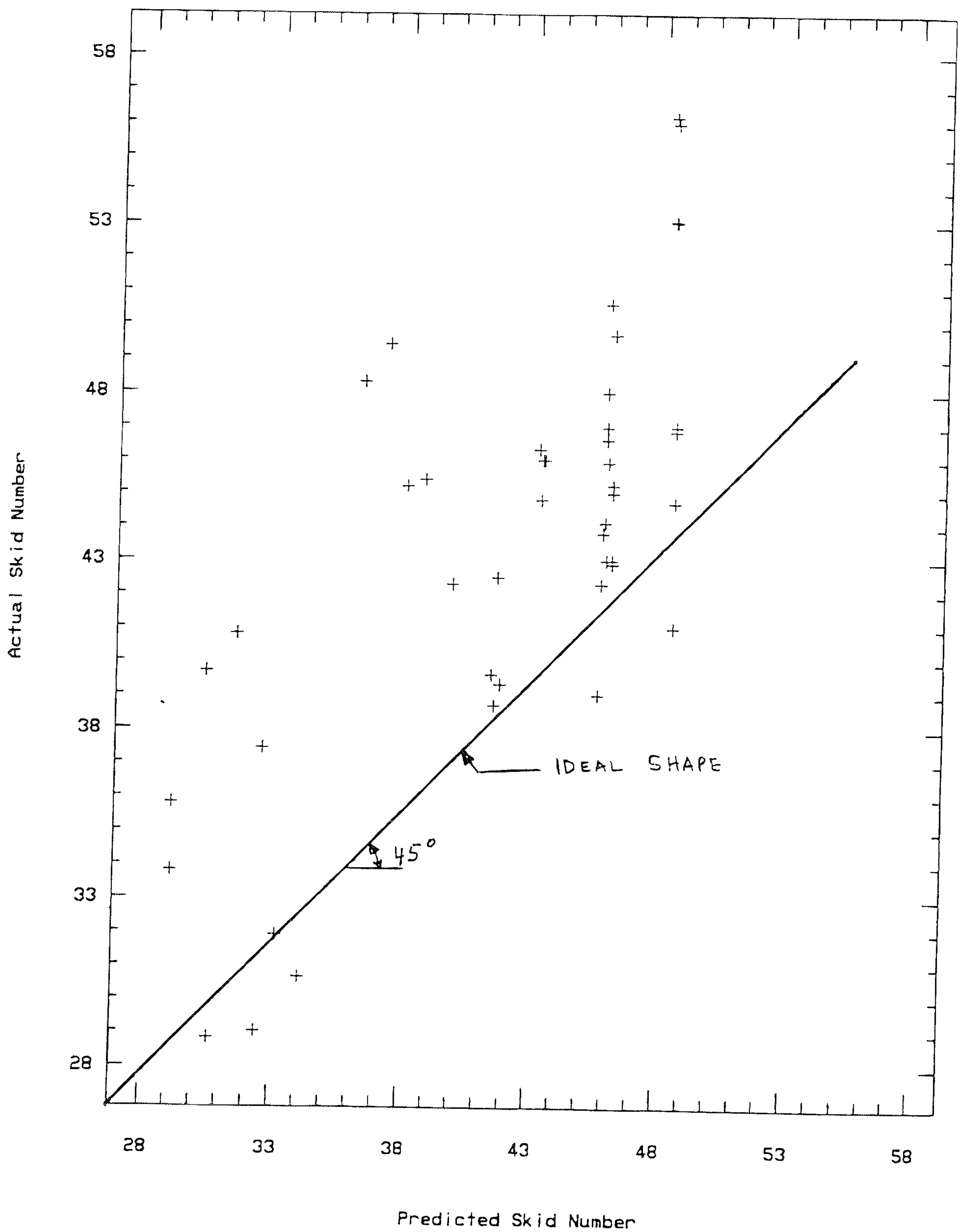
The two measures SE and R^2 are used to compare the two models. Figure 5 shows scattergrams depicting the observed versus predicted skid resistance for multiple linear regression and neural network for pavement Type E for in-sample data (i.e., the same data used in deriving the models). Figure 6 also shows scattergrams for the two model types but for out-of-sample data (i.e., field data that were not part of the data set used in developing the models). A subset of the data used in generating Figure 6 is shown in Table 1. The ideal shape in all scattergrams would be a straight line with a slope of 45 degrees that crosses the origin as shown in both Figures 5 and 6. Both graphs show deviations from the ideal, but it is evident that neural network yields much better in-sample and out-of-sample fitness than multiple linear regression in both cases. This is reflected in the SE and R^2 values found in Table 2. The neural network model gives significantly lower SE and higher R^2 values than the multiple linear regression model for in-sample and out-of-sample data. A subset of the out-of-sample data presented in Table 1 also shows smaller percentage absolute errors for the values predicted by the neural network model compared with those predicted by the multiple linear regression model.

SUMMARY AND CONCLUSIONS

This paper examined the development and predictive capabilities of multiple linear regression and neural network models for skid resistance on original bituminous concrete pavements containing no overlays. The surface courses for all pavements in this category consisted of ConnDOT-designated Class 1 PMB material mix. Hence, the models developed assumed that variation in skid resistance for the pavement type studied is due to external factors rather than surface course material characteristics. The main variables found significant in explaining skid resistance using the multiple linear regression model include the pavement regional location and the mean accumulated AADT based on the pavement surface age. For the neural network model, skid resistance was predicted using the pavement surface age, pavement regional location, and the accumulated AADT as the independent or explanatory variables. The two models were compared using in-sample and out-of-sample data, and in both cases the neural network was capable of fitting better models to the data than multiple linear regression.

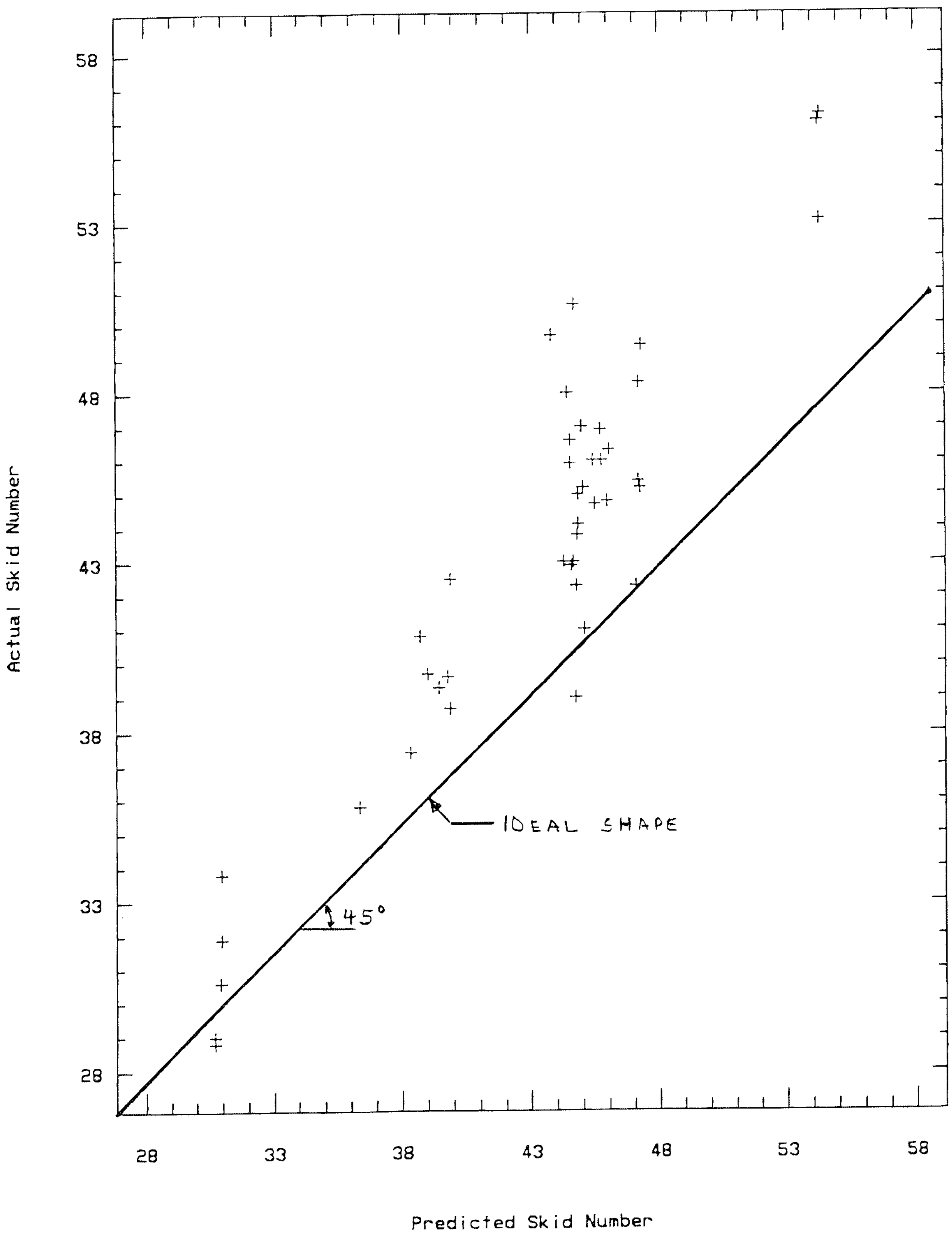
Multiple linear regression methods have proved to be useful tools in a variety of pavement studies and continue to be used routinely. However, like other classical statistical techniques for prediction, they reach their limitations in applications in which nonlinearities occur in the data set. Transforming variables to make the data linear theoretically can enable linear regression to be as accurate as any statistical model. However, achieving that goal in problems of any complexity requires skill, insight, and persistence. If nonlinearities are not found and fixed, linear regression will do nothing to help. Since all the variables must be understood as an interrelated group, the use of linear regression on complex problems tends to be arduous, expensive, and potentially error-prone. This may explain why models developed with linear regression techniques tend to be static in their application; they are hardly updated although variable characteristics change and new variables emerge with the passage of time. Neural network models, on the other hand, are dynamic in their applications. They can be updated easily in less time, and they allow the data to speak for themselves rather than imposing any a priori functional form or shape on the data.

The results of the skid resistance analysis presented in this paper indicate that the potential of neural network as a prediction tool in pavement studies appears excellent and should be explored using more comprehensive data bases. Several issues, however, need significant investigation; for example, the effect of training sample size on network stability, computing time, network architecture, and ways to spot network model overfitting.



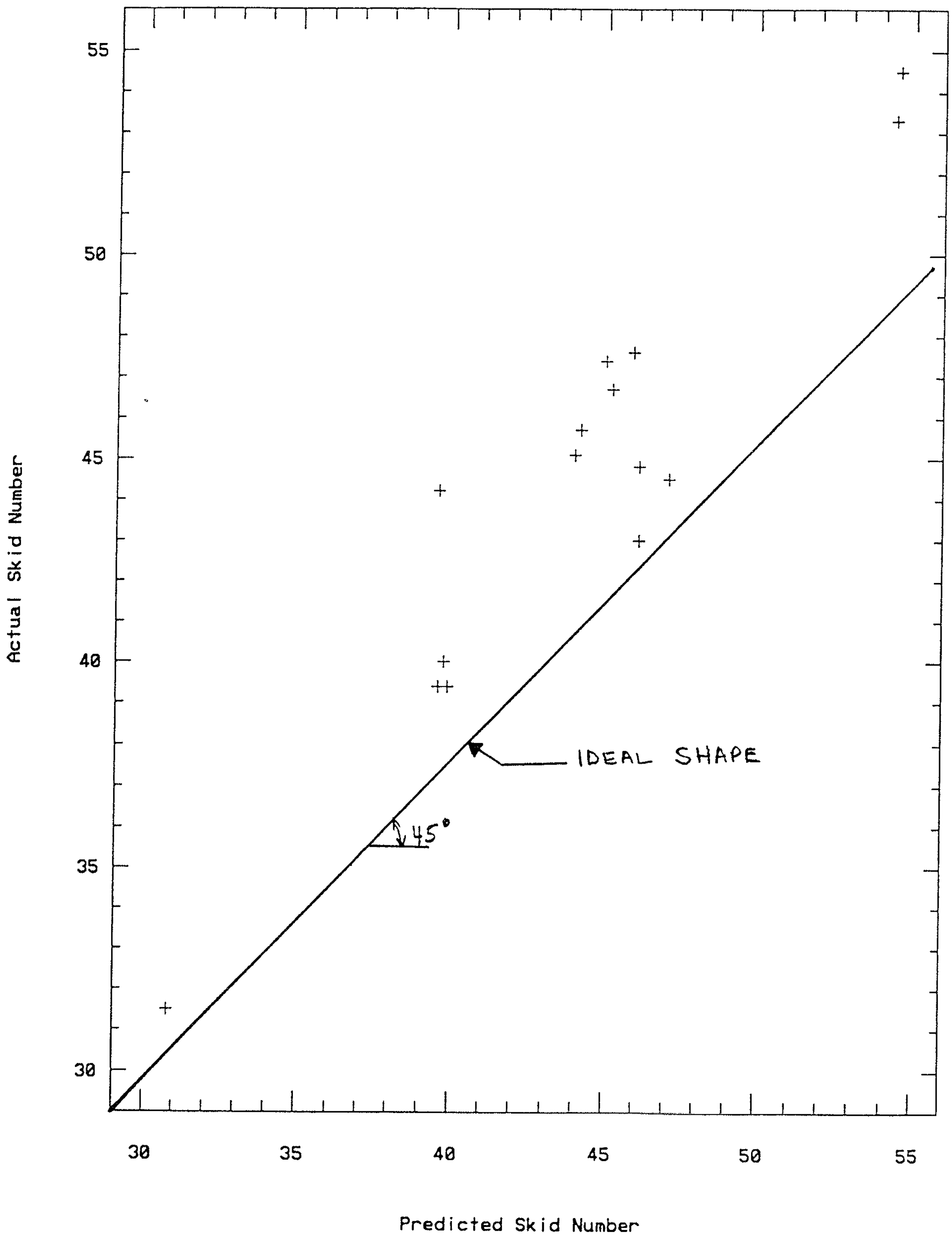
S.E.=3.92; R-squared = 0.63

FIGURE 5 Scattergram comparison of MLR (*this page*) and NN (*next page*) models for in-sample data. (*continued on next page*)



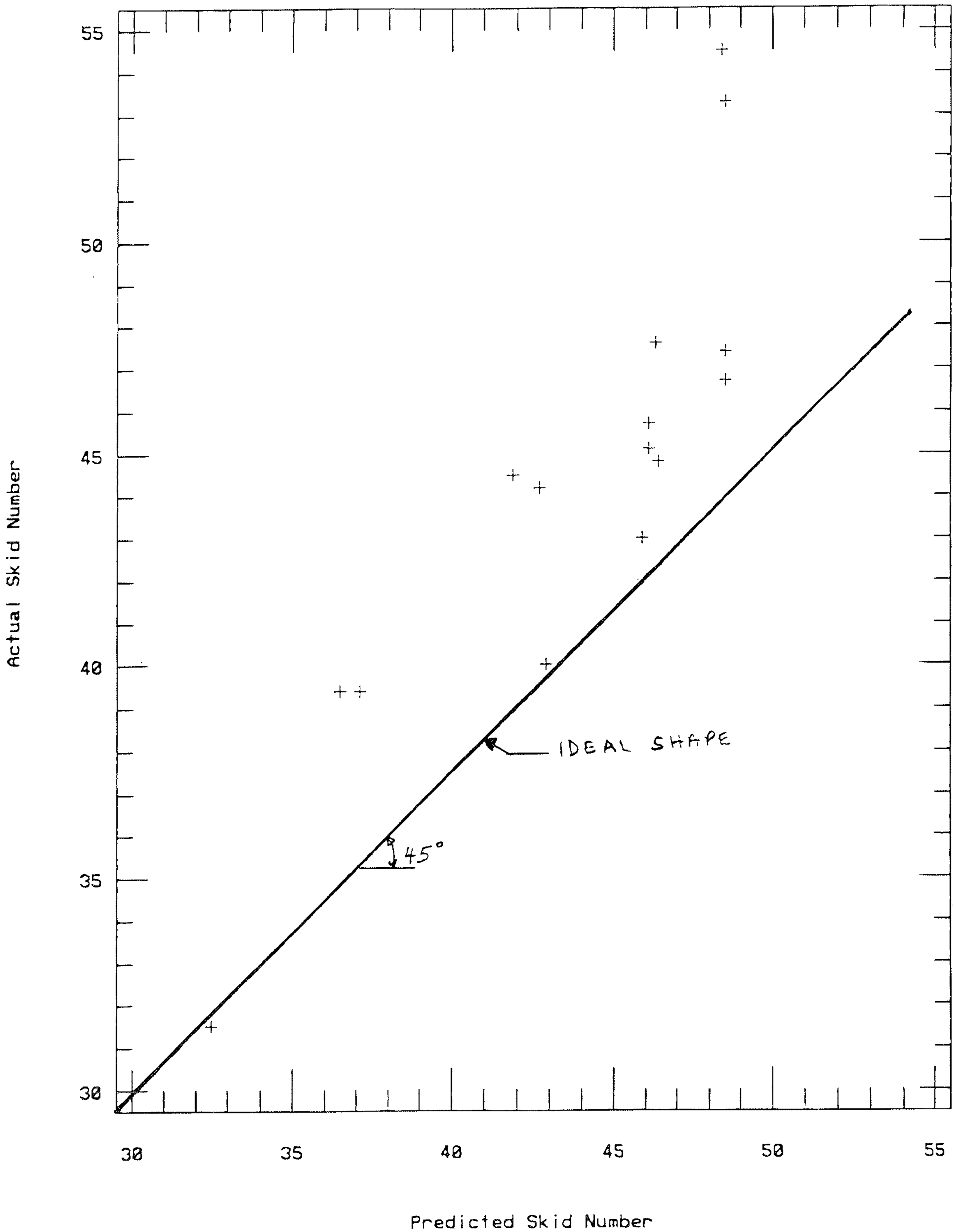
S.E.=2.36; R-Squared=0.87

FIGURE 5 *(continued)*



S.E.=1.94; R-Squared=0.89

FIGURE 6 Scattergram comparison of NN (*this page*) and MLR (*next page*) models for out-of-sample data. (*continued on next page*)



S.E.=3.13; R-Squared=0.71

FIGURE 6 (*continued*)

TABLE 1 Subset of Out-of-Sample Actual Versus Predicted Data for MLR and NN Models

ACTUAL SKID NUMBER	NN MODEL PREDICTED SKID NUMBER	MLR MODEL PREDICTED SKID NUMBER	% Error for NN Model	% Error for MLR Model
39.4	39.9	36.5	1.3	7.4
53.3	54.4	48.5	2.1	9.0
46.7	45.2	48.5	3.1	3.9
54.5	54.5	48.4	0.0	11.2
31.5	30.8	32.5	2.8	3.2
39.4	39.6	37.1	0.5	5.8
40.0	39.8	42.9	0.5	7.3

TABLE 2 Comparison of NN and MLR Models

DATA SET TYPE	MODEL TYPE	S.E VALUE	R^2 VALUE
IN-SAMPLE	NN	2.36	0.87
	MLR	3.92	0.63
OUT-OF-SAMPLE	NN	1.94	0.89
	MLR	3.13	0.71

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