Optimal Mixed Bus Fleet for Urban Operations

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A model is developed for optimizing vehicle sizes on multiple route operations in bus service. The demand characteristics are specified with regular discrete distributions that can realistically represent the variations over time. The total operator and user cost, which does not include the capital cost of vehicles in one-size operations on one route or multiple routes, is minimized by using a classical analytic optimization. Total system cost, which includes the capital cost of buying vehicles in mixed-fleet operation on one route or multiple routes, is minimized by using the numerical method in which the multidimensional optimization algorithm is applied. The optimized variables, which could be decided sequentially, are vehicle sizes, optimal headways, and operating fleet size. Computer programs are designed for optimization and sensitivity analysis. Numerical examples are presented for one-size and mixed-fleet operation on one route and multiple routes with discrete demand periods.

Optimal vehicle size is a very important issue in bus operations. Often, only one vehicle size is selected based on the peak-hour demand. The same vehicle size is then used on each route and in each time period. This is not necessarily economical, because if large vehicles are always used, user costs are high in off-peak periods, and if small vehicles are used, operator costs are high in peak periods. Mixed-fleet operation might be preferable to conventional one-size operation when the passenger demands differ widely between different time periods or among different routes. There will be a threshold in the passenger volume indicating whether to use one or two vehicle sizes. If analysis shows that mixed-fleet operation is already better than one-size operation on a single route with multiple demand periods, then mixed-fleet operation should be even more advantageous on multiple route systems, because the demand level will likely be significantly different on different routes and in different time periods.

A number of attempts have been made to optimize one vehicle size in bus operation. Most approaches have studied certain idealized problems by analytic methods. They have considered the optimal vehicle size generally with only one size and one demand period. They sought to determine the optimal vehicle size, service frequencies, and fleet size that should be used to carry passengers from distributed origins to a single destination, satisfying certain requirements. In this section the literature in the area of bus operation is reviewed. The focus is on the design or operation of a bus service in which users are served by one route or multiple routes.

Navin (*1*) developed a simple mathematical model to optimize bus size and vehicle productivity. Since London's city transit has been shown to be operating at almost optimal occupancies of 38 and 18 passengers during peak and off-peak periods, respectively, Navin's model tried to duplicate these results as well as observations of conventional commuter routes. The equations in Navin's study may be manipulated to give the "mathematically optimal" passenger productivities, vehicle occupancy and fleet size.

White and Turner (*2*) summarized recent developments in intensive minibus service in Britain. They developed a cost comparison of minibus versus conventional bus service with one-size operation on one route, and calculated the levels of demand and population density necessary to support minibus service. The total cost excluded user cost, and included capital cost, fuel consumption, maintenance cost, and labor cost. The researchers identified the needs for minibus operation throughout the day, but without vehicle size optimization.

An analytic model including vehicles size as a decision variable was developed by Oldfield and Bly (*3*). The objective in their simple bus line model was to maximize the total social benefit on one route. The passenger demand varied with the generalized cost of travel according to a constant elasticity. The concluded that operating cost increased linearly with bus size, and that the optimal size varied with the square root of demand. The analytic results indicated that the optimal bus size is smaller than the current British practice. Their study suggested an optimal capacity of about 60 seats, assuming that cost varied with size according to the average costs of current British bus operations.

Jansson (*4*) also developed an analytic model for vehicle size. His objective function value was to minimize total social costs, including the cost of passenger waiting time and ride time and the operating costs. He obtained the optimal frequency with the "square root formula" (which states that the frequency of service should be approximately proportional to the square root of the number of passengers carried on a bus line) and determined the optimal bus size by the peak demand and optimal frequency. Jansson accounted for different service frequencies in the peak and off-peak periods for the same vehicle size, and compared costs for peak and off-peak periods in numerical examples. A linear function for bus operating cost, $B = a + bS$, was used. This linear function will also be used in the present study.

Chang and Schonfeld (*5*) investigated the temporally integrated bus systems with analytic models in which fixed-route services are provided during higher-demand periods while flexible-route services are provided during lower-demand periods. They assumed demands to be fixed and uniformly distributed over time within each specified period. The researchers obtained optimized vehicle sizes by minimizing average cost per trip and compared them for fixed route, flexible route, and integrated systems. Their numerical results indicated that the optimal vehicle sizes (37 seats per vehicle) in an integrated system were a compromise between the optimal

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vehicle sizes for pure fixed-route (48 seats per vehicle) and pure flexible-route (17 seats per vehicle) services.

Bly and Oldfield (*6*) considered competition between minibus service and regular bus service. Their study discussed two cases: minibus operation on routes physically separated from existing large-bus services, and minibus services sharing routes with existing large-bus services. By assuming the minibus flow as a proportion of total peak bus flow, they determined the net benefit value using some specified minibus sizes on the London Transport routes. They suggested that minibuses would do well without necessarily attempting to segment the market, because most passengers were likely to accept the first bus to arrive even if it charged a little more. This shows the advantage of minibus service.

Nairn (*7*) used simulation techniques to develop and estimate the level of service, cost, and revenue for dial-a-ride services, fixed route services and mixed fleet services in a city with a population under 100,000. One conclusion was that the alternate dial-a-ride and fixed route services (mixed-fleet operation), comprising a mixed bus fleet of 45-seat vehicles for the peak period and 12-seat vehicles for the off-peak period, had lower net costs than either dial-a-ride or fixed-route services alone. They mentioned that a mixture of large and small vehicles had the potential to provide a high level of service at relatively low cost.

In summary, no previous study was found that optimized mixed bus fleet operations on multiple routes or in multiple periods. Therefore, this study focuses on the optimal mixed bus fleet for urban operation.

SYSTEM DEFINITION

The system analyzed in this study is shown in Figure 1. This study develops an optimization model for one-size operation and mixed-fleet operation in a bus system the demands of which vary over time according to discrete distributions. A fixed-route bus service network is considered in numerical examples. Each route's length, speed, and demand during each time period is different, The discrete demand on each route is shown in Figure 2. All variables and the typical values used in their numerical analysis are defined in Table 1.

The optimal vehicle sizes for mixed-fleet operation on multiple routes are the main focus of sensitivity analyses, which identify the relations between the decision variables (vehicle sizes) and various parameters.

Four main questions are addressed in this study. First, what is the threshold demand level between one-size and mixed-fleet operation (i.e., when is the mixed-fleet operation preferable)? Second, what vehicle sizes are optimal in one-size and mixed-fleet operations? Third, what is the optimal headway on each route in each time period? Fourth, what should be the size and composition of the total bus fleet?

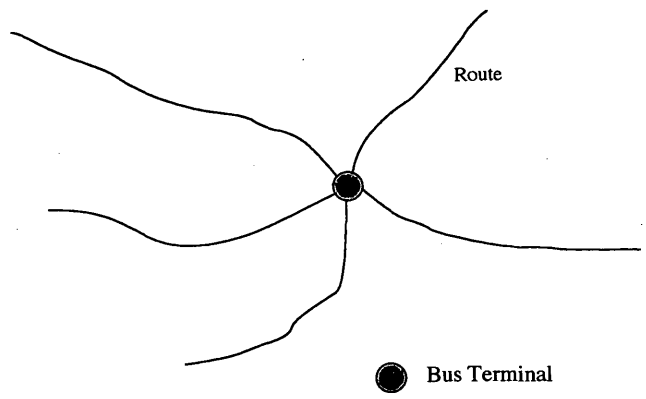

FIGURE 1 Analyzed system.

In describing the analytic models for the various operations, the following assumptions are made:

1. Conventional fixed-route and fixed-schedule services are provided on all routes.
2. The time required for boarding and exiting buses is included in the average speed.
3. Demands (passenger volumes) are specified with regular discrete distributions along the time periods and are invariant with price or service quality.
4. The cost of transferring vehicles among routes is negligible.

TOTAL COST FUNCTION

The objective function of these models is to minimize the sum of user and supplier cost over a full day. (A week or year may also be represented.) Both of these cost components have a major influence on the quality of bus service. Users desire frequent service on bus routes to reduce the waiting cost at bus terminals. A supplier is interested in providing the service while minimizing his cost.

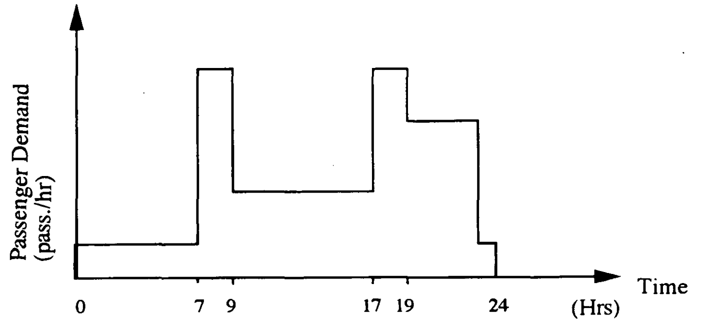

FIGURE 2 Typical passenger demand on one route in one day.

TABLE 1 Variable Definitions

Symbol	Definition	Baseline Value
a	fixed cost coefficient in vehicle operating cost ($/veh.hr.)	25.0
B	bus hourly operating cost ($/veh.hr.)	-
b	variable cost coefficient in vehicle operating cost ($/veh.seat hr.)	0.25
C	total cost ($/hr.)	-
C_o	operating cost ($/hr.)	-
C_p	the daily capital cost ($/day)	-
C_{rt}	total cost of route r in time period t ($/hr.)	-
C_{rts}	total cost of route r in period t with vehicle size s ($/hr.)	-
C_{s1}	total cost with vehicle size s1 ($/hr.)	-
C_{s2}	total cost with vehicle size s2 ($/hr.)	-
C_T	total system cost ($/day)	-
C_t	total cost in period t ($/hr.)	-
C_u	user cost ($/hr.)	-
C_v	user in vehicle cost ($/hr.)	-
C_w	user waiting cost ($/hr.)	-
c	fixed cost coefficient in vehicle capital cost ($/veh.)	16,000
D	route length (miles)	-
D_r	the length of route r (miles)	-
d	the average travel distance of boarding passengers (miles)	-
d_r	average travel distance of boarding passengers of route r (miles)	-
e	variable cost coefficient in vehicle capital cost ($/veh.seat)	2,400
G	vehicle capital cost ($/veh.)	-
H	vehicle headway (hrs.veh.)	-
H_{rt}	vehicle headway of route r in period t (hrs/veh.)	-
H_{rts}	vehicle headway of route r in period t with size s (hrs/veh.)	-
H_t	vehicle headway in period t (hrs/veh.)	-
i	interest rate	-
K	capital recovery factor with interest rate i and period N	0.1359
k	number of vehicle sizes	2
m	number of time periods	18
N	the operating fleet size (vehicles)	-
N_{rts}	the fleet size of route r in period t with size s (vehicles)	-
N_{ts}	the fleet size in period t with size s (vehicles)	-
N_s	the fleet size with size s (vehicles)	-
n	number of routes	4
Q	the passenger demand (pass.hr.)	-
Q_{rt}	the demand of route r in period t (pass.hr.)	-
Q_t	the demand in period t (pass.hr.)	-
q	the peak point demand along a route (pass.hr.)	-
q_{rt}	the peak point demand of route r in period t (pass.hr.)	-
q_t	the peak point demand in time period t (pass.hr.)	-
S	vehicle size (seats/veh.)	-
S_S	size for type s vehicles (seats/veh.)	-
T	the average economical life for buses (years)	-
V	average speed (mph)	-
V_t	average speed in time period t (mph)	-
V_{rt}	average speed of route r in time period t (mph)	-
υ_υ	time value of passenger in-vehicle time ($/pass.hr.)	6
υ_w	time value of passenger waiting time ($/pass.hr.)	10
*	superscript indicating optimal value	-

Reducing user cost increases supplier cost and vice versa. The choice of optimal vehicle sizes greatly affects the user and supplier costs. With large size(s), the supplier is favored since the corresponding headways are larger and fewer vehicles are operated. Conversely, the small vehicles will favor the user since the corresponding headways and wait times will be smaller. When the passenger demand varies over time periods, the optimal vehicle size should also vary over time. The purpose of this study is to identify the optimal vehicle size(s) which minimize the total cost of users and suppliers over a set of periods, such as one daily cycle.

The variables used in this model formulation are defined in Table 1. The two main cost components are the operator costs (C_o) and user costs (C_u). The operator cost is the product of the required fleet size, N, and hourly operating bus cost, B.

$$C_{mo} = NB = 2DB/VH \tag{1}$$

The user costs, C_u, consist of waiting cost and in vehicle cost:

$$C_u = C_{uw} + C_{uv} = (2v_wQH/2) + (2v_vQd/V) \tag{2}$$

The total cost, C, is the sum of operator cost and user cost:

$$C = C_o + C_U = 2DB/VH + v_wQH + 2v_vQd/V \tag{3}$$

A linear bus hourly operating cost function of the type used by Jansson (1980) is also used here:

$$B = a + bS \tag{4}$$

The above equation will be used to optimize the vehicle size S.

With the capacity constraint, $H \leq S/q$, the total cost function becomes:

$$C = [2Dq(a + bS)/VS] + v_wSQ/q + 2v_vQd/V \tag{5}$$

The vehicle size (S) that minimizes the total cost (C) can be found by setting the derivative of C with respect to S equal to zero and solving.

$$\delta C/\delta S = (-2aDq/VS^2) + v_wQ/q = 0 \tag{6}$$

The second-order derivative of C is as follows.

$$\delta^2C/\delta S^2 = 4aDq/VS^3 \tag{7}$$

Because all variables in Equation 7 are positive, the second-order derivative of C with respect to S is positive. Therefore, Equation 6 will yield the S value for a minimum rather than maximum total cost. The optimal vehicle size (S^*) is as follows.

$$S^* = (2aDq^2/v_wVQ)^{0.5} \tag{8}$$

S^* = the optimal vehicle size (seats per vehicle) for one route where total cost is minimized.

Therefore, the corresponding optimal headway that satisfies demand is as follows.

$$H^* = S^*/q = (2aD/v_wVQ)^{0.5} \tag{9}$$

The fleet size of each route is as follows.

$$N = 2D/VH^* \tag{10}$$

As determined above, the optimal vehicle size (S^*), the practical optimal headway (H), and fleet size (N) are only suitable for one specified time period on each route.

Actually, there are different passenger volumes in different periods. Often, a bus company will use the peak period volume to determine the vehicle size (S) and the fleet size (N). If only one vehicle size (large) is used, based on the peak period passenger volume, the user cost will be higher in off-peak periods because the headway with the large vehicle will be too large. Conversely, there will be higher operating costs in peak periods if the small vehicle size is used, based on the off-peak volume.

One Size–Multiple Routes–Multiple Periods

If one-size operation is used on multiple routes in multiple periods, the procedures for determining the optimal vehicle size should be modified as follows.

For the capacity requirement ($H_{rt} \leq S/q_{rt}$) the total cost function becomes the following.

$$C_T = \sum_{r=1}^{n}\sum_{t=1}^{m} \{[2D_rq_{rt}(a + bS)/V_{rt}S] + (v_wSQ_{rt}/q_{rt}) + (2v_vQ_{rt}d_r/V_{rt})\} \tag{11}$$

where C_T = the total cost for all routes and all times periods (dollars per day).

The vehicle size (S) that minimizes the total cost (TC) can be found by setting the derivative of TC with respect to S equal to zero and solving.

$$\delta C_T/\delta S = \left(\sum_{r=1}^{n}\sum_{t=1}^{m} - 2aD_rq_{rt}/V_{rt}S^2\right) + \left(\sum_{r=1}^{n}\sum_{t=1}^{m} v_wQ_{rt}/q_{rt}\right) = 0 \tag{12}$$

The second derivative of C_T is as follows.

$$\delta C_T/\delta S^2 = \sum_{r=1}^{n}\sum_{t=1}^{m} 4aD_rq_{rt}/V_{rt}S^3 \tag{13}$$

Because all variables in Equation 13 are positive, the second derivative of TC with respect to S is positive. Therefore, Equation 13 will yield the S value for a minimum rather than maximum total cost. The optimal vehicle size (S^*) is as follows.

$$S^* = \left[\sum_{r=1}^{n}\sum_{t=1}^{m} (2aD_rq_{rt}/V_{rt}) / (\sum_{r=1}^{n}\sum_{t=1}^{m} v_wQ_{rt}/q_{rt}\right]^{0.5} \tag{14}$$

where S^* = the optimal vehicle size for one route (seats per vehicle) where total cost is minimized.

Therefore, the corresponding optimal headway of route r in time period t that satisfies the demand is as follows.

$$h_{rt} = S/q_{rt} \tag{15}$$

$$h_{rt} = \left[\sum_{r=1}^{n}\sum_{t=1}^{m} (2aD_rq_{rt}/V_{rt})/(\sum_{r=1}^{n}\sum_{t=1}^{m} v_wQ_{rt}/q_{rt}\right]^{0.5} / q_{rt} \tag{15a}$$

where h_{rt} = the corresponding headway of route r in time period t (hrs/vehicle).

The optimal headway for each route in each demand period which minimizes the total cost can be found by setting equal to zero the derivative with respect to headway (H_{rt}) of the total cost of route r at time period t (C_{rt}).

$$C_{rt} = 2D_r(a + bS)/V_{rt}H_{rt} + v_wQ_{rt}H_{rt} + 2v_wQ_{rt}d/V_{rt}) \tag{16}$$

$$\delta C_{rt}/\delta H_{rt} = [-2D_r(a + bS^*)/V_{rt}H_{rt}^2] + v_wQ_{rt} = 0 \tag{17}$$

The second-order derivative of C_{rt} is as follows.

$$\delta^2C_{rt}/\delta H_{rt}^2 = 4D_r(a + bS^*)/V_{rt}H_{rt}^3 \tag{18}$$

Because all variables in Equation 18 are positive, the second-order derivative of C_{rt} with respect to H_{rt} is positive. Therefore, Equation 18 will yield the H_{rt} value for a minimum rather than maximum total cost. The optimal value of H_{rt} is as follows.

$$H^*_{rt} = [2D_r(a + bS^*)/Q_{rt}\nu_w V_{rt}]^{0.5} \tag{19}$$

The practical optimal headway H_{rt} is either the optimal headway H_{rt}* from Equation 19, which minimizes costs, or the maximum headway h_{rt} from Equation 15a, which satisfies demand, whichever is smaller.

$$H_{rt} = \text{Min}\,\{[2D_r(a + bS^*)/Q_{rt}\nu_w V_{rt}]^{0.5},$$
$$\left[\sum_{r=1}^{n}\sum_{t=1}^{m}(2aD_r q_{rt}/V_{rt})/\left(\sum_{r=1}^{n}\sum_{t=1}^{m}\nu_w Q_{rt}/q_{rt}\right)\right]^{0.5} / q_{rt}\} \tag{20}$$

The fleet size of route r in time period t is as follows.

$$N_{rt} = 2D_r/V_{rt}H^*_{rt} \tag{21}$$

These equations are used to obtain the optimal vehicle size by minimizing the sum of operating cost and user cost, but do not include the capital cost of the vehicles. Therefore the optimal vehicle sizes, which can be obtained by the analytic solutions of Equations 8 and 14, are only the approximate solutions, the objective function of which does not include the capital cost. The reason the analytic solutions cannot be obtained if the capital cost is included in the objective function is as follows: the capital cost is a function of the fleet size required and the vehicle size, while the fleet size is also a function of vehicle size. The fleet size that has to be available for any vehicle size is the maximum fleet required for that size through all the periods. This maximum function cannot be differentiable.

Capital Cost

In mixed-fleet operation, if two vehicle sizes are used on one route, the capital cost will be higher because the operator must buy two kinds of vehicles. However, if two sizes are used on multiple routes, the capital cost may be lower because the operator can share the vehicles on different routes in different time periods.

In general, the operating fleet size on each route in each period with the specified vehicle size can be formulated as follows.

$$N_{rts} = 2D_r/V_{rt}H_{rts} \tag{22}$$

The total operating fleet size for all routes in each time period with the specified vehicle sizes can be expressed as follows.

$$N_{ts} = \sum_{r=1}^{n} 2D_r/V_{rt}H_{rts} \tag{23}$$

The minimum fleet size required with the specified vehicle size is as follows.

$$N_s = \operatorname*{Max}_{t=1}^{m}\,(N_{ts}) \tag{24}$$

The capital cost function for vehicles can be formulated as follows.

$$G = c + eS_s \tag{25}$$

The capital cost per day can be formulated as follows.

$$C_p = \sum_{s=1}^{k}\{[(c + eS_s)(A/P,i,T)/365]^*N_s\} \tag{26}$$

where $(A/P, i, T) = k =$ the capital recovery factor with interest rate i and time period T.

Two Sizes–Multiple Routes–Multiple Periods

Because there are advantages in using two different sizes of vehicle to operate only one route when the demand level is extremely different in different time periods, it is worth attempting two-vehicle-size operation on multiple routes (large vehicles for high-demand periods and small vehicles for low-demand periods). The total cost of mixed-fleet operation on multiple routes with the route r, time period t, and vehicle size s_s can be obtained from the following equation.

$$C_{rts} = [2D_r q_{rt}(a + bS_s)/V_{rt}S_s] + \nu_w S_s Q_{rt}/q_{rt} + 2\nu_v Q_{rt} d_r/V_{rt} \tag{27}$$

The total cost of a bus system with multiple routes and multiple demand periods can be formulated as follows.

$$C_t = C_p + \sum_{\text{all } r}\sum_{\text{all } t}\sum_{\text{all } s} C_{rts} = C_p$$
$$+ \sum_{r=1}^{n}\sum_{t=1}^{m}\sum_{s=1}^{k}[2D_r q_{rt}(a + bS_s)/V_{rt}S_s]$$
$$+ \nu_w S_s Q_{rt}/q_{rt} + 2\nu_v Q_{rt} d_r/V_{rt} \tag{28}$$

The capital cost (C_p) can be obtained from Equations 22–26. The optimal vehicle size (S^*) should be the size with the minimum total cost of all time periods (C_t). The corresponding optimal headway and operating fleet size may be found with Equations 20 and 21.

Demand Boundary Among Different Vehicle Sizes

Conceptually, large vehicles should be used in high-demand periods and small vehicles should be used in the low-demand periods. If two prespecified vehicle sizes are used on a single route or on multiple routes, conceptually, a boundary expressed in terms of demand might indicate when the larger size should be used instead of the smaller size. According to Equation 8, q^2D/QV should serve as a very good combined factor for the boundary between using large or small vehicles.

If two prespecified vehicle sizes operate on one route in one specified time period, the total cost with respect to the two different vehicle sizes can be formulated as follows.

$$C_{s1} = [2Dq(a + bS_1)/VS_1] + \nu_w S_1 Q/q + 2\nu_v Qd/V \tag{29}$$

$$C_{s2} = [2Dq(a + bS_2)/VS_2] + \nu_w S_2 Q/q + 2\nu_v Qd/V \tag{30}$$

When Equations 29 and 30 are set to be equal, the following equation can be obtained.

$$q^2D/QV = v_w S_1 S_2/2a \quad (31)$$

Therefore, the boundary variable q^2D/QV can be obtained by Equation 31. By comparing the q^2D/QV of each route in each time period with the boundary $v_w S_1S_2/2a$, the choice of a large or small vehicle can be made for each route and in each time period. It is very important to note that the boundary q^2D/QV is only a function of the time value of passengers, vehicle sizes, and the fixed cost coefficient in vehicle operating cost function (Equation 8 shows the same relations). Clearly, when vehicle sizes are larger, the boundary should be higher. When the time value of passengers is higher, the boundary will also be higher, because it will favor the small vehicles. It is also interesting to note that if the fixed cost coefficient in the vehicle operating cost function increases, it favors the vehicles and the boundary decreases.

In numerical examples in this study, the maximum load on the route (q) is set to be equal to the total passengers boarding on that route, Q. (That would happen, for instance if all passengers go to the central business district in the morning and return in the evening.) Therefore Equation 31 can be simplified as follows.

$$QD/V = v_w S_1 S_2/2a \quad (31a)$$

OPTIMIZATION FOR MIXED FLEET

When two vehicle sizes are operated, a multidimensional optimization method is needed to optimize vehicle sizes. The objective function includes a maximum choice function. This cannot be solved analytically, because it cannot be differentiated with respect to the decision variables (vehicle sizes). A quasi-Newton method with finite-difference gradient has been used in this study to find an approximate initial solution for the optima.

IMSL routine UMINF was chosen for the multidimensional optimization in this study. This routine used a quasi-Newton method with finite-difference gradient to find the minimum of a function $[f(x)]$ of n variables, and is documented in the user's manual of *Fortran Subroutines for Mathematical Applications* (*8*). To determine the best vehicle size combination (S_1^*, S_2^*), corresponding headways, and fleet sizes on multiple routes, the procedures are as follows.

1. Choose the highest passenger volume and the lowest passenger volume from the periods in one day for all routes.
2. Initialize two vehicle sizes, using the highest and lowest passenger volumes by Equation 8 to be the upper and lower bounds of the possible vehicle sizes.
3. Determine the passenger-demand boundary between the large and small vehicle sizes with Equation 31.
4. Find the corresponding headway on each route in each period with Equation 20.
5. Determine the total fleet size for multiple routes with the combination of different vehicle sizes using Equations 22–26.
6. Calculate the total cost for multiple routes for all periods with Equation 28.
7. Use the UMINF quasi-Newton program to search for a two-vehicle-size combination with a smaller total cost (repeat Procedures 3–7) until the best combination with the minimum total system cost is found.
8. Use the optimal two-vehicle-size combination to obtain the corresponding optimal headway (H_{rt}^*) with Equation 20 and the operating fleet size (N_{rt}) with Equation 21.

In mixed-fleet operation, the fleet size for each vehicle size can be adjusted by violating the boundary (and using the "wrong" size across the boundary); doing so may reduce the total cost by reducing total fleet size and capital cost. The adjustment procedures are as follows:

1. Rank periods in order of decreasing demand as shown in Figure 3.
2. Referring to Figure 3, identify the boundary route (z_t) with the demand closest to the boundary in period t.
3. Substitute small or large vehicles on route z_t in period t.
4. Recompute the total fleet size (N_{ts}) in period t according to Equations 22–26.
5. Check whether the size substitution is necessary (N_{ts} must be greater than the fleet size in other periods).
6. If the size substitution is necessary, compute the total cost with the new total fleet size.
7. If the new cost is less that the old total cost, accept the substitution for new fleet size.
8. Repeat Procedures 1–6 for next boundary route in the next period.

The concept for assigning vehicles to each route in each period is shown in Figure 3. Only one boundary is shown since only two vehicle sizes are considered for the mixed-fleet operation in this study. If n sizes are used there will be $(n - 1)$ boundaries.

NUMERICAL EXAMPLE

The numerical examples for the various operations shown below are simplified as follows:

1. There are only two different passenger-demand periods in one day (peak demand for 4 hours and off-peak demand for 14 hours);
2. There are only two kinds of speed in 1 day (peak and off-peak); and
3. Only one or two vehicle sizes are operated.

In peak periods, the speed is lower, and the demand is higher than in off-peak periods. By using the demand, speed, and distance information from Table 2 the numerical results shown in Table 3 are obtained for four cases of bus operation.

The optimal two-vehicle-size combination is (33, 20) seats per vehicle, and the total fleet consists of 15 large vehicles and 29 small vehicles. The total fleet size (44 vehicles) is lower than the total fleet size for one-size (48 vehicles).

In these multiple-route examples, two-size operation will reduce total system cost by \$753 per day, compared to one-size operation. Not only is the operation and user cost lower (the difference is \$746 per day) but the capital cost is lower as well (the difference is \$7 per day), because the fleet size is reduced. The reason is that multiple routes can share the large or small vehicles in different periods.

If sizes are independently optimized for each route, mixed-fleet operation will reduce total system cost by \$65 per day. Compared

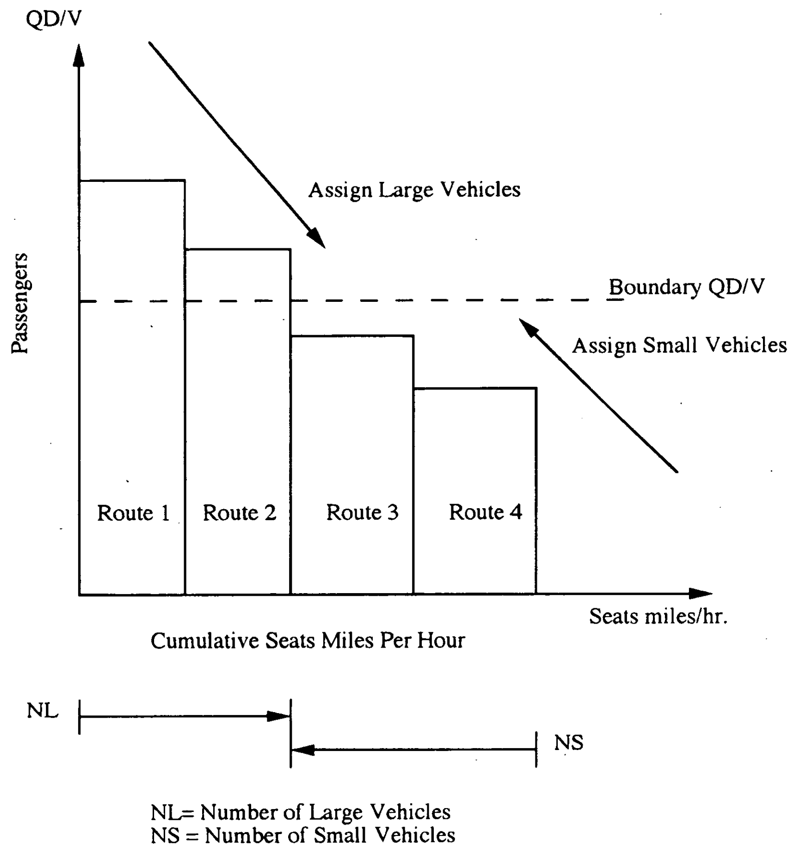

FIGURE 3 Procedures in assigning vehicles to each route in each period.

to one-size operation, the operation and user cost is significantly reduced by $609 per day; however, the capital cost increases by $544 per day in two-size operation because more, and different, vehicles are needed.

It is interesting to note that the total cost of one (different) independently optimized size on each route is less than the total cost of a systemwide size because no compromise among routes is necessary. For each route the vehicle size can be adjusted to demand. Similarly for mixed-fleet operation, the independent operation on each route is slightly better than the same systemwide two-size combination operation on all multiple routes.

TABLE 2 Demand, Speed, and Distance Information for Multiple Route System

Route	Distance (Kms)	Speed (kph)		Demand (pass./hr.)	
		Peak	Off-peak	Peak	Off-peak
1	16	40	48	200	100
2	20	40	48	300	150
3	16	40	48	150	60
4	24	40	48	400	200

The UMINF quasi-Newton program is designed to optimize continuous functions and its solutions are real numbers. Actually, possible vehicle sizes should be integer number of seats. Thus, the nearby integer solutions should be checked to obtain the optimal sizes. In the numerical example, the optimal vehicle sizes on multiple routes were (32.876, 20.221), from which (33, 20) was found to be the optimal integer solution.

These numerical results show that the mixed-fleet operation is preferable when the peak period demand is significantly different from the off-peak demand. To identify the threshold demand in choosing between one-size or mixed-fleet operation, the off-peak demands were fixed for all routes, and the peak demands were increased by multiples of the off-peak demand. The mixed-fleet operation is preferable for the data in Table 2. To determine the effects of passenger demand on the choice between one-size or mixed fleet operation, a very low off-peak demand (40 passengers per hour) was assumed on each route for both one-route and multiple route operation. The threshold ratio of peak demand to off-peak demand for multiple route operation is identified in Figure 4.

Figure 4 shows that the threshold ratio of peak demand to off-peak demand for multiple-route operation is 1.92. This shows that mixed-fleet operation is preferable on multiple-route operation when the demand variation is higher.

Figure 5 shows the relation between the passenger demand boundary and the total cost for mixed-fleet operation on multiple routes. This relation is not smooth because the demands on four

TABLE 3 Comparison of One-Size and Two-Size Operation on Multiple Routes

Items	One Size Each Route	One Size Systemwide	Two Sizes Each Route	Two Sizes Systemwide
R	10-40	10-40	10-40	10-40
S^*	-	22	-	(33, 20)
$S1^*$	18	-	(21, 14)	-
$S2^*$	22	-	(24, 15)	-
$S3^*$	16	-	(20, 11)	-
$S4^*$	27	-	(36, 23)	-
$H1_p^*$	0.090	0.110	0.105	0.100
$H2_p^*$	0.073	0.073	0.080	0.067
$H3_p^*$	0.107	0.128	0.127	0.127
$H4_p^*$	0.068	0.055	0.090	0.083
$H1_f^*$	0.140	0.143	0.138	0.141
$H2_f^*$	0.116	0.116	0.107	0.115
$H3_f^*$	0.180	0.184	0.176	0.183
$H4_f^*$	0.126	0.110	0.115	0.100
N^*	-	48	-	(15, 29)
$N1^*$	9	8	(8, 5)	-
$N2^*$	11	11	(11, 7)	-
$N3^*$	8	7	(7, 4)	-
$N4^*$	18	22	(14, 9)	-
C_o	29,904	31,029	30,877	30,424
C_{uw}	25,528	24,946	23,946	24,805
C_{uv}	58,160	58,160	58,160	58,160
C_{uo}	113,592	114,135	112,983	113,389
C_p	1,185	1,230	1,729	1,223
C_t	114,777	115,365	114,712	114,612

routes and two time periods are discrete. This relation should change to an approximately continuous U-shape when the number of routes or number of time periods of each route, or both, increases significantly.

CONCLUSIONS AND RECOMMENDATIONS

The models presented herein may be used to determine the optimal vehicle sizes for mixed-fleet operation on single or multiple routes. The main advantages of these models over previous studies include the following points.

1. These models can deal with mixed-fleet operations.
2. They can deal with vehicle size, fleet combinations, and vehicle assignment by including capital, operation, and user costs in objective function.
3. They can deal with demand variation over multiple periods.
4. They can be used to assign vehicles and determine the optimal headways for operation on multiple routes, and can help optimize vehicle and fleet sizes for planning purposes.

General conclusions and specific finding are presented as follows.

1. It may be advantageous to use two vehicle sizes on a single route when the demand level is very different in different time periods.
2. It is preferable to use a mixed fleet on multiple routes when demand variation over time is significant.
3. The required fleet size for mixed-fleet operation may be lower than the fleet size for one-size operation, thereby reducing the capital cost.
4. Using the boundary demand developed in this study to choose between different size vehicles, the operator can easily assign the vehicles to each route in each time period.
5. The results of the sensitivity analyses show that the proposed optimization algorithm can provide consistent and reasonable responses to various changes.

These relations can provide the operators with good guidelines for designing or operating bus routes efficiently. The mathematical

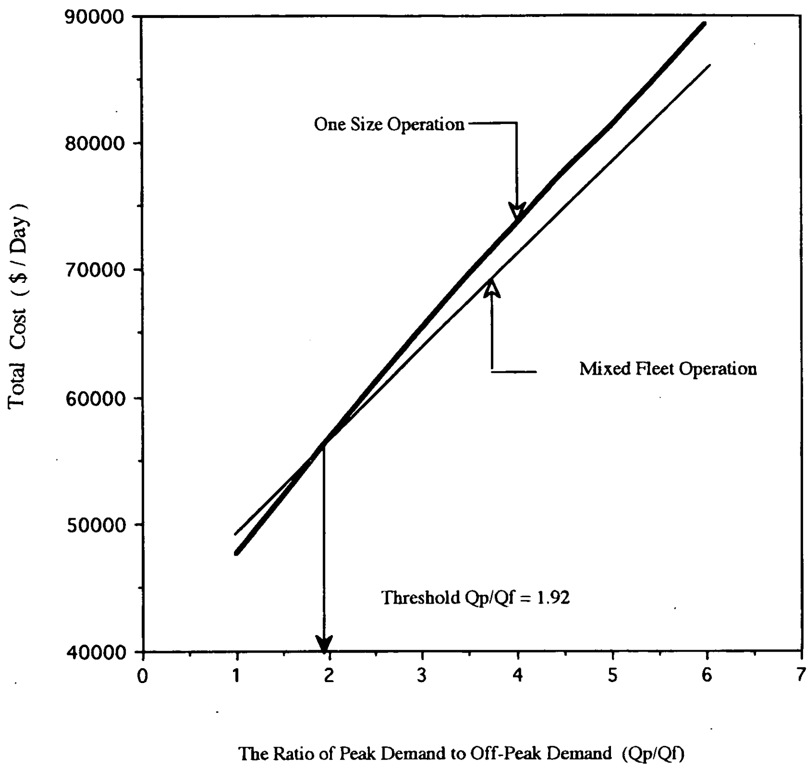

FIGURE 4 Threshold passenger-demand ratio in using one-size or mixed-fleet operation on multiple routes.

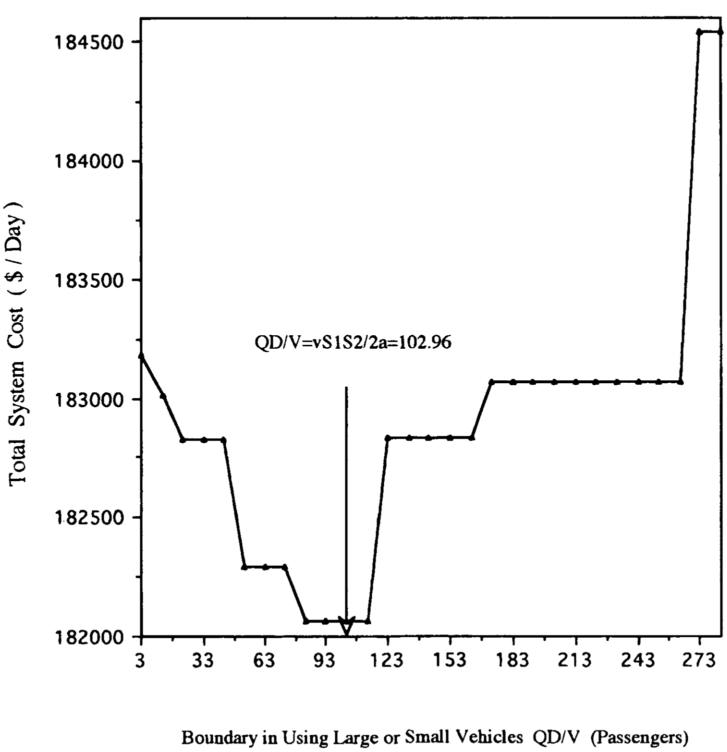

FIGURE 5 Relationship between passenger-demand boundary and total cost of mixed-fleet operation on multiple routes.

relations developed here provide useful guidelines for optimizing the vehicle sizes for one-size or mixed-fleet operations. The output from this approach can be used for planning or operating purposes. This is particularly useful to planners and operators contending with difficult factors, such as the rules for assigning vehicles on multiple routes.

Several possible extensions of the numerical analyses and analytic model developed in this research can be suggested.

The validity of some assumptions should be reexamined. Passenger demand is assumed to be inelastic in this research. A model with inelastic demand cannot properly address fare policy or optimize systems for objectives that include consumer surplus. Therefore, the passenger demand should vary with the amount of the fare and the level of service provided. The objective function should be modified to maximize profit or maximize social welfare (net social benefit).

Other assumptions can also be relaxed. For instance, the analyzed system could be used as a transfer terminal in which the transfer cost should be included in the total cost function. Another possibility is that the analyzed system could be extended to flexible route operation or integrated bus systems, in which fixed services are provided during higher demand periods while flexible-route services are provided during lower demand periods.

Only two sizes for mixed-fleet operation are considered in this study. Although more than two sizes may not be worth attempting because of the difficulties in operation and maintenance, more complex mixtures are still worth investigating.

The total fleet size formulated here is only an absolute minimum. In practice, total fleet size should include spare vehicles, which are needed for bus schedule reliability and for vehicle maintenance. The optimal number of reserve vehicles should also be considered in further studies.

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