Economic Characteristics of Multiple Vehicle Delivery Tours Satisfying Time Constraints

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Since deregulation of the aviation industry, a substantial body of literature has emerged analyzing the economic structure of passenger carrier operations. By comparison, a paucity of literature exists that addresses the economics of air freight transportation. This study contributes to filling that void by assessing the economic structure of ground-side freight distribution for air express carriers. To do so, we develop an "engineering" cost model of the ground-side distribution process. This circumvents the problem that appropriate historical performance data is not available with which to develop an "economic" cost model and affords greater flexibility and accuracy than the more frequently applied econometric based cost models. The cost model is developed by first employing a mathematical heuristic to design and locate freight delivery subregions employed by freight carriers operating under time constraints. The results of the design heuristic are then used to create a model that incorporates costs of overcoming distance, stopping costs, marginal freight distribution costs, and fixed vehicle costs. It is then used to demonstrate that ground-side freight distribution operations exhibit significant economies of scale and profound economies of density. Furthermore, it is indicated that increasing the delivery time constraint decreases distribution costs. However, this decrease in costs must be tempered with the trade-off that increasing the delivery time constraint could decrease the market available to the carrier.

The air cargo industry was deregulated in November 1977, 1 year before deregulation of the passenger airline industry. At that time, air cargo was primarily transported in the bellies of passenger aircraft with the notable exception of the cargo transported by Flying Tigers, a successful international air freight forwarder. Door-to-door delivery was uncommon, and overnight delivery was the exception, not the norm.

The industry changed dramatically after deregulation, as Federal Express Corporation, a small package express carrier, emerged and rapidly grew to dominate the air freight industry. Federal Express' rapid growth eventually led to their purchase of Flying Tigers [see Sigafous (1) and Trimble (2)]. Attracted by Federal Express' rapid rise to dominance and success, several other specialized air freight carriers emerged including UPS, Airborne Express, and DHL. By the mid-1980s the air cargo industry was dominated by these service oriented carriers, forming the organizational structure that exists today in the aviation industry: specialized carriers that focus on either cargo or passenger transportation.

One factor contributing to the rapid rise of dedicated air freight carriers was the apathy of passenger carriers toward air cargo following deregulation. However, there is reason to believe that the passenger carriers' apathy has ended. Shaw (3) reports that five major U.S. passenger carriers (American Airlines, Delta Air Lines, Northwest Airlines, United Airlines, and USAir) have joined forces with Roadway Package System (RPS), a national ground carrier, to provide door-to-door delivery of freight and compete with integrated freight carriers. Such an alliance has far-reaching implications. If passenger carriers can effectively compete with dedicated freight carriers, there will be a continued need for joint-use airports (i.e., both freight and passenger carriers using the same airports). If they cannot compete, there will be an increased specialization of services (specializing either in freight or passenger transportation) and, consequently, an increased need for specialized airports.

To assess how competitive dedicated freight carriers and combination carriers can be (from an economic perspective), we must be able to quantify the operational cost of freight delivery. One accepted way to do this is to develop an "economic" cost model using industrywide or carrier specific data to calibrate econometric cost and/or production functions [Kiesling and Hansen (4)]. Unfortunately, data are very limited, particularly for dedicated freight carriers and for specific delivery operations such as ground-side distribution, to support such an analysis. As a result, we employ another possible approach, which is to develop an "engineering" cost model of the more specific operations of air freight carriers. The results presented in this paper are the first step in developing such a model. (Whereas the final goal is to develop engineering cost models of system-wide operations, this study addresses only ground-side transportation costs.)

Ground-side pickup and delivery operations are a crucial battleground in the competition between specialized and combination freight carriers. One reason is that pickup and delivery operations are the interface wherein customers judge the level of service received. As delivery deadlines attest, one critical factor in defining the level of service is time. Air express customers pay premium rates for the timely transport of goods, both in the sense that pickup and delivery deadlines are reliably met, and in the more general sense that freight can be delivered as early as possible in the business day and be picked up as late as possible in the business day. The determination of pickup and delivery deadlines is one decision variable that affects the level of service provided. As will be indicated in this report, however, it is also a decision variable that significantly affects the costs incurred by the freight carrier—shorter time constraints raise the operational costs to the carrier. This trade-off will prove to be a crucial element in the competition between freight carriers.

The design and operation of multiple-vehicle delivery systems (such as those described above) have been analyzed by numerous authors. Daganzo (5) explores the impact that zone shape has on tour building strategies and ultimately on tour lengths. Daganzo (6) presents a strategy for designing distribution problems in which N...
points must be visited by a fleet of vehicles operating under the con-
straint of a maximum of $C$ stops per vehicle. Newell and Daganzo
(7,8) expand this work further by considering larger delivery areas
wherein line-haul distances are significantly greater than local travel
distances. Newell (9) modifies the analysis to consider the move-
ment of valuable goods. In all of the above studies, vehicles are con-
strained by capacity. Relatively little has been done on the design
of multiple vehicle delivery systems constrained by time. One such
study, by Langevin and Soumis (10), does consider this problem,
but only for ring-radial networks and a centrally located depot. Han
(11) also focuses primarily on ring-radial networks in developing
routing strategies for multiple vehicle delivery problems.

This study explores the design of multiple-vehicle delivery sys-
tems constrained by time (not vehicle capacity or dispatch fre-
cuency), and applies the design process to several different types
of cities. Section 1 presents the basic distribution process that is
employed by air freight carriers, and defines the basic design prob-
lem. Section 2 applies the design process to linear cities and cities
with $L_i$ metrics. Section 3 discusses the impact of “fast roads” on
the design of delivery subregions, and extends the design process to
allow for fast roads. Section 4 uses the results from Sections 1–3 to
estimate the average unit cost of transporting freight on the ground-
side distribution system, and demonstrates the crucial role that
pickup and delivery time constraints play in ground-side distribution.

MINIMIZING DELIVERY COSTS

Simply stated, the air freight carrier’s goal in designing its ground-
side distribution system is to visit all pickup and delivery points in
the city at minimal cost. Two constraints determine how many vehi-
cles are required to accomplish this task. First, vehicle weight and
volume constraints are likely exceeded before all points in a city can
be visited, even in small cities. It follows that the next best solution
is to fully use delivery vehicles by visiting as many points as possi-
ble before weighing-out (meeting the vehicle’s weight limit) or cub-
ing-out (meeting the vehicle’s volume limit). However, the time-
sensitive nature of delivery deadlines precludes the vehicles from
even visiting enough points to reach vehicle capacity, meaning that
time is the second, and as it turns out the binding, design constraint.
The solution is to divide the delivery area into subregions, the size
of which are determined by the maximum number of points that a
single delivery vehicle can visit in the allotted time. This is equiva-
lent to minimizing the number of delivery subregions in the delivery
area.

To illustrate analytically, consider the following simplified
ground-side delivery cost function facing a freight carrier:

$$TC = dC_s + Tc_l + NC_i$$  \hspace{1cm} (1)

where

$$C_s = \text{cost per mile traveled},$$
$$C_l = \text{cost per hour of labor},$$
$$C_i = \text{fixed cost per vehicle}.$$ 

First, consider the cost per distance term of Equation 1. Let $N$
be the number of delivery subregions required to visit all pickup and
delivery points, $n$, in a city. Each delivery tour consists of a line-haul
portion (the distance from the terminal to the nearest point in the
tour), and a local travel portion (the distance required to visit all
pickup and delivery points in the subregion). For a given number of
pickup and delivery points in a city, increasing $N$ by one increases
the total line-haul distance traveled by an amount on the order of the
average distance from the terminal to all pickup and delivery points
in the city. However, it decreases the total local distance traveled by
approximately the average travel distance between pickup and deliv-
ery points. Because line-haul trips are almost always much longer
than the average distance between pickup and delivery points, it fol-
low that the total travel distance increases with $N$. Thus, to minimi-
ize the cost of overcoming distance, we would want to minimize $N$.

The relationship between $N$ and the labor cost of delivery follows
a similar vein. Let $T_l$ be the total labor (hours) required to service
all pickup and delivery points in the city, which is comprised of the
total time required to travel (both line-haul and local travel) and the
total time required to handle and process freight at each pickup and
delivery point. The latter is constant regardless of the size of $N$.
Since the total travel time is directly proportional to the total dis-

tance traveled, it is obvious that the total travel time also increases
with $N$. Thus, the labor costs are comprised of fixed and variable
(with $N$) components, which are minimized by minimizing $N$.

Finally, it is clear that if one delivery vehicle is assigned to each
subregion, the vehicle cost is also minimized by minimizing $N$.
These transformations allow the cost function to be rewritten:

$$TC = f_1(N)C_s + f_2(N)C_l + NC_i$$  \hspace{1cm} (2)

where $f_1(N) > 0$. Therefore, to minimize costs, carriers should min-
imize the number of delivery subregions required, subject to the
constraint that all points are visited in time $T$.

DESIGN OF MULTIVEHICLE DELIVERY ZONES

Designing delivery subregions is a detailed, and case specific, activ-
ity. Results differ with changes in the terminal location or the under-
lying transportation metric. The design process remains the same,
however, as formalized below.

Let $T$ be the amount of time allotted to visit all points in the deliv-
ery area (city). Only one delivery vehicle visits each subregion in
time $T$. For the delivery process, $T$ includes the time required
to travel to the delivery subregion and visit all points in the subregion.
For the pickup process, it includes the time to visit all points in the
subregion and return to the terminal. (For cost estimating purposes,
both line-haul trips must be included.)

We can analytically express the design constraint by defining
three time quantities: the line-haul time, $T_1$, which is time required
to travel from/to the terminal to/from the delivery subregion; the
handling time, $T_2$, which is the time required to transport freight
to/from the customer from/to the vehicle; and the local travel time,
$T_3$, which is the time required to travel the local streets between
pickup and delivery points. The sum of these activities must be less
than or equal to $T$ for all delivery subregions:

$$T_1 + T_2 + T_3 \leq T$$  \hspace{1cm} (3)

This basic constraint holds true for all transportation metrics and
city shapes analyzed in the remainder of this section, wherein several
different scenarios are analyzed. For simplicity of demonstration, a
linear city is analyzed first. The design process is then expanded and
applied to cities with $L_i$ transportation metrics, a scenario that is
much more realistic than linear or ring-radial cities. Finally, the
impact of fast roads on the design of delivery subregions is considered, providing the most realistic design framework possible.

Linear City

To formalize and demonstrate the design process, a linear city of length \( D_c \) is considered first. A terminal is located at one end of the city, and all points to be visited are randomly distributed across its length. Delivery subregions are nonoverlapping zones of length \( d_i \), located a distance \( D_i \) from the terminal, as shown in Figure 1. Subregions are located adjacent to one another, so that \( \sum d_i = D_c \).

The line-haul time, \( T_1 \), is the time required to travel between the terminal and the nearest edge of the subregion. By assuming an average velocity, \( v \), the line-haul travel time to subregion \( i \) is simply \( T_1 = D_i / v \).

The handling time, \( T_2 \), for a vehicle of the \( i \)th subregion is the time required to perform the delivery or pickup tasks at all points in the subregion. Such a task includes parking the vehicle, walking to the appropriate location, processing the required paper work, handling the package, and returning to the vehicle. To obtain the total handling time in subregion \( i \), we assume that the handling time per stop, \( \tau \), is constant on the average, which we then multiply by the total number of points in the subregion. If \( \delta \) is the customer density (number of points per unit length), then the expected number of points in subregion \( i \) is \( \delta d_i \). Thus, the total handling time of zone \( i \) is \( T_2 = \delta d_i \tau \).

The third element of the time constraint is the local travel time, which is the time required to travel between all points in a specific zone. When the number of points in a subregion is sufficiently large, the distance traveled is closely approximated as the length of the subregion. If there are few points in the subregion, however, it may be deemed necessary to reduce the travel distance by one half the expected distance between points, \( 1/(2\delta) \). Assuming there is a sufficiently large number of points in the subregion, the local travel time in subregion \( i \) is \( T_3 = d_i \).

Having defined all three tasks, the time constraint facing vehicles in subregion \( i \) can be rewritten:

\[
\frac{D_i}{v} + \delta d_i \tau + \frac{d_i}{v} \leq T
\]

In designing the subregions, the underlying goal is to minimize the number of vehicles required, which is equivalent to maximizing the number of points per subregion. The design process is begun by considering the outermost delivery zone, subregion 1. Its optimal length, \( d_1^* \), is determined by replacing the line-haul distance, \( D_1 \), in the first term of the time constraint with the line-haul distance to the first zone, \( D_i = d_1 \), and solving for \( d_1^* \):

\[
d_1^* = \frac{T_v - D_1}{\delta \tau v}
\]

It should be noted that \( d_i \), and all remaining calculations of \( d_i \), can be solved in this manner only by assuming that time is the binding constraint. By substituting \( [D_i = (d_i + d_{i+1})] \) for \( D_i \) and \( d_i \) for \( d_i \) in the time constraint, \( d_i^* \) is easily determined. Solving recursively, an expression emerges that allows us to design zone \( i \) (for \( i = 2, \ldots, n \)) where \( n \) is the total numbers of zones required to cover the entire city:

\[
d_i^* = \frac{T_v - D_i + \sum_{j=2}^{i} d_j^*}{\delta \tau v}
\]

The zone adjacent to the terminal, zone \( n \), is simply the remaining length of the city,

\[
d_n^* = D_i - \sum_{j=2}^{n} d_j^* - 1
\]

It will be less than the length determined by the above design equation.

Thus, given a linear city of length \( D_c \) and customer density \( \delta \), we can optimally design all subregions. We simply begin with the above expression for \( d_1^* \), which gives the optimal size and location of the outermost subregion. Then, knowing the length \( d_1^* \), we can determine the length of all other zones (\( i = 2, \ldots, n - 1 \)) recursively with the above expression for \( d_i^* \).

L_1 Metrics

A linear city is clearly an unrealistic representation of any city that would be included in air freight networks. However, the design process that applies to the hypothetical linear cities also applies to two dimensional cities. Since U.S. cities rely primarily on rectangular \( (L_1) \) transportation metrics, we need to adapt the design process to apply to such metrics. In the following pages, we apply the design approach to cities with \( L_1 \) metrics when the terminal is located in the city center, on the edge of the city, and in one corner of the city.

Terminal in City Center

First, consider a delivery area with a centrally located terminal, as shown in Figure 2(left). For analysis purposes, the delivery area shape is approximated as a square oriented at 45° to a fine orthogonal transportation grid \( (L_1) \), a shape dictated by the equal-travel time contours (the locus of all points that can be reached in a given amount of time). The size of the delivery area is defined by \( D_c \), which is the travel distance to the outermost corner or edge of the city. All points to be visited are distributed randomly through-
out the area with a constant density, \( \delta(x, y) = \delta \). Vehicles travel at speed \( v \) throughout the city.

The first step in designing delivery subregions is to build equip-travel time contours from the depot. For this scenario, the contours are squares centered at the depot at 45° to the metric’s preferred directions. Daganzo (12) indicates that delivery subregions should be rectangular in shape and should be oriented perpendicular to these contours, as shown in Figure 2(left).

As before, the outermost delivery subregions are designed first, followed by the subregions in bands progressively closer to the terminal. Vehicles are again bound by a time constraint that includes the line-haul time, \( T_l \), the local travel time, \( T_2 \), and the handling time, \( T_3 \). Letting \( D_i \) equal the distance to the inner contour of band \( i \), the line-haul travel time can be defined:

\[
T_l = \frac{D_i - \sum_{T} D_{i}}{v} = \frac{D_i - \sqrt{2} \sum_{T} D_{i}}{v}
\]  

(7)

The handling time, \( T_2 \), is the time required to perform the delivery and/or pickup tasks at all points in the subregion. Assuming that the required handling time per stop, \( \tau \), is constant on the average, then the total handling time is the product of the total number of points in the subregion and \( \tau \). Daganzo (12) illustrates that, for an \( L_1 \) metric and randomly scattered points, the tour length minimizing dimensions for delivery subregions are approximately:

Subregion width = \((6\delta)^{1/2}\)  

(8)

Subregion length = \(C(6\delta)^{-1/2}\)  

(9)

where \( C \) is the number of points in the subregion. The total number of points in a subregion, then, can be estimated by solving the second equation for \( C = l(6\delta)^{1/2} \). By so doing, the total handling time for a zone in band \( i \) is approximated:

\[
T_2 = \tau l(6\delta)^{1/2}
\]  

(10)

The local travel distance is approximated as the product of the number of points in the subregion, defined above, and the expected travel distance between two points in a subregion. Daganzo (12) indicates that the expected travel distance between points is \( k\delta^{1/2} \), where \( k \) is a dimensionless constant; approximately 0.82 for \( L_1 \) metrics and 0.57 for Euclidean metrics. Thus, the local travel distance is approximately \( l k\sqrt{6} \), and the local travel time in a subregion in band \( i \) is:

\[
T_3 = \frac{l k\sqrt{6}}{v}
\]  

(11)

As in the previous section, the design process begins with the outermost band of the city, which faces the following time constraint:

\[
T_1 + T_2 + T_3 \leq T
\]  

(12)

Solving the constraint gives the optimal length of the subregions in band number 1:

\[
l_1^* = \frac{T v - D_i}{(6\delta)^{1/2} \tau v + k\sqrt{6} - \sqrt{2}}
\]  

(14)

For design purposes, and particularly for cost estimating purposes, we also need to know how many delivery subregions are in each band. Having determined \( l_1^* \), we can calculate the average perimeter of the band and the total number of subregions in band \( n \):

\[
N_i = \frac{\text{average perimeter of band } i}{\text{optimal zone width}}
\]  

(15)

\[
N_i = \left[ \frac{4\sqrt{2}D_i - 4l_1^*}{\delta} \right]^{1/2}
\]  

(16)

where \([\cdot]^{*}\) is the nearest integer greater than the quantity in brackets.
The second band, or any subsequent band, is designed in a similar process, substituting $D_i - \sqrt{2} \sum I_i$, for $D_i$, in the line-haul expression. Repeating this process, the following recursive design equations emerge:

$$I^*_t = \frac{TV_t - D_t + \sqrt{2} \sum_{j=2}^{t} l^*_j}{(6\delta)^{1/2} \tau_v + k \sqrt{6} - \sqrt{2}}$$  \hspace{1cm} (17)$$

$$N_i = \left[ \frac{4 \sqrt{2} D_i - 8 \sum_{j=2}^{i} l^*_j - 4 l^*_i}{(6\delta)^{1/2}} \right]^*$$  \hspace{1cm} (18)$$

The above equations can be used iteratively to design each delivery subregion in the delivery area. Note, however, that this design process will result in irregular delivery bands (and zones) adjacent to the terminal. It may be necessary to "manually" adjust subregions boundaries to cover the area in consideration, either by expanding/contracting nearby subregions or adding another subregion. Whatever method is employed, the number of additional delivery zones required is small relative to the total number of zones required for the entire delivery region.

**Other Terminal Locations**

Air express terminals are typically located at local airports which, more often than not, are located at the perimeters of cities due to land and noise constraints. As a result, it is not always appropriate to assume that the terminal is in the city center. Two other terminal locations have been evaluated using the procedure just described; one with the terminal located in the corner of the city, and another with the terminal in the middle of the city edge. Letting $D_t$ equal the travel distance from the terminal to the furthest edge of the city, the Euclidean length of the city edges are $l_t = D_t/\sqrt{2}$.

When the terminal is located in the center of the city’s edge, the equi-travel time contours take on a peculiar shape. The outermost bands are simply formed by straight contours. But, halfway through the city, the contours take on a rectangular shape, as shown in Figure 2(b). As a result, additional notation is required; delivery subregions on the outermost contours are in bands 1 to $(t - 1)$, the transition band is band $t$, and the half diamond shaped contours form bands $(t + 1)$ to $n$. To determine the number of subregions in the transition band, $t$, the zone is divided into two parts; the "cross-piece" (which is equivalent to bands 1 through $(t - 1)$) and the "legs" which form the edge of the city. The number of subregions in the cross-piece is given by Equation 20, and the number of subregions in the legs is estimated as the dividend of the area of the two legs and the optimal area of a subregion located in band $t$. Then, the design equations can be expressed:

$$N_i = \left[ \frac{D_t}{(12\delta)^{1/2}} \right]^* \quad \text{for } i = (1, 2, \ldots, t - 1),$$  \hspace{1cm} (19)$$

$$N_i = \left[ \frac{D_t}{(12\delta)^{1/2}} + \frac{x}{(12\delta)^{1/2}} l^*_t (D_t + \sqrt{2} D_t) - 2 \sqrt{2} I^*_t - 4 \sum_{j=2}^{t} l^*_j \right]^* \quad \text{for } i = t,$$  \hspace{1cm} (20)$$

The third scenario (terminal in the city corner) has associated with it a set of design equations similar in nature to the city center scenario originally considered:

$$N_i = \left[ \frac{4 \sqrt{2} D_i - 8 \sum_{j=1}^{i} l^*_j + \sqrt{6} - \sqrt{2}}{6\delta} \right]^*$$  \hspace{1cm} (21)$$

$$N_i = \left[ \left\{ \frac{4 D_i^*}{6\delta} \right\}^* \right] \quad \text{for } i = n,$$

$$N_i = \left[ \left\{ \frac{4 D_i^*}{6\delta} \right\}^* \right] \quad \text{for } i = (2, \ldots, n - 1)$$  \hspace{1cm} (22)$$

is the distance between contour $t$ and the edge of the city.

**FAST ROADS**

The models presented in the previous sections assume that vehicles travel the same speed on all roads. Since city networks are combinations of local roads, arterials, and freeways, the aforementioned models must be expanded to allow for more than one travel speed. Newell (13) examines the impact that fast roads have on the shape of equi-travel time contours, demonstrating that a single fast road stretches the contours in the direction of the road. Kiesling (14) indicates that a grid of fast roads, which arguably exists in any major city, results in equi-travel time contours that are closely approximated by a contour oriented at 45° to the origin as in the previously analyzed case in which no fast roads are present.

The approximated equi-travel time contour is dependent on previously defined variables and two vehicle travel speeds, $v_f$ (fast) and $v_s$ (slow). If we assume that delivery vehicles travel fast on the line-haul portion of their delivery tour, and travel slow on local portions of the tour, the original time constraint can be rewritten:

$$\frac{(D_i - \sqrt{2} l_t)}{v_f} + l_t \tau (6\delta)^{1/2} + \frac{l_t k \sqrt{6}}{v_s} \leq T$$  \hspace{1cm} (26)$$

Solving the constraint as in Section 2, we can determine the optimal zone lengths:

$$l^*_i = \frac{TV_i - D_i}{(6\delta)^{1/2} \tau_v + \frac{k \sqrt{6} \nu_f}{\nu_s} - \sqrt{2}}$$  \hspace{1cm} (27)$$
The marginal costs are very small compared to other logistics costs and are frequently ignored. 

\[ l_i^* = \frac{T_{V_i} - D_i + \sqrt{\frac{1}{2} \sum_{j \neq i} l_j^*}}{(66)^{1/2} \tau_{V_i} + \frac{k \sqrt{6} v_i}{v_i} - \sqrt{2}} \]  

(28)

The above expressions are true no matter where the terminal is located. However, the definition of \( D_i \) changes for each scenario. Generally speaking, \( D_i \) is the travel distance from the terminal to the furthest point on the city boundary.

The optimal zone length is now a function of two speeds. But, what about the number of subregions in each band? None of the previously defined expressions change simply because, in all previously analyzed scenarios, the number of subregions in each band is not a function of \( v \). Thus, the design of all delivery subregions in metrics with fast roads is identical to the previously described process with the exception that the optimal zone length changes.

LOGISTIC COSTS OF GROUND-SIDE DELIVERY

The heuristic developed and demonstrated up to this point provides all of the information needed to locate and size delivery subregions which, in turn, allows us to begin analyzing ground-side pickup and delivery costs. In this section, a basic cost model is developed that incorporates four categories of logistics cost; costs of stopping, costs of overcoming distance, costs of carrying additional freight, and fixed vehicle costs.

Included in the time constraint that underpinned the development of our design algorithm is the time required to stop at an origin or destination and move the package to/from the delivery vehicle. Several costs are incurred each time the delivery vehicle stops including labor, vehicle depreciation costs and materials. The total cost of stopping is determined by assuming that the cost per stop, \( C_s \), is constant on the average. Including the entire ground-side delivery system, the total number of stops is the sum of city-wide stops and stops at the local terminal. Thus, the total stopping cost follows:

\[ \text{Stopping cost} = C_s \left( A\delta + \sum_{i=1}^{n} N_i \right) \]  

(29)

where \( A \) is the area of the region in question.

The cost per mile, \( C_m \), is also assumed constant on the average.

Recalling the need to include both line-haul trips in the cost formulation, the total distance traveled is easily determined:

\[ \text{Total distance} = 2 \sum_{i=1}^{n} D_i^* N_i + k \sqrt{6} \sum_{i=1}^{n} l_i^* N_i \]  

(30)

\[ \text{Distance cost} = C_d \left( 2 \sum_{i=1}^{n} D_i^* N_i + k \sqrt{6} \sum_{i=1}^{n} l_i^* N_i \right) \]  

(31)

where \( D_i^* = D_i - \sum_{j \neq i} l_j \) and \( D_i^* = 0 \).

We also include the added cost per item carried, \( C_m \), in the formulation. The total number of items carried is the product of the number of stops and the number of packages per stop, \( z \):

\[ \text{Marginal cost} = C_m (A\delta z) \]  

(32)

The marginal costs are very small compared to other logistics costs and are frequently ignored.

The final cost to include in this model is the fixed vehicle cost, \( C_f \). The total number of vehicles required to deliver freight throughout the city is assumed equivalent to the number of delivery subregions in the city:

\[ \text{Fixed vehicle cost} = C_f \left( \sum_{i=1}^{n} N_i \right) \]

The total cost of delivery, \( TC \), can then be expressed as the sum of the aforementioned costs:

\[ TC = C_s \left( A\delta + \sum_{i=1}^{n} N_i \right) + C_d \left( 2 \sum_{i=1}^{n} D_i^* N_i + k \sqrt{6} \sum_{i=1}^{n} l_i^* N_i \right) \]  

(34)

...To demonstrate the economic characteristics of this model, we consider a diamond-shaped city with an \( L_i \) metric and a terminal located in the lower corner. In such a case the city area, \( A \), is \( L_i^2 \) or \( D_i/2 \). Table 1 summarizes the parameter values assumed for the remainder of this section.

The results of the subregion design algorithm confirm \( a \) priori expectations about the subregion partitioning, that bands furthest from the terminal are narrowest \( (l_i = 1.74 \text{ km}) \), and bands closest to the terminal are widest \( (l_i = 6.13 \text{ km}) \) with the exception of band \( n \). It is easily shown that bands \( i \) to \( n \) + \( i \) increases by the constant percentage

\[ \left( \frac{(66)^{1/2} \tau_{V_i} + \frac{2v_i}{v_i} - \sqrt{2}}{v_i} \right)^{-1} \]

To assess the concepts of scale and density economies in ground-side delivery operations, the total cost formulation is used to determine average costs (total cost per package) of delivery under various assumptions. Scale economies are defined as a change in the average unit costs of production resulting from a change in output. If output is defined as the number of points visited by a carrier, which increases as a result of city growth \( (D_i \) increasing, \( ceteris paribus \)) or an expansion in the carriers delivery market, it is easily shown that there are diseconomies of scale. If \( \delta \) increases while holding \( D_i \) and all other variables constant (which is more accurately called economies of density), it is clear that there are profound economies of density in ground side freight distribution, as indicated by the decreasing average unit cost curve in Figure 3. Although the finding of such significant economies of density is not

<table>
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<tr>
<th>TABLE 1</th>
<th>Assumed Parameter Values for Demonstration of Cost Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>Assumed Values</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.39 to 9.67 stops/km² (1 to 25 stops/mi²)</td>
</tr>
<tr>
<td>( D_c )</td>
<td>40.2 km (25 mi)</td>
</tr>
<tr>
<td>( T )</td>
<td>1 to 4.5 hr</td>
</tr>
<tr>
<td>( v_f )</td>
<td>64.4 kph (40 mph)</td>
</tr>
<tr>
<td>( v_s )</td>
<td>32.2 kph (20 mph)</td>
</tr>
<tr>
<td>( t )</td>
<td>5 min</td>
</tr>
<tr>
<td>( k )</td>
<td>0.82</td>
</tr>
<tr>
<td>( z )</td>
<td>1 to 4.5 pkg/stop</td>
</tr>
<tr>
<td>( C_d )</td>
<td>$0.62/km ($1/mi)</td>
</tr>
<tr>
<td>( C_s )</td>
<td>$2/stop</td>
</tr>
<tr>
<td>( C_m )</td>
<td>$0.05/pkg</td>
</tr>
<tr>
<td>( C_f )</td>
<td>$1.25/veh</td>
</tr>
</tbody>
</table>
The trade-off between economies of density and "economies of time" can be illustrated two ways. First, we can assess the impact of varying $T$ on average unit costs, taking into account the decrease in available demand caused by an increase in $T$. Clearly, at $T = 0$, the maximum number of points are potentially served (although there is no way to service the pickup and delivery points in zero time). Furthermore, it is appropriate to assume that the entire market ($\delta A$) is available for time constraints up to 2 hr. For $T$ greater than approximately 2 hr, however, the number of points that can be serviced begins to decline for the previously discussed reasons. Eventually, no demand is available at $T = 9$ hr, the full business day. The available demand ($AD$) distribution could be represented as follows:

$$AD = \delta \left(1 - \left(\frac{e^{T-y}}{1 + e^{T-y}}\right)^x\right)$$  \hspace{1cm} (35)

where $x$ and $y$ are distribution shape parameters, assumed to be 2 and 4 for demonstration purposes. The available demand (as a function $T$) is illustrated in Figure 4. Substituting this "available demand" quantity into the total cost model (Equation 34) and varying $T$ from 0 to 9, Figure 5 illustrates that the cost minimizing time constraint is from 5 to 7 hr, but the improvement in costs over $T = 3$ is relatively small. Profit maximization is more important than cost minimization for a freight carrier, however, so a more appropriate way to view the effect of time on production strategy is to consider the impact that the time constraint has on carrier profits. Assuming an average price per package of $10, Figure 6 illustrates that the profits of a hypothetical carrier are maximized when the time constraint is approximately 1.5 hr (for this example).
FIGURE 4  Available customer density as a function of time constraint, $T$.

FIGURE 5  Average unit costs as a function of time constraint, $T$. 
CONCLUSIONS

Throughout this report, several aspects of the design of ground-side delivery systems have been explored. It was first indicated that the binding design constraint is that of time, not vehicle capacity constraints as assumed by most previous studies. It was further indicated that an appropriate way to approach the design of delivery subregions is to first define the time constraint as a function of line-haul time, handling time, and local travel time. Then, according to this time constraint, design the delivery subregions along the equi-travel time contours beginning in the outermost band and iteratively moving toward the terminal. Expressions were derived for the subregion dimensions, the number of subregions per band, and the location of the subregions for cities employing $L_1$ metrics with terminals located centrally, in one corner of the city, and in the middle of one edge of the city.

City street networks generally allow for more than one speed of travel. Fast roads, as they are often called, significantly change the shape of the equi-travel time contours that the design is based on. As a result, the impact of fast roads in a city network was explored. The design framework was then generalized to allow for two travel speeds; fast travel on line-haul trips and slower travel on local streets.

The results of the design process for a square city with an $L_1$ metric, two travel speeds, and a terminal in one corner, were then used to develop a total cost model of ground-side pickup and delivery operations. The model, in turn, was used to explore the economic cost structure of the delivery system. It was indicated that ground-side pickup and delivery operations exhibit significant economies of scale and profound economies of density. The results were highly robust with respect to changes in all design variables. It was also demonstrated that as the time constraint increases, the average unit cost decreases. However, increasing this decision variable results in a decrease in the market that is potentially captured by a competing carrier.

REFERENCES


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