Pavement Performance Modeling Program for Pennsylvania

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The Pavement Performance Modeling Program (PPMP) developed for the Pennsylvania Department of Transportation by Texas Research and Development Foundation is described. PPMP is a MicroSoft Windows-based computer tool to assist engineers in developing pavement performance models for use in pavement management systems and updating these models annually as new data are input into the data base. The program can build both deterministic models and probabilistic models for an individual or group of pavement segments for each maintenance and rehabilitation (M&R) treatment. The program allows the user to define pavement performance indexes, grouping variables, and M&R treatments. Grouping variables are those that influence performance and are thus accounted for in the analysis. They include annual average daily traffic or equivalent single axle load (ESAL), functional class, pavement structure depth, maintenance level, and others at the discretion of the user. For deterministic models, five forms of equations are included. The independent variables can be pavement age or ESAL. Data and models can be plotted on screen and analyzed. The results from sample runs are presented and discussed.

Modeling of pavement performance is absolutely essential to pavement management on all levels, from project to national network (*1*). Pavement performance models can be developed based on pavement historical data. It is realized that many factors (i.e., pavement surface type, maintenance and rehabilitation (M&R) treatment, traffic, subgrade type, construction material, maintenance level, environment, climate etc.) have effects on pavement performance.

Pavement performance models can be broadly divided into group models and segment models. A group is a set of pavement segments defined by one or more variables. These variables are called performance grouping variables. For example, if pavement type and annual average daily traffic (AADT) are selected as grouping variables, pavement type is divided into two levels, flexible and rigid; and AADT into three levels, light, medium, and heavy, giving a total of six groups ($2 \times 3 = 6$). In terms of the analysis methods used in modeling, performance models can also be divided into deterministic models and probabilistic models. In pavement performance modeling, the most popular method for building deterministic models is regression analysis. For probabilistic models, the Markov chain process is the preferred method.

Pavement performance prediction is the most technologically difficult portion of pavement management (*2*) because of (a) the uncertainties of pavement behavior under changeable traffic load, environment etc., (b) the difficulty of quantifying many factors affecting pavements, (c) the error associated with using discrete testing points to represent the total pavement area when estimating pavement condition, and (d) the nature of the subjective condition survey. To develop the best models from the available data and update these models as more data become available is one of the most important tasks for engineers and researchers in pavement management. The development of pavement performance models involves extensive effort to create a data file (or a data base) by joining and calculating data from original data files. Currently, most researchers use a single model form to produce pavement performance models for all types of pavements (*2–5*). One reason is that no specific software has been available to allow relatively easy manipulation of a historical data base and development of models. A single model form may produce reasonable results, but may not get the best results due to the nature or variability of pavement performance in the real world.

The MicroSoft Windows-based Pavement Performance Modeling Program (PPMP) has been developed for the Pennsylvania Department of Transportation (PennDOT). The program provides a computer tool to assist PennDOT engineers in developing pavement performance models for use in their pavement management system (PMS) and updating these models annually as new data are input into the data base. There are five basic forms of models included in the program, They allow the user to try different types of models and select the best fit model for a specific situation. In this paper, the data used for developing PPMP is discussed, the components of the program are presented, the procedure used to produce performance models for PennDOT is described, and the models developed from sample data are analyzed and evaluated.

DATA DESCRIPTION

The road network of the Commonwealth of Pennsylvania is divided into approximately 150,000 road segments. Most data are stored in the Roadway Management System and the Maintenance Operations and Resources Information System. PennDOT uses IBM's Information Management Systems as its primary data base management system with MVS/ESA as the operating system. RMS contains 32 data bases and over 600 computer programs that generate 221 different inquiry and data input screens at computer terminals throughout the state.

The required data files for pavement performance modeling include (a) segment inventory, (b) pavement rehabilitation history, (c) asphalt concrete (AC) surface condition, (d) portland cement concrete (PCC) surface condition, and (e) pavement-related minor maintenance. Table 1 lists the data used for pavement performance modeling.

The pavement history file stores up to 10 layers of information. There are more than 200 surface, base, and subbase types coded in the file. The distresses stored in both the AC pavement condition file and PCC pavement condition file are two-digit codes representing

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TABLE 1 Data Used in Pavement Performance Modeling Data

Data Table	Data Used In Pavement Performance Modeling
Segment Inventory	length, width, lane count, federal functional class, truck percent, AADT, ESAL
Pavement History	layer year, layer code, layer thickness
AC Surface Condition	excess asphalt, raveling/weathering, block cracking, transverse/longitudinal cracking, alligator cracking, edge deterioration, bituminous patchings, potholes, widening drop-off, profile distortion, IRI, PSR
PCC Surface Condition	joint seal failure, longitudinal joint spalling, transverse joint spalling, faulting, bloken slab, bituminous patch, surface defects, rutting, IRI, PSR
Maintenance Activity	activity year, activity code

the severity and density of the distress. A total of 23 activities, from patching to surface treatments, are coded in the minor maintenance file. All condition survey data from 1983 through 1993 (approximately 1.5 million records) are available for performance modeling.

SYSTEM COMPONENTS

The project has three major objectives: (a) create a research data base so the modeling can be done efficiently and effectively, (b) develop statistical analysis procedures for developing various types of models, and (c) design user-friendly user interface so different approaches can be tried to obtain the best model fitting a specific data set. To achieve these objectives, six modules are designed for PPMP: user definition, data base, method base, modeling, analysis, and output. Figure 1 illustrates the components of the program.

User Definition Module

The user definition module defines (a) the deduct values for converting distress severity and density codes into individual distress indexes, (b) M&R treatments and maintenance level, (c) performance indexes, and (d) grouping variables.

In this module, distress codes are converted to individual distress indexes when the raw data are imported to PPMP. M&R treatments can be classified generally as thin overlay, medium overlay, and thick overlay or reconstruction, or specifically as detailed surface material types. Maintenance levels can be divided into no maintenance (Level 1), some maintenance (Level 2), or heavy maintenance (Level 3) between two major rehabilitation treatments. Pavement performance can have a single index (such as a cracking or rutting index) or composite indexes (such as an overall pavement index). The selection of grouping variables is essential to performance modeling in that it determines the number of models and the significance of the models to some extent.

To be flexible, models can be built to reflect county by county, district by district, a mix of counties and districts, or the whole state. The advantage of dividing the state into small regions such as counties or districts is that climate factor can be taken into account indirectly, since the climate of the whole state is more diversified than that of a county or a district. The disadvantage is that more computation effort is needed, and, in some cases, no model can be obtained from lack of data.

Data Base Module

Currently, the five data files used for modeling are downloaded from the PennDOT primary roadway data base in ASCII format and then imported to PPMP. A research data base is created and may be updated on a year-by-year basis. In addition to the original five files, the PPMP data base consists of more than 30 additional data files, such as distress deduct values, performance indexes, grouping variables, performance models, and so forth. A master file is created by joining and calculating the data from the original five files.

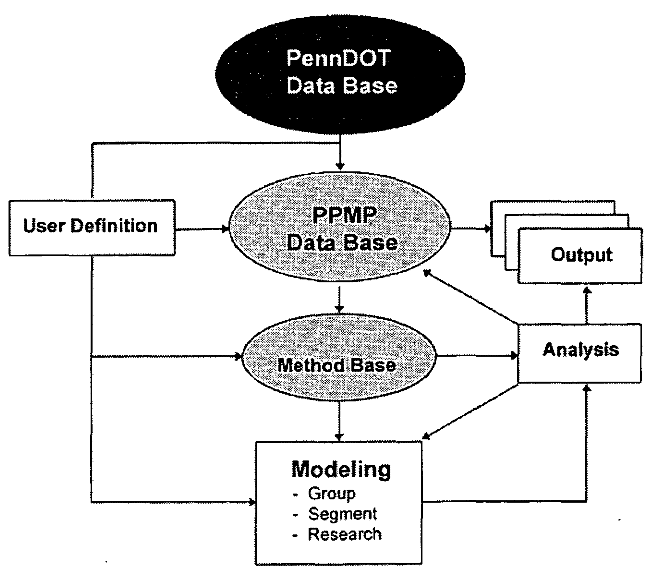

FIGURE 1 Components of PPMP.

Method Base Module

The method base is the key module of PPMP. It is the collection of various statistical analysis methods for pavement performance modeling. The current version of PPMP is composed of the following statistical methods: (a) least squares and constrained least squares methods for generating deterministic models, (b) probabilistic analysis for building Markov chain models, and (c) optimization algorithms for selecting best models. The method base allows the user to try different data transformation methods and types of models to get the best models possible.

Modeling Module

The modeling module provides two ways to build pavement performance models: group models and research models. A group model can be built once for all performance indexes, groups, and M&R treatments. A research model can be built with any combination of variables, for example, interstate highways, flexible pavements, heavy maintenance, AADT from 5,000 per lane to 10,000 per lane, and so forth. In any case, for deterministic models, the independent variables can be either age or cumulative equivalent single axle load (ESAL). Currently, the following forms of models are included in the program:

$$y = \alpha + \beta x \quad (1)$$
$$y = \alpha e^{-\beta x} \quad (2)$$
$$y = 100 - \alpha e^{\beta x} \quad (3)$$
$$y = 1/(\alpha + \beta x) \quad (4)$$
$$y = \alpha + \Sigma \beta x^{i} \; (i = 2 \ldots) \quad (5)$$

where

y = performance index;
x = independent variable, either pavement age or cumulative ESAL; and α, β, and β_i ($i = 2 \ldots$) = model parameters to be estimated.

The user of the program can build the foregoing models for any set of data; the model that fits the data best can then be selected.

For probabilistic modeling, the Markov chain model is included in the current version of the program. In building the Markov chain model, each performance index can be divided into a maximum of five states (e.g., excellent, good, fair, poor, and bad). It is assumed that pavements can change only to an equal or worse condition under routine maintenance in a period of 1 year (i.e., routine maintenance cannot improve the condition). The following equation is used to estimate the transition probability of the Markov chain model for any performance index after an M&R treatment is performed:

$$p_{ij}(k) = m_{ij}(k)/n_i(k) \quad (6)$$

where

$p_{ij}(k)$ = transition probability from state i to state j after M&R treatment k;
$m_{ij}(k)$ = number of segments moved from state i to state j in a period of 1 year after M&R treatment k; and
$n_i(k)$ = number of segments in state i before M&R treatment k.

Analysis Module

The purpose of the analysis module is to plot the raw data and the models built by the modeling module; analyze the data, outliers, and models; and select the best model. In some groups, for some performance indexes, the models built from the available data may be unrealistic. This module provides a practical tool for the user to determine whether the models can be used or adjusted, in addition to the test of statistical significance. In some cases, models cannot be obtained due to lack of data. From a network M&R planning point of view, models for some groups may be desired. In such cases, the models can be made subjectively based on the available models similar to these cases and supplemented with engineering judgment.

Output Module

The generic output module produces various reports for the performance models, such as listings, summaries, graphs, and so forth. In addition, it can also generate various file formats, such as ASCII, dBase, Paradox, and Excel, which can be accessed by network optimization programs and project life cycle cost analysis programs.

M&R TREATMENTS

In PPMP, an M&R treatment is the combination of a level of the thickness of a surface layer and the material type of the surface layer. The level of layer thickness can be divided into thin, medium, and thick, and may differ from one material type to another. The pavement type under a surface layer can be flexible or rigid if the surface layer is an overlay, or none if the surface is a new construction or reconstruction.

There are more than 160 types of surface layers used in Pennsylvania. If the average number of levels for all these layers is two, there are more than 320 M&R treatments ($160 \times 2 = 320$). Although the program allows the user to define unlimited M&R treatments, it may not be necessary to develop models for all the treatments.

Figure 2 depicts the screen for defining an M&R treatment. First, the surface layer groups are defined, the number of levels of the layer thickness is specified, and the limiting values for each level are determined. Currently, seven layer groups are used. ID2, ID3, FB1, FJ1, and FJ4 are flexible pavement surface layers; PCC and CRC are rigid pavement surface layers. The major differences among ID2, ID3, FB1, FJ1, and FJ4 are aggregate gradation and asphalt content. The structure numbers of ID2 and ID3 are 0.44; those of FJ1 and FJ4 are 0.2; and that of FB1 is 0.2 (*6*). Next, layer codes are grouped. This is done separately for AC pavements and PCC pavements.

PERFORMANCE INDEXES

The PPMP allows users to define their own performance indexes for modeling. In developing performance models for Pennsylvania, five performance indexes provided by PennDOT are used. All indexes range from 0 to 100, with 100 being excellent condition. SDI (Surface Distress Index), SFI (Surface Friction Index), and SI (Strength Index) are linear functions of condition ratings. RI (Ride Index) is a

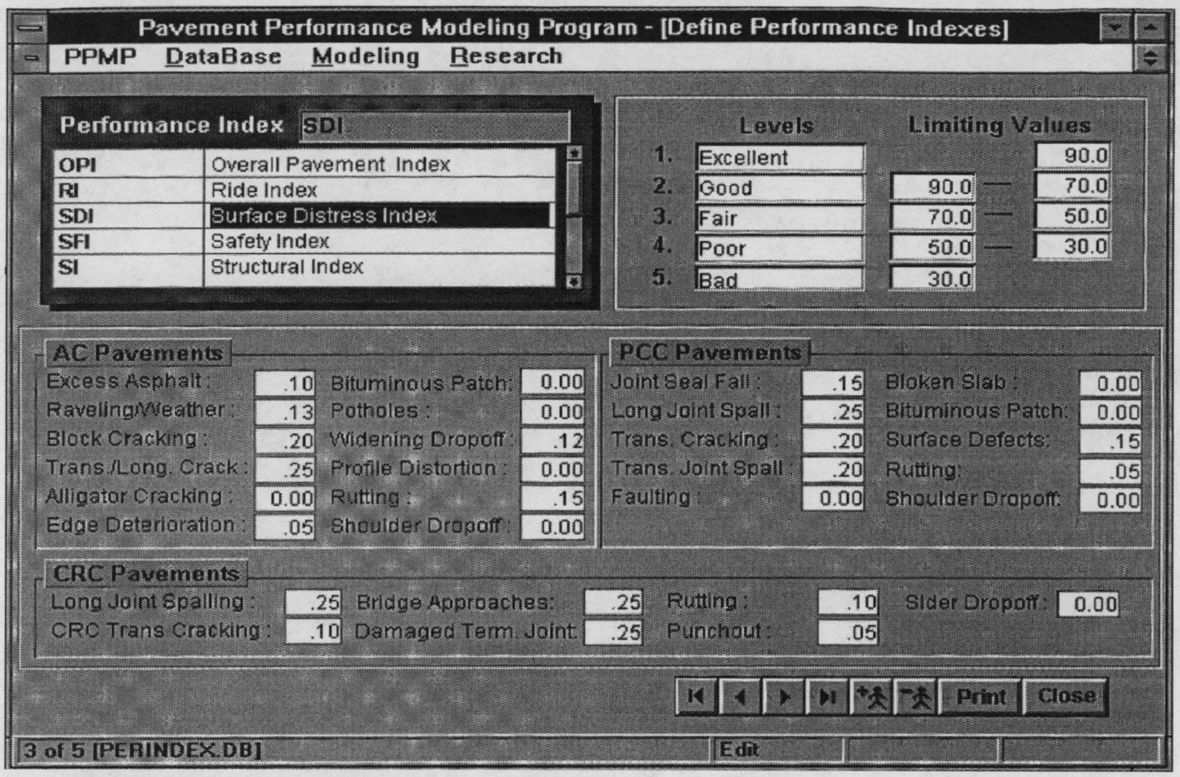

FIGURE 2 Performance definition screen.

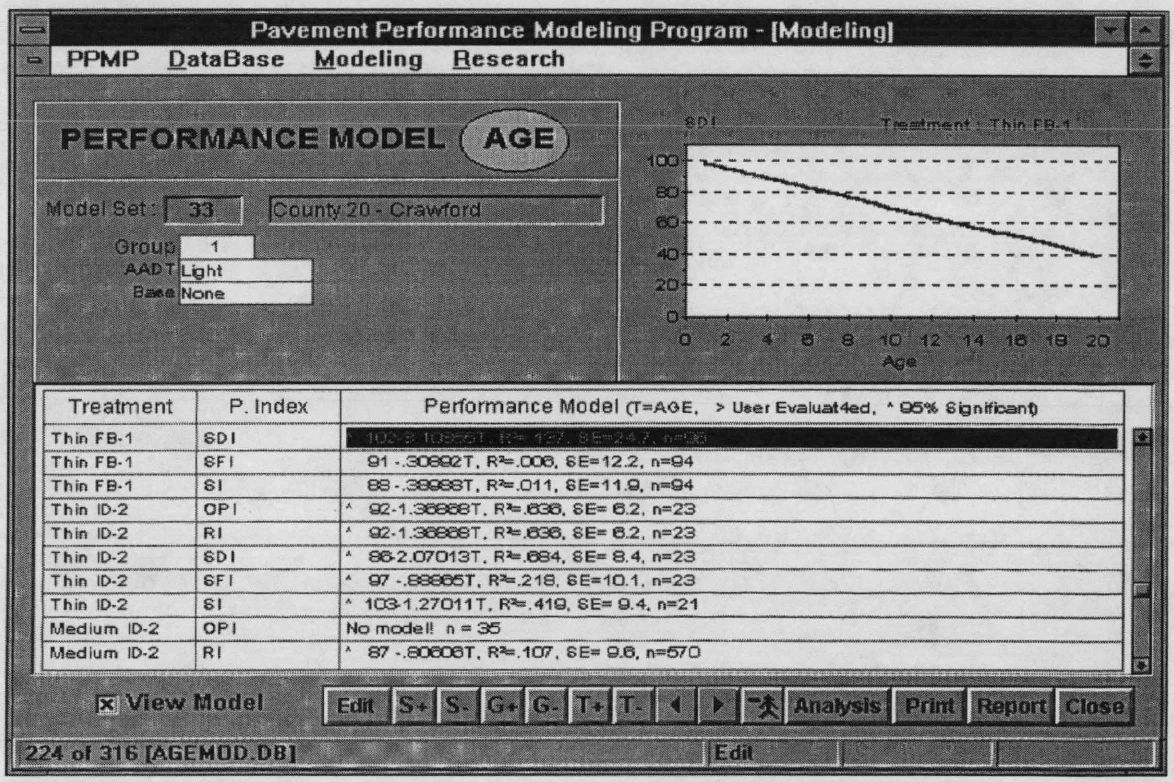

FIGURE 3 Deterministic models screen.

nonlinear function of roughness measurements; OPI (Overall Pavement Index) is a linear function of RI, SDI, SFI and SI (*7*). Figure 2 shows the pick list of the five performance indexes with that selected, thus showing its levels and definition as one of the indexes (SDI):

SDI = 0.1 (Excess Asphalt) + 0.13 (Raveling/Weathering) + 0.2 (Block Cracking) + 0.25 (Trans./Long. Cracking) + 0.05 (Edge Deterioration) + 0.12 (Widening Dropoff) + 0.15 (Rutting).

MODEL DEVELOPMENT

Figure 3 illustrates a portion of the group models developed using the default form of Equation 1. Shown on the upper part of the screen is the group definition for the current active record. The program identifies three types of models: (a) those marked with an asterisk (*) are statistically significant though the R^2 may be small; (b) those marked with nothing are not statistically significant; and (c) those marked "No model" indicate the slope parameter with a positive sign, which is unrealistic and unacceptable.

Figure 4 depicts the screen for comparing all types of models, and a model before and after outliers are removed. The degree (*n*) of polynomial models can be specified by the user. In cases where no model can be obtained, a user-defined model can be easily constructed.

It is important that a priori condition should be met by a performance model. If a performance model cannot meet a priori condition, it should be deleted from the model set. The a priori conditions for a performance model outlined by Lytton (*1*) can be used to evaluate the usability of a model. The most important a priori condition for the models built by this program are (a) The *initial value* of a model should be less than or equal to 100; (b) The *slope* of a model should be negative (for those with positive slopes, "No model" indications are presented as shown in Figure 3); and (c) the *final value* should be greater than or equal to zero.

ANALYSIS OF RESULTS

To test the significance of grouping variables and performance indexes, a set of runs with different combinations of grouping variables were made without removing any data points from the original data set. Significance level was set to $\alpha = 0.05$. Tables 2 and 3 list the grouping variables and the M&R treatments used in the analysis, respectively. For each run, two grouping variables—AADT and pavement type—are included because they are important for pavement performance modeling. Since there is no information about the relationship between pavement structural number and pavement age, the pavement depth of the whole structure (excluding the surface layer that is used as M&R treatment) is used as a grouping variable. It was found that, as a grouping variable, AADT is more sensitive than ESAL; and for all runs with cumulative ESAL as an independent variable, less than 20 percent of the models turned out to be significant.

Because the models of performance indexes against age are much better than those against ESAL, models against age are used for analysis. Table 4 lists the summary of the results of eight runs. In Table 4, Columns 2 through 6 list the grouping variables; Column

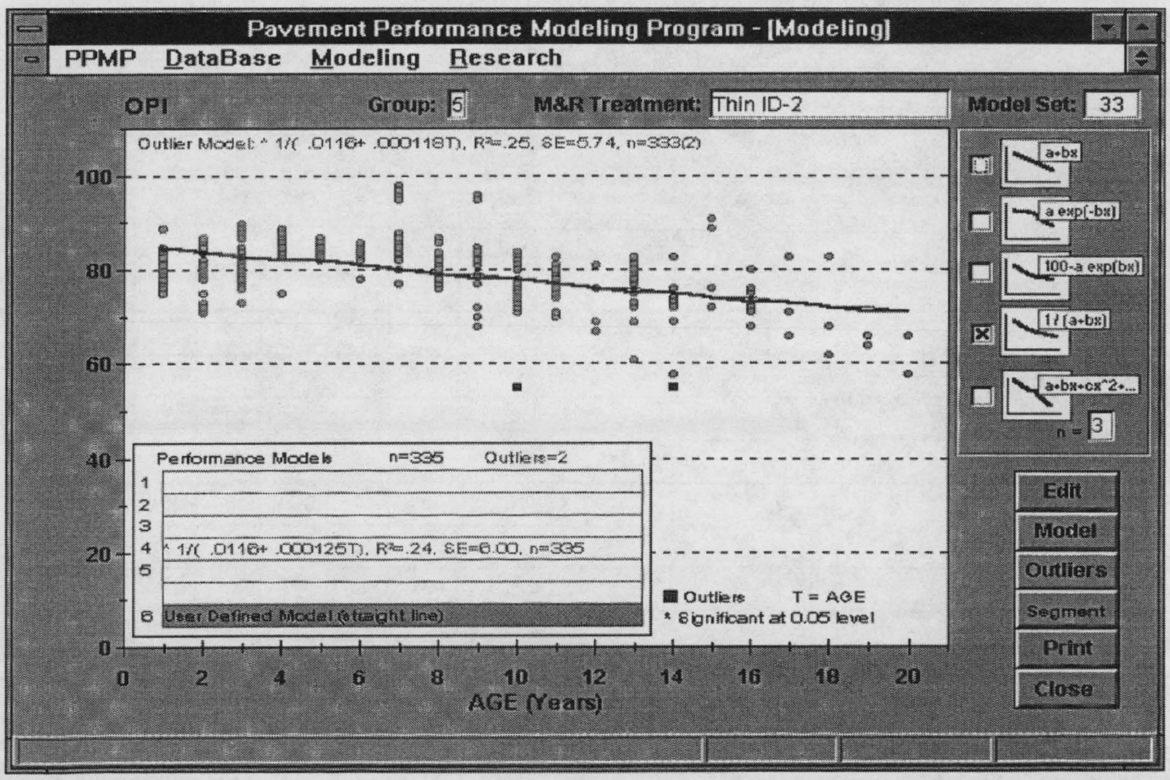

FIGURE 4 Deterministic analysis screen.

TABLE 2 Grouping Variables

Grouping Variables	Levels	Description
AADT	Light Medium Heavy	<1000 per lane 1000 - 4999 per lane ≥ 5000 per lane
Pavement Type	Flexible Rigid None	
Maintenance Level	Level 1 Level 2 Level 3	no maintenance minor maintenance such as patching major maintenance such as surface treatment
Functional Class	Rural Arterial Rural Collector Rural Local Urban Arterial Urban Collector Urban Local Ramp	
Structure Depth	Thin Medium Thick	<30 inches (76 cm) 30 - 49 inches (76 - 127 cm) ≥ 50 inches (127 cm)

TABLE 3 M&R Treatments

M&R Treatments	Levels	Description
ID2, and ID3	Thin Medium Thick	< 2 inches (5 cm) 2 - 5 inches (5 - 13 cm) ≥ 5 inches (13 cm)
FB1	Thin Thick	< 3 inches (7.6 cm) ≥ 3 inches (7.6 cm)
PCC	Thin Thick	< 8 inches (20 cm) ≥ 8 inches (20 cm)
FJ1, FJ4, CRC	One level	

TABLE 4 Number of Significant Models

No (1)	AADT (2)	Pavement Type (3)	Maintenance Level (4)	Functional Class (5)	Structure Depth (6)	No. of Models (7)	OPI (%) (8)	RI (%) (9)	SDI (%) (10)	SFI (%) (11)	SI (%) (12)	Average (%) (13)	Average $R^2 \geq 0.50$ (%) (14)
1	●	●				100	75	75	65	65	55	67	5
2	●	●	●			125	58	42	49	33	37	44	13
3	●	●		●		125	58	58	49	29	31	45	18
4	●	●			●	100	66	66	58	54	50	59	4
5	●	●	●	●		415	40	40	35	18	25	32	27
6	●	●	●		●	215	60	47	51	37	40	47	15
7	●	●		●	●	270	46	52	37	22	28	37	18
8	●	●	●	●	●	450	40	42	35	17	25	32	29

OPI = Overall Pavement Index
RI = Ride Index
SDI = Surface Distress Index
SFI = Surface Friction Index
SI = Strength Index

TABLE 5 Examples 1–4

Examples	Model Type	Original		After Outliers Removed		
		R-Square	SEE	No. Outliers	R-Square	SEE
1.	1) $\alpha+\beta x$	0.33	13.65	0		
Group #1	2) $\alpha e^{-\beta x}$	0.29	13.83	0		
SDI	3) $100-\alpha e^{\beta x}$	0.43	14.12	1	.44	11.64
Thin ID2	4) $1/(\alpha+\beta x)$	0.23	14.27	0		
N = 29	5) $\alpha+\sum \beta_i x^i$ (i=2...n)					
	n=2	0.36	13.62	1	0.48	10.89
	n=3	0.37	13.83	1	0.52	10.72
	n=4	0.38	13.92	**1**	**0.53**	**10.79**
	n=5	0.45	13.45	0		
	n=6	0.47	13.48	0		
2.	1) $\alpha+\beta x$	0.20	6.67	2	0.22	6.50
Group #2	2) $\alpha e^{-\beta x}$	0.21	6.70	2	0.23	6.53
OPI	3) $100-\alpha e^{\beta x}$	0.09	6.86	5	.10	6.46
Thin ID2	4) $1/(\alpha+\beta x)$	0.23	6.76	1	0.24	6.66
N = 471	5) $\alpha+\sum \beta_i x^i$ (i=2...n)					
	n=2	0.25	6.47	5	0.29	6.01
	n=3	0.25	6.47	5	0.29	6.01
	n=4	0.27	6.39	**5**	**0.30**	**5.98**
	n=5	0.28	6.38	5	0.30	5.98
3.	1) $\alpha+\beta x$	0.4	9.84	**4**	**0.62**	**6.19**
Group #3	2) $\alpha e^{-\beta x}$	0.34	9.77	4	0.62	6.21
SFI	3) $100-\alpha e^{\beta x}$	0.38	11.37	4	0.51	6.81
Thin ID2	4) $1/(\alpha+\beta x)$	0.27	9.80	3	0.47	7.24
N = 79	5) $\alpha+\sum \beta_i x^i$ (i=2...n)					
	n=2	0.46	9.40	3	0.54	6.86
	n=3	0.52	8.89	2	0.61	6.31
	n=4	0.52	8.94	3	0.62	6.28
	n=5	0.52	9.00	3	0.62	6.31
4.	1) $\alpha+\beta x$	0.29	6.87	2	0.34	5.83
Group #10	2) $\alpha e^{-\beta x}$	0.27	6.92	2	0.33	5.87
SDI	3) $100-\alpha e^{\beta x}$	0.32	7.06	2	0.32	5.93
Medium ID2	4) $1/(\alpha+\beta x)$	0.25	6.99	2	0.33	5.93
N = 153	5) $\alpha+\sum \beta_i x^i$ (i=2...n)					
	n=2	0.42	6.28	2	0.52	5.00
	n=3	0.44	6.16	**3**	**0.62**	**4.46**
	n=4	0.47	6.06	2	0.52	5.00

7 lists the total models obtained (number of models per index multiplied by the number of indexes); Columns 8 through 12 present the percentage of significant models for each performance index.

As indicated in Table 4, the number of significant models decreases, but the number of models with $R^2 \geq 0.5$ increases with an increase in the number of grouping variables. As far as performance indexes are concerned, OPI and RI are more significant than SDI, SFI, and SI. For groups with three grouping variables, the best combination is AADT, pavement type, and structure depth. For groups with four grouping variables, the best combination is AADT, pavement type, functional class, and structure depth. In general, structure depth is more significant than maintenance level and functional class.

To evaluate the types of models used for pavement performance modeling, M&R treatment ID2—"Thin ID2" and "Medium ID2" (Thick ID2 is unavailable) from the runs of No. 4 and No. 8 defined in Table 4—were selected for detail analysis. Tables 5 and 6 list the results of eight examples (Table 5 from No. 4 runs and Table 6 from No. 8 runs).

Tables 5 and 6 indicate that polynomial models built by the constrained least squares method perform much better than other types of models since any shape of performance curves can be generated using polynomial models. Polynomial models have been used successfully to build pavement performance models (*2,5*). It is obvious that R^2 increases and standard error of estimate (SEE) decreases with the increase of n before outliers are removed. Statistically, the

TABLE 6 Examples 5 and 6

Example	Model Type	Original		After Outliers Removed		
		R-Square	SEE	No. Outliers	R-Square	SEE
5.	1) $\alpha+\beta x$	0.54	7.15	4	0.66	5.03
Group #32	2) $\alpha e^{-\beta x}$	0.50	7.22	4	0.65	5.20
SDI	3) $100-\alpha e^{\beta x}$	0.43	9.02	5	0.41	5.63
Thin ID2	4) $1/(\alpha+\beta x)$	0.45	7.44	4	0.62	5.51
N = 190	5) $\alpha+\sum \beta_i x^i$ (i=2...n)					
	n=3	0.54	7.19	4	0.69	4.81
	n=4	0.59	6.78	4	**0.71**	**4.67**
	n=5	0.59	6.80	4	0.71	4.68
6.	1) $\alpha+\beta x$	0.48	5.75	0		
Group #32	2) $\alpha e^{-\beta x}$	0.51	5.81	0		
OPI	3) $100-\alpha e^{\beta x}$	0.19	6.26	0		
Thin ID2	4) $1/(\alpha+\beta x)$	**0.53**	**5.90**	0		
N = 204	5) $\alpha+\sum \beta_i x^i$ (i=2...n)					
	n=2	0.49	9.40	0		
	n=3	0.49	8.89	0		
	n=4	0.52	8.94	0		
	n=5	0.55	9.00	0		

larger the n, the better the model will be. It is difficult to estimate the best n for all cases. In most cases, reasonable models can be obtained with $n = 4$ after outliers are removed from the analysis, with the exception of Example 4 ($n = 3$). It can also be seen that polynomial models may not be the best models in some cases, as the results indicated in Example 3 and Example 6.

Of all the examples shown in Tables 5 and 6, R^2 increases greatly after outliers are removed from modeling, but care should be taken in removing outliers (*8*).

SUMMARY AND CONCLUSIONS

The Pavement Performance Modeling Program presented in this paper provides a powerful tool for developing pavement performance models for Pennsylvania. The program allows engineers and researchers to develop various performance models based on available data, to evaluate the data and the models, and to select the best model for use in a PMS.

The program is flexible enough to allow the user to define modeling scope, performance indexes, grouping variables, M&R treatments, and maintenance levels. Modeling scope can be a county, a district, a mix of counties and districts, or the whole state. Grouping variables include AADT or ESAL, functional class, pavement type, pavement structure depth, maintenance level, and so forth. The user can define individual performance indexes and comprehensive performance indexes. M&R treatments and maintenance level can be determined by grouping the detailed pavement surface types and the maintenance activities. For deterministic models, five types of models can be built, and outlier analysis can be performed. The program is user friendly with a graphical user interface in which the data and models can be plotted on screen and analyzed one by one.

From the preliminary analysis of the original data and the models developed using the program, it has been found that AADT is more significant than ESAL, and that the performance indexes OPI and RI are more significant than SFI, SDI, and SI. In general, polynomial models perform well in fitting the data, but they are not the best models in some cases.

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