Optimal Design of Maintenance Districts

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Two optimization models are presented and compared for use in partitioning a transportation network into service districts for which snow and ice control routes are subsequently designed. The models are used to redistrict a winter maintenance service area in northern Indiana. The service areas created by these models are shown to be compact with centralized locations of depots/garages, enhancing the efficiency of routes that can be designed to cover these areas.

Maintenance of the intrastate highway system is an enormous undertaking both financially and logistically. Typical activities for which state departments of transportation (DOTs) are responsible include crack sealing and pothole repair, painting and striping, pavement and facilities inspection, weed control and median maintenance, and snow removal and ice control. For many northern U.S. states, winter snow and ice control is the most resource intensive of all network maintenance activities.

Most maintenance activities are characterized by a service being performed according to a set schedule or following established service routes. Service routes are typically designed to cover a partition of the network assigned to a particular depot. One of the goals of DOTs is to provide the necessary service at the lowest cost without compromising quality.

The quality of snow removal routes in Indiana, for example, is evaluated based on the following criteria:

- Frequency of service. Based on the volume of average daily traffic (ADT), roads are categorized into three classes. A required frequency of service is specified for each.
- Quantity of deadhead travel. Travel by service vehicles is classified as either service travel, with the vehicle plowing snow off the road or spreading material, or deadhead travel, with the truck traversing the road segment to begin servicing another road segment. Total deadhead travel should be minimized.
- Class continuity. Each route should be homogeneous in class as far as possible to allow for a clear hierarchy in the importance of a particular route.

In addition to these three criteria, it is desirable that service is cost-effective in that the lowest possible number of vehicles is used.

In general, service routes are designed to best cover a predesignated partition. Partitions are not designed to best support the design of service routes. The premise of this research is that an efficient design of network partitions can greatly enhance the quality of service routes that can be designed. Specifically, the problem being addressed herein may be stated as follows:

> Given a fixed set of P service depots on a transportation network, find the optimal assignment of arcs to those depots as measured by the quality of service routes that can be designed to service all segments of that network.

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The problem of winter snow and ice control in the state of Indiana provides the context for this study. Although the focus for these models is winter snow and ice control, other districting problems, in which service is provided to network arcs, can be accommodated.

TRANSPORTATION NETWORK MAINTENANCE ENGINEERING

Many engineering management systems require decisions concerning the location of facilities and the allocation of workload. Brief summaries of some of the methods applicable to such problems and earlier studies focusing on improving road network maintenance operations, winter maintenance operations in particular, are presented.

Location-Allocation Models

Location-allocation models try to simultaneously (a) select locations for facilities and (b) assign workloads that are either continuously distributed in the area or specified on a network to those facilities so as to optimize some specified measurable criteria. Ghosh and Rushton (*1*) review several early works and present an overview of the methods developed in the last few decades to solve these problems. They discuss exact solution methods and heuristic methods both in continuous space and for networks.

The problem of locating p facilities in p potential sites so as to minimize the average weighted or unweighted distance between the facilities and the clients they serve is called the p-median problem. Many location-allocation problems can be posed as variations of the p-median problem. ReVelle and Swain (*2*) demonstrate a method by which the network p-median problem can be solved to optimality using linear programming. Hillsman (*3*) presents a unified linear model (ULM) for location-allocation analysis based on the structure of the p-median problem.

More recently, Densham and Rushton (*4*) have shown that the processing costs for most heuristics for location-allocation algorithms can be reduced by exploiting the spatial structure inherent in these problems. Rose et al. (*5*) outline a systematic analytic framework for examining decisions concerning the location and size of depots responsible for road maintenance in the state of Victoria, Australia. This system provided a basis for a defensible long-term depot location policy.

Operations Research and Winter Network Maintenance

Savas (*6*), Russell and Sorenson (*7*), and Cifelli et al. (*8*) describe some early attempts made to improve the snow removal operation. Cook and Alprin (*9*) and Tucker and Clohan (*10*) used simulation to predict whether the planned fleet size and deployment of equipment would function adequately under different conditions for urban snow removal settings.

The snow removal problem requires both the development of good routes and the assignment of these routes to depots. In practice, either clustering is done first and routing second, or vice versa. England (*11,12*) describes the methodology that is of the former type, and Reinert et al. (*13*) take the second approach.

Evans and Weant (*14*) describe the use of a computer-based routing system and indicate the advantages in using such a system. A more specialized multi-objective decision support system for computer-aided route design taking into account spatial network data, road classification, and direction restrictions is presented by Wang and Wright (*15*).

SPATIAL OPTIMIZATION APPROACH

The districting problem faced by DOTs can be formulated as a location-allocation problem. One variable is assigned to each decision that needs to be made (assignment of arcs to partitions, vehicles to depots, etc.). The goal of the model is expressed in terms of decision variables and is referred to as the objective function. The limitations within which a solution is sought can also be expressed as mathematical expressions termed as constraints. If a solution satisfying the constraints can be found, a feasible solution is said to exist. If the goal and all the constraint equations can be formulated as linear equations, the model is termed a linear program (LP). Several commercial software applications that can find the optimum solution to such systems rapidly are available.

In some cases, the value of certain decision variables should be integers for reasonable physical interpretation (for example, the number of vehicles). Variables depicting yes/no type decisions can be represented by binary variables that take the value of 0 or 1. Solutions to problems involving integer and binary variables tend to be more computationally intensive. Solutions to such problems are frequently found by a branch and bound enumeration scheme that involves relaxing the integer requirements and solving a series of LPs with upper or lower bounds set on the values of these variables until an integer solution is found. Tighter bounds on the range of values allowable for these integer variables helps reduce the number of enumerations required.

Two optimization models for solving the location-allocation problem as it relates to transportation network maintenance have been developed and are presented in the following sections. They can be used to develop partitions that are compact and have centralized locations of depots. Because solutions can be found within a reasonable time period, this approach can offer valuable support to a decision maker considering trade-offs between alternatives, modeled as problems with different limits on the constraints or as problems with slightly varying sets of constraints.

Discrete Variable Arc Partitioning Model (DVAP)

Network maintenance by the state of Indiana is administered out of service depots distributed throughout the state. The network partitions assigned to these depots are called service units. Four or five units are grouped together as subdistricts for administrative convenience. Four or five subdistricts are similarly grouped together and constitute a district. Each depot is responsible for different maintenance activities associated with the service unit assigned to it in addition to the routing considerations mentioned earlier. Designing each unit to be compact affords both accessibility and flexibility for those operations as well as in real-time snow removal operations. For instance, in the event of a breakdown, trucks servicing adjacent routes may assist by assuming extra loads more easily if the partitions are compact and well-connected.

A major goal in the design of service routes is to minimize the total amount of deadhead travel incurred during service. Except for very small units, which are not economically viable, it is impossible to design a set of routes incurring no deadhead. As the distance between the depot and the roads it services increases, it may become necessary to deadhead over larger distances to be able to complete service to a route within the set target time for that class of road. (Deadhead travel generally allows for higher travel speeds than service travel.) Further, deadhead is essentially a consequence of the location of the depot in relation to the network it services. A central location of the depot with access to many routes (such as a junction) should help reduce deadhead. Even where relocating the existing depots is infeasible, accessibility may be improved by repartitioning the network.

Maximum utilization of available resources is essential to efficient network maintenance. The number of service vehicles required in each unit is a function of the workload associated with that unit. Because fractional truck assignment is not physically possible, the workload assigned to each depot must be adjusted to require as close to an integer number of vehicles as possible. Also, it is reasonable to expect that eliminating wasteful travel in deadhead will help reduce the requirement for trucks. It must be noted that while these models try to minimize the estimated number of routes, the routes themselves are not being designed. It is assumed that routing will be done subsequently within each service unit. There is a relatively small range for the number of trucks required, with a lower limit specified by the kilometers of roadway (considering all lanes) requiring service and the plowing speed, assuming no deadhead. The maximum number of trucks worth considering is in the range of the number currently used. Thus, the knowledge implicit in past designs can be used to limit the branch-and-bound searches in terms of number of trucks.

The number of units that are needed to provide satisfactory service can vary between the number currently used and a lower limit based on (*a*) the number of kilometers requiring service in a region and (*b*) the number of kilometers that can be serviced out of a given depot. Currently all depots are designed such that no more than 13 to 15 routes can be serviced out of each.

Maximizing Compactness of Network Partitions

Requiring the networks to be compact implies that as many kilometers as possible are included within any given area. This in turn results in maximizing the connections within a given area. Let L_{ijp} be the sum of the distances from depot p to the endpoints i and j if the road segment having endpoints i and j is assigned to depot p for service, and 0 otherwise. Minimizing the sum of the shortest distances from depot p to the road segments that are serviced from depot p for all depots becomes a surrogate compactness measure.

$$\text{Minimize} \sum_{p} \sum_{(i,j)} L_{ijp} \tag{1}$$

Model Constraints

Consider a set of potential service depots indexed as p. Define N_p to be the number of trucks required to service partition p. The number of trucks assigned to a depot p must be sufficient to service the total length of road segments assigned to that depot. Because all routes may require service simultaneously, the number of service trucks is also equal to the number of routes. Vehicular resources necessary to service a given route depend on (*a*) the length (workload) of that route and (*b*) the quality of service required for that route.

The workload of a road segment (W_{ij}) is specified in terms of the total kilometers (all directions) of that segment. Consider a road segment having endpoints i and j (endpoints of a road segment might be intersections or vehicle turnaround areas). If X_{ijp} is a binary decision variable that assumes a value of 1 if a road segment having endpoints i and j is assigned to partition p, and 0 otherwise, then $\sum_{(i,j)\ pairs} W_{i,j}X_{ilp}$ is the total workload assigned to depot p.

The frequency of service that must be provided to a given road segment depends on the classification of that segment (based on historical ADT across that segment). The class of a given route is determined by the classification of the highest-classed road segment assigned to that route. Given fixed service and deadhead speeds, the length of a route that a truck services depends on the class of that route. For example, in the state of Indiana, every portion of a route that includes a Class 1 road segment (ADT of 5,000 or greater) must receive service (plowing and/or spreading of abrasives and chemicals) every 2 hr. Assuming a plowing speed of 32.2 km/hr (20 mi/hr), a truck assigned to a Class 1 route can cover 64.4 km (40 lane mi). Trucks assigned to Class 2 and 3 routes, which require service every 3 hr, can cover routes 96.6 km (60 lane mi) long. While all routes may not be designed to be homogeneous in class, calculating the number of trucks based on the kilometers of each class of road, accounting for deadhead by a suitably chosen factor, will provide an estimate of the number of trucks required. The quality of the estimate depends on the validity of the factor selected. Efforts will be made in the future to relate the factor to the location of the unit as well as the workload assigned to the partition associated with it.

Define N_p^1 to be the number of trucks needed for servicing all Class 1 routes serviced out of depot p, and similarly for N_p^2 and N_p^3. N_p is the total number of trucks required in the area. Let CL_p^1, CL_p^2, and CL_p^3 be the kilometers of Class 1, 2, and 3 roads (all lanes), respectively, assigned to partition p and dhf_p be the deadhead factor used for partition p. Based on such homogeneous routes, a lower bound on the number of trucks required for servicing all road segments assigned for service out of depot p is determined by the following model constraints:

$$\sum_{(i,j) \in Class\ 1} W_{ij} \cdot X_{ijp} - CL_p^1 \leq 0 \quad \forall p \tag{2}$$

$$\sum_{(i,j) \in Class\ 2} W_{ij} \cdot X_{ijp} - CL_p^2 \leq 0 \quad \forall p \tag{3}$$

$$\sum_{(i,j) \in Class\ 3} W_{ij} \cdot X_{ijp} - CL_p^3 \leq 0 \quad \forall p \tag{4}$$

$$40N_p^1 - dhf_p \cdot CL_p^1 \geq 0 \quad \forall p \tag{5}$$

$$60N_p^2 - dhf_p \cdot CL_p^2 \geq 0 \quad \forall p \tag{6}$$

$$60N_p^3 - dhf_p \cdot CL_p^3 \geq 0 \qquad \forall p \tag{7}$$

$$N_p - N_p^1 - N_p^2 - N_p^3 = 0 \qquad \forall p \tag{8}$$

Upper and lower bounds on the required number of trucks can be derived as explained earlier. Let *NUMT* be the maximum number of trucks to be used such that

$$\sum_p N_p \leq NUMT \tag{9}$$

Likewise, let *NUMU* be the number of units to be operative at any time:

$$\sum_p U_p = NUMU \tag{10}$$

Both *NUMT* and *NUMU* are selected to be within the ranges discussed earlier.

If CAP_p is the workload capacity of depot p, assumed known for all depots, then a set of model constraints may be included to ensure that service to a partition from depot p can only be provided if that depot is opened ($U_p = 1$), and may not exceed CAP_p:

$$\sum_{(i,j)} W_{ij} \cdot X_{ijp} - CAP_p \cdot U_p \leq 0 \qquad \forall p \tag{11}$$

The model must include a number of additional logical network constraints. First, each road segment identified for winter service must be assigned to exactly one service depot p:

$$\sum_p X_{ijp} = 1 \qquad \forall (i,j) \tag{12}$$

All arcs in a unit must be connected to allow design of routes having continuous stretches for plowing. To model this, imaginary flows through the network from depots to the nodes in the unit they serve are introduced. Figure 1 shows the pattern of imaginary flows through a small network. Only arcs assigned to a partition may be used to carry this flow. Define Y_{ijp} as the flow from i to j in the arc (i,j) in partition p, and define Y_{jip} as the flow from j to i in the same arc. These flows may be in either direction (but not both) and be of any quantity up to some assumed maximum flow (*MF*):

$$Y_{ijp} - MF \cdot X_{ijp} \leq 0 \qquad \forall (i,j), p \tag{13}$$

$$Y_{ijp} - MF \cdot X_{ijp} \leq 0 \qquad \forall (i,j), p \tag{14}$$

In this example, flow originates in a super node (0) and flows into the depots (4, 5, and 15). Flow leaves the system only through non-depot nodes to return to node 0. Flow into the system is represented by Y_{0dp}, where d is any depot node, and flow out of the system is represented by Y_{i0p}, where i is any non-depot node. At each node, the following flow balance constraints must be satisfied:

$$\sum_k Y_{kip} - \sum_j Y_{ijp} - Y_{i0p} = 0 \qquad \forall i \text{ non-depot nodes} \tag{15}$$

$$\sum Y_{kdp} - \sum_j Y_{djp} + Y_{0dp} = 0 \qquad \forall d \text{ depot nodes} \tag{16}$$

Each non-depot node must conduct at least some positive flow out of the system. This ensures that each node is connected to at least one partition. In this formulation, the flows ensure connectiv-

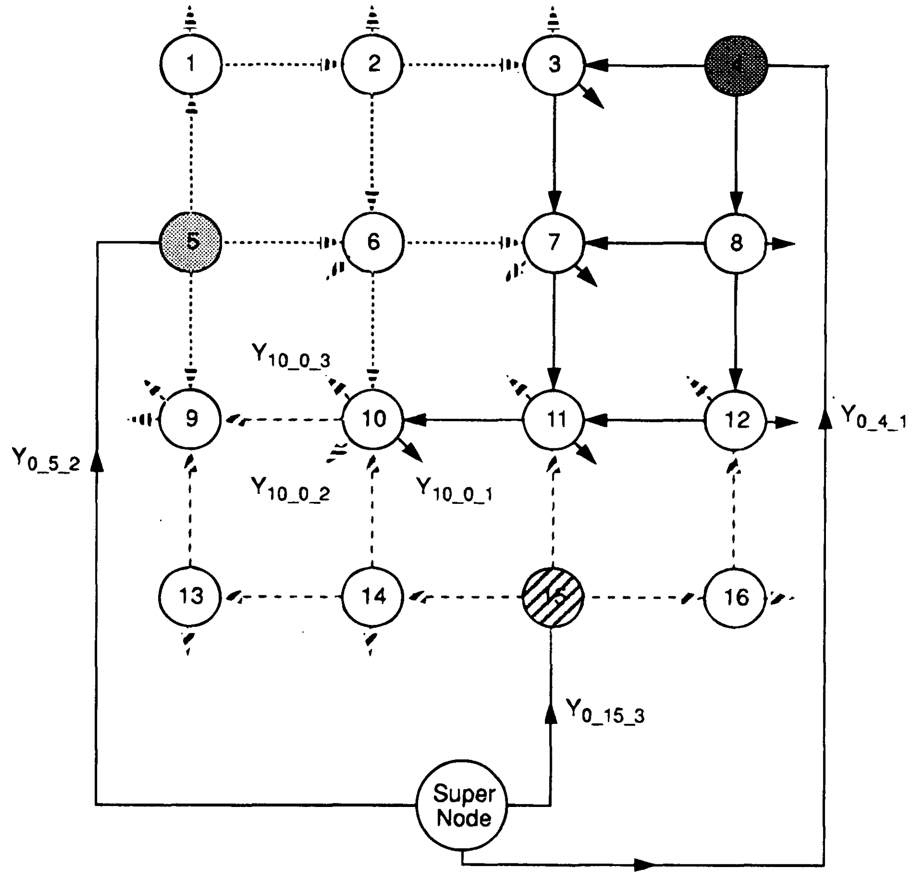

FIGURE 1 Connectivity enforced by imaginary flows in a hypothetical example.

ity, though they have no physical significance. The minimum flow is assumed to be 1:

$$\sum_{p} Y_{i0p} \geq 1 \qquad \forall i \text{ non-depot nodes} \tag{17}$$

The sum of flows into depots must be sufficient to meet the demands at all the nodes. Because each non-depot node has a demand of at least one unit, the total inflow should be greater than or equal to the number of non-depot nodes (*ND*):

$$\sum_{p} \sum_{d} Y_{0dp} \geq ND \qquad \forall d \text{ depot nodes} \tag{18}$$

Each depot is associated with a particular unit. For instance, Node 25 may correspond to a depot that services arcs assigned to Unit 2 (if open). If U_2 is 0 and the depot at Node 25 is not open, no arcs may be assigned to Unit 2. Node 25 cannot be the depot of any partition other than Unit 2. This is ensured by requiring any flow from the Supernode 0 into 25 to be zero for all partitions other than Unit 2. Thus

$$Y_{0dp} = 0 \tag{19}$$

if d is not in partition p.

Any arc can be assigned to a partition only when both ends are connected to that partition.

$$X_{ijp} - Y_{i0p} \leq 0 \qquad \forall (i,j), p \tag{20}$$

$$X_{ijp} - Y_{j0p} \leq 0 \qquad \forall (i,j), p \tag{21}$$

The sum of the shortest distances to both ends of an arc (i,j), L_{ijp} for all arcs (i,j) connected to the depot in partition p is nonzero only when the arc is assigned to that partition. The shortest distances to the ends i and j from the depot in unit p (SP_{ip} and SP_{jp}) are precalculated. L_{ijp} may be calculated as

$$L_{ijp} - SP_{ip} \cdot X_{ijp} - SP_{jp} \cdot X_{ijp} = 0 \qquad \forall (i,j), p \tag{22}$$

LMAX is the maximum of L_{ijp} values over all partitions:

$$LMAX - L_{ijp} \geq 0 \qquad \forall (i,j), p \tag{23}$$

LMAX should be less than some maximum permissible limit (*ML*):

$$LMAX - ML \leq 0 \tag{24}$$

The sum of L_{ijp} in any partition p ($SUML_p$) offers a means of comparing the quality of the compactness of one solution with another and is computed using

$$\sum_{(i,j)} L_{ijp} - SUML_p \leq 0 \qquad \forall p \tag{25}$$

The total cost for the selected number of trucks and units (*COST*) can be estimated as

$$COST - C^T \cdot \sum N_p - C^U \cdot \sum_{p} U_p = 0 \tag{26}$$

where C^T is the cost per truck and C^U is the cost per unit.

The complete model formulation follows.

Minimize $\sum_p \sum_{(ij)} L_{ijp}$

Subject to

$$\sum_p X_{ijp} = 1 \qquad \forall(i,j)$$

$$\sum_{(i,j)} W_{ij} \cdot X_{ijp} - U_p \cdot CAP_p \leq 0 \qquad \forall p$$

$$\sum_{(i,j) \in Class\ 1} W_{ij} \cdot X_{ijp} - CL_p^1 \leq 0 \qquad \forall p$$

$$\sum_{(i,j) \in Class\ 2} W_{ij} \cdot X_{ijp} - CL_p^2 \leq 0 \qquad \forall p$$

$$\sum_{(i,j) \in Class\ 3} W_{ij} \cdot X_{ijp} - CL_p^3 \leq 0 \qquad \forall p$$

$$40N_p^1 - dhf_p \cdot CL_p^1 \geq 0 \qquad \forall p$$

$$60N_p^2 - dhf_p \cdot CL_p^2 \geq 0 \qquad \forall p$$

$$60N_p^3 - dhf_p \cdot CL_p^3 \geq 0 \qquad \forall p$$

$$N_p - N_p^1 - N_p^2 - N_p^3 = 0 \qquad \forall p$$

$$\sum_p N_p \leq NUMT$$

$$\sum U_p = NUMU$$

$$Y_{ijp} - MF \cdot X_{ijp} \leq 0 \qquad \forall(i,j), p$$

$$Y_{ijp} - MF \cdot X_{ijp} \leq 0 \qquad \forall(i,j), p$$

$$\sum_k Y_{kip} - \sum_j Y_{ijp} - Y_{i0p} = 0 \qquad \forall i \text{ non-depot nodes}, p$$

$$\sum_k Y_{kdp} - \sum_j Y_{djp} + Y_{0dp} = 0 \qquad \forall d \text{ depot nodes}, p$$

$$\sum_p Y_{i0p} \geq 1 \qquad \forall i$$

$$\sum_p \sum_d Y_{0dp} \geq ND \qquad \forall d \text{ depot nodes}$$

$$Y_{0dp} = 0 \qquad d \text{ not in partition } p$$

$$X_{ijp} - Y_{i0p} \leq 0 \qquad \forall(i,j), p$$

$$X_{ijp} - Y_{j0p} \leq 0 \qquad \forall(i,j), p$$

$$L_{ijp} - SP_{ip} \cdot X_{ijp} - SP_{jp} \cdot X_{ijp} = 0 \qquad \forall(i,j), p$$

$$LMAX - L_{ijp} \geq 0 \qquad \forall(i,j), p$$

$$LMAX - ML \leq 0$$

$$\sum_{(i,j)} L_{ijp} - SUML_p \leq 0 \qquad \forall p$$

$$COST - C^T \cdot \sum_p N_p - C^U \cdot \sum_p U_p = 0$$

$$U_p, X_{ijp} \in (0,1)$$

$$Y_{ijp}, Y_{i0p}, Y_{0dp}$$

$$N_p, N_p^1, N_p^2, N_p^3 \in \{integers\}$$

where

CAP_p = capacity of partition p,
CL_p^k = number of class k kilometers in partition p,
$COST$ = total cost of alternative,
C^T = cost of a truck,
C^U = cost of unit operations,
dhf_p = deadhead factor for partition p,
L_{ijp} = sum of the shortest distances to the node i and the node j from the depot in the partition p to which the arc (i,j) is assigned,
$LMAX$ = maximum of all L_{ijp},
MF = maximum imaginary flow in any arc,
ML = maximum allowable $LMAX$,
ND = number of non-depot nodes,
N_p = total number of trucks in partition p,
N_p^k = number of trucks for class k routes in partition p,
$NUMT$ = number of trucks chosen to service the area,
$NUMU$ = number of units chosen to be operative,
SP_{ip} = shortest path to i from depot in partition p,
$SUML_p$ = sum of L_{ijp} in partition p,
U_p = 1 if depot p is open and 0 otherwise,
W_{ij} = workload associated with arc (i,j),
X_{ijp} = 1 if arc (i,j) is assigned to depot p and 0 otherwise,
Y_{ijp} = flow in arc (i,j) assigned to partition p,
Y_{0dp} = flow into depot d from Supernode 0, and
Y_{i0p} = flow out of non-depot node i as a result of flow in arcs assigned to unit p.

Continuous Variable Arc Partitioning Model (CVAP)

Realizing that proximity considerations are sufficient in most cases to ensure connectivity within a unit's boundary, the flow constraints (Equations 13 through 21) can be relaxed as well as the requirement that X_{ijp} be a binary variable. In this formulation, variable X_{ijp} is the fraction of arc (i,j) assigned to partition p. Some interpretation as well as adjustment of the values of X_{ijp}'s may be required to understand which portion of the arc to assign to which partition as well as when an arc is assigned to more than two partitions. Using the same symbols (except for X_{ijp}), the complete formulation is included below.

Minimize $\sum_p \sum_{(i,j)} L_{ijp}$

Subject to

$$\sum_p X_{ijp} = 1 \qquad \forall(i,j)$$

$$\sum_{(i,j)} W_{ij} \cdot X_{ijp} - U_p \cdot CAP_p \leq 0 \qquad \forall p$$

$$\sum_{(i,j) \in Class\ 1} W_{ij} \cdot X_{ijp} - CL_p^1 \leq 0 \qquad \forall p$$

$$\sum_{(i,j) \in Class\ 2} W_{ij} \cdot X_{ijp} - CD_p^2 \leq 0 \qquad \forall p$$

$$\sum_{(i,j) \in Class\ 3} W_{ij} \cdot X_{ijp} - CL_p^3 \leq 0 \qquad \forall p$$

$$40N_p^1 - dhf_p \cdot CL_p^1 \geq 0 \qquad \forall p$$

$$60N_p^2 - dhf_p \cdot CL_p^2 \geq 0 \qquad \forall p$$

$$60N_p^3 - dhf_p \cdot CL_p^3 \geq 0 \qquad \forall p$$

$$N_p - N_p^1 - N_p^2 - N_p^3 = 0 \qquad \forall p$$

$$\sum_p N_p \leq NUMT$$

$$\sum_p U_p = NUMU$$

$$L_{ijp} - SP_{ip} \cdot X_{ijp} - SP_{jp} \cdot X_{ijp} = 0 \qquad \forall (i,j), p$$

$$LMAX - L_{ijp} \geq 0 \qquad \forall (i,j), p$$

$$LMAX - ML \leq 0$$

$$\sum_{(i,j)} L_{ijp} - SUML_p \leq 0 \qquad \forall p$$

$$COST - C^T \cdot \sum_p N_p - C^U \cdot \sum_p U_p = 0$$

$$U_p \in (0,1)$$

$$X_{ijp} \geq 0$$

$$N_p, N_p^1, N_p^2, N_p^3 \in \{integers\}$$

RESULTS

Both of the models presented in the previous section can be used to (a) select a prespecified number of depot locations from a set of potential sites (nodes on the network) and assign arcs to these depots or (b) repartition the network among existing depots. Service routes based at each depot are designed through a separate modeling process. The overall goal is to develop network partitions that best support the development of "good" routes. The quality of partitioning is measured in terms of compactness and the size of the fleet required.

A real data set representing an area served by four depots in the La Porte district of Indiana was used (63 nodes, 79 arcs) in testing both models. Mathematical programming formulations were generated using these data and solved using the CPLEX Mixed Integer Optimizer with barrier code, Version 2.1.

Discrete Variable Arc Partitioning Model

In the DVAP model presented above, the parameter *LMAX* was defined as the maximum allowable value for *L* (the sum of the shortest distances to the ends of the arc from the depot to which they are assigned). The selection of an appropriate value is essential in eliminating sites that cannot serve all areas adequately.

Figure 2 shows the existing service territories that are currently being used in an area in La Porte District. The total compactness measure is 2,600 km (1,615 mi), and *LMAX* is 84 km (52.2 mi). Figure 3 shows the service territories as suggested by DVAP for the same region. The total compactness measure is 2,236.9 km (1,389.4 mi) with *LMAX* restricted to being less than or equal to 56 km (35 mi). Through visual inspection of the service areas designed, it can be seen that this model can be quite effective in developing compact partitions.

DVAP uses a deadhead factor to account for deadhead in each partition. The deadhead in any partition would likely depend on the location of the depot, the portion of network assigned to it, and the number and quality of routes designed. A model that predicts the

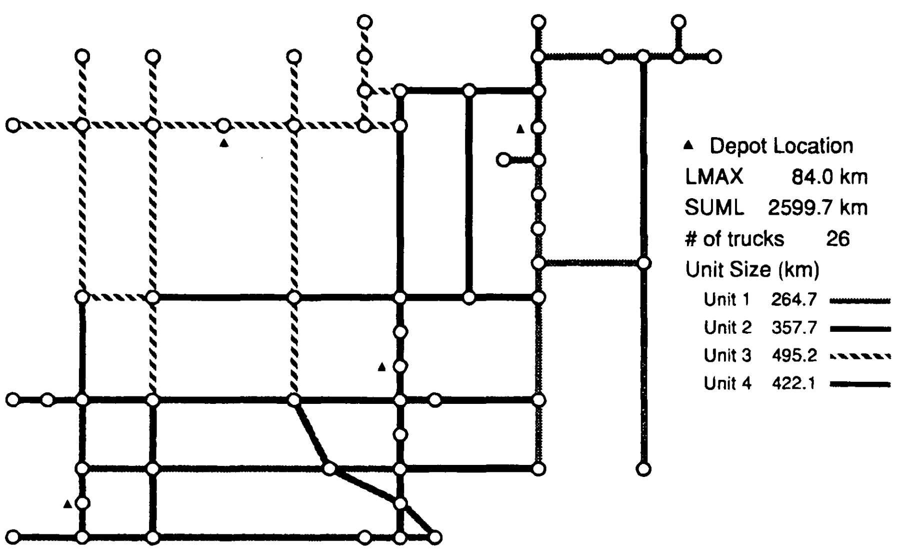

FIGURE 2 Existing service partitions in the test area.

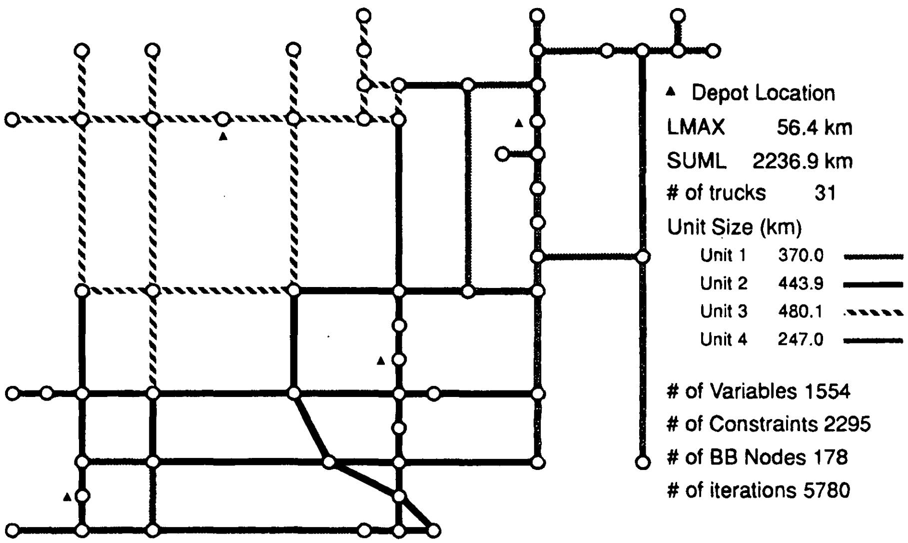

FIGURE 3 Partitions for the test area designed by the DVAP model.

deadhead factor prior to the development of partitions cannot consider the variation of deadhead with the size of the partition. Even if it is assumed that deadhead will not vary much within the range of partition sizes that are to be considered (400 to 565 service km or 250 to 350 service mi), the pattern of connectivity within the partition cannot easily be incorporated into the deadhead factor prediction. The estimate of the number of trucks allows for deadhead. Its accuracy also depends on the accuracy of the deadhead estimate.

An accurate prediction of the number of routes to be used is essential to decision makers because the costs associated with the number of routes are recurring. The minimum number of routes determined to be essential by the model (31) is greater than what is currently being used (26). This is partly because this model assumes that routes are homogeneous in class. In the current version of DVAP, mileage of each road class is used to calculate the number of trucks. Such homogeneous routes are, in general, not practical because homogeneous road segments are not necessarily contiguous and/or the workload for some road classes requires a fractional number of trucks. In practice, routes consisting of arcs of more than one type are designed. In estimating the number of trucks, therefore, it may be more practical to consider nonhomogeneous routes assuming an average route length, which is closer to 2 hr if there is a dominance of Class 1 roads and closer to 3 hrs otherwise.

Another factor that has led to overestimating the number of trucks is that the higher speed available for a truck that is deadheading has not been incorporated. The length of routes considered, 64.4 or 96.6 km (40 lane mi or 60 lane mi) are based on the service speed. These factors will be considered in future studies.

Continuous Variable Arc Partitioning Model

The same data set was used with the continuous variable version of the arc partitioning model, resulting in partitions as shown in Figure 4. Relaxing the integrality restrictions on arc assignment through the use of CVAP model reveals interesting characteristics of the trade-off between the operational practicality of a solution and the computational burden of the model. A comparison of solutions is summarized in Table 1.

The solution using CVAP on the same real data set used previously for DVAP produces the same solution. However, a feasible solution was found using the CVAP model and restricting the number of trucks to 26, the actual number of vehicles currently being used (Figure 4). A visual comparison of the two solutions presented in Figures 3 and 4 suggests that the two are quite close in terms of compactness and central depot locations. But from an operations standpoint, the CVAP solution would seem to be inferior to the DVAP solution for two reasons: (a) several arc assignments are fractional (arcs with two different shadings in Figure 4) and (b) one partition is not contiguous (upper right in Figure 4). Such solutions would require some means of adjustment, manual or otherwise. Some interpretation is necessary to decide which fraction of the arc is assigned to which partition. However, the cost associated with the CVAP solution clearly dominates that of DVAP (the cost of 26 vehicles instead of 31).

As expected, the CVAP solution is cheaper to implement than the DVAP solution from a computational standpoint as well. The continuous model required 670 variables and 751 constraints as compared with the 1,554 variables and 2,295 constraints required for the discrete formulation. CVAP found the solution with 172 branch-and-bound nodes and 484 iterations as compared with 178 branch-and-bound nodes and 5,780 iterations with DVAP. The continuous formulation can provide fairly good solutions more rapidly. The compactness requirement is, to a large extent, sufficient to enforce connectivity.

CONCLUDING REMARKS

Comparing the results obtained using CVAP and DVAP, it appears that the level of discretization of the network requires further consideration. CVAP uses the same set of equations to estimate the number of trucks required and is able to find feasible solutions

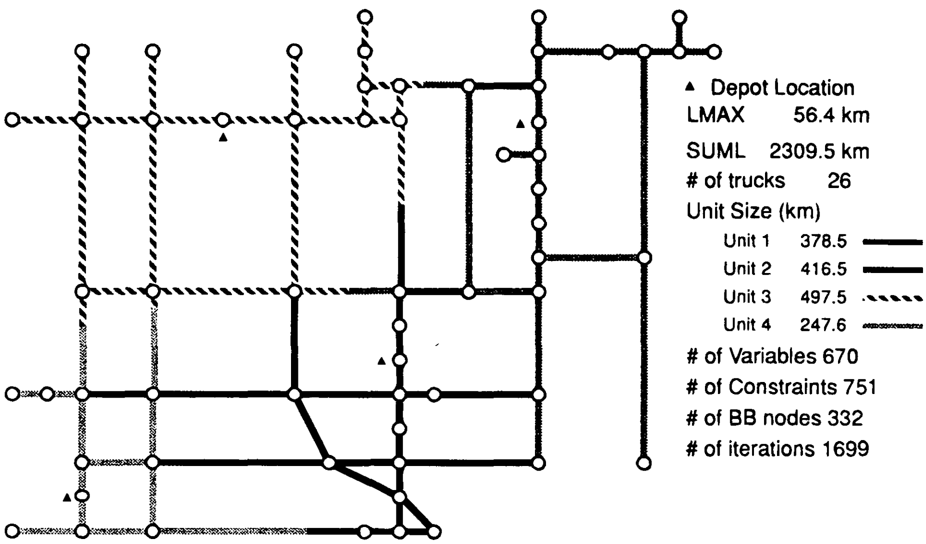

FIGURE 4 Partitions for the test area designed by the CVAP model.

TABLE 1 Comparison of DVAP- and CVAP-Generated Partitions for the Test Case

Model	Number of Variables	Number of Constraints	Number of Branch & Bound Nodes	Number of Iterations	Number of Trucks
DVAP	1554	2295	178	5780	31[a]
CVAP	670	751	172	484	31[b]
CVAP	670	751	332	1699	26[c]

a.Number of trucks necessary for a solution to exist.
b.Feasible solution does exist for a lesser number of trucks (c).
c.Number of trucks restricted to a maximum of 26.

requiring fewer trucks than the DVAP model. However, the solution using fewer trucks (Figure 4) requires fractional assignment of arcs. If a trade-off between the number of trucks and compactness was being studied using DVAP and the current data set, this solution would not have been considered. Yet if the network discretization had been such that those split arcs were in fact considered as two adjacent smaller arcs, DVAP might have been able to find a comparable solution. The selected network representation could influence the quality of solutions found and should therefore be chosen carefully. It is possible that solutions using CVAP could provide information to make such a choice.

Both models show some promising directions for future research and practical applications. While the solutions generated by the DVAP model are integer feasible, the computational effort needed to solve the model is significant. The CVAP model is simple and easy to solve, and may be considered if heuristics are to be used to account for routing considerations and deadhead. Additional studies on test cases may suggest enhancements to the models as well as suitability of these models to find solutions to larger districting problems. Evaluation of the quality of solutions through the routing phase of design is essential.

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