Estimating Intersection Turning Movement Proportions from Less-Than-Complete Sets of Traffic Counts

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Estimated turning movement proportions are used in a number of traffic simulation and traffic control procedures to predict the turning movement flows at intersections. Historically, these proportions have been estimated by manual counting, but the ongoing deployment of real-time adaptive traffic control strategies indicates that the ability to automatically estimate these proportions from traffic detector data is becoming increasingly important. When it is possible to count the vehicles both entering and exiting at each of an intersection's approaches, methods based on ordinary least squares can produce usable estimates of the turning movement proportions, but when the number or placement of the detectors does not support complete counting, these methods fail. The feasibility of estimating turning movement proportions from less-than-complete sets of traffic counts is assessed, and the statistical properties of less-than-complete count estimates are compared with estimates generated from complete counts. It turns out that estimation from less-than-complete counts can be done as long as the detector configuration satisfies an identifiability condition. A numerical test is presented to assess whether or not this condition is satisfied, along with some simple rules for designing detector configurations that are likely to satisfy this condition. A Monte Carlo experiment suggests that estimates generated from less-than-complete counts can be more variable than those generated from complete counts.

A commonly used representation of the demand for travel on a bounded network of urban streets requires specifying (*a*) the arrival flows at each input point on the boundary of the network, and (*b*) the turning movement proportions at each of the network's intersections. Both arrival flows and turning movement proportions may vary in time. When coupled with a method for estimating the travel times on street segments, knowledge of the arrival flows and turning movement proportions allows a traffic engineer to predict the turning movement flows at each intersection in the network, and these in turn are needed to evaluate the effectiveness of all but the most simple intersection signal control plans. Not surprisingly, this representation of demand has a long history of practical application, including use by classical methods for computing pretimed controls for isolated intersections (e.g., Webster's method), the *Highway Capacity Manual*'s method for evaluating level of service at intersections and along arterials (*1*), and computer models used for off-line optimization and evaluation of timing plans for networks of intersections (e.g., TRANSYT, NETSIM). More recently, on-line adaptive control schemes (e.g., SCAT, CARS) have also used this representation.

In the past, a major limitation on the timely updating of signal control plans was that the only reliable method for estimating the turning movement flows was time-consuming and costly manual counting. This limitation became even more burdensome when one wished to adapt a control plan in real time, and often it led to reliance on a stored library of "typical" turning movement patterns, which were determined by off-line counting. It is no surprise, then, that over the past 15 years, a number of researchers have investigated methods for estimating turning movement proportions automatically from the traffic count data collected by real-time traffic control systems, which typically are gathered using detectors embedded in the pavement. Almost without exception, however, this work has assumed that it is possible to count the total number of vehicles entering the intersection from each of its approaches as well as the total number of vehicles exiting from each exit leg. For example, the intersection of two two-lane, two-way streets would require a minimum of eight detectors. It is now well-established that when time series of an intersection's input and output counts are available, estimation methods based on ordinary least squares will produce usable estimates of the turning movement proportions, both off-line and in real time (*2–6*). However, such a rich density of detectors tends to be the exception rather than the norm, at least in the United States, and the slow application of automatic turning movement estimation in the United States can in part be blamed on the added expense imposed by the additional detectors. The functional specifications for real-time traffic adaptive control systems (RT-TRACS), recently prepared for FHWA, explicitly recognizes this limitation by calling for a maximum of 20,000 detectors for a total of 5,000 intersections.

Before proceeding, it is useful to specify more completely the relation between this paper and past work. For the case in which counters are placed at each entry and exit point of an intersection, it has been recognized that the problem of estimating turning movement flows or turning movement proportions from the counts is a special case of the more general problem of estimating an origin-destination (OD) matrix from traffic counts, and reviews of this problem can be found elsewhere (*7–9*). As noted by Davis (*10*), OD estimation methods can be classified as either over- or underdetermined, depending on whether the traffic count data at hand are sufficient to produce a unique estimate of the OD elements. For underdetermined approaches, an infinite number of OD estimates consistent with the count data will exist, and one of these is selected by specifying a prior estimate of the OD matrix and then selecting as the new estimate the OD matrix that is consistent with the count data and "closest" to the prior estimate (*7*).

Three general approaches to underdetermined OD estimation have appeared to date, defined primarily by how they define "closeness" to the prior estimate: the information minimizing (IM)

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approach developed by Van Zuylen and Willumsen (*11*) and Bell (*12*), the weighted least squares (WLS) approach initiated by Maher (*13*) and Cascetta (*14*), and a maximum likelihood (ML) approach described by Speiss (*15*). Speiss also assumes that the prior estimate comes from a survey with known sampling properties. All of these approaches are subject to the criticism that despite more than 15 years of research, none has been shown to yield estimators that are consistent, in the statistical sense of becoming increasingly accurate as the amount of traffic count data become arbitrarily large. In fact, Davis and Nihan (*16*) have shown that an underdetermined least squares OD estimator remains underdetermined, and hence not consistent, even with an infinite time series of traffic count data. The IM, WLS, and ML approaches all have specializations to the problem of estimating an intersection's turning movement flows from traffic counts, and Maher (*17*) has provided a concise summary of these methods, where he found that for a particular computational example, these three approaches tended to produce similar estimates. As with general OD estimators, the underdetermined methods for estimating intersection turning flows will fail if a good prior estimate is not available, so they are unable to "bootstrap" good estimates from traffic count data alone. This dependence on prior information makes them particularly ill-suited for real-time implementation.

The limitations of underdetermined approaches were described by Cremer and Keller (*2*), who also described the first overdetermined method for estimating intersection turning movement proportions. Here it was assumed that time-series data of the intersection's entering and exiting counts were available, and an estimate of the turning movement proportions was coupled with the entering counts to produce predictions of the exiting counts. Those values of the turning movement proportions that minimized a measure of error between the predicted and observed exit counts were then selected as the best estimates. Subsequent papers (*3–5*) located this work within the framework of the systems identification paradigm (*18–20*), and general results on systems identification have been used to show not only that ordinary least-squares estimates of turning movement proportions are consistent (*5*), but also that consistent estimates of more general OD matrices can be computed from time series of traffic counts (*10*). A particular advantage of the systems identification approach is that real-time implementation of the estimation algorithms is often straightforward.

When considering the problem of estimating turning movement proportions for a network of intersections and complete entry and exit counts are not available, the estimation problem is no longer a special case of OD estimation, and to date no underdetermined methods have been proposed for this problem. When time series of traffic counts are available, however, the overdetermined estimation problem again falls within the systems identification paradigm, for which a reasonably general statistical theory (*20*) and real-time implementations (*18*) have been described. This paper considers the problem of estimating intersection turning movement proportions in networks where time series of traffic counts are available from automatic traffic detectors but the number or placement of the detectors may not be sufficient for the standard least-squares estimation methods. Although it is recognized that method of moments, least squares, and ML approaches are applicable to this problem, the focus will be on a nonlinear least squares (NLS) approach because (*a*) it leads to a straightforward generalization of the methods that use complete sets of counts, and (*b*) the basic ideas behind this approach can be developed with the least amount of statistical jargon. Thus the authors believe that the NLS approach is more likely to be accessible to interested practitioners. The primary focus in this paper is on determining feasibility, so the authors concentrate on off-line computation of the turning proportion estimates and simply note that on-line versions of NLS estimation, using state-space models, have been described in the literature (*18,21*). This restriction to off-line methods is justified by the fact that an approach that performs poorly off-line will also perform poorly on-line, and the pathologies of an approach are usually easier to diagnose off-line.

TRAFFIC FLOW MODEL

To date, all methods for automatic estimation of turning movement proportions have used prediction error minimization methods, in which one first specifies a model for predicting the intersection's exit counts using the intersection's input counts and a trial set of turning movement proportions. One then selects as the estimated proportions those values that minimize some measure of the difference between the predicted and the actual exit counts. The prediction model thus is essential for estimating, or identifying the turning proportions. The first requirement then is a prediction model that is capable of handling several intersections simultaneously and that allows for a variety of detector configurations.

Consider a set of street intersections surrounded by a cordon boundary. Traffic counters are located at each point where traffic can enter the cordon area; they count the number of vehicles crossing into the cordon area at that point. Suppose there are m of these input counters, and let $q_i(t)$ equal the traffic count at input counter i during time interval t, $i = 1, \ldots, m$.

Next, suppose the streets within the cordon have been divided into s sections, or compartments, according to the following rules:

1. Traffic flow within a compartment is unidirectional,
2. The stop lines at intersections always mark the downstream boundaries of a compartment, and
3. The exit line on an intersection leg always marks the upstream boundary of a compartment.

A segment of a two-way street connecting two intersections must be divided into at least two compartments, one for each direction, with the compartment boundaries being the intersection stop and exit lines. These two compartments may be divided further. At a total of n compartment boundary points are placed additional detectors that count the number of vehicles crossing that boundary point. Call these the output detectors, and let $y_j(t)$ equal the number of vehicles crossing output detector j during time interval t, $j = 1, \ldots, n$.

Next, let

$x_k(t)$ = number of vehicles in compartment k at beginning of time interval t;

$\mathbf{q}(t), \boldsymbol{x}(t), \mathbf{y}(t)$ = m-, s-, and n-dimensional vectors, respectively, containing individual elements $q_i(t), x_k(t)$, and $y_j(t)$;

b_{lk} = proportion of vehicles currently in compartment l that desire entry into compartment k, if compartment l is adjacent to compartment k, or 0, if compartment k is not adjacent to compartment l;

$\boldsymbol{b}$ = d-dimensional vector containing turning movement proportions;

$p_k(\mathbf{x})$ = proportion of vehicles that can physically exit compartment k during time interval t, as a function of the current distribution of vehicles in the system;

g_{ki} = 1, if input counter i is at the upstream boundary of compartment k, and 0, otherwise:

The distribution of vehicles over the compartments then evolves in time according to the mass balance equations

$$x_k(t+1) = \{1 - p_k[\mathbf{x}(t)]\}x_k(t) + \sum_l x_l(t)p_l[\mathbf{x}(t)]b_{lk} + \sum_i g_{ki}q_i(t) \quad k = 1,\ldots,s \tag{1}$$

Thus the quantity $x_l(t)\,p_l\,[\mathbf{x}(t)]$ gives the number of vehicles actually exiting compartment l during time interval t, and these are then distributed to the neighboring compartments in proportion to the b_{lk}, with $\Sigma_k b_{lk} = 1.0$. At this point no assumptions are made concerning specific functional forms for the exit probabilities $p_k(\mathbf{x})$, but note that plausible forms can be derived from traffic flow models, so that the quantity $x_k p_k$ behaves like a traffic flow—that is, as the product of space-mean speed and traffic density *(22–24)*. Additional generality can be achieved by letting these exit functions depend explicitly on time or on the destination compartment as well as the origin compartment, or on other dynamic variables, such as compartment mean speeds, making this class of models roughly coextensive with macroscopic traffic models based on continuum theory. Such enhancements do not affect the main conclusions of this paper, but they tend to obscure the drift of the argument with notational complexities and so will not be dealt with here. It is noted, though, that actual application requires specification of the exit functions.

Finally, for a given sequence of input counts $\mathbf{q}(1), \mathbf{q}(2), \ldots, \mathbf{q}(N)$ and a given vector of turning movement proportions $\mathbf{b}$, predicted output counts can be generated by solving the mass balance equations recursively while computing the predicted output counts via

$$\hat{y}_j(t,\mathbf{b}) = \begin{cases} x_k(t)p_k[\mathbf{x}(t)] & \text{if detector } j \text{ counts exits from compartment } k \\ \sum_l x_l(t)p_l[\mathbf{x}(t)]b_{lk} & \text{if detector } j \text{ counts entries into compartment } k \end{cases} \tag{2}$$

Equations 1 and 2 define a nonlinear state-space model: the first describes the state dynamics and the second gives predictions of the observations.

The simplest example of such a model would be a network consisting of a single intersection and its adjacent compartments, with the input counters located at the upstream boundaries of the intersection's approaches, the output counters located at the intersection's exit points, and $p_k(\mathbf{x}) = 1.0$ for all k and $\mathbf{x}$. Since each proportion b_{kl} corresponds to exactly one input/output pair, these can be reindexed as b_{ij}, and they give the intersection's turning movement proportions as defined elsewhere *(2–6)*. In this case, given the input counts, the prediction of an output count is given by the simple linear relationship

$$\hat{y}_j(t,\mathbf{b}) = \sum_i b_{ij}q_i(t-1) \tag{3}$$

and constrained ordinary least squares (CLS) estimates of the turning movement proportions can be computed by minimizing the sum of squares function

$$S_1(\mathbf{b}) = \sum_t \sum_j [y_j(t) - \sum_i b_{ij}q_i(t-1)]^2 \tag{4}$$

subject to the constraints

$$0 \le b_{ij} \le 1.0 \tag{5a}$$

$$\sum_j b_{ij} = 1.0 \qquad i = 1,\ldots,m \tag{5b}$$

This problem is well-defined as long as the matrix

$$\mathbf{Q} = \sum_t \mathbf{q}(t)\mathbf{q}(t)^T \tag{6}$$

is nonsingular. This is the basic model used by Cremer and Keller *(2,3)* and Nihan and Davis *(4,5)* in developing their numerous variants of least-squares estimators of turning movement proportions, whereas letting $p_k \le 1$ produces the platoon dispersion model proposed by Bell *(6)* to account for travel time lags between the input and output counters.

IDENTIFIABILITY OF TURNING MOVEMENT PROPORTIONS

Returning now to the nonlinear prediction model defined in Equations 1 and 2, for a given sequence of input counts and an estimate of the turning movement proportions $\mathbf{b}$, this model can be used to generate a sequence of predicted output counts, which in turn can be used to compute the sum-of-squares function

$$S_2(\mathbf{b}) = \sum_t [\mathbf{y}(t) - \hat{y}(t,\mathbf{b})]^T[\mathbf{y}(t) - \hat{y}(t,\mathbf{b})] \tag{7}$$

where $\hat{y}(t, \mathbf{b})$ denotes the vector of predicted outputs produced by Equation 2. The dependence of the predicted outputs on the unobserved state vector $\mathbf{x}(t)$ makes $\hat{y}(t, \mathbf{b})$ a nonlinear function of the turning movement proportions, so that attempting to minimize S_2 with respect to $\mathbf{b}$ leads to an NLS problem. This can be solved using any of a number of standard routines as long as the problem is well-defined, in the sense that at least a locally unique minimizing value of $\mathbf{b}$ exists. It may be, though, that the number or placement of the output detectors is not sufficient to produce a well-defined problem, leading to a situation analogous to the underdetermined OD estimation problem.

The problem of determining in advance whether a data collection experiment will support estimation of a model's parameters is an example of the system identifiability problem, to which a substantial research effort has been devoted *(24,25)*. It is straightforward to verify that when the output count predictions are differentiable functions of the turning movement proportions (which is true for prediction model used here), and when there exists a vector $\mathbf{b}_0$ that produces "good" predictions (in the sense that the prediction errors are uncorrelated with the input counts), then the problem will be well-defined as long as the matrix $\mathbf{J}(\mathbf{b})^T\mathbf{J}(\mathbf{b})$ is nonsingular, where

$$\mathbf{J}(\mathbf{b}) = \begin{bmatrix} \frac{\partial \hat{y}_1(1,\mathbf{b})}{\partial b_1} & \cdots & \frac{\partial \hat{y}_1(1,\mathbf{b})}{\partial b_d} \\ \cdots & \cdots & \cdots \\ \frac{\partial \hat{y}_n(N,\mathbf{b})}{\partial b_1} & \cdots & \frac{\partial \hat{y}_n(N,\mathbf{b})}{\partial b_d} \end{bmatrix} \tag{8}$$

is the Jacobian matrix giving the derivatives of the predicted output counts with respect to the turning movement proportions. In practice, one would test whether or not a particular configuration of output counters will support identification of the turning movement proportions by computing the determinant of $\mathbf{J}(\mathbf{b})^T\mathbf{J}(\mathbf{b})$ at a sample of values for **b**, using a typical sequence of input counts. Analytic expression for the partial derivatives appearing in $\mathbf{J}(\mathbf{b})$ is not needed, as these can be evaluated numerically; as long as one can generate an *a priori* reasonable set of input counts, no actual data are needed to perform these tests. This makes this test suitable for use in designing detector configurations. A justification for testing only a few sample values for **b** is given by a result attributable to Eisenfeld (*26*): suppose $\mathbf{J}(\mathbf{b})$ is a polynomial function of **b** (as is the case for the prediction model described by Equations 1 and 2). Then if there exists one value **b** such that the determinant of $\mathbf{J}(\mathbf{b})^T J(\mathbf{b})$ does not equal 0, the determinant of $\mathbf{J}(\mathbf{b})^T\mathbf{J}(\mathbf{b})$ does not equal 0 for almost all values of **b.**

Experience from the identification of compartment models in biology and medicine indicates that this property, known as local identifiability, is useful for determining which data collection configurations can support parameter estimation (*24,25*).

DESIGN OF IDENTIFIABLE DETECTOR CONFIGURATIONS

The Jacobian test provides a method for assessing the ability of a given detector configuration to provide enough information for estimating turning movement proportions, but it provides no guidance as to how one might arrive at plausible configurations in the first place, nor does the test indicate how to correct an unidentifiable configuration. Ideally one would like to have identifiability conditions that are both necessary and sufficient, where the necessary conditions give guidance on how to design the detector configuration while the sufficient conditions verify that the design is in fact adequate. In the current state of the art, useful necessary and sufficient conditions have yet to be found, even for linear, time-invariant models. For linear models, however, there do exist necessary conditions that indicate how to avoid certain common reasons for nonidentifiability, and although the traffic model described previously is nonlinear, because of the dependence of the exit flows on the current traffic distribution $\mathbf{x}(t)$, it shares many of the structural features of linear models, becoming a time-invariant linear model when the exit probabilities are constant. Thus it can be recommended that following the conditions for linear systems should provide good starting points for designing identifiable detector configurations for the nonlinear model.

- A configuration of detector placements will be said to produce an input-reachable model if there is a route to each compartment from at least one input detector. Similarly, the configuration is output-reachable if there exists a route from each compartment to at least one output detector.
- A pair of turning movement proportions will be called inseparable if every route connecting an input detector to an output detector that involves one of these turning movements also involves the other.

For linear models, it has been established that models that are not input- and output-reachable are unidentifiable whereas two inseparable parameter values will be identifiable only in special cases (*25*). Thus input and output reachability and separability can be regarded as highly desirable properties for a detector configuration, and for very simple networks it is usually possible to verify input and output reachability and separability by inspecting a graphical representation of the network (*25*). For larger networks, input and output reachability can be verified by computing reachability matrices for the network (*27*), but separability is more difficult to check. The task becomes much simpler if the network shows the graph theoretic properties of strong connectedness and degree-2 vulnerability. [By strongly connected, the authors mean that it is possible to travel from any internal compartment to any other internal compartment; by degree-2 vulnerability, they mean that the network remains strongly connected even if any one of its turning movements is forbidden. Roberts (*27*) gives a more detailed discussion of these properties.]

- *Proposition.* Suppose a network of intersections is bounded by a cordon line, with no internal origins or destinations. Suppose the network is strongly connected and degree-2-vulnerable and that detectors are placed so that a complete cordon count of both entering and exiting vehicles is achieved. Then this detector placement is both input- and output-reachable and separable.
- *Proof.* Since the vehicles entering from the cordon line must enter an internal compartment, and since the vehicles exiting at the cordon line must exit from an internal compartment, strong connectivity implies input and output reachability. Now let $(k, l_1, l_2 \ldots, l_r l)$ denote a sequence of compartments that when traversed, form a route from input point k to output point l. Let $[(k, l_1)(l_1, l_2), \ldots, (l_r, l)]$ denote the sequence of turning movements used in traversing this route, and select any two turning movements from this sequence, denoting them by (l_a, l_b) and (l_α, l_β). Since the network is degree-2-vulnerable, it is possible to forbid movement $(l_\alpha l_\beta)$ and still construct one route from an input from an input point to compartment l_a and another route from compartment l_b to an output point. Joining these routes with the movement (l_a, l_b) creates a route from an input to an output that uses (l_a, l_b) but not (l_α, l_β), so the configuration is separable.

Although it is easy to construct networks that are not strongly connected (the network shown in Figure 1 is an example), the authors believe that most well-designed street systems should have this property. For if a network is not strongly connected, it will be possible to divide it into two or more components, some of which are inaccessible from others (*27*). That is, a vehicle that is one part of the network can find it impossible to travel to other parts. Degree-2 vulnerability also appears plausible but less general, so that some networks will have this property and some will not. One exception would arise from a T-intersection formed by two one-way streets, where, for instance, vehicles turn left from the cross of the T into the stem of the T. Forbidding this left turn would make it impossible to enter the stem of the T (and hence destroy the network's strong connectivity), and this also makes it impossible to construct a route using a movement exiting the stem of the T without using this left turn. The solution for this problem would be to place an additional output detector to count vehicles entering the stem of the T, so that routes terminating at this detector would separate the left turn into the stem of the T from the movements exiting the stem. Finally, for networks with internal origins or destinations, placing detectors to count the vehicles exiting or entering these points will convert them to "internal" cordon points, and the preceding results will still hold.

To summarize, a detector configuration that is input- and output-reachable and separable is not guaranteed to be identifiable, but it

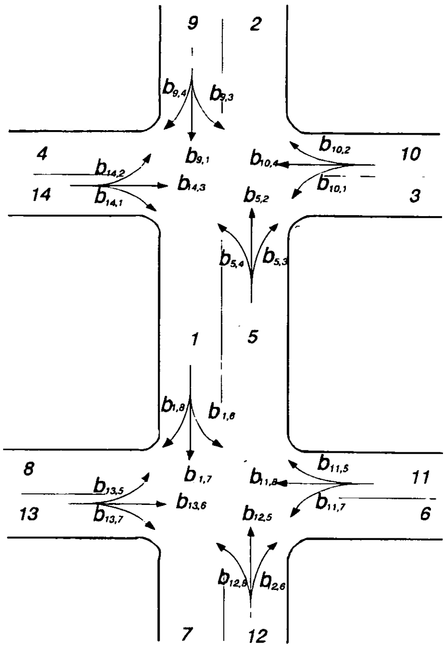

FIGURE 1 Simple signalized network.

will avoid two common causes of nonidentifiability. If a network is strongly connected and this connectivity is relatively invulnerable to disruption, then a complete cordon count will give an input- and output-reachable and separable configuration. Finally, internal compartments that can be entered from only one other internal compartment are likely to cause separability problems unless additional detectors are used.

MONTE CARLO EXPERIMENT

A system that is identifiable in the preceding sense is one for which the data collection configuration will not, by itself, prevent estimation of the turning movement parameters. However, the quality of the resulting estimates will depend at least in part on factors such as quality and quantity of the available data, the algorithm used to solve the NLS problem, and the choice of NLS as opposed to some other estimation approach, such as method of moments or ML. A comprehensive answer to the questions raised here is not available, but to illustrate these issues, consider the simple network depicted in Figure 1, showing two intersections of two-way streets. The various compartments are numbered from 1 to 14, and the figure also shows the 24 separate turning movement proportions, indexed according to their exit and entry compartments. Since for any given approach the proportions for left turns, right turns, and through movements must add up to 1.0, there are in fact only 16 linearly independent turning movement parameters in this network, and the vector **b** containing these independent parameters will have dimension $d = 16$. Figure 2 shows two different configurations of detector placements for this network. Placement Scenario 1 corresponds to the complete detectorization assumed by the linear model for estimating turning movement proportions, and Scenario 2 corresponds to a cordon count placement. It is straightforward to verify that under Scenario 2, the detector configuration is both input- and output-reachable and separable.

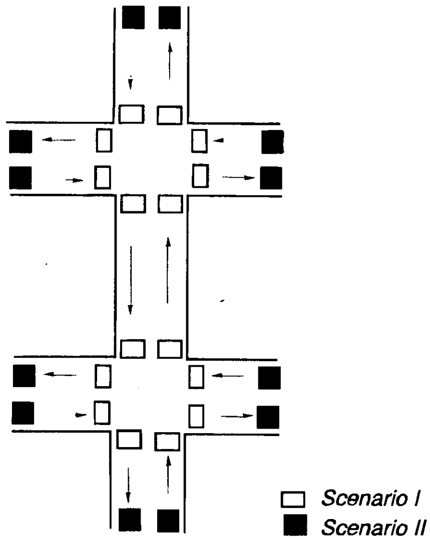

FIGURE 2 Configurations of detector placements.

The primary objective of this paper was to generate a sample of turning proportion estimates computed by minimizing the nonlinear sum of squares function S_2 and then to compare it with a sample of estimates generated by minimizing the linear least-squares function S_1. To this end, simulated traffic counts for both the Scenario 1 and the Scenario 2 detectors were generated using a stochastic version of the prediction model described by Equations 1 and 2. Simulated input counts at each of the six input points for time interval t were generated as Poisson outcomes with time-varying means $\bar{q}_i(t)$, and the number of vehicles exiting compartment k during interval t was generated as binomial random variable with parameters $x_k(t)$, $p_k[\mathbf{x}(t)]$. The exiting vehicles were then allocated to adjacent compartments as multinomial random outcomes with classification probabilities b_{kl}. The exit probability functions were of the same form as those presented and tested elsewhere (*23,24*) to describe freeway traffic flow, but with free-flow speeds, capacities, and jam densities selected to make them more representative of arterial travel. The traffic signal at each intersection was given a standard two-phase timing plan, with a 60-sec cycle length and 30 sec of green allocated to each phase (i.e., no yellow intervals were used). The effect of red time on a movement was simulated by setting the exit probability to 0.0 during the red interval. Fifty simulated data sets were generated, each consisting of 180 1-min traffic counts for each of the detectors depicted in Figure 2. Under Scenario 1, it was assumed that data from the white detectors were available, and estimates of the turning movement proportions were computed using the equality-constrained least-squares algorithm (*28*). Under Scenario 2, it was assumed that data from the black detectors were available, and predicted values for the cordon output detectors were computed recursively using the prediction model described in Equations 1 and 2, with the 1-min input detector counts as inputs. This recursion was implemented as a subroutine called by the NAG optimization routine E04JBF (*29*), which computed those estimates of the turning movement proportions that minimized the nonlinear sum-of-squares function S_2. For the nonlinear estimation, only the left and right turning proportions at each approach were treated as independent parameters, with the through proportion then being computed as $b_{\text{through}} = 1 - b_{\text{left}} - b_{\text{right}}$.

As noted earlier, even when a detector configuration supports identification of the turning movement proportions, the statistical properties of these estimates remain to be assessed. The least-squares estimates generated by an identifiable configuration may still show enough bias or variability to limit their practical usefulness. Estimated turning proportions were computed for each of the 50 simulated data sets, giving a pseudorandom sample of the estimates under each scenario. Table 1 presents the results of this experiment.

The mean columns in Table 1 give the average, across the 50 data sets, of the estimates for that parameter, whereas the "std" column gives the standard deviation of the estimates. The "t" columns give the t-statistic testing the hypothesis that the sample average for that parameter is equal to its true value (*i.e.*, a test for whether that estimate is biased). For each approach, the "true" parameter values used in generating the simulated data were $b_{\text{through}} = 0.6$, $b_{\text{left turn}} = 0.3$, and $b_{\text{right turn}} = 0.1$. For the NLS estimates, the t-statistics for the through movements are omitted since they are actually deterministic functions of the estimates for the right and left turn proportions. The results for the CLS estimates are consistent with those reported by Nihan and Davis (*5*), being unbiased with moderately low standard deviations. As would be expected, the NLS estimates show an increase in variability, because the NLS estimator is working with less information than the CLS estimator. The first set of NLS estimates also shows a substantial number of instances of bias, but this appears to be due in large part to numerical difficulties experience by E04JBF. In 21 of 50 instances, E04JBF terminated with a message indicating that it was unable to satisfy all convergence criteria; in the remaining 29 cases, satisfactory convergence was achieved. Computed means, standard deviations, and t-statistics for only those cases showing satisfactory convergence are displayed in the three rightmost columns of Table 1, and these show removal of a number of instances of bias. This result suggests that careful attention to the numerical properties of one's optimization algorithm may result in improved estimator performance.

From a practical standpoint, probably the most interesting result is the increased variability shown by NLS estimates when compared with CLS estimates. To interpret this, the results in Table 1 suggest that with 180 1-min traffic counts, roughly 95 percent of the time one could expect to have an estimate of $b_{5,4}$ that would fall in the interval [0.25, 0.35], whereas with NLS one would need the interval [0.18, 0.42] for the same degree of confidence. A similar result is shown for each of the turning movement proportions. Thus, shifting to fewer detectors does not guarantee something for nothing. The cost savings can be offset by a loss of precision.

CONCLUSION

The first objective of this paper was to assess the feasibility of estimating intersection turning movement proportions from automatic traffic counts, when the number or placement of the detectors cannot provide complete counts for each intersection. It was determined that such estimation was possible for detector configurations

TABLE 1 Results of Monte Carlo Experiments

Para-	CLS			NLS (1)			NLS (2)		
meters	mean	std	t	mean	std	t	mean	std	t
$b_{5,4}$	0.3015	0.0274	0.38	0.2978	0.0584	0.27	0.3011	0.0613	0.10
$b_{5,2}$	0.5926	0.0312	1.67	0.5769	0.0535	-	0.5854	0.0526	-
$b_{5,3}$	0.1059	0.0297	1.40	0.1254	0.0426	4.21*	0.1136	0.0402	1.82
$b_{9,3}$	0.2994	0.0190	0.21	0.2913	0.0259	2.38*	0.2981	0.0264	0.40
$b_{9,1}$	0.5953	0.0308	1.08	0.6045	0.0495	-	0.5985	0.0398	-
$b_{9,4}$	0.1052	0.0232	1.60	0.1041	0.0401	0.73	0.1035	0.0306	0.61
$b_{10,1}$	0.3096	0.0453	1.50	0.3080	0.1120	0.50	0.3139	0.1029	0.73
$b_{10,4}$	0.5888	0.0404	1.97	0.5825	0.1095	-	0.5832	0.1094	-
$b_{10,2}$	0.1016	0.0338	0.34	0.1095	0.0710	0.94	0.1029	0.0657	0.23
$b_{14,2}$	0.3067	0.0331	1.44	0.2819	0.0577	2.21*	0.2812	0.0583	1.74
$b_{14,3}$	0.5970	0.0296	0.72	0.5883	0.0525	-	0.5908	0.0564	-
$b_{14,1}$	0.0963	0.0323	0.81	0.1298	0.0612	3.44*	0.1280	0.0681	2.22*
$b_{12,8}$	0.3032	0.0286	0.80	0.3106	0.0415	1.80	0.3160	0.0429	2.01
$b_{12,5}$	0.5933	0.0264	1.80	0.5751	0.0544	-	0.5711	0.0611	-
$b_{12,6}$	0.1035	0.0287	0.85	0.1144	0.0442	2.30*	0.1129	0.0498	1.39
$b_{1,6}$	0.3010	0.0343	0.21	0.2838	0.0416	2.76*	0.2842	0.0432	1.97
$b_{1,7}$	0.5973	0.0424	0.45	0.5859	0.0477	-	0.5852	0.0515	-
$b_{1,8}$	0.1017	0.0315	0.38	0.1303	0.0422	5.08*	0.1306	0.0501	3.29*
$b_{11,7}$	0.3020	0.0214	0.65	0.2993	0.0280	0.18	0.3011	0.0292	0.21
$b_{11,8}$	0.5975	0.0243	0.72	0.5468	0.0490	-	0.5494	0.0362	-
$b_{11,5}$	0.1005	0.0191	0.19	0.1474	0.0377	8.90*	0.1495	0.0399	6.68*
$b_{13,5}$	0.3038	0.0295	0.91	0.3162	0.0643	1.78	0.3131	0.0518	1.36
$b_{13,6}$	0.6002	0.0384	0.04	0.6036	0.0549	-	0.6046	0.0389	-
$b_{13,7}$	0.0960	0.0414	0.69	0.0802	0.0553	2.54*	0.0823	0.0536	1.78

providing a requisite minimum amount of information. The authors described a numerical test of whether a given pattern of detector placements could provide this information and recommended a minimal placement pattern that is likely (but not guaranteed) to produce adequate information. Overall, it appears plausible that there is more information about turning movement proportions in limited detector configurations than is being used.

The second objective was to obtain some idea of the effects on the statistical properties of turning movement estimates that result from a reduced detector configuration. A Monte Carlo study using a simple two-intersection network showed a noticeable increase in a tendency toward bias and in estimate variability when one shifted from a complete set of counts to cordon counts. This suggests that minimal identifiable detector configurations might not provide the precision needed for real-time tracking of turning movement proportions. If full detectorization is not possible, one could begin with a minimal configuration, such as cordon counters, and add as many detectors as is economically possible. One compromise might be to divide a large network into a number of smaller cordoned areas, allowing some detectors to do double duty on the boundary between two areas. Doing so would also facilitate direct verification of input and output reachability and separability.

For practitioners, the fact that a residual amount of uncertainty remains in the estimates of the turning movement proportions, even after processing 3 hr of data, should cause them to question the standard practice of "certainty equivalent" control, in which estimated quantities are used as if they were known with certainty. For a given identifiable detector configuration, some of this uncertainty might be eliminated by switching to a more efficient estimation approach, such as ML; the feasibility of such a switch is currently under investigation. It does not appear likely, however, that all uncertainty can be eliminated, and genuinely optimal control of traffic signal systems may need to take uncertainty into explicit account.

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