# Three-Dimensional Response of Deeply Buried Profiled Polyethylene Pipe 

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#### Abstract

Three-dimensional finite element stress analysis is used to examine the response of profiled polyethylene (PE) pipe under various burial conditions. Radial, circumferential, and axial normal stresses are examined for three pipes of different diameter buried at various depths in different soil materials. The implications for PE pipe design are examined in relation to arching, time-dependent pipe response, and tensile rupture. The study found that circumferential stresses are predominantly compressive and can be predicted reasonably well using conventional twodimensional analysis. Tensile axial stresses, which develop in the inner liner of the pipe, cannot be evaluated using three-dimensional analysis. These tensions are greatest at the spring line and may be an important performance limit for lined corrugated PE pipe under deep burial. The pipes considered have a peak tension of not more than half the AASHTO allowable value at 22-m burial in very good quality (SW95) backfill material or 11 m in the same soil at lower density (SW85).


In the design of HDPE pipe for storm water sewer applications, several performance limits must be considered. Besides established limits such as buckling (1) and excessive deflection (2), the maximum circumferential bending stresses in the pipe are often considered in order to prevent tensile yield or rupture of the pipe (3). However, for a complex three-dimensional structure such as a lined corrugated HDPE pipe, there is no guarantee that the maximum tensile or compressive stresses are in the circumferential direction. A three-dimensional analysis is required to examine the distribution of radial and axial stresses in the buried pipe, according to Moore and Hu in another paper in this record.

A theoretical study of the three-dimensional stress state of a buried HDPE pipe is presented. Pipes with diameters of 300,460 , and 760 mm are examined. The pipe profiles consist of an annular corrugated section with an internal liner that provides improved hydraulic properties. Three-dimensional finite element analyses of the pipe-soil system were used to detect stresses in the pipe under various burial depths and for various backfill materials.

After defining the buried pipe problem and describing the threedimensional finite element analysis, the authors examine the response of a $460-\mathrm{mm}$ pipe buried 11 m within a granular soil embankment. The circumferential stresses that were predicted using the three-dimensional finite element analysis and the conventional two-dimensional analysis are compared, and radial and axial stresses are examined. The nature of the local three-dimensional pipe deformation is considered in order to explain the axial stresses that occur. Pipe stresses are then examined for other burial depths, soil materials, pipe diameters, and HDPE moduli. A discussion of the significance of the three-dimensional pipe response in relation to various performance limits is also presented.

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## PROBLEM DEFINITION

Figure $1(a)$ is a sketch of the the pipe-soil system that was studied. A circular pipe with annular corrugation and smooth internal liner is buried deeply within an earth embankment (this is generally the most conservative case, as the loads that develop on a pipe in an embankment burial condition will be more than those for the trench case where shear stresses develop on the sides of the trench, which provide some support to the column of soil over the pipe). The geometry of the pipe profile illustrates the three-dimensional nature of the problem Figure $1(b)$. The stresses that must be examined act in the circumferential $\sigma_{\mathrm{t}}$, radial $\sigma_{\mathrm{r}}$, and axial $\sigma_{z z}$ directions as shown.

The usefulness of two-dimensional linear and nonlinear elastic analyses of buried pipe systems has been well established through the work of Katona (4) and others. Undertaking a full, threedimensional analysis of the buried pipe system, however, is a more formidable task. To understand the three-dimensional stress distribution through the pipe, it is essential to accurately model the corrugated pipe profile. Recognizing that the central axis of the annular pipe is an axis of symmetry [Figure $1(b)$ ], the three-dimensional pipe geometry can be efficiently modeled using axisymmetric finite element theory. It is then possible to obtain a reasonable first approximation of the stresses in the pipe provided the correct soil stiffness and stress state are modeled near the pipe.

The annular axisymmetric structure shown in Figure 1 is assumed to be buried in a region of elastic soil subjected to biaxial stress field (uniform vertical stress $\sigma_{v}$ and uniform horizontal stress $\sigma_{\mathrm{h}}$ ). Residual stresses are neglected and it is assumed that the pipe has been installed with a uniform backfill envelope around the full circumference (the impact of local bending at the pipe invert associated with variable support under invert and haunches is neglected). The research indicates that the spring line is the critical location for tensile stress in the pipe and that the use of uniform support for invert and haunches should not significantly affect those results.

The finite element analysis so performed (5) is really the threedimensional equivalent of the well-known, two-dimensional elastic solutions [such as those of Burns and Richard (6) and Hoeg (7)], which are used as design tools in various buried pipe industries [e.g., CANDE (8) and the design practice of the German Waste Water Association ATV (9)]. These use the plane strain continuum theory to determine the response of a uniform thickness tube buried in a uniform elastic soil.

Table 1 contains details of the three pipe profiles to be considered. The section properties have been calculated by numerical integration of each of the three pipe profiles.

Tables 2 and 3 describe the soil materials selected. For the embankment, three alternatives represent the kind of embankment materials that could be expected in the field. They include (a) one


FIGURE 1 Geometry of buried HDPE pipe: (a) exposed view, (b) detail of pipe with annular corrugation and smooth internal liner.
well-graded granular material compacted to 85 percent of the AASHTO T-99 maximum dry density (classified as SW85), (b) one silty material compacted to 90 percent of the AASHTO T-99 maximum dry density (classified as ML90), and (c) one clayey soil compacted to 90 percent of the AASHTO T-99 maximum dry density (classified as CL90). Two well-graded granular backfill materials were considered: one at 85 percent of the AASHTO T- 99 maximum dry density (classified as SW85) and one at 95 percent (classified as SW95). The latter material has a stiffness similar to a crushed rock backfill, which shows that these backfill materials represent the full range of materials that should normally be used in the field. The backfill zone is assumed to extend 300,300 , and 380 mm from the sides of pipes with diameters of 300,460 , and 760 mm , respectively. The density and coefficient of lateral earth pressure are also given for the SW95 material, because this soil is used as an embankment soil for one analysis (see the section, Pipe Response for One Typical Installation).

The hyperbolic soil parameters given in Table 2 were used to estimate the elastic soil modulus. Table 3 shows estimates of stress and secant (not tangent) modulus at three depths within the backfill and embankment materials for embankments constructed from the SW85, ML90, and CL90 soils. These estimates are judged to be "average" values for the soil materials and stress values considered,

TABLE 1 Circumferential Pipe Properties per Unit Length

| diameter | area <br> mm | $I_{x x}$ <br> $\mathrm{~mm}^{2} / \mathrm{mm}$ |
| :---: | :---: | :---: |
| $\mathrm{mm}^{4} / \mathrm{mm}$ |  |  |$|$| 305 | 5.4 | 541 |
| :---: | :---: | :---: |
| 457 | 7.5 | 1606 |
| 762 | 9.8 | 5211 |

but are neither unique nor exact. Elastic soil modulus is not $E^{\prime}$, the empirical model-dependent parameter used in the Spangler deflection equation. Instead, it is a real physical quantity that can be measured in the laboratory and for which data exist based on extensive laboratory testing (10).

The selected pipe burial depths ( $3.6,11$, and 22 m ) exceed the typical range for pipes of these diameters. Poisson's ratio for the soil has been estimated as 0.3 . The interface between the pipe and the soil is assumed to be a "zero-slip" or "bonded" condition. For twodimensional analysis, the "zero-slip" assumption leads to the largest bending stresses in the structure (6).

Three HDPE modulus values were examined. In accordance with AASHTO design recommendations, a short-term value of 760 MPa and a long-term value of 152 MPa were selected to indicate the possible extremes. An intermediate modulus value of 310 MPa also was examined. The selection of this intermediate modulus and the significance of short-and long-term HDPE moduli will be discussed in more detail in a subsequent section. Most of the analyses were performed using the pipe modulus value of 152 MPa . A value of 0.4 was assumed for Poisson's ratio of the HDPE (trial analyses using both 0.4 and 0.46 revealed that this parameter can affect the results by about 10 percent).

## PIPE RESPONSE FOR ONE TYPICAL INSTALLATION

Before undertaking a parametric study to examine the limits of pipe stress for a range of burial conditions, pipe diameters, and effective pipe moduli, it is instructive to examine in detail the pipe response for one particular burial condition; namely, at a depth of 11 m in an embankment constructed from SW95 well-graded granular material with the same SW95 material used as backfill [this use of a uniform soil allows direct comparisons with the two-dimensional analysis of Hoeg (7)]. The 460-mm-diameter pipe was used for this first phase of the study.

TABLE 2 Soil Materials Including Density $\gamma$ Coefficient of Lateral Earth Pressure $K_{o}$ and Hyperbolic Modulus Data

| material | classfctn | $K$ | $n$ | $R_{f}$ | $c$ | $\phi_{0}$ | $\Delta \phi$ | $K_{0}$ | density <br> kPa |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| well graded <br> granular | SW95 | 950 | 0.6 | 0.7 | 0 | $48^{\circ}$ | $8^{\circ}$ | 0.43 | 21.7 |
| well graded <br> granular | SW85 | 450 | 0.35 | 0.8 | 0 | $38^{\circ}$ | $2^{\circ}$ | 0.43 | 21.7 |
| well com- | ML90 | 200 | 0.26 | 0.89 | 24 | $32^{\circ}$ | 0 | 0.47 | 18.1 |
| pacted silt |  |  |  |  |  |  |  |  |  |
| well com- | CL90 | 75 | 0.54 | 0.94 | 48 | $17^{\circ}$ | $7^{\circ}$ | 0.71 | 15.7 |
| pacted clay |  |  |  |  |  |  |  |  |  |

Figure 2 shows the finite element mesh used for the analysis near the $460-\mathrm{mm}$ diameter pipe. This is a two-dimensional mesh, with variations in stress and displacement around the pipe circumference modeled using harmonic terms. Actually, the mesh stretches five pipe diameters away from the pipe, but the stresses within the pipe and the soil directly adjacent to it are the principal concern. Six noded linear strain triangles ( 800 for the full mesh) are used to model the pipe and the soil material surrounding it. A mesh refinement study was conducted and revealed that the use of finer meshes would affect the stress predictions by a few percentage points for large stress values, or by as much as 10 kPa for low stress levels (those less than about 30 kPa ). For consistency, all solutions reported for the $460-\mathrm{mm}$-diameter pipe were performed using the mesh shown, and the solutions given later for the pipes of $300-\mathrm{mm}$ and $760-\mathrm{mm}$ diameter are based on very similar meshes.

TABLE 3 Vertical and Horizontal Stress in Different Embankment Soils and Secant Moduli (MPa) in Embankment and Backfill Soils at Various Depths

| embkmi <br> soil | depth | $\sigma_{v}$ <br> kPa | $\sigma_{h}$ <br> kPa | embkmt <br> modulus | SW85 <br> modulus | SW95 <br> modulus |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SW85 | 3.6 m | 99 | 43 | 28 | 28 | 53 |
| SW85 | 11 m | 238 | 102 | 37 | 37 | 88 |
| SW85 | 22 m | 476 | 205 | 47 | 47 | 132 |
| ML90 | 3.6 m | 83 | 39 | 14 | 28 | 51 |
| ML90 | 11 m | 199 | 93 | 16 | 37 | 85 |
| ML90 | 22 m | 396 | 186 | 19 | 47 | 127 |
| CL90 | 3.6 m | 72 | 51 | 5 | 34 | 62 |
| CL90 | 11 m | 172 | 123 | 7 | 46 | 105 |
| CL90 | 22 m | 345 | 245 | 10 | 58 | 158 |

Figures 3 to 5 show contours of circumferential stress $\sigma_{t}$, radial stress $\sigma_{\mathrm{r}}$, and axial stress $\sigma_{z z}$ for the pipe and for the soil near the pipe. All stress values given in the figures and reported in subsequent sections are tension positive.

Circumferential stress has long been a concern of pipe designers trying to limit tensions associated with pipe bending. Figures $3(a)$, 3(b), and 3(c) show the distributions of circumferential stress at the pipe crown or invert; the spring line; and the quarter points (haunches and shoulders). A conventional two-dimensional analysis of the pipe [e.g., Hoeg (7)] can be used to estimate the distributions of hoop thrust and circumferential bending moment, as well as circumferential stresses in the pipe wall. At $11-\mathrm{m}$ depth with soil modulus 88 MPa and HDPE modulus 152 MPa , the resulting two-dimensional predictions of circumferential stress at the pipe centroid (neutral axis) are $30,1,100$, and 570 kPa , respectively. These values are very similar to those calculated using the three-dimensional finite element analysis. The three-dimensional analysis also reveals local variations away from the neutral axis as follows:

- Stress in the section farthest from the pipe axis increases as one would expect at the crown-invert position, but remains close to the neutral axis values at the other two pipe locations. These extreme fiber stresses are less affected by bending than would be expected from calculations based on two-dimensional analysis. The mass of soil adjacent to the pipe at this location appears to be acting with the HDPE material to carry much of the bending stress.
- A stress concentration occurs where the corrugation section and the lining intersect.
- Stresses in the sections of lining spanning the corrugation decrease below the neutral axis values at each point around the pipe circumference. This is associated with the local bending that occurs here. This bending is examined later in this section in relation to the axial stress distributions shown in Figure 5.

Radial stress values are all relatively low. Two-dimensional analysis of the system indicates that the radial stress at the neutral axis is about 14 kPa , which is consistent with what is shown in Figure 4 . This is only about 8 percent of the vertical overburden pressure ( 172 kPa ). This high amount of "positive arching" is associated


FIGURE 2 Finite element mesh close to the $\mathbf{4 6 0}$-mm-diameter pipe.


FIGURE 3 Circumferential pipe stresses at (a) crown-invert, (b) spring lines, and (c) quarter points.


(b)

FIGURE 4 Radial pipe stresses at (a) crown-invert, (b) spring lines, and (c) quarter points.
with circumferential shortening of the pipe, which also will be discussed in more detail later.

The radial stresses increase somewhat in the area where the corrugation section and the lining intersect, with the maximum and minimum values in relatively close proximity. Of particular interest are the small regions of radial tension predicted within the pipe.

The axial stress distributions shown in Figure 5 reveal that tensile stresses do develop in the pipe liner. These stresses are the result of local bending and have been explained and investigated by Moore and Hu elsewhere, and by Selig (11) in relation to pipe response under hoop compression.

The tensions in the liner that develop close to the linercorrugation junction represent an important performance limit for lined corrugated pipe under very deep burial.

## DIFFERENT DEPTHS AND BACKFILLS

A parametric study was performed to determine how minimum and maximum stresses are affected by burial depths and backfill quality. The results are shown in Table 4 for the $460-\mathrm{mm}$ pipe with

HDPE modulus of 152 MPa and with the ML90 embankment soil (closest to the soil used in real embankments). For each burial depth, backfill type, stress direction, and pipe location, the largest tensile (positive) stress is shown above the largest compression (negative). Where the upper value is negative, this represents the lowest compressive stress value (no tension occurs in that case).

Also shown are estimates of the pipe deformation for each of the pipe burial conditions considered. Changes in vertical pipe diameter $\Delta D_{\mathrm{v}}$ and horizontal pipe diameter $\Delta D_{\mathrm{h}}$ are given, in addition to percent changes in diameter.

The results reveal that:

- Stresses within the pipe decrease as soil stiffness is increased (this is consistent with well-known trends for buried flexible and rigid pipe).
- Pipe deformations decrease as backfill stiffness is increased (this also is consistent with what is expected for the flexible pipe, in which the pipe deformations are controlled predominantly by the soil, not the pipe itself.
- Stresses in the pipe increase with burial depth, but at a rate that is less than linear. As soil depth increases the soil stiffness also


(b)

FIGURE 5 Axial pipe stresses at (a) crown-invert, (b) spring lines, and (c) quarter points.
increases so that additional "positive arching" somewhat reduces the resulting loads.

- The only tensile circumferential stresses that occur develop at the crown of pipes buried deeply in the stiffer backfill; the magnitude of these tensions is quite small (the largest tension is 160 kPa ).
- Tensile radial stresses do develop in the pipes, but are of relatively low magnitude (less than 410 kPa ) except at the spring line of the pipe deeply buried with lower density backfill.
- Axial tensions $\sigma_{z z}$ develop in the liner of all of the pipes. They are highest at the spring lines (for the SW95 backfill, maximum tensions range from 1 MPa to 2.7 MPa for $3.6-\mathrm{m}$ and $22-\mathrm{m}$ burial, respectively).


## PIPE MODULUS

A parametric study was performed to examine how minimum and maximum stresses and pipe deformations are affected by HDPE modulus. The results are shown in Table 5 for the $460-\mathrm{mm}$ pipe buried 11 m within an ML90 embankment with SW95 backfill. Pipe moduli of 760,310 , and 152 MPa are considered; these are effec-
tive (or "secant") moduli over the time period for which the load is applied.

Time-dependent analysis using viscoelastic material models has demonstrated that for a parallel plate test performed on an HDPE pipe over a period of a few minutes, the effective pipe modulus is about 390 MPa (12). For a pipe burial with a depth of 3.6 m over 1 hr or less (a rapid case), a modulus value of about 310 MPa applies. Modulus for the pipe as it responds to this earth load over most of its design life ranges from 152 to 310 MPa , generally occurring closer to the lower value. Incremental HDPE pipe modulus for live loads will be much closer to 760 MPa , depending on the rate at which the live load is applied and then removed. The net pipe response at a particular time involves adding the short-term live load response to the long-term response to dead load.
Table 5 indicates that stresses are highest with an HDPE modulus of 760 MPa , decreasing substantially with decreases in the effective HDPE modulus. This is not surprising given that there is circumferential shortening in the pipe and, therefore, "positive arching"; hence, decreases in effective pipe modulus lead to additional "positive arching" (i.e., more of the overburden load is redis-

TABLE $4 \quad 460-\mathrm{mm}$ Pipe Response at Crown, Spring Line, and Quarter Points for Burial at Various Depths in Two Different Backfills; Minimum and Maximum Stresses Given, Tension Positive

| burial <br> condtn | $\sigma_{r r}$ <br> crown | $\sigma_{t t}$ <br> crown | $\sigma_{z z}$ <br> crown | $\sigma_{r r}$ <br> sprgl | $\sigma_{t t}$ <br> sprgl | $\sigma_{z z}$ <br> sprgl | $\sigma_{r r}$ <br> qrtr | $\sigma_{t t}$ <br> qrtr | $\sigma_{z z}$ <br> qrtr | $\Delta D_{v \mathrm{~mm}}$ <br> $(\% D)$ | $\Delta D_{h} \mathrm{~mm}$ <br> $(\% D)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SW85 | 40 | -40 | 220 | 220 | -70 | 1400 | 130 | -60 | 820 | -4.1 | -0.2 |
| 3.6 m | -150 | -340 | -280 | -710 | -2200 | -1700 | -410 | -1300 | -990 | $(0.9 \%)$ | $(0.0 \%)$ |
| SW85 | 60 | -70 | 370 | 450 | -160 | 2800 | 250 | -140 | 1600 | -8.1 | -0.3 |
| 11 m | -270 | -590 | -450 | -1400 | -4400 | -3400 | -800 | -2500 | -1900 | $(1.8 \%)$ | $(0.1 \%)$ |
| SW85 | 80 | -140 | 550 | 750 | -300 | 4700 | 420 | -280 | 2600 | -13.5 | -0.5 |
| 22 m | -430 | -920 | -680 | -2400 | -7500 | -5700 | -1300 | -4200 | -3200 | $(3.0 \%)$ | $(0.1 \%)$ |
| SW95 | 10 | -10 | 80 | 170 | -50 | 1000 | 90 | -50 | 560 | -3.0 | 0.1 |
| 3.6 m | -90 | -170 | -100 | -520 | -1700 | -1300 | -280 | -880 | -670 | $(0.7 \%)$ | $(0.0 \%)$ |
| SW95 | 0 | 40 | 20 | 280 | -100 | 1800 | 140 | -120 | 890 | -5.1 | 0.3 |
| 11m | -220 | -290 | -150 | -890 | -2800 | -2100 | -450 | -1400 | -1100 | $(1.1 \%)$ | $(0.1 \%)$ |
| SW95 | 40 | 160 | 80 | 410 | -200 | 2600 | 200 | -220 | 1300 | -7.4 | 0.5 |
| 22m | -460 | -640 | -320 | -1300 | -4100 | -3100 | -640 | -1200 | -1500 | $(1.6 \%)$ | $(0.1 \%)$ |

tributed into the surrounding ground). This beneficial effect has been observed in the field for profiled HDPE pipes (13).

Two-dimensional analysis can be performed for the soil-pipe system using the same geometrical and material data used to generate the results in Table 5. This indicates that as the modulus decreases, the average radial stress applied to the pipe decreases from about 32 percent to about 9 percent of the vertical overburden pressure, with a value of 16 percent for modulus 310 MPa .

These data have important consequences for HDPE pipe design. Current design practice is based largely on procedures developed for the metal pipe industry. The basis of these procedures is the ring compression theory of White and Layer (14), which asserts that the full overburden load acts on the pipe.

Clearly the ring compression theory is not valid for HDPE pipes because there is substantial positive arching and only a fraction of the overburden load reaches the pipe. For the case considered, the
ring compression theory leads to overestimates of pipe stress levels by a factor of $100 / 16=6$ in the short term and $100 / 9=11$ in the long term. As can be seen, this is very conservative.

From the pipe deflection data, it appears that small increases in pipe deflection are associated with the changes in HDPE modulus. From the $310-\mathrm{MPa}$ to $152-\mathrm{MPa}$ modulus reduction, there is a 10 percent increase in pipe deflection. This small increase is a direct result of the action of the granular backfill, which is largely controlling the pipe deflection. Again, this is consistent with field observation (13).

These results have implications for the labels used to describe the time-dependent HDPE response. The observed behavior can be labeled as "stress relaxation" (decreases in stress with time for HDPE kept at constant strain) or "creep" (increases in deformation with time for HDPE kept at constant stress). Although the actual behavior lies somewhere between these two simplified conditions,

TABLE 5 460-mm Pipe Response at Crown, Spring Line, and Quarter Points for Different HDPE Moduli; Burial Depth 11 m; ML90 Embankment Soil; SW95 Backfill; Minimum and Maximum Stresses Given, Tension Positive

| pipe <br> modulus | $\sigma_{r r}$ <br> crown | $\sigma_{\mathrm{tt}}$ <br> crown | $\sigma_{z z}$ <br> crown | $\sigma_{r r}$ <br> sprgl | $\sigma_{\mathrm{tt}}$ <br> sprgl | $\sigma_{z z}$ <br> sprgl | $\sigma_{\mathrm{rr}}$ <br> qrtr | $\sigma_{\mathrm{tt}}$ <br> qrtr | $\sigma_{z z}$ <br> qrtr | $\Delta D_{v \mathrm{~mm}}$ <br> $(\% D)$ | $\Delta D_{h \mathrm{~mm}}$ <br> $(\% D)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 760 | 100 | -60 | 660 | 960 | -100 | 6000 | 540 | -110 | 3300 | -3.6 | 0.4 |
| MPa | -640 | -1400 | -820 | -3000 | -9600 | -7300 | -1700 | -5200 | -4000 | $(0.8 \%)$ | $(0.1 \%)$ |
| 310 | 30 | 0 | 160 | 500 | -110 | 3200 | 270 | -120 | 1700 | -4.6 | 0.3 |
| MPa | -230 | -470 | -290 | -1600 | -5100 | -3800 | -850 | -2600 | -2000 | $(1.0 \%)$ | $(0.1 \%)$ |
| 152 | 0 | 40 | 20 | 280 | -110 | 1800 | 140 | -120 | 890 | -5.1 | 0.3 |
| MPa | -220 | -290 | -150 | -890 | -2800 | -2100 | -450 | -1400 | -1100 | $(1.1 \%)$ | $(0.1 \%)$ |

it is most similar to the "stress relaxation" condition in that the soil keeps the pipe deformations fairly constant, and the principal effect of the time-dependent HDPE response is a beneficial reduction in internal stresses.

The predictions for pipe deflection that result from the present work are consistent with field data but are very different from what would be predicted using the Spangler equation for pipe deflection. The problem with the Spangler equation is that it ignores the substantial positive arching associated with circumferential shortening and uses an $E$ ', or "soil spring" model, which is generally not adjusted to account for the real stiffness of the backfill surrounding the pipe. A better approach would be to use two-dimensional elastic continuum analysis for the pipe-soil system. Improved estimates of pipe stresses and deformations would be the result:

- The elastic continuum solution includes the possibility of circumferential shortening and thus can predict the positive arching that occurs in the field. The assumption that the full overburden stress is active across the pipe (an approach that works well for flexible metal pipes, which experience little circumferential shortening) is not appropriate for the HDPE pipes examined in this report.
- The elastic continuum solution demonstrates that it is quite normal for horizontal diameter change to be different from the ver-
tical diameter change (this difference is not an indication of a perceived problem, such as "squaring."
- The elastic continuum solution leads to the conclusion that decreases in effective HDPE modulus are beneficial. Because of historical circumstances, the design of HDPE pipe is currently based on ring compression theory and the Spangler equation. Unfortunately, this has led to the incorrect conclusion that it is beneficial to use the higher AASHTO modulus in design.


## DIFFERENT EMBANKMENT SOILS

A parametric study was performed to examine how minimum and maximum stresses and pipe deformations are affected by the embankment material. The results are shown in Table 6 for the $460-\mathrm{mm}$ pipe buried at various depths within SW85, ML90, and CL90 embankments with SW95 backfill. Pipe modulus of 152 MPa was used.
It appears that the stresses and pipe deformation are not greatly affected by the embankment material (Table 6). The trends are complicated somewhat because although there are higher vertical load levels in the denser granular embankment, there is increased positive arching, which offsets this effect.

TABLE $6 \quad 460-\mathrm{mm}$ Pipe Response at Crown, Spring Line, and Quarter Points for Burial at Various Depths in Various Embankment Soil Materials; HDPE Modulus 152 MPa; SW95 Backfill; Minimum and Maximum Stresses Given, Tension Positive

| embkmt <br> depth |  | $\begin{gathered} \sigma_{t t} \\ \text { crown } \end{gathered}$ | $\begin{gathered} \sigma_{2 x} \\ \text { crown } \end{gathered}$ | $\begin{gathered} \sigma_{r r} \\ \text { sprgl } \end{gathered}$ | $\begin{gathered} \sigma_{t t} \\ \text { sprgl } \end{gathered}$ | $\begin{gathered} \sigma_{x z} \\ \text { sprgl } \end{gathered}$ | $\begin{gathered} \sigma_{\mathrm{rr}} \\ \mathrm{qrtr} \end{gathered}$ | $\begin{aligned} & \sigma_{\mathrm{tt}} \\ & \mathrm{qrtr} \end{aligned}$ | qrir | $\Delta D_{v}$ mm (\%D) | $\Delta D_{h \mathrm{~mm}}$ <br> (\%D) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| CL90 | 40 | -40 | 250 | 130 | -40 | 800 | 80 | -40 | 520 | -2.0 | -0.5 |
| 3.6 m | -130 | -380 | -310 | -410 | -1300 | -960 | -270 | -820 | -640 | (0.5\%) | (0.1\%) |
| CL90 | 60 | -90 | 360 | 200 | -90 | 1300 | 130 | -100 | 810 | -3.6 | -0.8 |
| 11m | -220 | -560 | -440 | -640 | -2000 | -1500 | -410 | -1300 | -1000 | (0.8\%) | (0.2\%) |
| CL90 | 70 | -120 | 460 | 270 | -180 | 1700 | 180 | -200 | 1100 | -4.8 | -1.0 |
| 22m | -440 | -730 | -560 | -880 | -2800 | -2100 | -560 | -1700 | -1300 | (1.1\%) | (0.2\%) |
| ML90 | 10 | -10 | 80 | 170 | -50 | 1000 | 90 | -50 | 560 | -3.0 | 0.1 |
| 3.6 m | -90 | -170 | -110 | -520 | -1700 | -1300 | -280 | -880 | -670 | (0.7\%) | (0.0\%) |
| ML90 | 0 | 40 | 20 | 280 | -110 | 1800 | 140 | -120 | 890 | -5.1 | 0.3 |
| 11m | -220 | -290 | -150 | -890 | -2800 | -2100 | -450 | - 1400 | $-1100$ | (1.1\%) | (0.1\%) |
| ML90 | 40 | 160 | 80 | 410 | -200 | 2600 | 200 | -220 | 1300 | -7.4 | 0.5 |
| 22m | -460 | -640 | - 320 | -1300 | -4100 | -3100 | -640 | -2000 | -1500 | (1.6\%) | (0.1\%) |
| SW85 | 10 | -20 | 80 | 160 | -80 | 1000 | 80 | -70 | 540 | -2.8 | 0.0 |
| 3.6 m | -100 | -160 | -110 | -500 | -1600 | -1200 | -270 | -850 | -660 | (0.6\%) | (0.0\%) |
| SW85 | 10 | 0 | 70 | 260 | -190 | 1700 | 130 | -170 | 870 | -4.6 | 0.0 |
| 11m | -240 | -230 | -140 | -840 | -2600 | -2000 | -440 | -1400 | -1000 | (1.0\%) | (0.0\%) |
| SW85 | 0 | 50 | 40 | 380 | -340 | 2400 | 200 | -240 | 1200 | -6.6 | 0.1 |
| 22m | -490 | -510 | -280 | -1200 | -3900 | -2900 | -620 | -2000 | -1500 | (1.5\%) | (0.0\%) |

TABLE 7 Pipe Response at Crown, Spring Line, and Quarter Points for Various Pipe Diameters at HDPE Modulus of 152 MPa; SW95 Backfill; ML90 Embankment; Minimum and Maximum Stresses Given, Tension Positive

| diameter <br> depth | $\sigma_{r r}$ <br> crown | $\sigma_{t t}$ <br> crown | $\sigma_{x z}$ <br> crown | $\sigma_{r r}$ <br> sprgl | $\sigma_{t t}$ <br> sprgl | $\sigma_{x z}$ <br> sprgl | $\sigma_{r r}$ <br> qrtr | $\sigma_{t t}$ <br> qrtr | $\sigma_{z z}$ <br> qrtr | $\Delta D_{v \mathrm{~mm}}(\% D)$ | $\Delta D_{h \mathrm{~mm}}$ <br> $(\% D)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 300 mm | 20 | -20 | 120 | 150 | 0 | 1000 | 80 | -40 | 580 | -1.8 | -0.1 |
| 3.6 m | -100 | -210 | -220 | -770 | -1800 | -1800 | -430 | -1000 | -1000 | $(0.6 \%)$ | $(0.0 \%)$ |
| 300 mm | 20 | -10 | 130 | 250 | 10 | 1700 | 130 | -60 | 900 | -2.8 | -0.1 |
| 11 m | -230 | -290 | -220 | -1300 | -3100 | -3000 | -680 | -1600 | -1600 | $(0.9 \%)$ | $(0.0 \%)$ |
| 300 mm | 10 | 0 | 120 | 350 | 20 | 2400 | 190 | -90 | 1300 | -3.8 | -0.1 |
| 22 m | -480 | -570 | -290 | -1800 | -4400 | -4300 | -950 | -2300 | -2300 | $(1.3 \%)$ | $(0.0 \%)$ |
| 460 mm | 10 | -10 | 80 | 170 | -50 | 1000 | 90 | -50 | 550 | -3.0 | 0.1 |
| 3.6 m | -90 | -170 | -100 | -520 | -1600 | -1200 | -280 | -870 | -660 | $(0.7 \%)$ | $(0.0 \%)$ |
| 460 mm | 0 | 40 | 20 | 280 | -100 | 1700 | 140 | -120 | 880 | -5.1 | 0.3 |
| 11 m | -220 | -280 | -150 | -880 | -2800 | -2100 | -440 | -1400 | -1100 | $(1.1 \%)$ | $(0.1 \%)$ |
| 460 mm | 30 | 160 | 80 | 400 | -200 | 2600 | 190 | -220 | 1200 | -7.4 | 0.5 |
| 22 m | -460 | -630 | -320 | -1300 | -4100 | -3100 | -630 | -2000 | -1500 | $(1.6 \%)$ | $(0.1 \%)$ |
| 760 mm | 10 | -10 | 40 | 130 | 10 | 900 | 70 | -60 | 480 | -5.6 | 0.2 |
| 3.6 m | -90 | -150 | -80 | -330 | -1400 | -910 | -160 | -720 | -480 | $(0.7 \%)$ | $(0.0 \%)$ |
| 760 mm | 10 | 80 | 20 | 220 | 10 | 1500 | 110 | -120 | -750 | -9.1 | 0.2 |
| 11 m | -210 | -330 | -160 | -570 | -2300 | -1600 | -280 | -1100 | -770 | $(1.2 \%)$ | $(0.1 \%)$ |
| 760 mm | 60 | 250 | 140 | 320 | 20 | 2300 | 150 | -230 | 1100 | -13.5 | 0.9 |
| 22 m | -440 | -750 | -350 | -830 | -3500 | -2300 | -390 | -1600 | -1100 | $(1.8 \%)$ | $(0.2 \%)$ |

## DIFFERENT PIPE DIAMETERS

The final parametric study was performed to examine how minimum and maximum stresses and pipe deformations are affected by the pipe diameter. The results are shown in Table 7 for the $300-, 460$-, and $760-\mathrm{mm}$ pipes buried at various depths within an ML90 embankment with SW95 backfill. Pipe modulus of 152 MPa was used.
Again, the data indicate that the pipe diameter is not a particularly significant parameter. This is simply a reflection of the fact that the pipe profiles have been designed to give approximately equal performance under similar burial conditions. The compressive circumferential, axial, and radial stresses are greatest in the $300-\mathrm{mm}$ pipe, whereas the tensile stresses are slightly higher in the $460-\mathrm{mm}$ pipe than in either of the other diameters. The very similar results for each of the pipe diameters supports the use of the $460-\mathrm{mm}$ pipe for most of this three-dimensional buried pipe study.

## DISCUSSION AND CONCLUSIONS

It is clear that the three-dimensional bending effects in lined corrugated pipes cannot be predicted using two-dimensional analysis. While the maximum compressive stresses are in the circumferential
direction and could be estimated using two-dimensional theory, compressions and tensions develop in the axial and radial directions that cannot be predicted using two-dimensional plane strain theory.

The analysis has shown that at the spring line of a $460-\mathrm{mm}$ storm water pipe buried 11 m within dense granular backfill, a local axial tension of about 1.7 MPa can develop. As burial depth increases or backfill stiffness decreases, the magnitude of this local axial tension rises. The zone of tension is located within the liner, and the corrugated component of the pipe profile is essentially unaffected.

Comparing (a) the AASHTO short-term tensile strength of 20.7 MPa ( $3,000 \mathrm{psi}$ ) with short-term stress values (i.e., values calculated using short-term HDPE modulus) and (b) the long-term tensile rupture stress of $6.2 \mathrm{MPa}(900 \mathrm{psi})$ with the long-term stress values (i.e., values calculated using long-term HDPE modulus), it appears that the three pipes considered in this report have local stress not more than half the allowable value at 22 m burial in very good quality (SW95) material, or 11 m in the same soil at lower density (SW85).

The analyses suggest that increases in allowable burial depths may be possible for the profiled HDPE pipes in relation to the expected performance for deflection and local bending stress. This is conditional on a careful construction of the soil envelope, sufficient soil quality to maintain stability against buckling, and successful comparisons with field data to confirm the validity of the idealized soil-structure interaction model used in this study.

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## REFERENCES

1. Moore, I. D., and E. T. Selig. Use of Continuum Buckling Theory for Evaluation of Buried Plastic Pipe Stability. In Special Technical Publication 1093, Buried Plastic Pipe Technology, ASTM, Philadelphia, 1990, pp. 344-359.
2. Janson, L. E., and J. Molin. Design and Installation of Underground Plastic Sewer Pipes. Proc., International Conference on Underground Plastic Pipes, New Orleans, 1981, pp. 79-88.
3. Chambers, R. E., and T. J. McGrath. Structural Design of Buried Plastic Pipes. Proc., International Conference on Underground Plastic Pipes, New Orleans, 1981, pp. 10-25.
4. Katona, M. G. Allowable Fill Heights for Corrugated Polyethylene Pipe. Transportation Research Record 1191, 1988, pp. 30-38.
5. Moore, I. D. Local Strain in Corrugated Pipe: Experimental Meásurements to Test a Numerical Model. Journal of Testing and Evaluation, ASTM, Vol. 22, No. 2, March 1994, pp. 132-138.
6. Burns, J. Q., and R. M. Richard. Attenuation of Stresses for Buried Cylinders. Proc., Symposium on Soil-Structure Interaction, ASTM, University of Arizona, 1964, pp. 379-392.
7. Hoeg, K. Stresses Against Underground Structural Cylinders. Journal of Soil Mechanics and Foundation Engineering, ASCE, Vol. 94, No. SM4, 1968, pp. 833-858.
8. Katona, M. G. CANDE: A Modern Approach for the Structural Design and Analysis of Buried Culverts. Report FHWA-RD-77-5, FHWA, U.S. Department of Transportation, Oct. 1976.
9. ATV: The German Waste Water Association. Guidelines for Static Calculation of Drainage Conduits and Pipelines. Regulation DK 628.22/083, (Gesellschaft zur Forderung der Abwassertechnik, E.V.), 1984.
10. Selig, E. T. Soil Properties for Plastic Pipe Installations. In Special Technical Publication 1093, Buried Plastic Pipe Technology, ASTM, Philadelphia, 1990, pp. 141-158.
11. Selig, E. T., L. C. DiFrancesco, and T. J. McGrath. Laboratory Test of Buried Pipe in Hoop Compression. Dave Eckstein, ed. Special Technical Publication 1222, Buried Plastic Pipe Technology, ASTM, Philadelphia, 1994; pp. 119-132.
12. Moore, I. D., and F. Hu. Linear Viscoelastic Models for HDPE. Canadian Journal of Civil Engineering, (under review), 1995.
13. Hashash, N. M. A. Design and Analysis of Deeply Buried Polyethylene Drainage Pipes. Ph.D. thesis. Department of Civil Engineering, The University of Massachusetts at Amherst, 1991.
14. White, H. L., and J. P. Layer. The Corrugated Metal Conduit as a Compression Ring. HRB Proc., Vol. 39, 1960, pp. 389-397.

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