Seismic Pushover Analysis Using AASHTO Guide Specifications for LRFD Seismic Bridge Design

Elmer E. Marx, Alaska Department of Transportation and Public Facilities
Michael Keever, California Department of Transportation
Learning Objective

• Given the plans for a conventional bridge pier, calculate the lateral force-displacement (pushover) response of the system necessary for the seismic design of bridges in accordance with the AASHTO Guide Specifications for LRFD Seismic Bridge Design (SGS).

• Focus is *pushover analysis* not modeling, seismic displacement demand or capacity design
Overview

• Flexural mechanics refresher
• Generation of Moment-Curvature ($M - \phi$)
• Material models and failure strains
• Hand checking $M - \phi$
• Analytical plastic hinge length, $L_p$
• Force-Displacement pushover example
• Hand checking force-displacement
• Design Example – Two circular column bent
Why Pushover?

- SGS is primarily a displacement-based approach

- More rational approach than force-based method of AASHTO LRFD

- Seismic Design Category D (SDC D) requires pushover analysis
Why Pushover?

• Need to verify that the seismic displacement demand, $\Delta^L_D$, exceed the displacement capacity, $\Delta^L_C$

• Results can be used to help create the seismic model and subsequent displacement demands

• Provides for a better understanding of bridge response and behavior
Flexural Mechanics

- Relationship between force, shear, moment, curvature, slope and deflection
  - Integration
  - Moment area
  - Energy methods
  - Stiffness methods
Force – Displacement Relationship

\[ P = load \]

\[ V = \int P \cdot dx \]

\[ M = \int V \cdot dx \]
Force – Displacement Relationship

\[ M = \int V \cdot dx \]

\[ \varphi = \frac{M}{EI} \]
Force – Displacement Relationship

\[ \varphi = \frac{M}{E \cdot I} \]

\[ \theta = \int \varphi \cdot dx \]

\[ \Delta = \int \theta \cdot dx \]
Example - Cantilever

\[ P \]

\[ V = P \]

\[ \theta_L = \frac{\varphi_o \cdot L}{2} \]

\[ \Delta_L = \frac{\varphi_o \cdot L^2}{3} \]

\[ M_o = P \cdot L \]

\[ \varphi_o = \frac{M_o}{E \cdot I} = \frac{P \cdot L}{E \cdot I} \]
Compatibility

• Bernoulli-Euler assumption – sections that are plane (linear) before bending remain plane after bending

• Perfect bond between the steel reinforcing bars and surrounding concrete

• Material (constitutive $\sigma-\varepsilon$) models are representative of actual material response (confinement, strain hardening, buckling, spalling, cracking, creep, strain-rate, shear, etc.)
Moment – Curvature (\( M - \phi \))

\[
\frac{dx}{R} = \frac{\varepsilon_c \, dx}{c} = \frac{\varepsilon_s \, dx}{d-c}
\]

\[
\frac{1}{R} = \frac{\varepsilon_c}{c} = \frac{\varepsilon_s}{d-c}
\]
Moment – Curvature ($M - \phi$)

- For small angles, curvature ($\phi$) may be calculated as:

$$\phi = \varepsilon_c / c = \varepsilon_t / (D - c)$$

$\phi = \text{strain} / \text{distance from neutral axis}$
Moment – Curvature \( (M - \phi) \)

- From equilibrium:

\[ P = F_{SC} + F_{CC} + F_{CU} - F_{ST} \]

- Summing moments about the neutral axis

\[ M = CG_{SC} \cdot F_{SC} + CG_{CC} \cdot F_{CC} + CG_{CU} \cdot F_{CU} + CG_{ST} \cdot F_{ST} + P \cdot (D/2 - c) \]
Generation of Moment-Curvature Relationship

1. Select a strain at the compression face of column $\varepsilon_c$
2. Guess a value for the neutral axis depth $c$
3. Determine the material stresses for the assumed strain profile
4. Calculate $P$ as the sum of the internal forces
5. Does $P$ equal the applied axial load?
   - Yes: Calculate $M$ as the sum of the forces about the neutral axis
   - No: Calculate the curvature $\phi = \varepsilon_c / c$
6. Do failure strains $\varepsilon_{cu}$ or $\varepsilon_{su}$ occur?
   - Yes: Done
   - No: Repeat steps 1-5
Moment – Curvature ($M - \phi$)

Predicted Response

Idealized Bilinear Response used in SGS
Idealized $M - \phi$ Response

- Used for design purposes
- *First yield* Moment and Curvature, $M_y$ and $\phi_y$
- Effective stiffness, $E_{ce} * I_{eff} = M_y / \phi_y$
- *Idealized yield* Moment and Curvature, $M_p$ and $\phi_{yi}$
- Expected nominal moment, $M_{ne}$ at $\varepsilon_c = 0.003$
- Balance the area above and below the curve
SGS Material Models

- **Concrete**: Mander et al model for unconfined and confined condition

- **Reinforcing steel**: elastic - perfectly plastic with strain hardening

- **Other**: prestressing steel (see SGS)

- Any function / model that accurately captures the material $\sigma$-$\varepsilon$ relationship is acceptable
SGS Material Models - Concrete

- Stress-strain relationship for concrete

\[ f_c = \frac{f'_{cc}xr}{r - 1 + x^r} \]

where:

\[ f'_{cc} = f'_{ce} \left( 2.254 \sqrt{1 + \frac{7.94 f'_{l}}{f'_{ce}}} - \frac{2 f'_{l}}{f'_{ce}} - 1.254 \right) \]

\[ r = \frac{E_{ce}}{E_{ce} - E_{sec}} \]

\[ x = \frac{\varepsilon_c}{\varepsilon_{cc}} \]

Mander, Priestley, Park 1988
SGS Material Models - Concrete

(continue)

\[ \varepsilon_{cc} = 0.002 \left[ 1 + 5 \left( \frac{f'_{cc}}{f'_{ce}} - 1 \right) \right] \]

\[ E_{ce} = 1900 \sqrt{f'_{ce}} \]

\[ E_{sec} = \frac{f'_{cc}}{\varepsilon_{cc}} \]
Confinement of circular columns

\[ \rho_s = \frac{4A_{sp}}{D's} \]

\[ f'_l = K_e f_l \approx 0.95 f_l \]

\[ f_l = \frac{2A_{sp}f_{yh}}{D's} = \frac{\rho_s f_{yh}}{2} \]

Mander, Priestley, Park 1988
Confinement of rectangular columns

\[ f_l' \approx \frac{K_e (\rho_x + \rho_y) f_{yh}}{2} \approx \frac{0.8 (\rho_x + \rho_y) f_{yh}}{2} \]

\[ \rho_x = \frac{A_{sx}}{sH_c} \]

\[ \rho_y = \frac{A_{sy}}{sB_c} \]
SGS Material Models - Concrete

\( \varepsilon_c \) = strain in concrete (IN/IN)
\( f_c \) = stress in concrete corresponding to strain \( \varepsilon_c \) (KSI)
\( f'_{ce} \) = expected nominal compressive strength (KSI)
\( f_{yh} \) = nominal yield stress of transverse reinforcing (KSI)
\( \varepsilon_{su}^R \) = reduced ultimate tensile strain of transverse bars (IN/IN)
\( A_{sx} \) = total area of transverse bars in the “x” axis (IN²)
\( A_{sy} \) = total area of transverse bars in the “y” axis (IN²)
\( A_{sp} \) = area of hoop/spiral bar for circular column (IN²)
\( H_c \) = confined core dimension in the “y” axis (IN)
\( B_c \) = confined core dimension in the “x” axis (IN)
\( D' \) = diameter of spiral or hoop for circular column (IN)
\( s \) = pitch of spiral or spacing of hoops or ties (IN)

Mander, Priestley, Park 1988
SGS Material Models - Concrete
SGS Material Models - Concrete

• Effective confinement factor for **circular** columns

\[
K_e = \left(1 - \frac{s'}{2D'}\right)^n \left(1 - \frac{\rho_{cc}}{1}\right) \sim 0.95
\]

• Effective confinement factor for **rectangular** columns

\[
K_e = \left[1 - \sum_{i=1}^{N} \frac{(w_i')^2}{6b_d d_c}\right] \left(1 - \frac{s'}{2b_c}\right) \left(1 - \frac{s'}{2d_c}\right) \left(1 - \frac{\rho_{cc}}{1}\right) \sim 0.80
\]
SGS Material Models - Concrete

\( s' = \) clear distance between transverse bars = \( s - d_{bh} \) (IN)
\( d_{bh} = \) diameter of transverse reinforcing bar (IN)
\( D' = \) centerline diameter of hoop or spiral (IN)
\( \rho_{cc} = \frac{A_{st}}{A_{cc}} \)
\( A_{st} = \) total area of longitudinal reinforcement (IN\(^2\))
\( A_{cc} = \) area of confined concrete core (IN\(^2\))
\( n = 1 \) for continuous spiral
\( n = 2 \) for individual hoops
\( w_i' = \) clear distance between adjacent tied longitudinal bars (IN)
\( b_d = \) confined core dimension in the longer direction (IN)
\( d_c = \) confined core dimension in the shorter direction (IN)
\( N = \) number of spaces between longitudinal bars

Chen 2003
SGS Material Models - Concrete

- Stress-strain curves for concrete

- Spalling strain, $\varepsilon_{sp}$, not to exceed 0.005

- Confined concrete crushing strain, $\varepsilon_{cu}$, critical component

- Recommend $\varepsilon_{cu} < 0.02$ for design purposes

SGS 2011
SGS Failure Strain - Concrete

• Confined concrete crushing strain limit, $\varepsilon_{cu}$

\[
\varepsilon_{cu} = 0.004 + \frac{1.4 \rho_s f_{yh} \varepsilon_{su}}{f'_{cc}} < 0.025
\]

where:

$\rho_s = \text{transverse reinforcement ratio (}\rho_x + \rho_y \text{ for rect.)}$

$f_{yh} = \text{nominal yield stress of transverse steel}$

$\varepsilon_{su} = \text{rupture strain of transverse steel} \sim 0.09 \text{ to } 0.12$

$f'_{cc} = \text{confined concrete compressive stress}$

Mander, Priestley, Park 1988
Figure 4. Longitudinal Steel (#6 and #7 Bars) Sample Tensile Test Results
SGS Material Models - Steel

- Stress-strain relationship for reinforcing steel

Elastic \( \varepsilon_s \leq \varepsilon_{ye} \quad f_s = E_s \varepsilon_s \)

Plastic \( \varepsilon_{ye} \leq \varepsilon_s \leq \varepsilon_{sh} \quad f_s = f_{ye} \)

Strain hardening \( \varepsilon_{sh} \leq \varepsilon_s \leq \varepsilon_{su} \quad f_s = f_{ue} \left[ 1 - (f_{ue} - f_{ye}) \left( \frac{\varepsilon_{su} - \varepsilon_s}{\varepsilon_{su} - \varepsilon_{sh}} \right)^2 \right] \)

Priestley, Calvi, Kowalsky 2007
# SGS Failure Strains - Steel

- Reinforcing steel failure strain, $\varepsilon_{su}^R$

<table>
<thead>
<tr>
<th>Property</th>
<th>Notation</th>
<th>Bar Size</th>
<th>ASTM A706</th>
<th>ASTM A615 Grade 60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specified minimum yield stress (ksi)</td>
<td>$f_y$</td>
<td>#3 - #18</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>Expected yield stress (ksi)</td>
<td>$f_{ye}$</td>
<td>#3 - #18</td>
<td>68</td>
<td>68</td>
</tr>
<tr>
<td>Expected tensile strength (ksi)</td>
<td>$f_{ue}$</td>
<td>#3 - #18</td>
<td>95</td>
<td>95</td>
</tr>
<tr>
<td>Expected yield strain</td>
<td>$\varepsilon_{ye}$</td>
<td>#3 - #18</td>
<td>0.0023</td>
<td>0.0023</td>
</tr>
<tr>
<td>Onset of strain hardening</td>
<td>$\varepsilon_{sh}$</td>
<td>#3 - #8</td>
<td>0.0150</td>
<td>0.0150</td>
</tr>
<tr>
<td></td>
<td></td>
<td>#9</td>
<td>0.0125</td>
<td>0.0125</td>
</tr>
<tr>
<td></td>
<td></td>
<td>#10 - #11</td>
<td>0.0115</td>
<td>0.0115</td>
</tr>
<tr>
<td></td>
<td></td>
<td>#14</td>
<td>0.0075</td>
<td>0.0075</td>
</tr>
<tr>
<td></td>
<td></td>
<td>#18</td>
<td>0.0050</td>
<td>0.0050</td>
</tr>
<tr>
<td>Reduced ultimate tensile strain</td>
<td>$\varepsilon_{su}^R$</td>
<td>#4 - #10</td>
<td>0.090</td>
<td>0.060</td>
</tr>
<tr>
<td></td>
<td></td>
<td>#11 - #18</td>
<td>0.060</td>
<td>0.040</td>
</tr>
<tr>
<td>Ultimate tensile strain</td>
<td>$\varepsilon_{su}$</td>
<td>#4 - #10</td>
<td>0.120</td>
<td>0.090</td>
</tr>
<tr>
<td></td>
<td></td>
<td>#11 - #18</td>
<td>0.090</td>
<td>0.060</td>
</tr>
</tbody>
</table>

SGS 2011
SGS Material Models - Steel

- ASTM A 706 Grade 60 v. ASTM A 615 Grade 60

<table>
<thead>
<tr>
<th>Property</th>
<th>ASTM A 706 Grade 60</th>
<th>ASTM A 615 Grade 60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile strength, min, psi [MPa]</td>
<td>80 000 [550]</td>
<td>90 000 [620]</td>
</tr>
<tr>
<td>Yield strength, min, psi [MPa]</td>
<td>60 000 [420]</td>
<td></td>
</tr>
<tr>
<td>Yield strength, max, psi [MPa]</td>
<td>78 000 [540]</td>
<td></td>
</tr>
<tr>
<td>Elongation in 8 in. [200 mm], min, %</td>
<td></td>
<td>60 000 [420]</td>
</tr>
</tbody>
</table>

Bar Designation Nos.

- 3, 4, 5, 6 [10, 13, 16, 19] 14
- 7, 8, 9, 10, 11 [22, 25, 29, 32, 36] 12
- 14, 18 [43, 57] 10

**A** Tensile strength shall not be less than 1.25 times the actual yield strength.
Quasi-static v. Cyclic Discussion

- Quasi-static constitutive models for cyclic response

Figure 6. Lateral Force vs. Top Column Displacement Response

\[ \phi_y \approx 2.25 \times 0.0024 = 0.00054 \]
\[ \phi_u \approx 0.03 = 0.00025 \]
\[ M_o \approx (0.05 + 0.025 + \frac{P_{Ce}}{E_o}) \cdot 5 = 11.75 \text{ kNm} \]
Moment – Curvature Example

- Find the $M-\phi$ for the column shown below

- ASTM A 706 Grade 60

- $f'_c = 4$ KSI

- $D = 48$ IN

- $L = H_o = 20$ FT

- #5 hoop @ 4IN

- 2 IN clr.

- 20 #11

- 940 KIP
Moment – Curvature Example

- Diameter, $D = 48$ IN
- Gross Area, $A_g = \pi D^2/4 = 1810$ IN$^2$
- 20 #11 $\Rightarrow A_{st} = 20*1.56 = 31.2$ IN$^2$
- $\rho_l = A_{st} / A_g = 0.01724 = 1.724\%$
- #5 hoop ($d_{sp} = 0.625$ IN and $A_{sp} = 0.31$ IN$^2$)
- Hoop spacing = 4 IN pitch ($s = 4$ IN)
- Clear cover, $cov = 2$ IN over transverse bars
- Core diameter, $D' = D - 2*cov - d_{sp} = 43.375$ IN
- $\rho_s = 4*A_{sp} / (D' * s) = 0.00715 = 0.715\%$
- ASTM A 706 Grade 60 so $f_{ye} = 68$ KSI, $f_{ue} = 95$ KSI
- $f'_{c} = 4$ KSI so $f'_{ce} = 5.2$ KSI
- $P = 940$ K Axial Load Ratio, ALR = $P / (f'_{ce} * A_g) = 0.1$
Moment – Curvature Example

• Confined concrete crushing strain limit, $\varepsilon_{cu}$

$$
\varepsilon_{cu} = 0.004 + \frac{1.4 \rho_s f_{yh} \varepsilon_{su}}{f'_{cc}} = 0.01232
$$

where:

$$
\rho_s = 4 * A_{sp}/(D'*s) = 0.00715
$$

$$
f'_{l} = K_e * \rho_s * f_{yh}/2 \sim 0.95*0.00715*60/2 = 0.204 \text{ KSI}
$$

$f_{yh} = 60 \text{ KSI}$ (use nominal for horizontal steel)

$\varepsilon_{su} = \varepsilon^R_{su} = 0.09$ for #5 hoops

$$
f'_{cc} = f'_{ce}\left(2.254\sqrt{1 + \frac{7.94 f'_{l}}{f'_{ce}} - \frac{2 f'_{l}}{f'_{ce}} - 1.254}\right) = 6.49
$$

• Spreadsheet demo then check with commercial
### M — φ Spreadsheet Example

#### Slice Summary

<table>
<thead>
<tr>
<th>Slice Number</th>
<th>Distance from Top</th>
<th>Dist to kd</th>
<th>Strain</th>
<th>Distance from D/2</th>
<th>Width Steel Out</th>
<th>Width Steel In</th>
<th>Area of Steel</th>
<th>Width Unconfined</th>
<th>Width Confined</th>
<th>Area Unconfined</th>
<th>Area Confined</th>
<th>Unconfined Stress</th>
<th>Confined Stress</th>
<th>Steel Stress</th>
<th>Unconfined Force</th>
<th>Confined Force</th>
<th>Steel Force</th>
<th>Unconfined Moment</th>
<th>Confined Moment</th>
<th>Steel Moment</th>
<th>Compression Total</th>
<th>Tension Total</th>
<th>Compression Moment</th>
<th>Tension Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.00</td>
<td>13.67</td>
<td>-0.014890</td>
<td>24.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>1</td>
<td>0.24</td>
<td>13.63</td>
<td>-0.014644</td>
<td>23.76</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>0.48</td>
<td>13.39</td>
<td>-0.014384</td>
<td>23.52</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>3</td>
<td>0.72</td>
<td>13.15</td>
<td>-0.014141</td>
<td>23.28</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>4</td>
<td>0.96</td>
<td>12.91</td>
<td>-0.013873</td>
<td>23.04</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>5</td>
<td>1.20</td>
<td>12.67</td>
<td>-0.013612</td>
<td>22.80</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>6</td>
<td>1.44</td>
<td>12.43</td>
<td>-0.013359</td>
<td>22.56</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>7</td>
<td>1.68</td>
<td>12.19</td>
<td>-0.013096</td>
<td>22.32</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>8</td>
<td>1.92</td>
<td>11.95</td>
<td>-0.012844</td>
<td>22.08</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**TOTAL**

- integrated area: 31.20
- actual rebar area: 31.20
- total integrated area: 1808.88
- integrated/actual adjustment: 100.0%
- actual column area: 1809.56
- integrated/actual adjustment: 100.0%

#### Summary of Numerical Analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neutral axis depth, kd</td>
<td>-13.87 IN</td>
</tr>
<tr>
<td>Equilibrium force sum total, P</td>
<td>-940.0 K</td>
</tr>
<tr>
<td>P/Ag*Ece</td>
<td>-0.10</td>
</tr>
<tr>
<td>Mid height, H/2</td>
<td>24 IN</td>
</tr>
<tr>
<td>Mid height - kd, eccentricity of P</td>
<td>10.1 IN</td>
</tr>
<tr>
<td>Compressive resultant, C</td>
<td>2482.3 K + P/2 = -2012.4</td>
</tr>
<tr>
<td>Tensile resultant, T</td>
<td>1542.36 K + P/2 = 2012.4</td>
</tr>
<tr>
<td>Lever arm, jD = dD</td>
<td>25.60 IN</td>
</tr>
<tr>
<td>Total Moment</td>
<td>54088 K-FT</td>
</tr>
<tr>
<td>Moment</td>
<td>54088 K-FT</td>
</tr>
<tr>
<td>Curvature, phi</td>
<td>0.00107744 IN</td>
</tr>
<tr>
<td>Curvature, phi</td>
<td>0.00107744 K-FT</td>
</tr>
<tr>
<td>Verification using approximate methods</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>48 IN</td>
</tr>
<tr>
<td>Asl</td>
<td>31.20 IN^2</td>
</tr>
<tr>
<td>Ag</td>
<td>1810 IN^2</td>
</tr>
<tr>
<td>rho_0</td>
<td>1.7242</td>
</tr>
<tr>
<td>M</td>
<td>50340891 K-IN^2 check at first yield</td>
</tr>
<tr>
<td>M</td>
<td>11619 IN^4 check at first yield</td>
</tr>
<tr>
<td>lefF</td>
<td>0.045</td>
</tr>
<tr>
<td>leff/F</td>
<td>11%</td>
</tr>
<tr>
<td>M_p</td>
<td>4599 K-FT 54713 K-IN</td>
</tr>
<tr>
<td>Approx idealized yield curvature</td>
<td>0.0001099 IN</td>
</tr>
<tr>
<td>Approx ultimate curvature</td>
<td>0.0012933 IN</td>
</tr>
<tr>
<td>T</td>
<td>2.7 Ast*Eye</td>
</tr>
</tbody>
</table>
Moment – Curvature Example

- Spreadsheet summary results for ALR = 0.1

<table>
<thead>
<tr>
<th>Curvature (1/IN)</th>
<th>Moment (K-IN)</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00000000</td>
<td>0</td>
<td>no moment condition</td>
</tr>
<tr>
<td>0.00004712</td>
<td>26265</td>
<td>half way to yield 0.001172</td>
</tr>
<tr>
<td>0.00008669</td>
<td>39554</td>
<td>Onset of first yield 0.002345</td>
</tr>
<tr>
<td>0.00012245</td>
<td>45265</td>
<td>compression strain 0.002</td>
</tr>
<tr>
<td>0.00020743</td>
<td>49638</td>
<td>compression strain 0.003</td>
</tr>
<tr>
<td>0.00029382</td>
<td>50817</td>
<td>compression strain 0.004 - spalling</td>
</tr>
<tr>
<td>0.00037104</td>
<td>49805</td>
<td>Onset of strain hardening 0.0115</td>
</tr>
<tr>
<td>0.00058923</td>
<td>51142</td>
<td>compression strain 0.008</td>
</tr>
<tr>
<td>0.00064380</td>
<td>51614</td>
<td>steel strain 0.02</td>
</tr>
<tr>
<td>0.00107460</td>
<td>54089</td>
<td>Concrete core crushing 0.01232</td>
</tr>
</tbody>
</table>
Idealized Moment – Curvature

• Elastic-perfectly plastic relationship is defined as:

\[ M_i = \text{idealized moment values} = E_{ce} I_{eff} \times \phi < M_p \]
\[ M_p = \text{idealized plastic moment (solving for this)} \]
\[ \phi = \text{actual calculated curvature from } M-\phi \text{ results} \]
\[ E_{ce} I_{eff} = \text{effective stiffness at first yield} = \frac{M_y}{\phi_y} \]
\[ \Delta(M\phi) = \text{actual - idealized} = (\Delta M) \times (\Delta \phi) \]

\[ \Sigma \Delta(M\phi) = \text{sum of difference} = 0 \text{ by changing } M_p \]

• Spreadsheet demo then check with commercial
# Idealized Moment – Curvature

<table>
<thead>
<tr>
<th>Predicted Curvature 1/IN</th>
<th>Predicted Moment K-IN</th>
<th>Idealized Moment K-IN</th>
<th>Difference between M-phi curves</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000000000</td>
<td>0</td>
<td>0</td>
<td>0.1123</td>
<td>no moment condition</td>
</tr>
<tr>
<td>0.00004712</td>
<td>26265</td>
<td>21498</td>
<td>0.0943</td>
<td>half way to yield 0.001172</td>
</tr>
<tr>
<td>0.00008669</td>
<td>39554</td>
<td>39554</td>
<td>-0.1097</td>
<td>Onset of first yield 0.002345</td>
</tr>
<tr>
<td>0.00012245</td>
<td>45265</td>
<td>51398</td>
<td>-0.3354</td>
<td>compression strain 0.002</td>
</tr>
<tr>
<td>0.00020743</td>
<td>49638</td>
<td>51398</td>
<td>-0.1011</td>
<td>compression strain 0.003</td>
</tr>
<tr>
<td>0.00029382</td>
<td>50817</td>
<td>51398</td>
<td>-0.0839</td>
<td>compression strain 0.004 - spalling</td>
</tr>
<tr>
<td>0.00037104</td>
<td>49805</td>
<td>51398</td>
<td>-0.2017</td>
<td>Onset of strain hardening 0.0115</td>
</tr>
<tr>
<td>0.00058923</td>
<td>51142</td>
<td>51398</td>
<td>-0.0011</td>
<td>compression strain 0.008</td>
</tr>
<tr>
<td>0.00064380</td>
<td>51614</td>
<td>51398</td>
<td>0.6262</td>
<td>steel strain 0.02</td>
</tr>
<tr>
<td>0.00107460</td>
<td>54089</td>
<td>51398</td>
<td></td>
<td>Concrete core crushing 0.01232</td>
</tr>
</tbody>
</table>

Sum Total Difference under curve = 0.0000

\[(E)_{\text{eff}} = \frac{M_y}{\phi-y} = 456,259,058 \quad \text{K-IN}^2 \quad \text{defined at onset of first yielding of extreme tension bar}
\]

\[M_p = \frac{51398}{\phi-y} \quad \text{K-IN} \quad \text{alter until area differential area is zero}
\]

\[\phi-y = \frac{M_p}{(E)_{\text{eff}}} = 0.0001127 \quad \text{K-IN} \quad \text{idealized curvature}\]
Moment – Curvature Example

• Idealized $M - \phi$ relationship using more data points

\[ P = 940 \text{ K} \]
\[ \phi_{yi} = 0.0001132 \, 1/\text{IN} \]
\[ \phi_u = 0.001075 \, 1/\text{IN} \]
\[ M_p = 51626 \, \text{K-IN} \]
\[ E_{ce \text{eff}} = 456.3E6 \, \text{K-IN}^2 \]
Moment – Curvature Example
Moment – Curvature Example

\[ P = 940 \text{ K} \]
\[ \phi_y = 0.0001109 \text{ 1/IN} \quad 2\% \]
\[ \phi_u = 0.001073 \text{ 1/IN} \quad 0.2\% \]
\[ M_p = 50460 \text{ K-IN} \quad 2\% \]
\[ E_{ce} I_{eff} = 454.85\text{E}6 \text{ K-IN}^2 \quad 0.3\% \]

Spreadsheet Comparison
\[ \phi_y = 0.0001132 \text{ 1/IN} \]
\[ \phi_u = 0.001075 \text{ 1/IN} \]
\[ M_p = 51626 \text{ K-IN} \]
\[ E_{ce} I_{eff} = 456.3\text{E}6 \text{ K-IN}^2 \]
Approximate Methods

- the use of curvature ($\phi$) in design is uncommon
- don’t have a good “feel” for curvature values
- simple method to check the computer results
- can help with iterative processes
- *use these approximations with due skepticism*
Approximate Methods - $\phi_{yi}$

- For a rough check of conventional circular reinforced concrete column sections:

$$\phi_{yi} \sim 2.25 \frac{\varepsilon_{ye}}{12B_o} \sim \frac{1}{2300}B_o$$

$$\phi_{yi} \sim 2.25 \frac{\varepsilon_{ye}}{D} \sim \frac{1}{190}D$$

where:

$\phi_{yi} = $ idealized yield curvature (1/IN)

$B_o = $ column diameter (FT)

$D = $ column diameter (IN)

$\varepsilon_{ye} = $ expected yield strain $\sim 0.002345$ (IN/IN)

Priestley, Calvi, Kowalsky 2007
Approximate Methods - $\phi_{yi}$

• For a rough check of conventional **rectangular** reinforced concrete column sections:

$$
\phi_{yi} \sim 2.1^* \varepsilon_{ye}/12B_o \sim 1/2500B_o
$$

where:

$\phi_{yi} = \text{idealized yield curvature (1/IN)}$

$B_o = \text{column width in loaded direction (FT)}$

$\varepsilon_{ye} = \text{expected yield strain} \sim 0.002345 \text{ (IN/IN)}$

*Priestley, Calvi, Kowalsky 2007*
Approximate Methods - $\phi_u$

• And for a *very rough check* of conventional **circular** or **rectangular** reinforced concrete column sections with good confinement:

$$\phi_u = \min \left( \frac{\varepsilon_{cu}}{c_c}, \frac{\varepsilon_{su}^R}{d-c} \right) \sim \frac{\varepsilon_{su}^R}{12B_o}$$

where:

- $\phi_u =$ ultimate curvature (1/IN)
- $\varepsilon_{cu} =$ ultimate confined concrete strain (1/IN)
- $\varepsilon_{su}^R =$ reduced ultimate tensile strain (IN/IN)
- $c_c =$ neutral axis to edge of confined core (IN)
- $d-c =$ neutral axis to extreme tension bar (IN)
- $B_o =$ column diameter / width (FT)
Effective Stiffness - $E_{ce} * I_{eff}$

$$E_{ce} * I_{eff} = M_y / \phi_y$$

where:

$E_{ce} = $ Expected modulus of elasticity for concrete

$M_y = $ moment at first yield (expected materials at first yield)

$\phi_y = $ curvature at first yield (expected materials at first yield)

So,

$$I_{eff} = M_y / (\phi_y * E_{ce})$$

And typically,

$$0.7 < I_{eff} / I_g < 0.3$$
Approximate Methods – $I_{eff}/I_g$

$I_{eff}/I_g \approx 0.2 + 0.1*\rho_l + 0.5*(P/f'_{ce} * A_g)$

where:

$\rho_l$ = longitudinal reinforcement ratio in %

$= A_{st}/A_g * 100 < 3\%$ practical limit

$P$ = axial load – keep below $0.2*A_g*f'_{ce}$

SGS 2011
Approximate Methods - $M_p$

$$M_p \sim D^3 \times [0.05 + 0.2 \rho_l + P/(f'_{ce} A_g)]$$

where:

- $M_p$ = idealized plastic moment for \textbf{circular} section (K-IN)
- $D$ = column diameter (IN) $> 30$ IN
- $\rho_l$ = longitudinal reinforcement ratio in \%
  - $= A_{st} / A_g \times 100$ $< 4\%$ but $3\%$ is practical limit
- $P$ = axial load on column (K) $< 0.2 \times f'_{ce} A_g$
- $f'_{ce}$ = expected concrete strength (KSI)
- $A_g$ = gross area of column (IN$^2$)

- Assumes ASTM A 706 Grade 60 reinforcing steel and $f'_{ce} = 5.2$
Approximate Methods - $M_p$

$$M_p \sim b*d^2 * [0.15 + 0.25*\rho_l + 1.5*P/(f'_{ce}*A_g)]$$

where:

- $M_p = \text{idealized plastic moment for rectangular section (K-IN)}$
- $d = \text{column depth in direction of loading (IN) > 30 IN}$
- $b = \text{column width (IN) > 30 IN}$
- $\rho_l = A_{st}/A_g*100 = \text{longitudinal reinforcement ratio in %}$
- $P = \text{axial load on column (K) < 0.2*f'_{ce}*A_g}$
- $f'_{ce} = \text{expected concrete strength (KSI)}$
- $A_g = b*d = \text{gross area of column (IN^2)}$

• Assumes ASTM A 706 Grade 60 reinforcing steel and $f'_{ce} = 5.2$
Moment – Curvature Check

• Diameter, $D = 48$ IN
• $I_g = D^4 \pi / 64 = 260576$ IN$^4$
• $\rho_l = A_{st} / A_g = 0.01724 = 1.724\%$
• Axial Load Ratio, ALR = $P / (f'_{ce} * A_g) = 0.1$ \([P \sim 940 \text{ K}]\)
• $\varepsilon_{ye} = f_{ye} / E_s = 68/29000 = 0.002345$ \(\varepsilon_{su}^R = 0.06 \text{ (#11)}\)

\[
\phi_{yi} \sim 2.25 \times 0.00234/48 = 0.000110 \, \text{1/IN v. 0.0001132 1/IN}
\]

\[
\phi_u \sim 0.06/48 = 0.00125 \, \text{1/IN v. 0.001075 1/IN}
\]

\[
l_{eff} \sim (0.2 + 0.1 \times 1.724 + 0.5 \times 0.1) \times 260576 = 0.42 \times 260576
\]

\[
l_{eff} \sim 109963 \, \text{IN}^4 \, v. 105307 \, \text{IN}^4 \, 4\%
\]

\[
M_p \sim 48^3 (0.05 + 0.2 \times 1.724 + 0.1)
\]

\[
M_p \sim 54713 \, \text{K-IN v. 51626 K-IN} \, \, 6\% 
\]
Moment – Curvature Check

Moment-Curvature Design Example

Approximate Method
Predicted Response
Idealized Response
XTRACT™ Response
• Calculate deflections using $M-\phi$ result

• Elastic deformation component, $\Delta_{yi}$

• Plastic deformation component, $\Delta_p$

• Foundation deformation component – important but not specifically addressed in this workshop

• Conservative to neglect shear deformations
Force - Displacement
Analytical Plastic Hinge Length - $L_p$

- Approximation used to simplify analysis
- Converts (integrates) curvature to rotation
- Includes a moment gradient part (integration), tension shift (diagonal cracking) and a strain penetration part (yielding into cap, footing or shaft)
- Calibrated to the failure condition only and modification may be needed for full strain-displacement response
Analytical Plastic Hinge Length - $L_p$

$L_p = k * L + L_{sp} > 2 * L_{sp}$

$L_p = 0.08 * L + 0.15 * f_{ye} * d_{bl} > 0.3 * f_{ye} * d_{bl}$

where:

$k = 0.2*(f_{ue}/f_{ye} - 1) \leq 0.08$

$L = \text{length of column from point of maximum moment to the point of moment contraflexure (IN)}$

$L_{sp} = \text{strain penetration component} = 0.15*f_{ye}*d_{bl} \text{ (IN)}$

$f_{ye} = \text{expected yield stress of longitudinal bars (KSI)}$

$f_{ue} = \text{expected tensile strength (KSI)}$

$d_{bl} = \text{diameter of longitudinal column bars (IN)}$
Predicted Force – Displacement

\[ \Delta_y \sim \frac{1}{3} \phi_y (L + L_{sp})^2 \]

\[ \Delta(M, \phi) \sim \Delta_y \left( \frac{M}{M_y} \right) + (\phi - \phi_y) L_p \left( L - L_p / 2 \right) \]

where:

- \( \phi_y \) = curvature at first yield
- \( L \) = column height
- \( L_p \) = analytical plastic hinge length
- \( L_{sp} \) = strain penetration
- \( \phi \) = curvature at point of interest
- \( M_y \) = moment at first yield
- \( M \) = moment associated with \( \phi \)
- \( F \) = force associated with \( \Delta = \frac{M}{L} \)

Priestley, Calvi, Kowalsky 2007
Idealized Force – Displacement

\[ \Delta_y \sim \frac{1}{3} \phi_y \phi_y (L+L_{sp})^2 \]

\[ \Delta^L_C = \Delta_y + \Delta_p \sim \Delta_y + (\phi_u - \phi_y) \phi_y \phi_y (L-L_p/2) \]

where:

- \( \phi_y \) = idealized yield curvature
- \( \Delta_y \) = idealized yield displacement
- \( \Delta_p \) = plastic displacement capacity
- \( L \) = column height
- \( L_p \) = analytical plastic hinge length
- \( L_{sp} \) = strain penetration
- \( \phi_u \) = ultimate curvature
- \( \Delta^L_C \) = ultimate displacement
- \( F_p \) = plastic force \( = \frac{M_p}{L} \)

Priestley, Calvi, Kowalsky 2007
Idealized Force - Displacement

• So the deformation values for the example problem are:

\[ \Delta_{yi} = \frac{1}{3} \times 0.0001132 \times (240 + 14.38)^2 = 2.44 \text{ IN} \]

\[ \Delta_{C} = \Delta_{yi} + (0.001075 - 0.0001132) \times 33.58 \times (240 - 33.58/2) = 9.62 \text{ IN} \]

where:

\[ \phi_{yi} = 0.0001132 \text{ 1/IN} \]

\[ L = 20 \text{ FT} = 240 \text{ IN} \]

\[ L_{sp} = 0.15 \times 68 \times 1.41 = 14.38 \text{ IN} \]

\[ L_{p} = 0.08 \times 240 + 14.38 = 33.58 \text{ IN} > 28.8 \text{ IN} \]

\[ \phi_{u} = 0.001075 \text{ 1/IN} \]

\[ M_{p} = M_{yi} = M_{u} = 51626 \text{ K-IN} \]

\[ F_{p} = \frac{M_{p}}{L} = \frac{51626}{240} = 215 \text{ KIP} \]

\[ \mu_{D} = \frac{\Delta_{C}}{\Delta_{yi}} = \frac{9.62}{2.44} = 3.94 \]
The idealized response for the single column example

Approximate $M_p$, $\phi_{yi}$ and $\phi_u$

Idealized Spreadsheet $M_p$, $\phi_{yi}$ and $\phi_u$

Predicted Response from Spreadsheet calculated $M-\phi$
Approximate Methods

• As with $M-\phi$, we prefer a simplified method to check the computer results

• simple method to check the computer results

• could use the closed-form AASHTO equations to start but they are developed for specific target ductility / strain limits

• use these approximations with due skepticism
Approximate Methods - $\Delta_{yi}$

- For a rough check of conventional **circular** reinforced concrete column sections:

  \[ \Delta_{yi} \sim \frac{1}{3} \phi_{yi} (12L + 0.15f_{ye}d_b)^2 \sim L^2 / 42B_o \]

where:

- $\Delta_{yi} =$ idealized yield displacement (IN)
- $L =$ contraflexure to plastic hinge distance (FT)
- $d_b =$ diameter of longitudinal column bar (IN)
- $\phi_{yi} =$ idealized yield curvature $\sim 2.25 \varepsilon_{ye} / B_o (1/IN)$
- $B_o =$ column diameter (FT)
- $f_{ye} =$ expected yield stress (KSI)
Approximate Methods - $\Delta_y$:

- For a rough check of conventional **rectangular** reinforced concrete column sections:

  $$\Delta_y \sim \frac{1}{3} \phi_y (12L + 0.15f_{ye}d_b)^2 \sim \frac{L^2}{45B_o}$$

where:

- $\Delta_y$ = idealized yield displacement (IN)
- $L$ = contraflexure to plastic hinge distance (FT)
- $d_b$ = diameter of longitudinal column bar (IN)
- $\phi_y$ = idealized yield curvature $\sim 2.1\varepsilon_{ye}/12B_o$ (1/IN)
- $B_o$ = column width in direction of loading (FT)
- $f_{ye}$ = expected yield stress (KSI)
Approximate Methods - $\Delta_C^L$

• And for a very rough check of conventional reinforced concrete columns with $L/B_o > 4$:

$$\Delta_C^L \sim \Delta_{yi} + (\phi_u - \phi_{yi}) \times L_p \times (12 \times L - L_p / 2) \sim L^2 / 10B_o$$

where:

- $\Delta_C^L$ = local displacement capacity (IN)
- $L$ = contraflexure to plastic hinge distance (FT)
- $L_p$ = analytical plastic hinge length (IN)
- $\phi_u$ = ultimate curvature $\sim \varepsilon_{su}^R / 12B_o$ (1/IN)
- $\phi_{yi}$ = idealized yield curvature (1/IN)
- $B_o$ = column diameter / width (FT)
Force-Displacement Check

• Diameter, \( D = 48 \) IN \( \Rightarrow B_o = 4 \) FT

• Height, \( L = H_o = 20 \) FT

• \( M_p \sim 54713 \text{K-IN}/12 = 4560 \text{ K-FT} \) (see previous check)

\[
\Delta_{yi} \sim \frac{L^2}{42B_o} = \frac{20^2}{(42*4)} = 2.4 \text{ IN v. } 2.4 \text{ IN}
\]

\[
\Delta_{Lc} \sim \frac{L^2}{10B_o} = \frac{20^2}{(10*4)} = 10 \text{ IN v. } 9.6 \text{ IN}
\]

\[ F_p = \frac{M_p}{L} = \frac{4560}{20} = 228 \text{ KIP v. } 215 \text{ KIP } \]
Force - Displacement

- Idealized response for the single column example

Approximate $M_p$, $\Delta y_i$ and $\Delta L_C$

Spreadsheet calculated $M_p$, $\phi_{yi}$ and $\phi_u$
What about Double Curvature?

- Effective column height, $L$, is taken from the maximum moment location to the contraflexure point.
- Then add the displacement results for each part.
What about $P-\Delta$ Effects?

From equilibrium and summing moments about the base of the column:

$$M_p = F_p * H_o + P * \Delta$$
What about P-Δ Effects?

• The moment left to resist lateral forces becomes,

\[ M'_p = M_p - P \Delta \]

\[ F'_p = \frac{M'_p}{H_o} \]

• But when using an idealized elastic perfectly-plastic force-displacement relationship check (SGS method)

\[ P_{dl} \Delta_r \leq 0.25 \times M_p \]
What about P-Δ Effects?

- Adjusted response for the single column example
What about P-\(\Delta\) Effects?

• Rearranging the \(P-\Delta\) limit the maximum permissible deflection for the single column example

\[ \Delta_r \leq 0.25 \times \frac{M_p}{P_{dl}} \]

\[ \Delta_r \leq 0.25 \times 51626 \text{ K-IN} / 940 \text{ K} = 13.7 \text{ IN} \]

• In this case, the \(P-\Delta\) limit is greater than the calculated \(\Delta^L_C\) value

• With more confinement, the \(P-\Delta\) limit may govern
Local Ductility v. Global Ductility

- Use *local* member displacements

\[ \Delta_D^L < \Delta_C^L \]

\[ \mu_D = \Delta_D^L / \Delta_{yi} \]

where:
- \( \Delta_D^L \) = Local member deformation demand
- \( \Delta_C^L \) = Local member deformation capacity
- \( \mu_D \) = local member displacement ductility demand
- \( \Delta_{yi} \) = idealized yield deformation
Local Ductility v. Global Ductility

- Must remove the deformation components associated with non-column deformation demands
Design Example

- $D = 48$ IN
- $H_o = 24$ FT
- $Z = 32$ FT
- $d = 4$ FT
- $2$ IN clr.
- 20 #11
- #5 hoop @ 4IN
- 940 KIP
- Rigid cap and footings
Design Example

• Assume that the point of contraflexure is at column mid-height

\[ L_1 = L = \frac{H_o}{2} \]

\[ L_2 = L = \frac{H_o}{2} \]
Design Example – Predict Response

- Use approximate methods to predict expected response
- Diameter, $D = 48$ IN $\Rightarrow B_o = 4$ FT
- Height, $H_o = 24$ FT so for the transverse direction, $L = 12$ FT
- $P = P_{DL} \pm P_{EQ}$
- $P_{EQ} = (M_{p-L} + M_{p-R}) \times (1 + d / H_o) / Z$ \(\text{why?}\)
- Use $P = P_{DL} = 940$ K for first iteration

- $M_p \sim 54720$K-IN/12 = 4560 K-FT (see previous calculations)

\[ \Delta y_i \sim 2 \times L^2 / 42B_o = 2 \times 12^2 / (42 \times 4) = 1.71 \text{ IN} \]

\[ \Delta L_C \sim 2 \times L^2 / 10B_o = 2 \times 12^2 / (10 \times 4) = 7.2 \text{ IN} \]

\[ F_p = (2 \times M_{p-L} + 2 \times M_{p-R}) / H_o \sim 4 \times M_p / 24 = 760 \text{ KIP} \ \text{why?} \]
Design Example - FBD

Moment Diagram – slope of line is the shear – shear in cap beam is axial force in column
Design Example – Predict Response

- Perform a second iteration to verify initial assumptions

- \( P_{EQ} = (M_{p-L} + M_{p-R}) \times (1 + d/H_o) / Z \sim 2 \times 54720 \times 1.167 / 12 / 32 \sim 332 \text{ K} \)

- \( P = P_{DL} \pm P_{EQ} = 940 -/+ 332 = 608\text{K} \) and \( 1272\text{K} \) [compression]

- \( M_{p-L} \sim (0.05 + 0.2 \times 1.724 + 0.0643) \times 48^3 = 50808 \text{ K-IN} \)

- \( M_{p-R} \sim (0.05 + 0.2 \times 1.724 + 0.130) \times 48^3 = 58612 \text{ K-IN} \)

- \( P_{EQ} = (M_{p-L} + M_{p-R}) \times (1 + d / H_o) / Z \)
- \( P_{EQ} = (50808 + 58612)(1.167) / 32 / 12 = 332 \text{ K} \checkmark \)

- \( F_p = (2 \times M_{p-L} + 2 \times M_{p-R}) / H_o = 760 \text{ KIP} \checkmark \)
Design Example - Process

• Now use the refined analysis to determine the force-displacement response of the pier

• First calculation idealized $M-\phi$

• Then calculate $L_{sp}$ and $L_{p}$

• Then calculate the force-displacement for each column

• Then add the results of each column to find the total pier response
Design Example – Refined Analysis

Moment - Curvature

<table>
<thead>
<tr>
<th>P</th>
<th>(EI)eff</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>-608</td>
<td>435,542,753</td>
<td>K-IN^2</td>
</tr>
<tr>
<td>Mp</td>
<td>48172</td>
<td>K-IN</td>
</tr>
<tr>
<td>PHI-yi</td>
<td>0.00011060</td>
<td>1/IN</td>
</tr>
<tr>
<td>PHI-ult</td>
<td>0.0011844</td>
<td>1/IN</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>P</th>
<th>(EI)eff</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1272</td>
<td>491,250,000</td>
<td>K-IN^2</td>
</tr>
<tr>
<td>Mp</td>
<td>55174</td>
<td>K-IN</td>
</tr>
<tr>
<td>PHI-yi</td>
<td>0.00011231</td>
<td>1/IN</td>
</tr>
<tr>
<td>PHI-ult</td>
<td>0.009898</td>
<td>1/IN</td>
</tr>
</tbody>
</table>
Design Example – Refined Analysis

• Verify that the axial forces are reasonably close to the initial estimate

\[ P_{EQ} = (M_{p-L} + M_{p-R}) \times (1 + \frac{d}{H_o}) / Z \]

\[ P_{EQ} = (48172+55174) \times 1.167 / (12\times32) \sim 314 \text{ K v. 332k} \]

\[ P = P_{DL} - P_{EQ} = 940 - 314 = 626 \text{K v. 608K – close enough @ 3%} \]

\[ P = P_{DL} + P_{EQ} = 940 + 314 = 1254 \text{K v. 1272K – close enough @ 2%} \]

• Use the initial values since they are close
Design Example – Hinge Formation

- Order of hinge formation and hinge failure
Design Example – Displacement

- For the left / trailing side column:

\[ \Delta_{yi-L} = \frac{1}{3} \times 0.0001106 \times (144+14.38)^2 = 0.92 \text{ IN} \]

\[ \Delta_{L_{C-L}} = 0.92 + (0.0011844 - 0.0001106) \times 28.8 \times (144 - 28.8/2) = 4.92 \text{ IN} \]

where:

- \( \phi_{yi} = 0.0001106/\text{IN} \) (from \( M-\phi \) analysis)
- \( L = H_o / 2 = 24 \text{ FT} / 2 \times 12 = 144 \text{ IN} \)
- \( L_{sp} = 0.15 \times 68 \times 1.41 = 14.38 \text{ IN} \)
- \( L_p = 0.08 \times 144 + 14.38 = 25.9 \text{ IN} < 2 \times L_{sp} = 28.8 \text{ IN} \)
- \( \phi_u = 0.0011844 \text{ 1/IN} \) (from \( M-\phi \) analysis)
- \( M_p = 48172 \text{ K-IN} \) (from \( M-\phi \) analysis)
- \( F_{p-L} = M_p / L = 48172 / 144 = 335 \text{ KIP} \)
Design Example – Displacement

• For the right / right side column:

\[ \Delta_{yi-R} = \frac{1}{3} \times 0.00011231 \times (144+14.38)^2 = 0.94 \text{ IN} \]

\[ \Delta_{Lc-R} = 0.94 + (0.0009898 - 0.00011231) \times 28.8 \times (144-28.8/2) = 4.21 \text{ IN} \]

where:

\[ \phi_{yi} = 0.00011231/\text{IN} \ (\text{from } M-\phi \text{ analysis}) \]

\[ L = \frac{H_o}{2} = 24 \text{ FT} / 2 \times 12 = 144 \text{ IN} \]

\[ L_{sp} = 0.15 \times 68 \times 1.41 = 14.38 \text{ IN} \]

\[ L_p = 0.08 \times 144 + 14.38 = 25.9 \text{ IN} < 2 \times L_{sp} = 28.8 \text{ IN} \]

\[ \phi_u = 0.0009898 \ 1/\text{IN} \ (\text{from } M-\phi \text{ analysis}) \]

\[ M_p = 55174 \text{ K-IN} \ (\text{from } M-\phi \text{ analysis}) \]

\[ F_{p-R} = \frac{M_p}{L} = \frac{55174}{144} = 383 \text{ KIP} \]
# Design Example

<table>
<thead>
<tr>
<th></th>
<th>Left Column</th>
<th>Right Column</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial compression, $P$</td>
<td>608</td>
<td>1272</td>
</tr>
<tr>
<td>Effective stiffness, $(EI)_{eff}$</td>
<td>4.36E9</td>
<td>4.91E9</td>
</tr>
<tr>
<td>Plastic moment, $M_p$</td>
<td>48172</td>
<td>55174</td>
</tr>
<tr>
<td>Idealized yield curvature, $\phi_{yi}$</td>
<td>0.00011060</td>
<td>0.00011231</td>
</tr>
<tr>
<td>Ultimate curvature, $\phi_u$</td>
<td>0.0011844</td>
<td>0.0009898</td>
</tr>
<tr>
<td>Plastic curvature, $\phi_p$</td>
<td>0.001074</td>
<td>0.000877</td>
</tr>
<tr>
<td>Analytical Plastic Hinge Length, $L_p$</td>
<td>28.8</td>
<td>28.8</td>
</tr>
<tr>
<td>$\Delta_{yi}$ (per half of column height)</td>
<td>0.92</td>
<td>0.94</td>
</tr>
<tr>
<td>$\Delta_p$ (per half of column height)</td>
<td>4.00</td>
<td>3.27</td>
</tr>
<tr>
<td>$\Delta_u$ (per half of column height)</td>
<td>4.92</td>
<td>4.21</td>
</tr>
<tr>
<td>Plastic shear, $F_p$</td>
<td>335</td>
<td>383</td>
</tr>
</tbody>
</table>
Design Example – Combined Response

• Total effective lateral force

\[ F_p = F_{p-L} + F_{p-R} = 335K + 383K = 718K \]

• Idealized yield displacement

\[ \Delta_{yi} \sim \frac{(2 \Delta_{yi-L} + 2 \Delta_{yi-R})}{2} = 0.92 + 0.94 = 1.86 \text{ IN} \]

• Ultimate displacement (first hinge failure on right)

\[ \Delta^L_c = 2 \ast \Delta^L_{c-R} = 2 \ast 4.21 \text{ IN} = 8.42 \text{ IN} \]
Design Example – Idealized Response

Example 1 Transverse Force-Displacement

Combined

Right / Leading Column

Left / Trailing Column
Design Example – Refined Response

- Idealized force displacement for the two column pier

Approximate $M_p$, $\Delta y_i$ and $\Delta^L_C$

Spreadsheet

Idealized $M_p$, $\phi_{yi}$ and $\phi_u$
Design Example – Verification

• The maximum deflection based on $P$-$\Delta$ limits

$$\Delta_r \leq 0.25 \times 48172 \text{ K-IN} / 940 \text{ K} = 12.8 \text{ IN} \checkmark \text{ (per half)}$$

• And the displacement ductility capacity

$$\mu_D < \frac{\Delta_c}{\Delta_{yi}} = 8.42 / 1.86 = 4.5$$
Capacity Design

• Designer dictates the bridge response (e.g., column hinge response mechanism – Type I)

• Preclude all failure modes that would prevent the formation of the predicted force-deflection response including:
  – Shear failure inside and outside the plastic hinge region (not the same as analytical plastic hinge length, $L_p$)
  – Column-cap and column-footing joint failure
  – Cap beam, footing or shaft moment, shear, and axial overload / hinging
  – Footing overturning, sliding, uplift and rocking failure
  – Any other failure that occurs prior to reaching $\Delta_{LC}$
Thank you - Questions