

# Model-based Optimal Control of Autonomous Mobility-on-Demand Systems in Multi-modal Transportation Networks

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Mauro Salazar

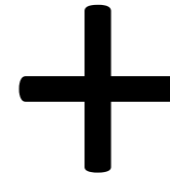
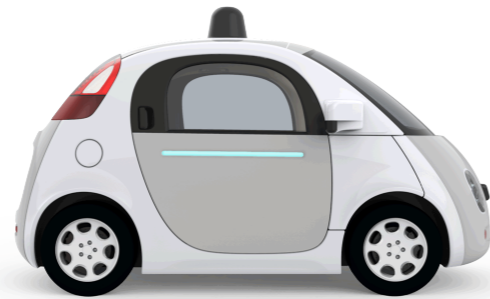
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Contributors: Ramon Iglesias, Matthew Tsao,  
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# Optimal Operation of Intermodal AMoD Systems

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## Vehicle Autonomy



## Car Sharing



# Optimal Operation of Intermodal AMoD Systems

## Vehicle Autonomy



## Car Sharing



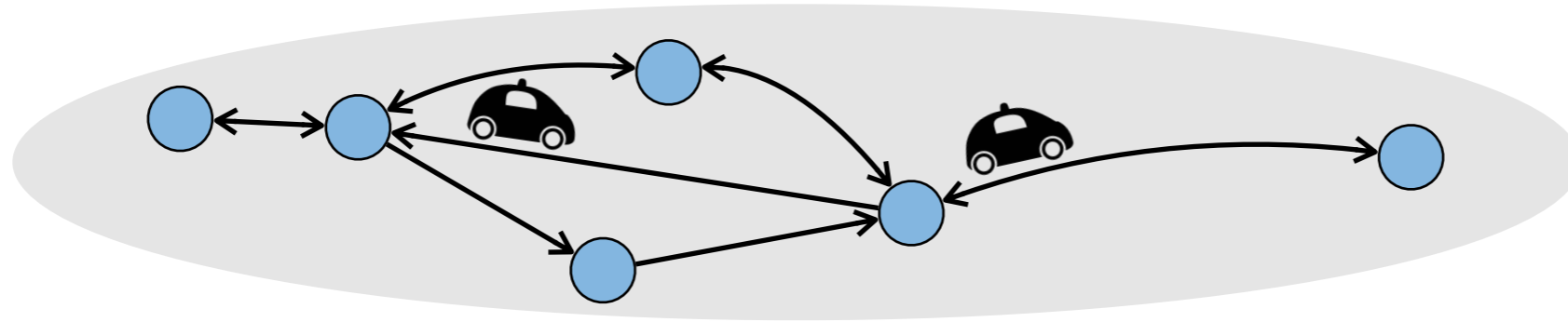
## Public Transit



# Intermodal Autonomous Mobility-on-Demand

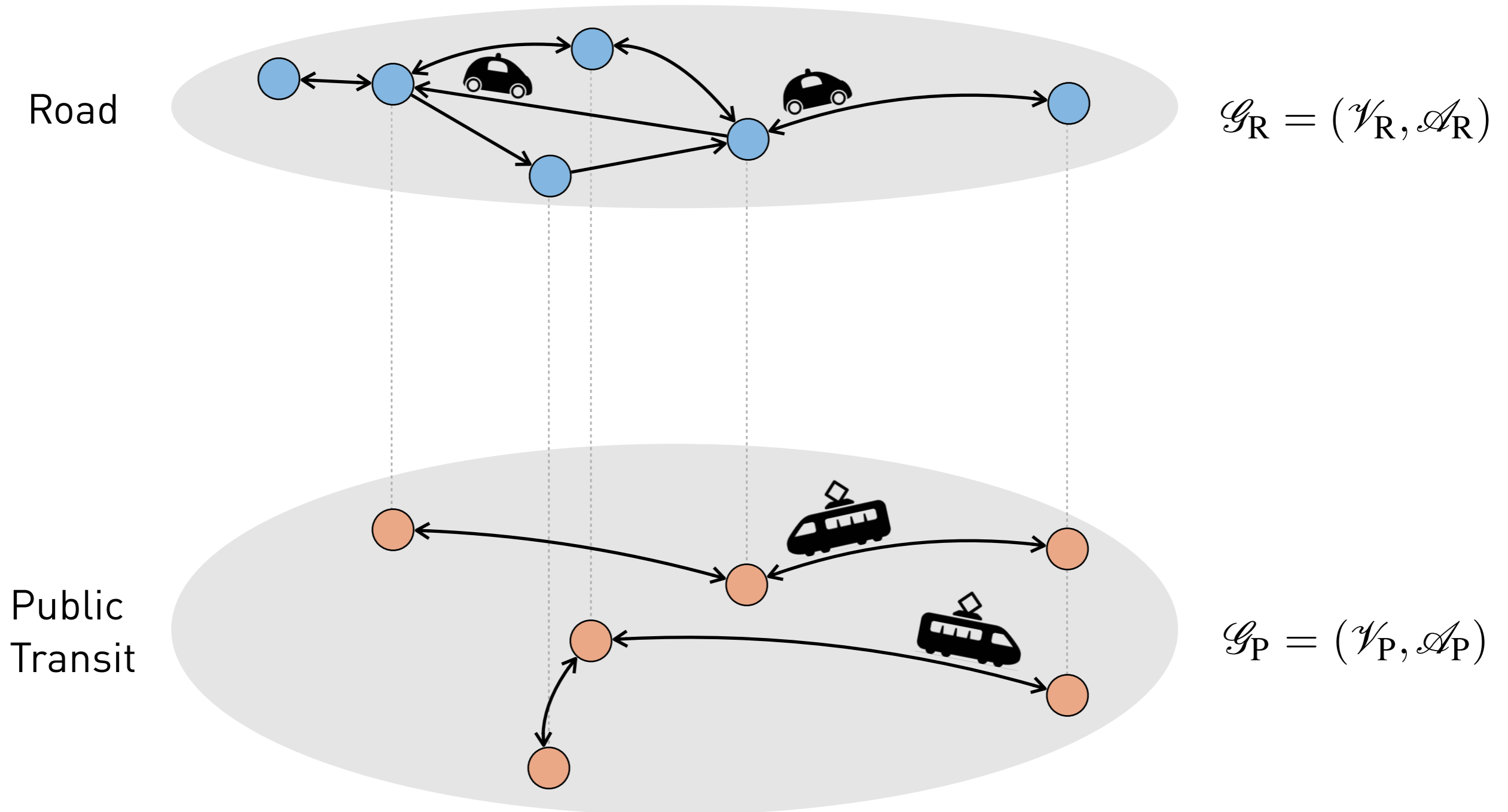
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Road



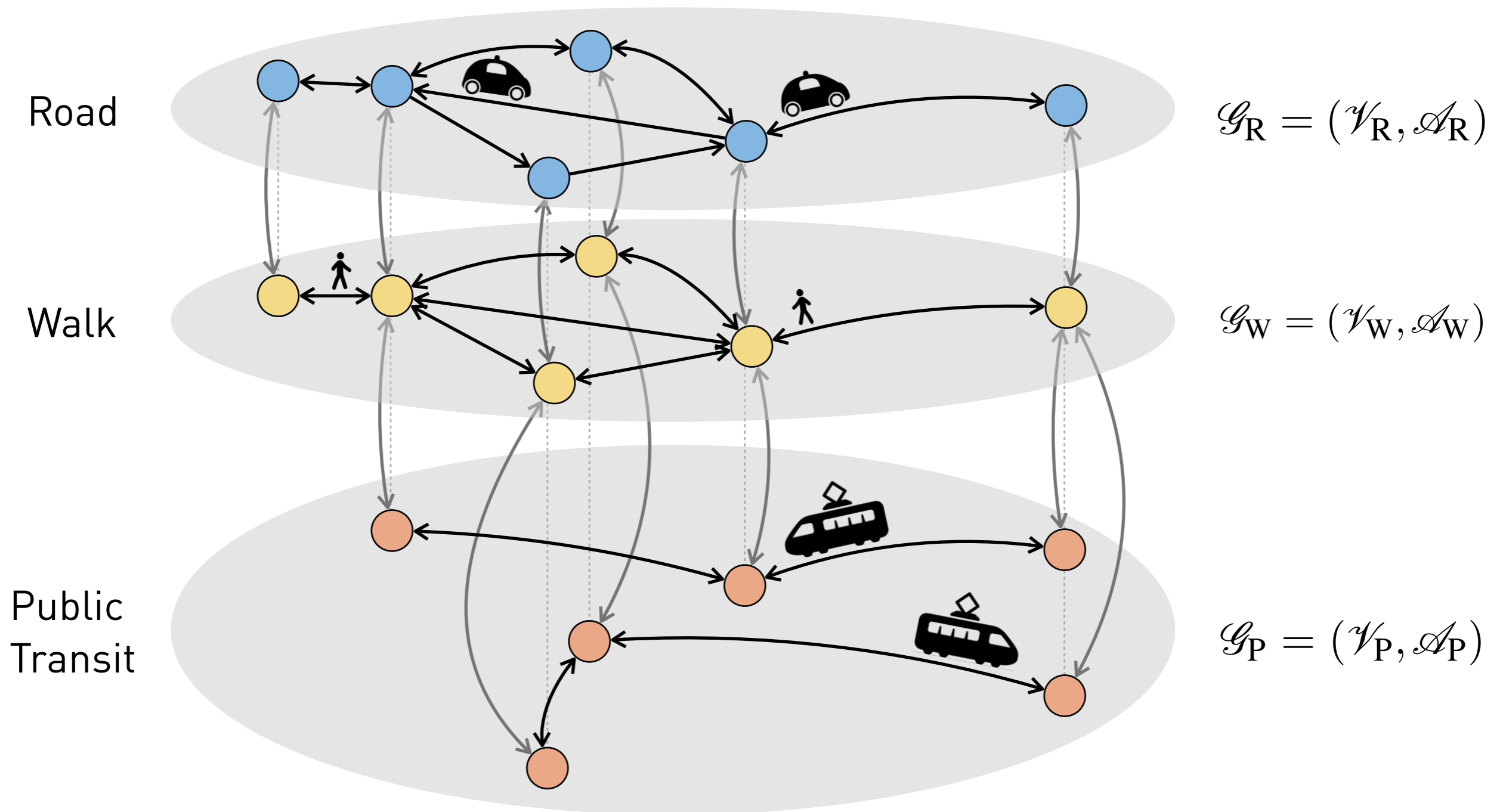
$$\mathcal{G}_R = (\mathcal{V}_R, \mathcal{A}_R)$$

# Intermodal Autonomous Mobility-on-Demand





# Intermodal Autonomous Mobility-on-Demand



# Network Flow Model

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## Advantages

- Highly scalable (LP)
- Very expressive

# Network Flow Model

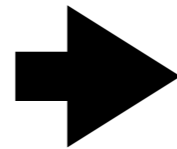
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## Advantages

- Highly scalable (LP)
- Very expressive

## Assumptions

- No stochasticity
- Continuum approximation
- One passenger per car



Stochastic process in expectation  
[Iglesias et al. 2018]

Flow decomposition and sampling

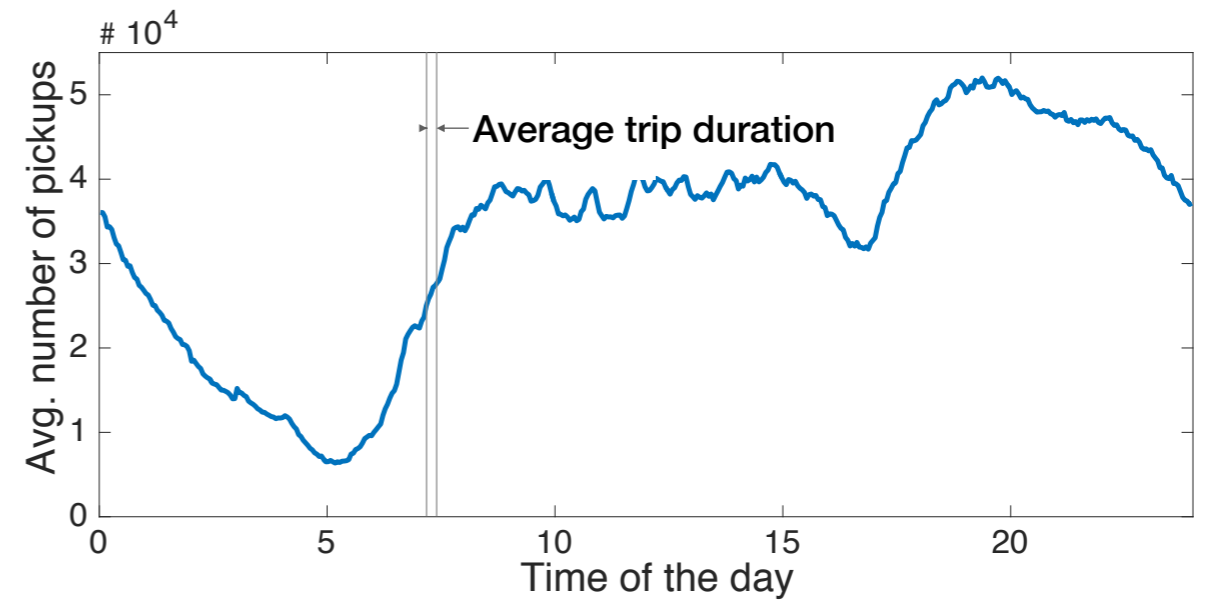
In line with current trends



# Network Flow Model - Assumptions

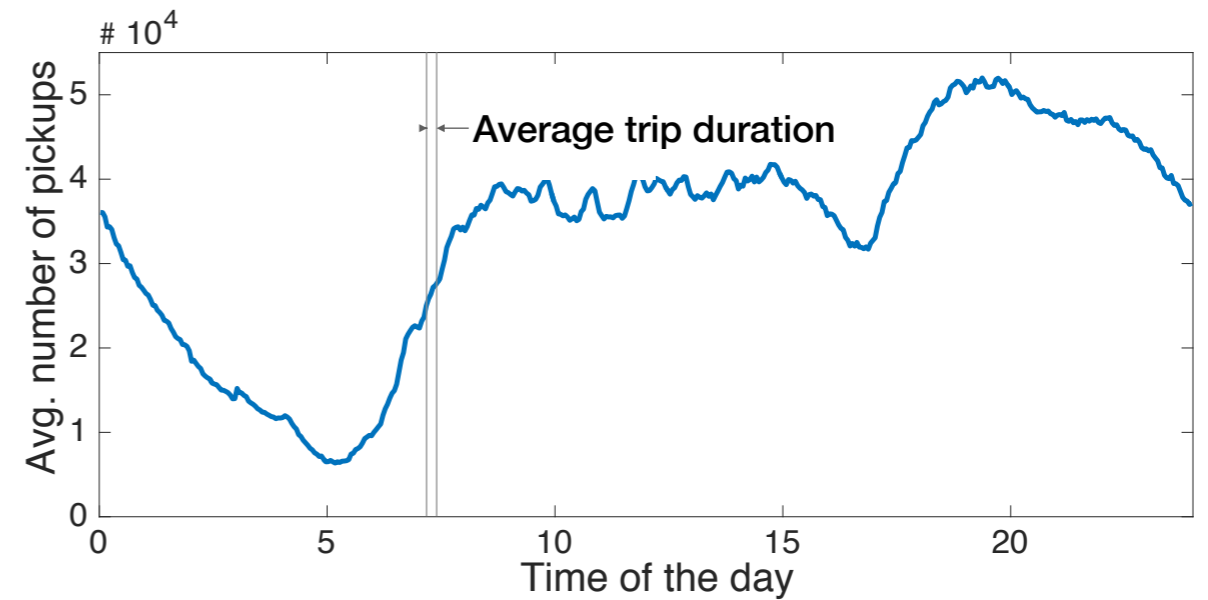
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- Demand is time-invariant

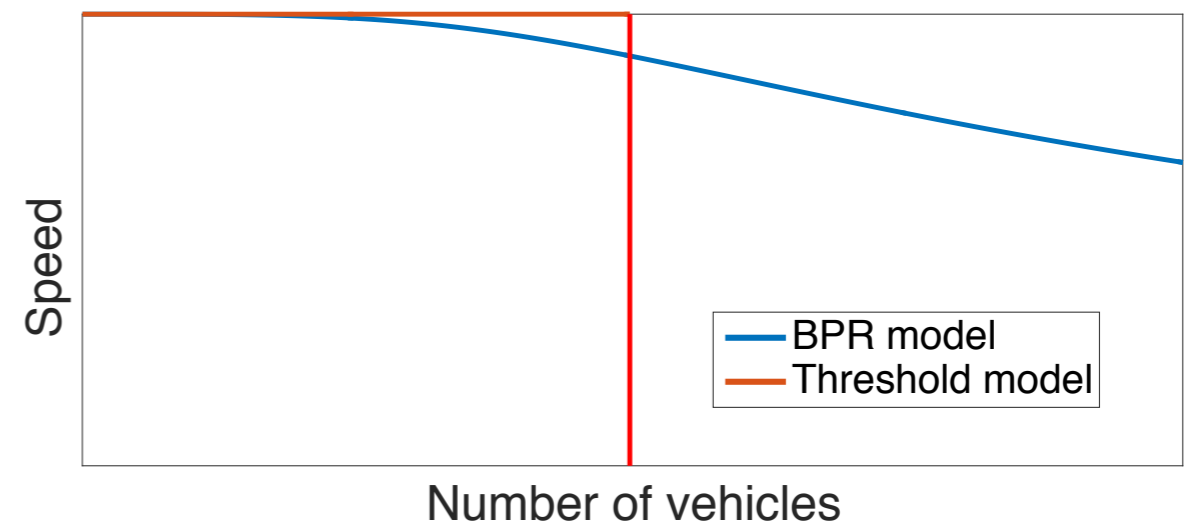


# Network Flow Model - Assumptions

- Demand is time-invariant



- Congestion as a threshold

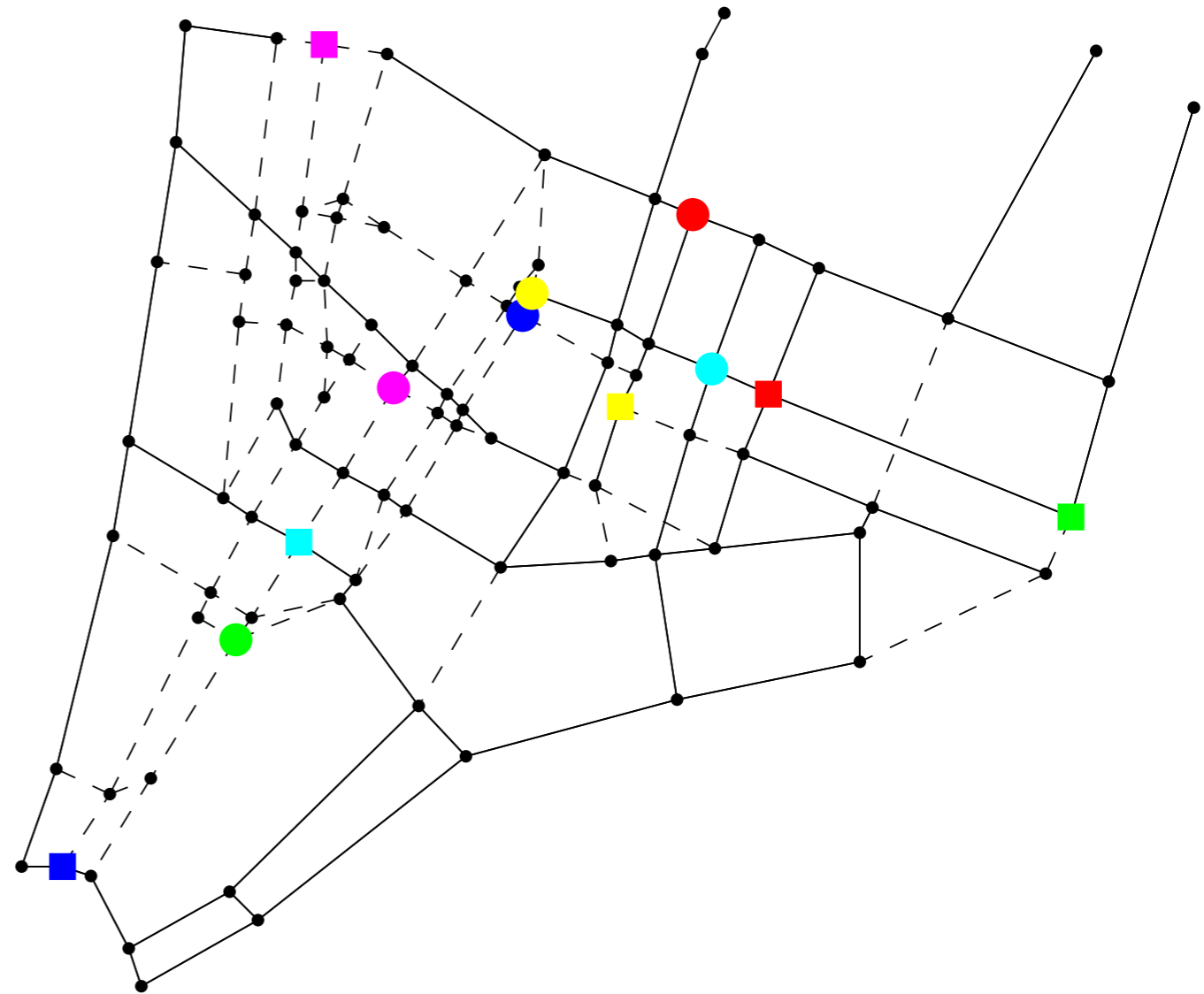


# Network Flow Model

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Transportation requests

- Origin
- Destination
- Rate of demand (customers/minute)



# Network Flow Model

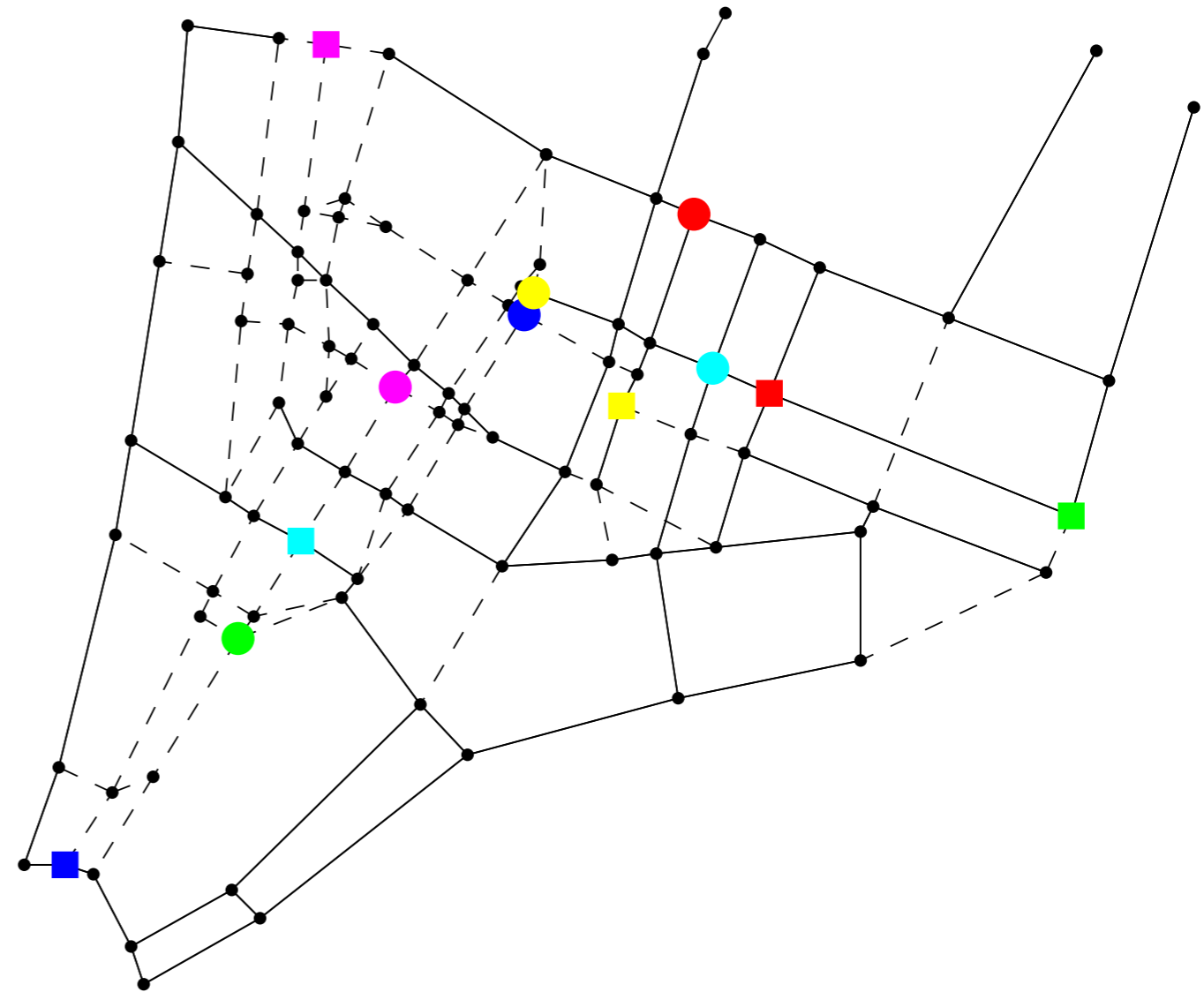
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## Transportation requests

- Origin
- Destination
- Rate of demand (customers/minute)

## Network model

- Nodes: intersections and stops
- Capacitated arcs: roads, walk, switch and public transit

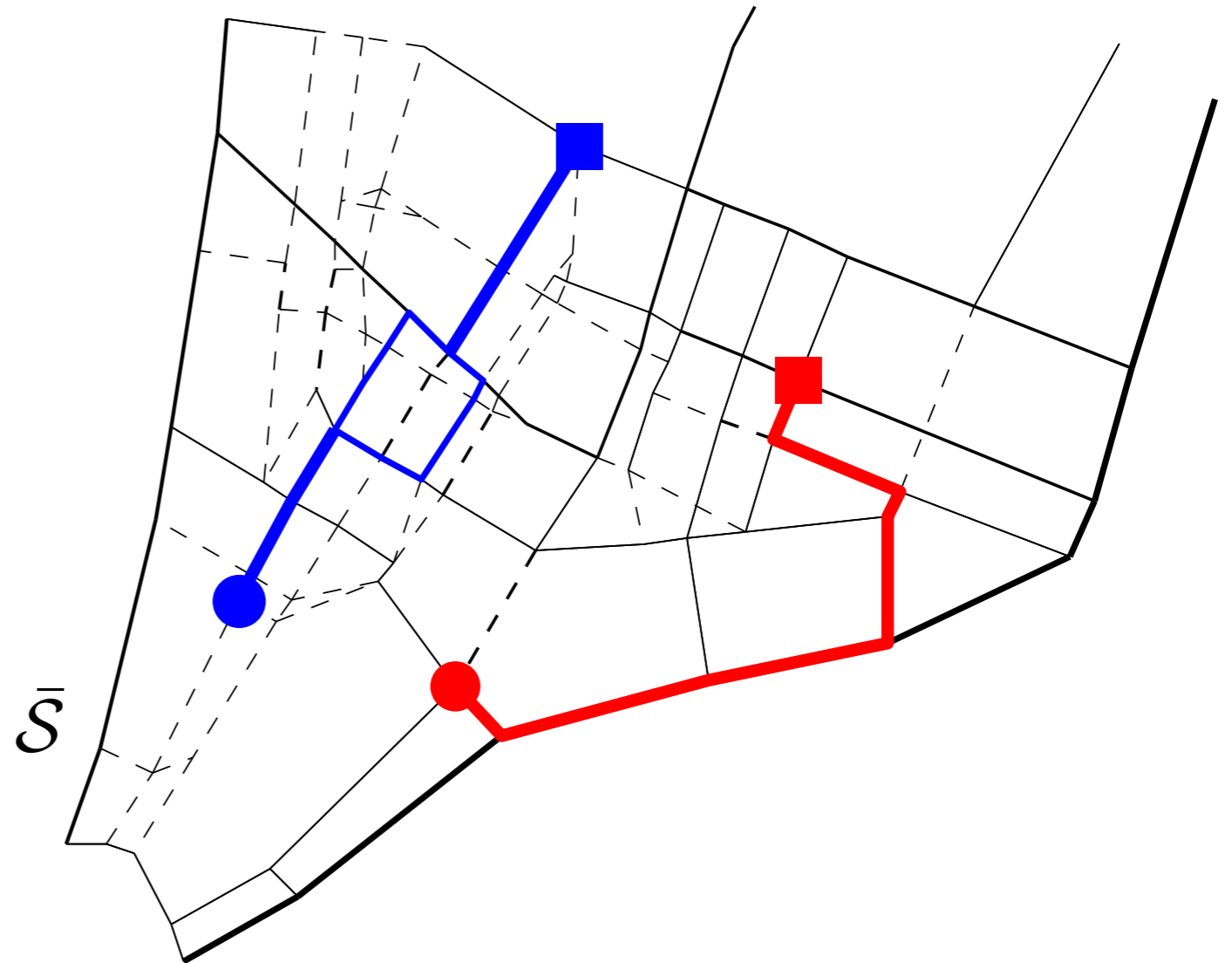


# Network Flow Model - Assumptions

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## Flows

- Customer flows  $f_m(i, j)$
- Rebalancing flows

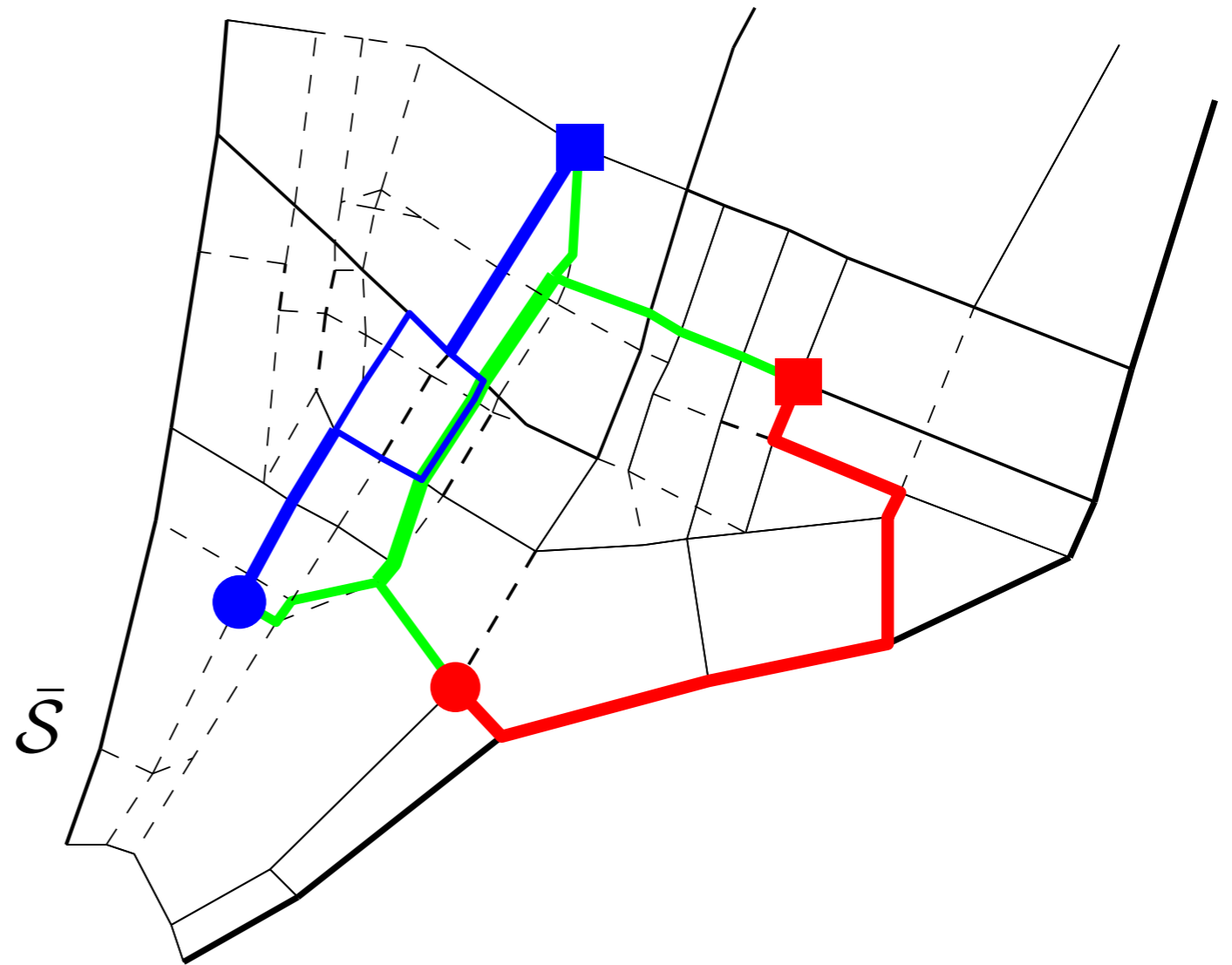


# Network Flow Model - Assumptions

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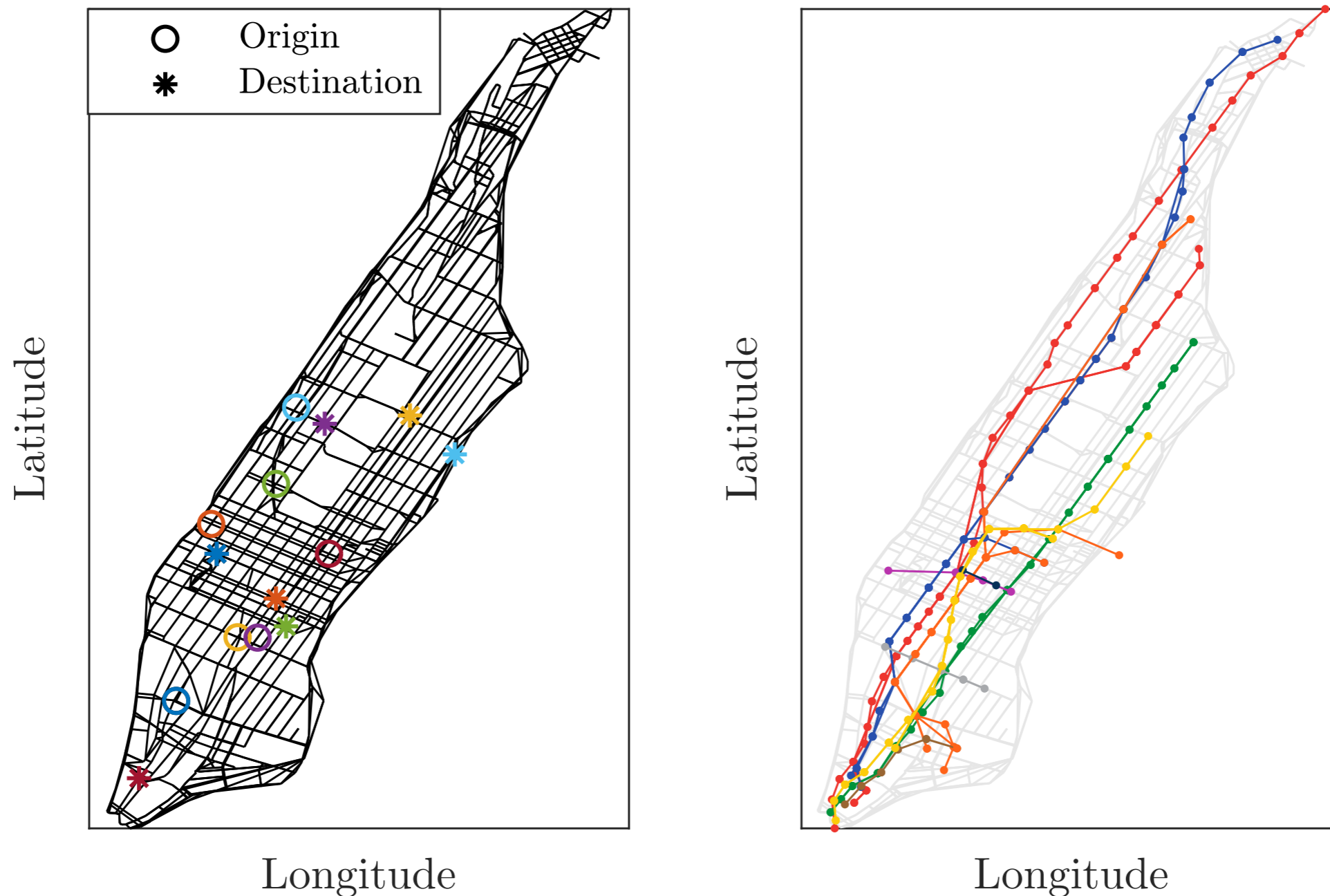
## Flows

- Customer flows
- Rebalancing flows  $f_0(i, j)$



# Intermodal AMoD - Full Graph - Manhattan

54000 taxi rides during rush hour, distributed in 6774 origin destination pairs

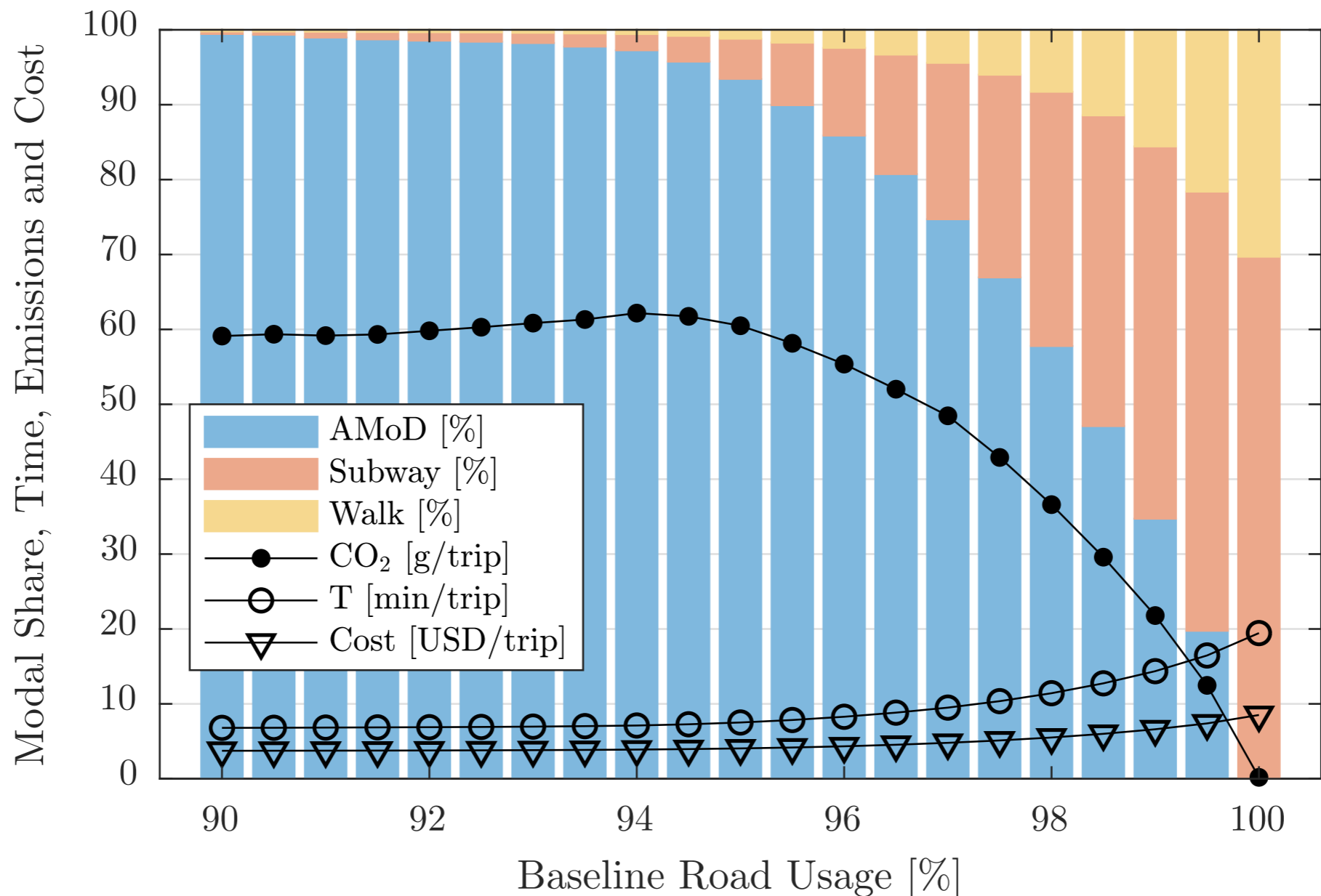


Compute optimal control strategies to maximise social welfare



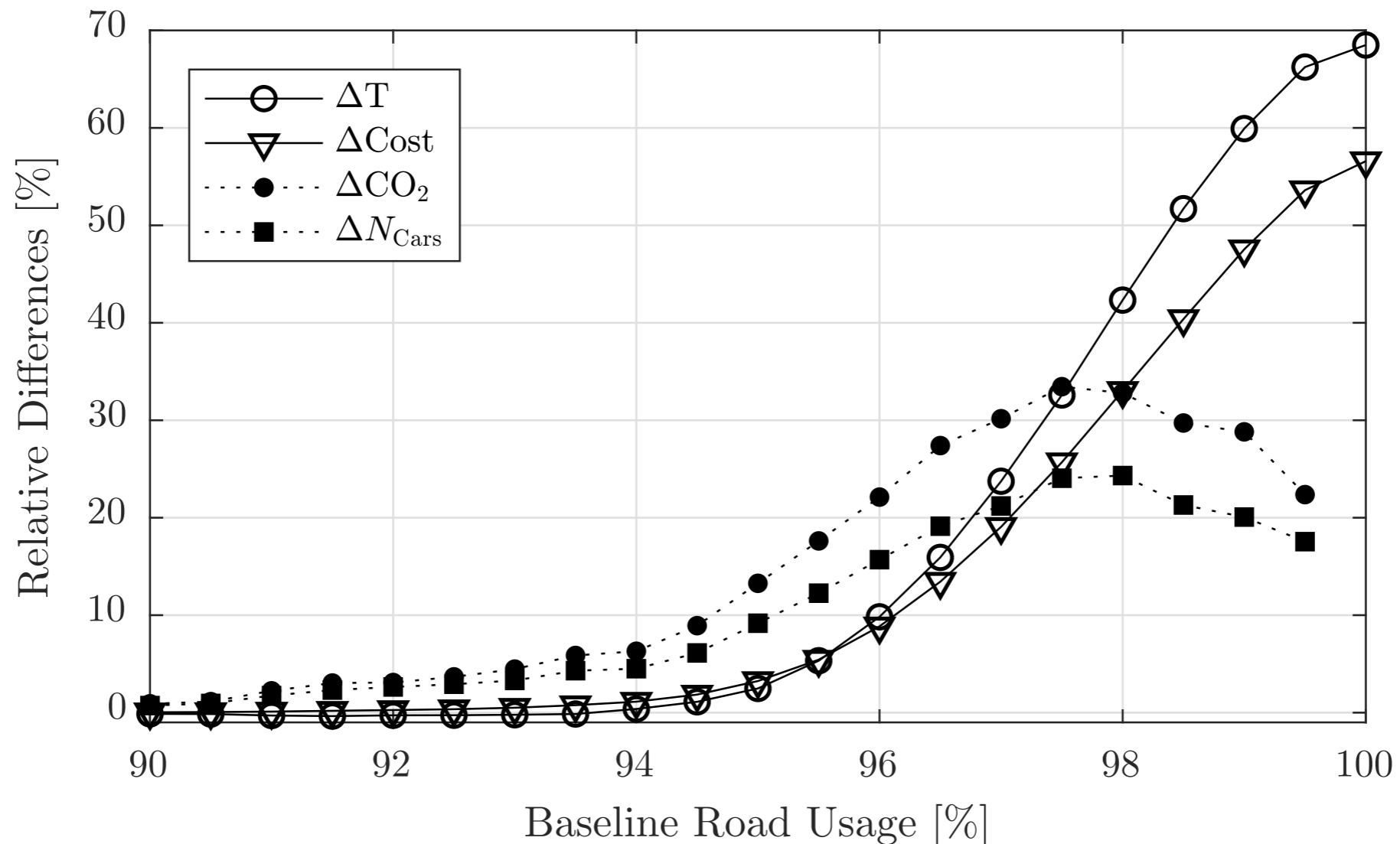
# Case Study - NYC

## I-AMoD - Optimal Control Policy for Different Road Capacities



# Case Study - NYC

## Pure AMoD VS I-AMoD - Optimal Control Policy for Different Road Capacities



Coordination with public transit significantly reduces **travel times, number of vehicles, emissions and cost!**

# Outlook

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- Real-time operational algorithms
- Stochastic effects: demand, congestion and delays
- Interaction with the power grid
- Human-centred metrics: comfort and switch-over costs

# Network Flow Model

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## Extended Graph

$$G = (\mathcal{V}, \mathcal{A}), \quad \mathcal{V} = \mathcal{V}_R \cup \mathcal{V}_P \cup \mathcal{V}_W, \quad \mathcal{A} = \mathcal{A}_R \cup \mathcal{A}_P \cup \mathcal{A}_W \cup \mathcal{A}_{RW} \cup \mathcal{A}_{PW}$$

## Conservation of Customers

$$\sum_{i \in \mathcal{V}} f_m(i, j) + \mathbf{1}_{j=o_m} \cdot \alpha_m = \sum_{k \in \mathcal{V}} f_m(j, k) + \mathbf{1}_{j=d_m} \cdot \alpha_m \quad \forall m \in \mathcal{M}, \forall j \in \mathcal{V}$$

## Conservation of Vehicles

$$\sum_{i \in \mathcal{V}_R} \left( f_0(i, j) + \sum_{m \in \mathcal{M}} f_m(i, j) \right) = \sum_{k \in \mathcal{V}_R} \left( f_0(j, k) + \sum_{m \in \mathcal{M}} f_m(j, k) \right) \quad \forall j \in \mathcal{V}_R$$

# Network Flow Model

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Capacity of Road and Public Transportation

$$f_0(i, j) + \sum_{m \in \mathcal{M}} f_m(i, j) \leq c_R(i, j), \quad \forall (i, j) \in \mathcal{A}_R$$

$$\sum_{m \in \mathcal{M}} f_m(i, j) \leq c_P(i, j), \quad \forall (i, j) \in \mathcal{A}_P$$

Objective Social Welfare: **time, operational costs and energy**

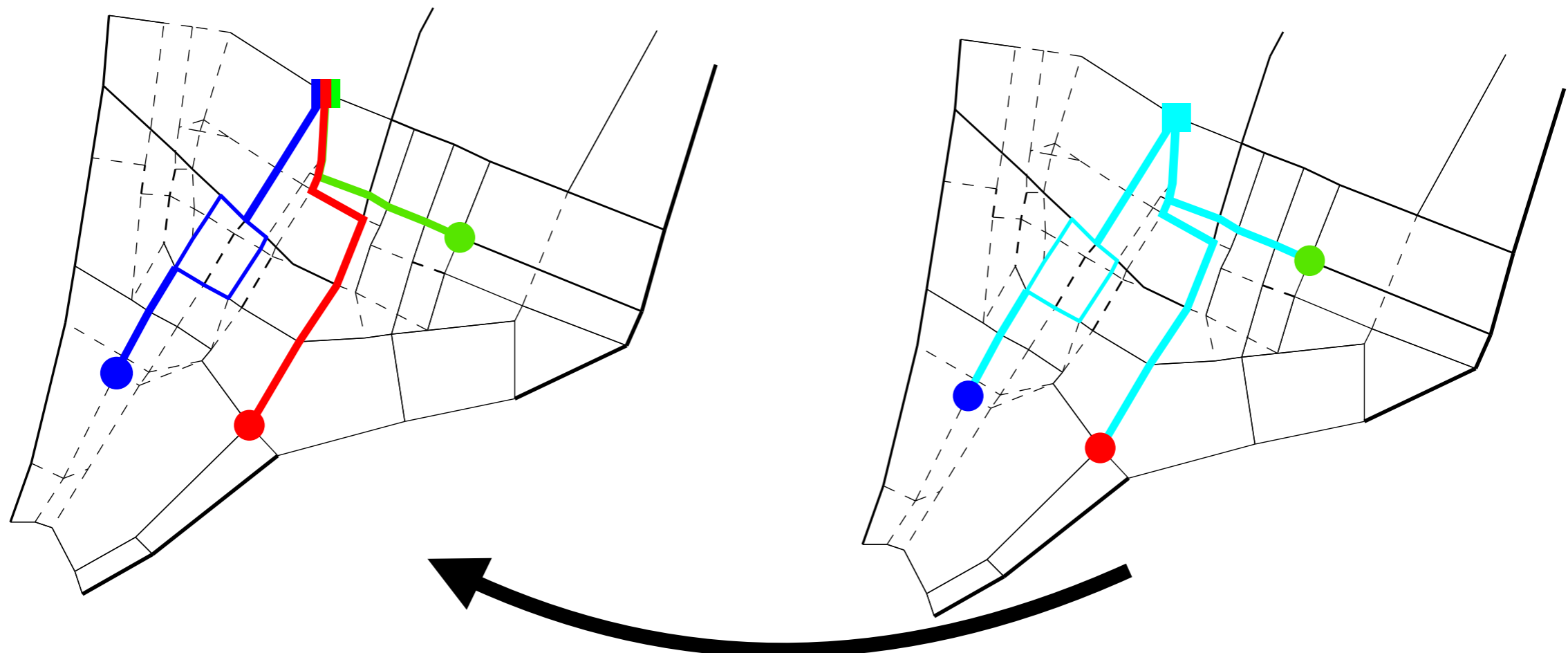
$$\begin{aligned} & \min_{f_m(i, j), f_0(i, j)} \sum_{(i, j) \in \mathcal{A}} \sum_{m \in \mathcal{M}} V_T \cdot t_{ij} \cdot f_m(i, j) \\ & + \sum_{(i, j) \in \mathcal{A}_R} (V_{D,R} \cdot d_{ij} + V_E \cdot e_{R,ij}) \cdot \left( f_0(i, j) + \sum_{m \in \mathcal{M}} f_m(i, j) \right) \\ & + \sum_{(i, j) \in \mathcal{A}_P} V_{D,P} \cdot d_{ij} \cdot \sum_{m \in \mathcal{M}} f_m(i, j) \end{aligned}$$

# Network Flow Model - Flow Bundling

Bundle flows with same destination

$$O(|\mathcal{V}|^2 \cdot |\mathcal{A}|) \approx 10^{10}$$

$$O(|\mathcal{V}| \cdot |\mathcal{A}|) \approx 10^7$$

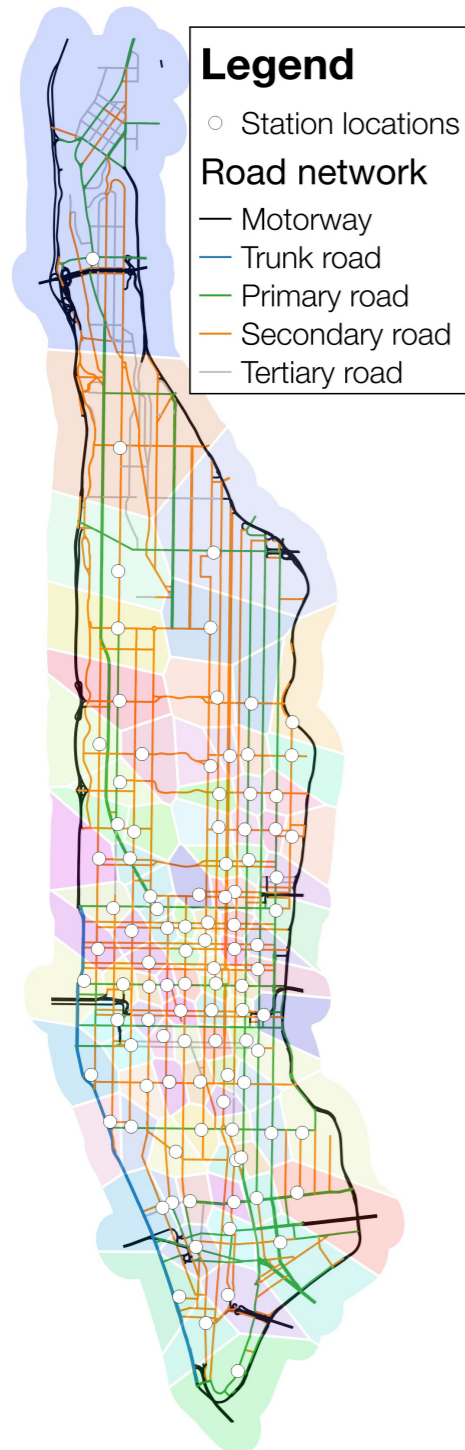


Flow decomposition algorithm

Theorem: Flow bundling is lossless [Rossi et al. 2018]

# Intermodal AMoD - AMoD and Pedestrian

## Manhattan Road Network - Data from OpenStreetMap



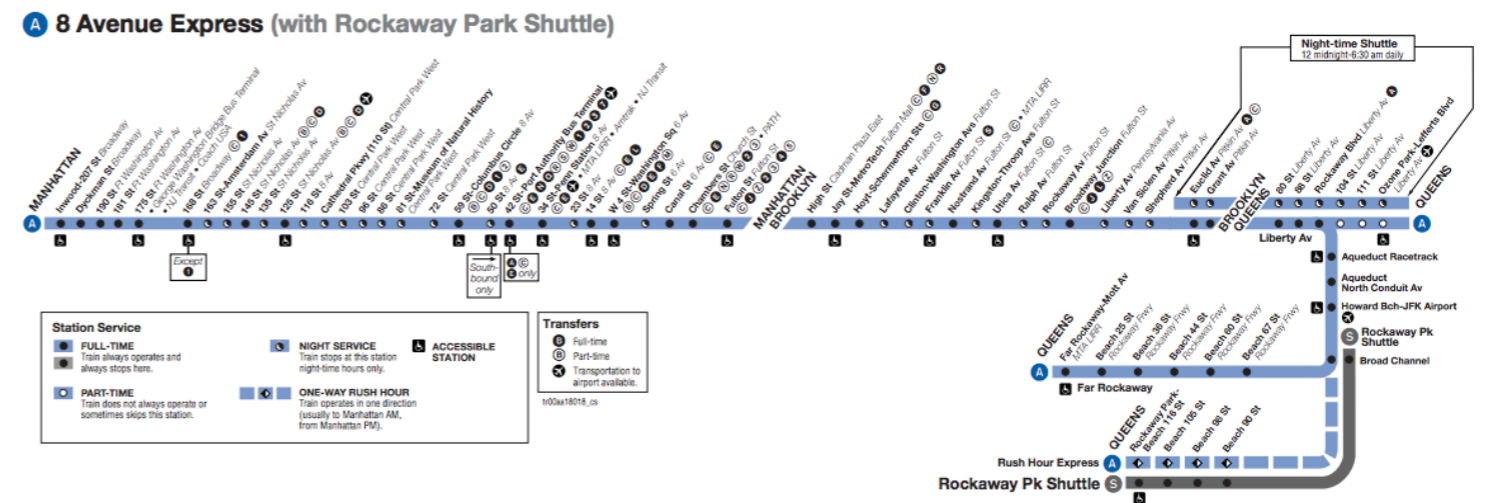
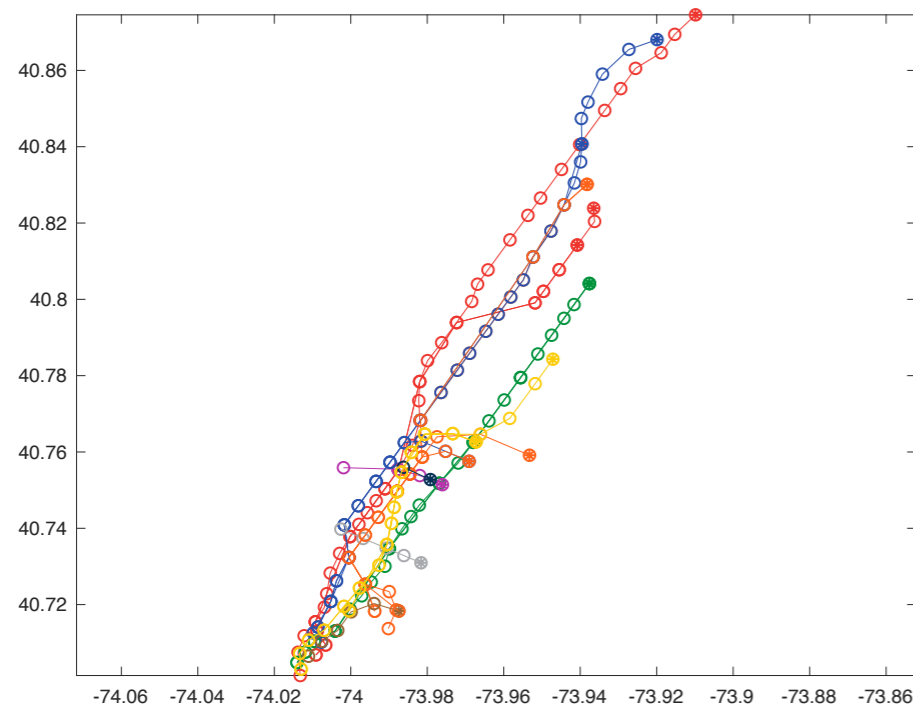
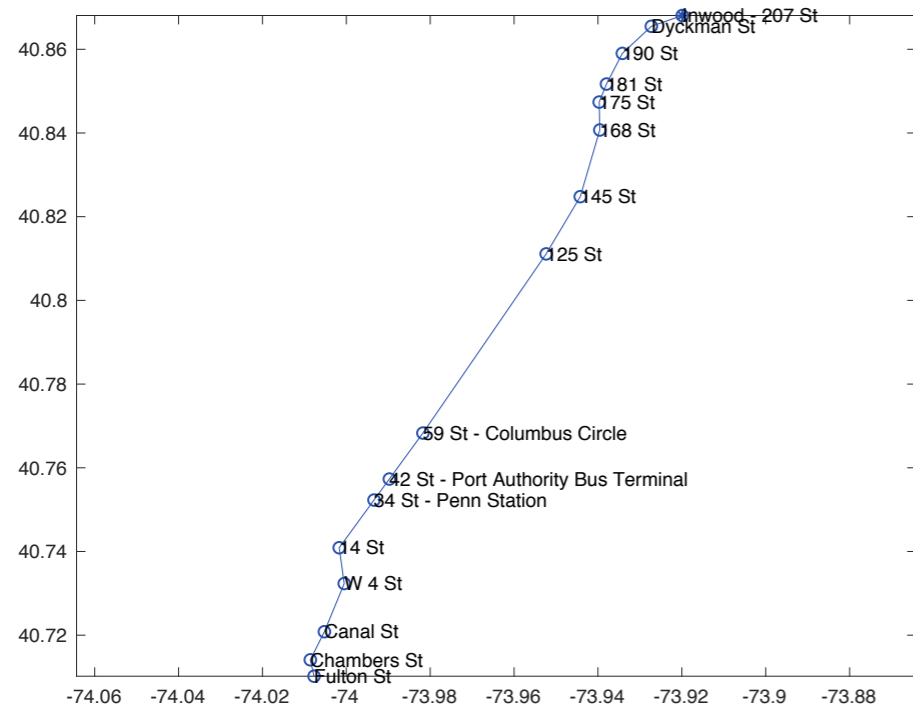
Consider 54000 taxi rides during rush hour, distributed in 6774 origin destination pairs

Since cabs are only a fraction of the vehicles, we will assume a baseline road usage of 90-100%



# Intermodal AMoD - Subway

Manhattan Subway Network - Data from [data.cityofnewyork.us](http://data.cityofnewyork.us) and [mta.info](http://mta.info)



**Weekday Service**  
**A Southbound**  
 From 207 St, Manhattan, to Lefferts Blvd/Far Rockaway/Rockaway Park, Queens

	207 St	Dyckman St	188 St	145 St	125 St	59 St	42 St	W 4 St	Chambers St	Fulton St	Jay St	Metrotech	Bowling Green	Euclid Av	Rocky Av	Lefferts Blvd	Herald Sq	Rockaway Pt. Shuttle	Rockaway Pt. Shuttle	Rockaway Pt. Shuttle	Rockaway Pt. Shuttle	
12:12	12:13	12:20	12:25	12:28	12:40	12:43	12:49	12:54	12:56	1:01	1:17	1:24	1:30	—	1:36	1:42	2:02	—	—	—	—	—
12:30	12:31	12:38	12:43	12:46	12:59	1:03	1:09	1:14	1:16	1:21	1:37	1:44	1:50	—	1:56	2:02	2:22	—	—	—	—	—
12:50	12:51	12:58	1:03	1:06	1:19	1:23	1:29	1:34	1:36	1:41	1:57	2:04	2:10	—	2:16	2:22	2:42	—	—	—	—	—
1:10	1:11	1:18	1:23	1:26	1:39	1:43	1:49	1:54	1:56	2:01	2:17	2:24	2:30	—	2:36	2:42	3:02	—	—	—	—	—
1:20	1:21	1:28	1:33	1:36	1:49	1:53	2:00	2:04	2:06	2:11	2:27	2:34	2:40	—	2:46	2:52	3:12	—	—	—	—	—
1:50	1:51	1:58	2:03	2:06	2:19	2:23	2:29	2:34	2:36	2:41	2:57	3:04	3:10	—	3:16	3:22	3:42	—	—	—	—	—
2:10	2:11	2:18	2:23	2:26	2:39	2:43	2:49	2:54	2:56	3:01	3:17	3:24	3:30	—	3:36	3:42	4:02	—	—	—	—	—
2:30	2:31	2:38	2:43	2:46	2:59	3:03	3:09	3:14	3:16	3:21	3:37	3:44	3:50	—	3:56	4:02	4:22	—	—	—	—	—
2:50	2:51	2:58	3:03	3:06	3:19	3:23	3:29	3:34	3:36	3:41	3:57	4:04	4:10	—	4:16	4:22	4:42	—	—	—	—	—
3:10	3:11	3:18	3:23	3:26	3:39	3:43	3:49	3:54	3:56	4:01	4:17	4:24	4:30	—	4:36	4:42	4:59	—	—	—	—	—
3:30	3:31	3:38	3:43	3:46	3:59	4:03	4:09	4:14	4:16	4:21	4:37	4:44	4:50	—	4:56	5:02	5:19	—	—	—	—	—
3:50	3:51	3:58	4:03	4:06	4:19	4:23	4:29	4:34	4:36	4:41	4:57	5:04	5:10	—	5:16	5:22	5:38	—	—	—	—	—
4:10	4:11	4:18	4:23	4:26	4:39	4:43	4:49	4:54	4:56	5:01	5:17	5:24	5:30	—	5:36	5:42	5:56	—	—	—	—	—
4:30	4:31	4:38	4:43	4:46	4:59	5:03	5:09	5:14	5:16	5:21	5:37	5:44	5:50	—	5:56	6:02	6:15	—	—	—	—	—
4:50	4:51	4:58	5:03	5:06	5:19	5:23	5:29	5:34	5:36	5:41	5:57	6:04	6:10	—	6:16	6:22	6:35	—	—	—	—	—
5:09	5:11	5:18	5:21	5:25	5:34	5:38	5:42	5:46	5:48	5:53	6:09	6:16	6:22	—	6:28	6:36	6:49	—	—	—	—	—

**Express service in Manhattan and Brooklyn begins:**

5:26	5:27	5:34	5:38	5:41	5:50	5:52	5:58	6:02	6:04	6:09	6:22	6:26	6:32	—	6:38	6:44	6:57	—	—	—	—	—
5:41	5:42	5:49	5:53	5:56	6:05	6:08	6:13	6:18	6:20	6:25	6:32	6:37	6:43	—	6:49	6:55	7:02	7:15	—	—	—	—
5:54	5:55	6:02	6:06	6:09	6:17	6:20	6:25	6:30	6:32	6:37	6:49	6:53	6:59	—	7:05	7:12	7:26	—	—	—	—	—
6:03	6:05	6:12	6:16	6:19	6:27	6:30	6:35	6:40	6:42	6:47	6:59	7:03	7:09	—	7:15	—	—	—	—	—	—	—
6:13	6:15	6:22	6:26	6:29	6:37	6:40	6:45	6:51	6:53	6:58	7:10	7:05	7:11	—	7:17	7:24	7:38	—	—	—	—	—
6:18	6:20	6:27	6:31	6:34	6:42	6:47	6:52	6:57	6:59	7:04	7:16	7:11	7:17	—	7:23	7:30	7:44	—	—	—	—	—
6:27	6:29	6:36	6:40	6:43	6:51	6:54	6:59	7:04	7:06	7:11	7:23	7:27	7:33	—	7:39	7:46	8:00	—	—	—	—	—
6:34	6:36	6:44	6:47	6:51	6:59	7:02	7:08	7:13	7:15	7:20	7:32	7:37	7:43	—	7:49	7:56	8:10	—	—	—	—	—
6:42	6:44	6:52	6:55	6:59	7:07	7:10	7:15	7:20	7:22	7:27	7:39	7:43	7:49	—	7:55	8:02	8:16	—	—	—	—	—

**Then every 6-9 minutes to Rockaway Blvd with less frequent service to either Lefferts Blvd or Far Rockaway until:**

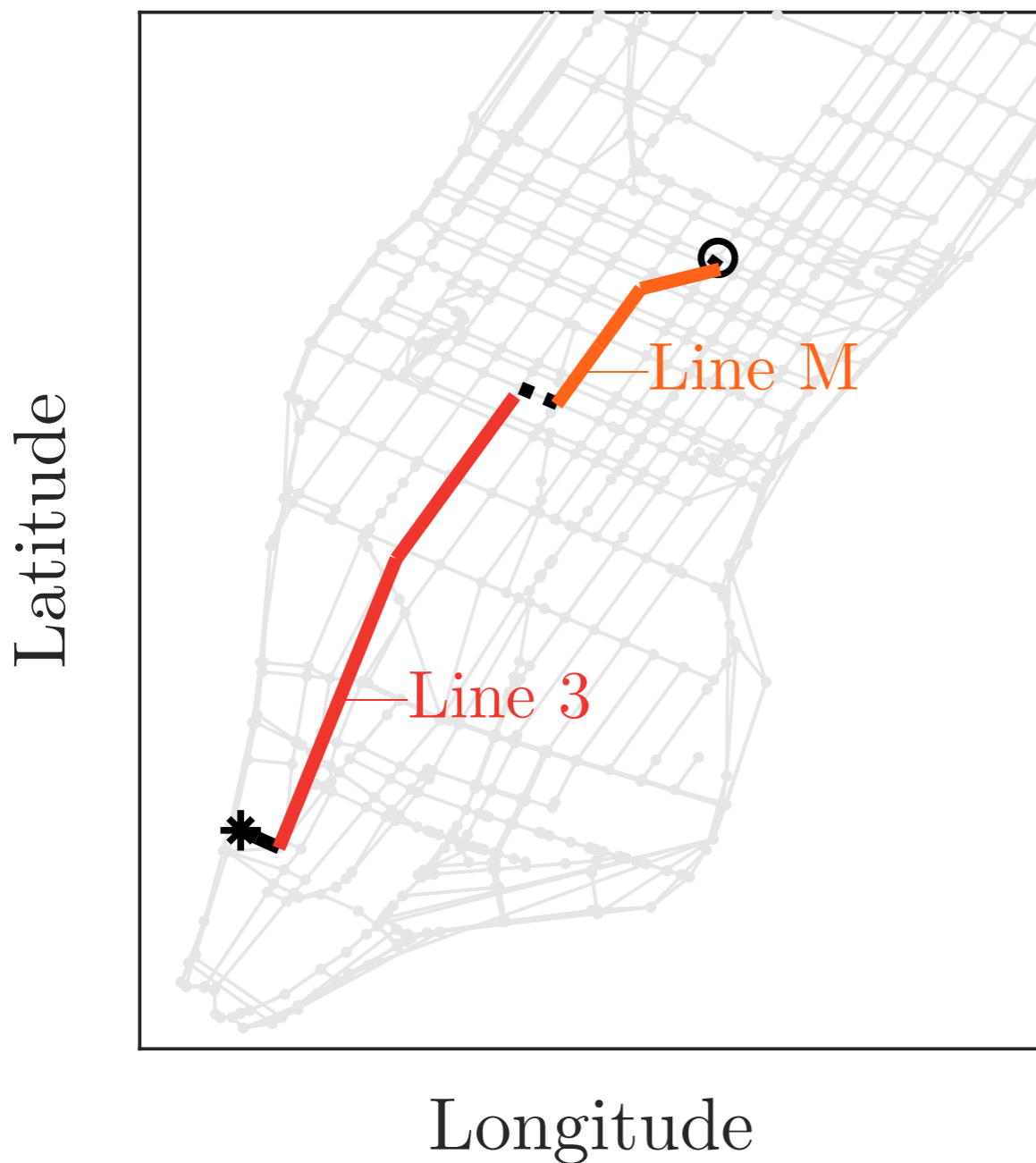
9:38	9:40	9:48	9:51	9:55	10:03	10:06	10:12	10:16	10:18	10:23	10:36	10:40	10:46	—	10:52	10:58	11:16	—	—	—	—	—
9:46	9:47	9:55	9:58	10:02	10:10	10:13	10:18	10:23	10:25	10:30	10:43	10:47	10:53	—	11:02	11:08	11:26	—	—	—	—	—
9:52	9:54	10:02	10:06	10:09	10:17	10:21	10:26	10:32	10:34	10:39	10:51	10:55	11:01	—	11:07	11:14	11:33	—	—	—	—	—
9:59	10:00	10:08	10:11	10:15	10:23	10:27	10:33	10:37	10:39	10:44	10:57	11:01	11:07	—	11:13	11:20	11:47	—	—	—	—	—
10:07	10:09	10:17	10:21	10:24	10:32	10:35	10:40	10:45	10:47	10:52	11:04	11:08	11:14	—	11:20	11:27	11:47	—	—	—	—	—
10:15	10:17	10:25	10:28	10:32	10:40	10:43	10:48	10:54	10:56	11:01	11:13	11:17	11:23	—	11:29	11:36	11:54	—	—	—	—	—
10:23	10:25	10:32	10:36	10:39	10:47	10:51	10:56	11:02	11:04	11:09	11:21	11:25	11:31	—	11:37	11:44	12:04	—	—	—	—	—
10:29	10:30	10:38	10:41	10:45	10:53	10:57	11:03	11:07	11:09	11:14	11:27	11:31	11:37	—	11:43	11:50	12:10	—	—	—	—	—
10:38	10:40	10:47	10:51	10:54	11:02	11:05	11:10	11:15	11:17	11:22	11:34	11:38	11:44	—	11:50	11:57	12:17	—	—	—	—	—
10:46	10:47	10:54	10:58	11:00	11:08	11:10	11:14	11:19	11:21	11:26	11:38	11:42	11:48	—	11:54	12:01	12:20	—	—	—	—	—

Very advanced format...

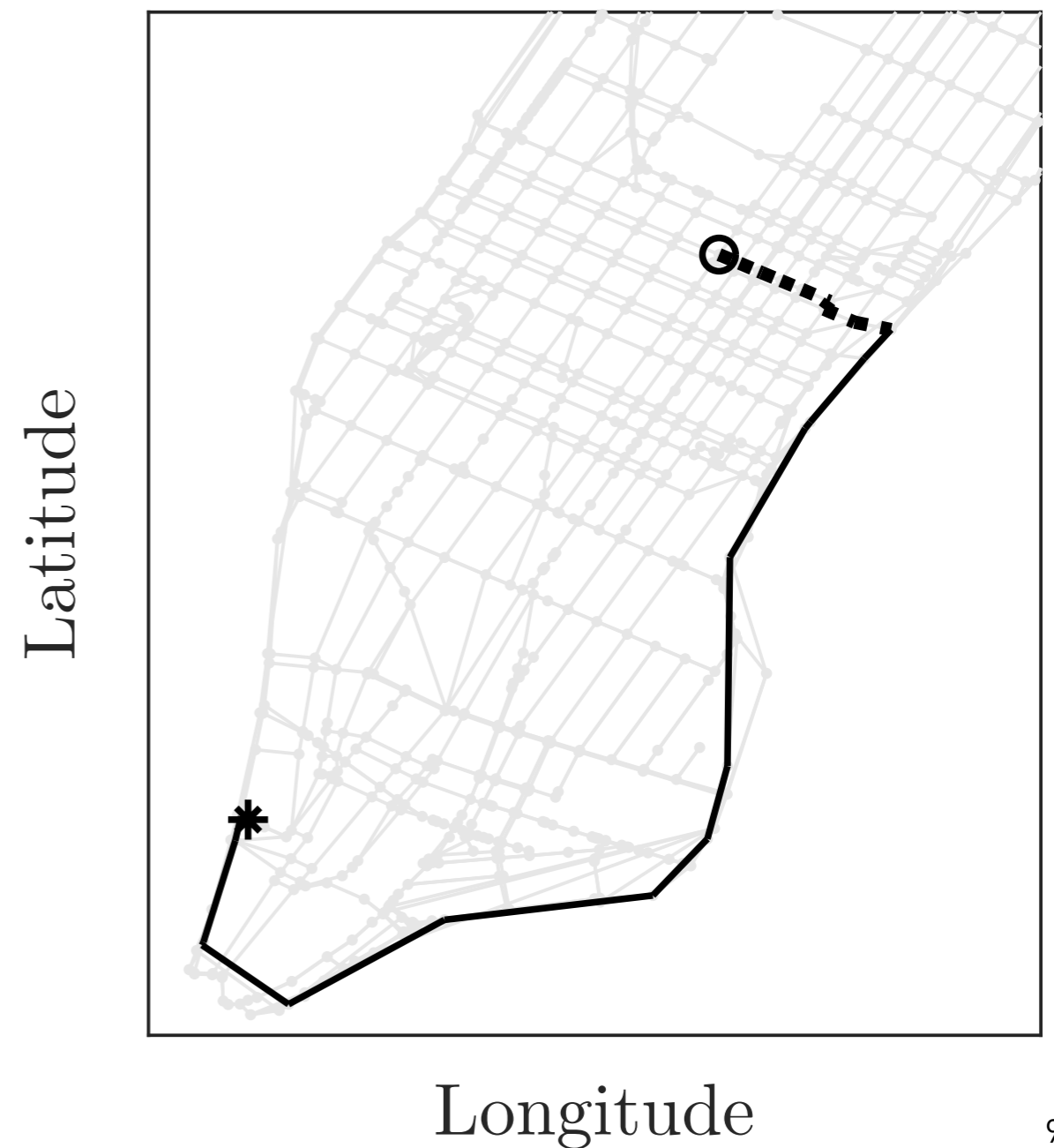
# Case Study - NYC - 98% Road Usage

Sample optimal paths

I-AMoD

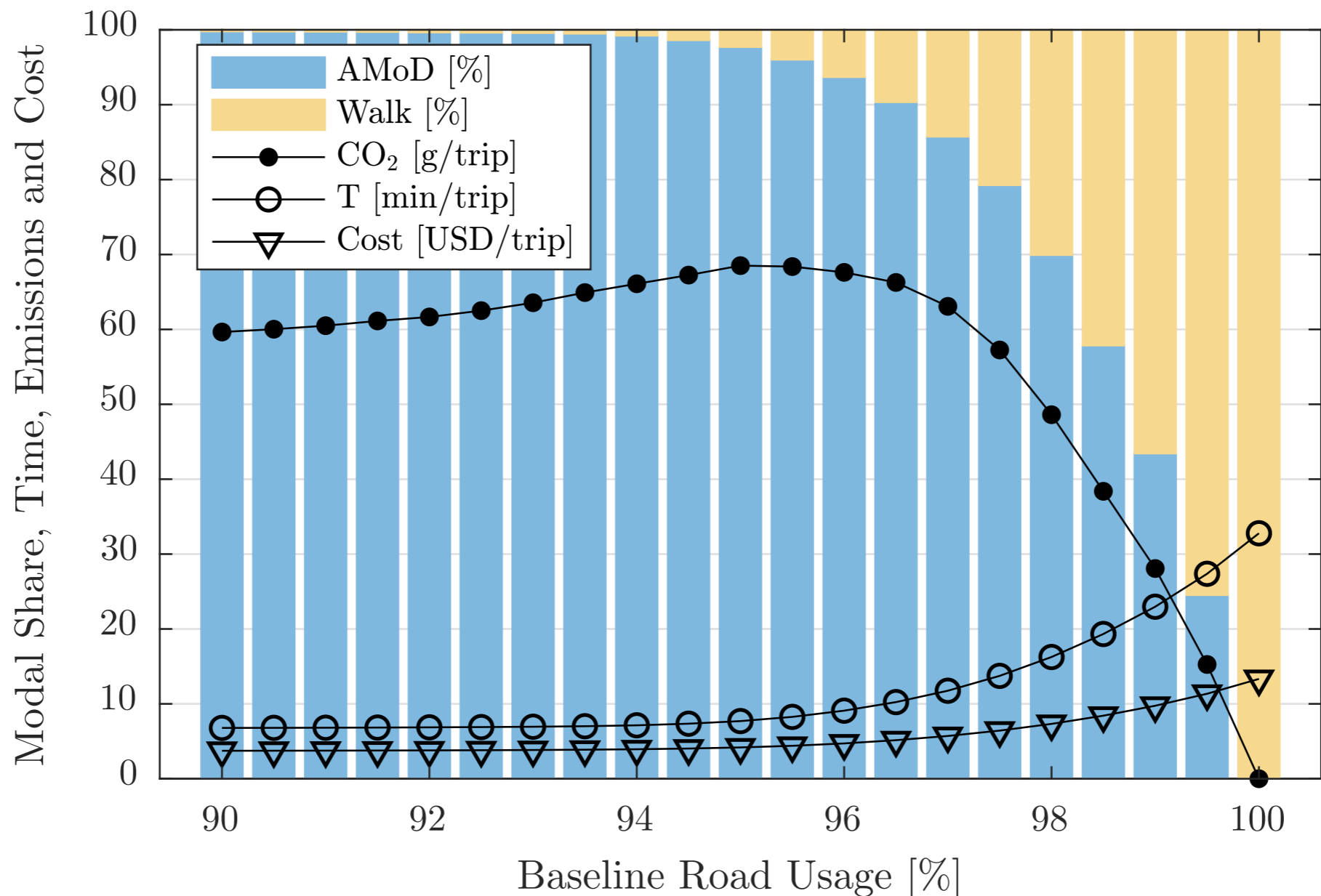


Pure AMoD



# Case Study - NYC

## Pure AMoD - Scan in road capacity



# A Socially Optimal Road Tolling Scheme

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Municipality wants to relieve congestion through road tolls

**The New York Times**

**N.Y. / REGION**

## ***Congestion Plan for Manhattan Gets Mixed Reviews***

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By WINNIE HU and VIVIAN WANG JAN. 19, 2018

Congestion constraint and dual multipliers

$$f_0(i, j) + \sum_{m \in \mathcal{M}} f_m(i, j) - c_R(i, j) \leq 0, \quad \forall (i, j) \in \mathcal{A}_R \quad \leftrightarrow \quad \mu_{c_R}(i, j) \geq 0$$

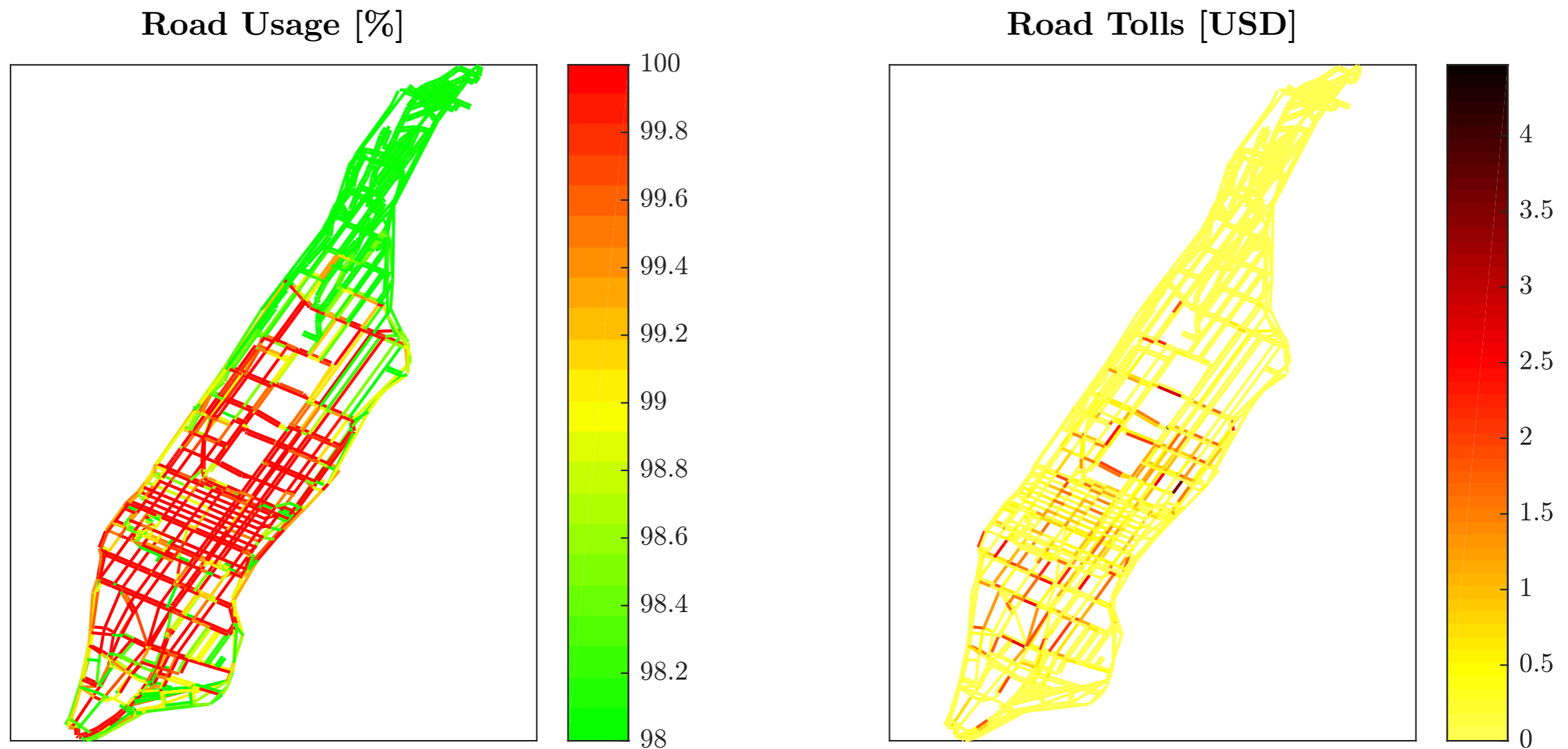
Optimal road tolls are the dual multipliers of the road congestion constraints

$$t_R(i, j) = \mu_{c_R}(i, j)$$

Theorem: Tolling scheme guarantees social optimum [Salazar et al. 2018]

# Case Study - NYC - 98% Usage

## I-AMoD - Tolling scheme



The average surcharge would be about **\$2** VS almost **\$6** with pure **AMoD**.  
In line with Cuomo's per-trip surcharge of **\$2-5!** [New York Times, Jan 2018]