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Think Forward

Gaussian Process Regression for Risk Analysis of Travel Demand Forecasts

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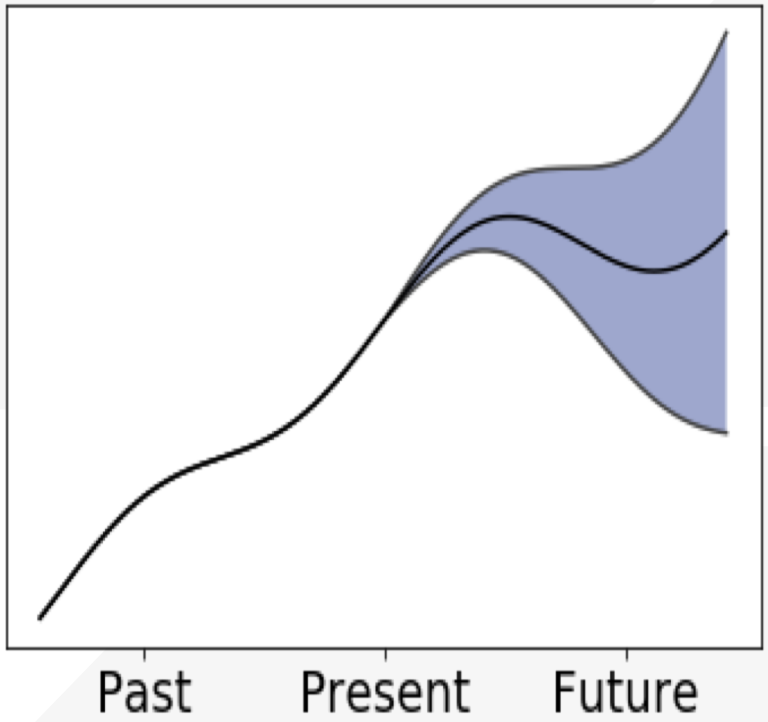
presented by

Cambridge Systematics, Inc.
Jeffrey Newman

with
Rachel Copperman, Jason Lemp, David Kurth
Boris Lipkin, *California High-Speed Rail Authority*
Matt Henley, *John Helsel, WSP*

Why Risk Analysis?

- The base forecast model generates a single **point estimate** forecast for future conditions.
- This estimate is **not precise**, because of incomplete or inaccurate representations of present or future inputs or assumptions embedded in the models.
- Robust planning and decision making processes instead will consider a **range of model forecasts**.



Risk Analysis Approach

these are important but let's assume these are already done

Today's Topic

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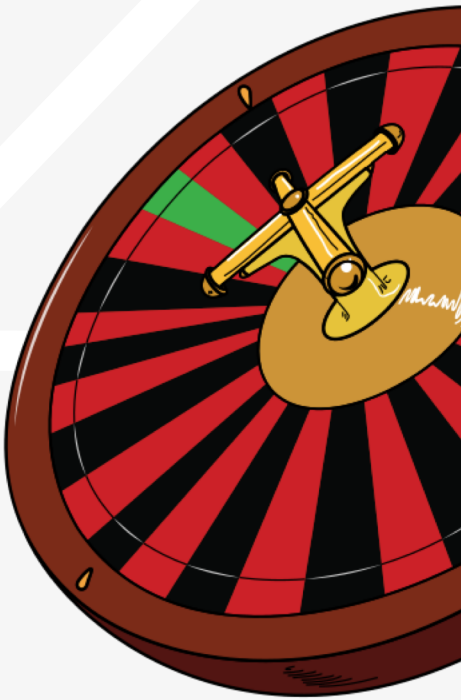
Monte Carlo Simulation

- To develop a robust distribution of outcomes, we want to run our model a lot: many thousands of times
- But our simulation model is complex, it takes a long time to complete a single experiment



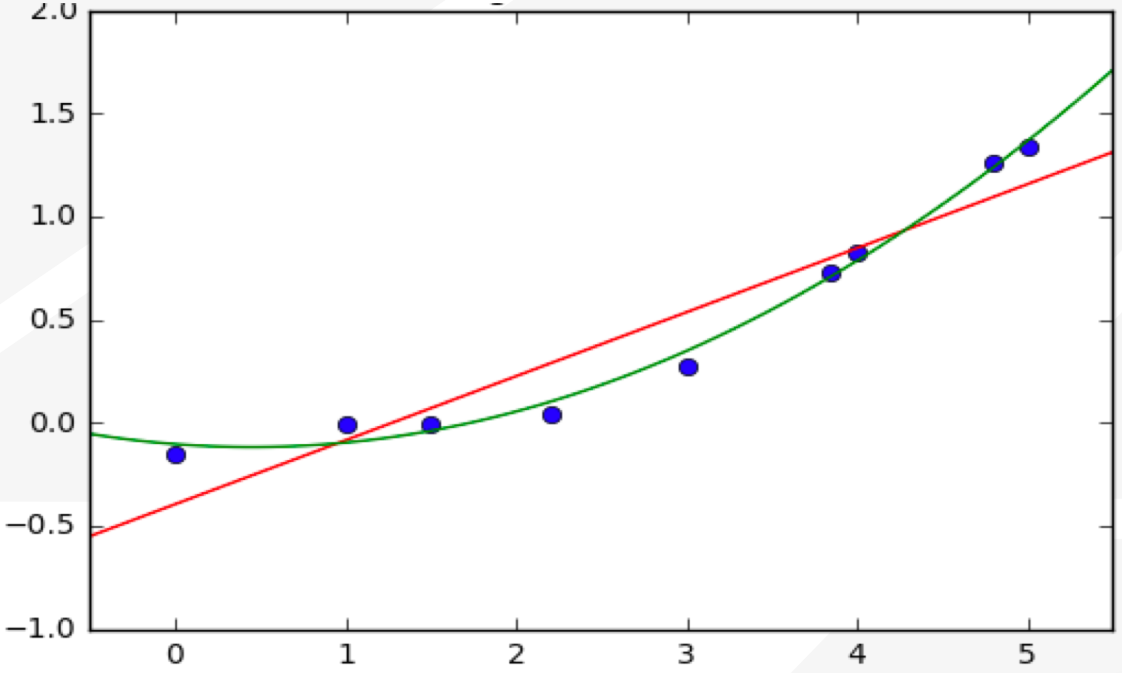
Monte Carlo Simulation

- To develop a robust distribution of outcomes, we want to run our model a lot: many thousands of times
- But our simulation model is complex, it takes a long time to complete a single experiment
- Solution: replace the model with a simpler one, which takes only fractions of a second to run

Enter the Meta-Model

- We replace the expensive simulation model with a fast regression meta-model.
- Common practice in transportation planning is to use a linear regression model:
  - Easy to implement
  - Exceptionally fast
  - Generally appears to have good fit
    - But is it really good enough?

An Illustration in One Dimension



Simple Linear Regression $R^2 = 0.90$

Polynomial Regression $R^2 = 0.99$

R^2 this high is typically regarded as a good fit…

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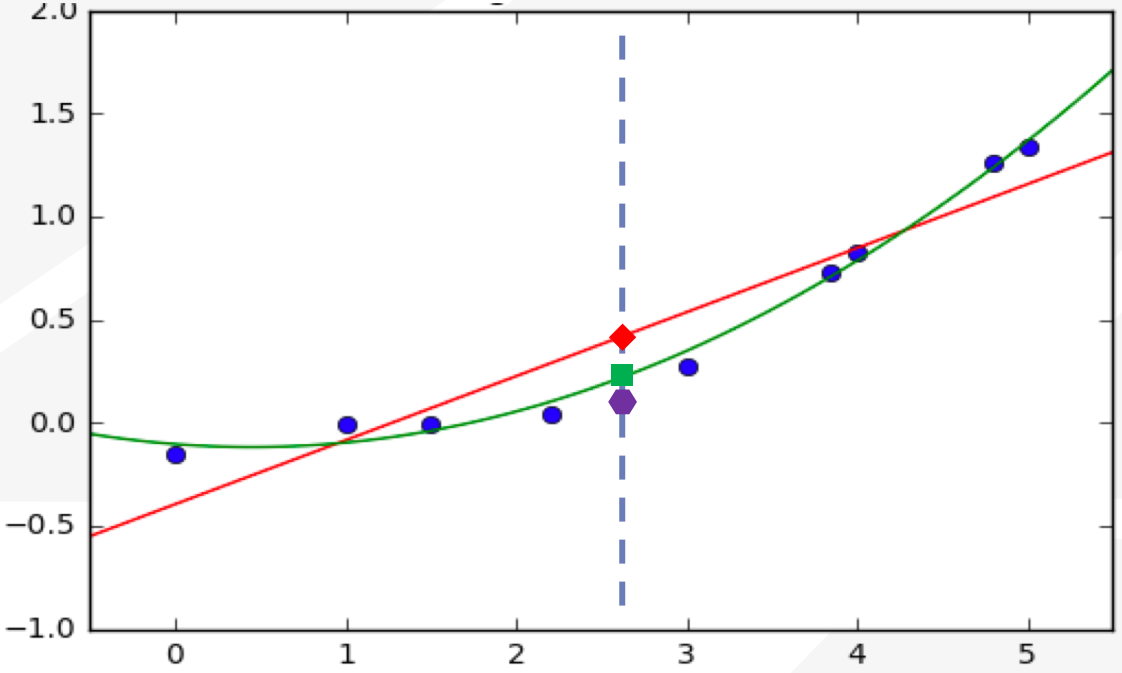
An Illustration in One Dimension

Suppose we want to predict at X=2.6



- Simple Linear Regression Y=0.43
- Polynomial Regression Y=0.26
- Probably the Real Value Y=0.21

… but there is still some remaining error

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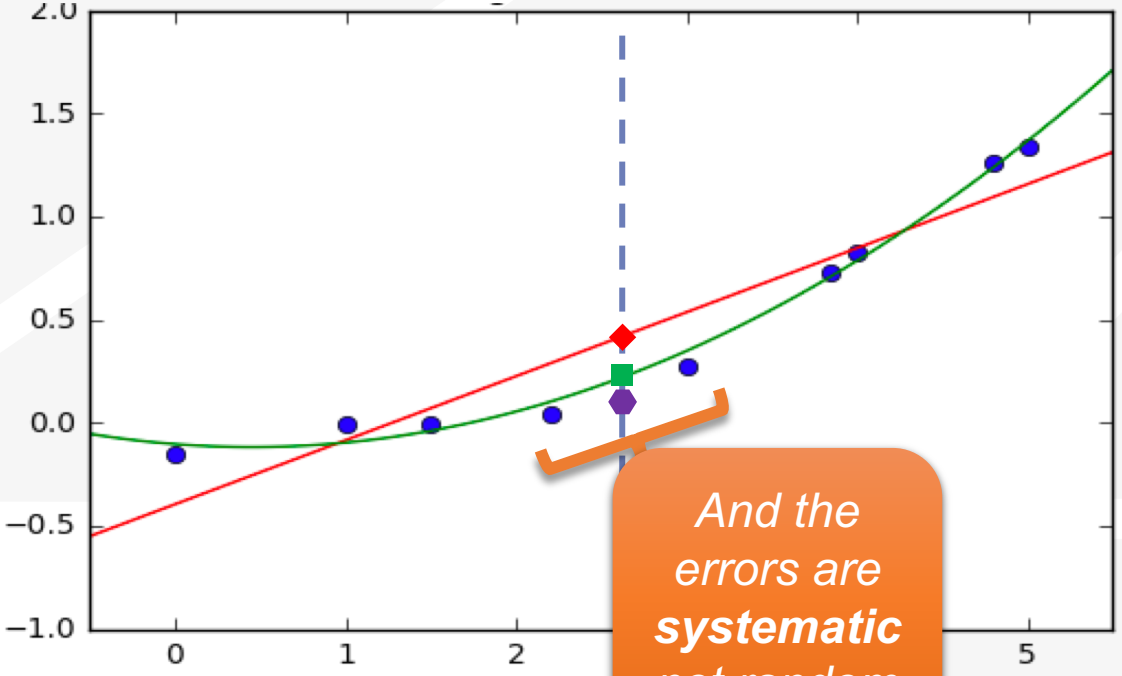
An Illustration in One Dimension

Suppose we want to predict at X=2.6



*And the errors are **systematic** not random*

- Simple Linear Regression Y=0.43
- Polynomial Regression Y=0.26
- Probably the Real Value Y=0.21

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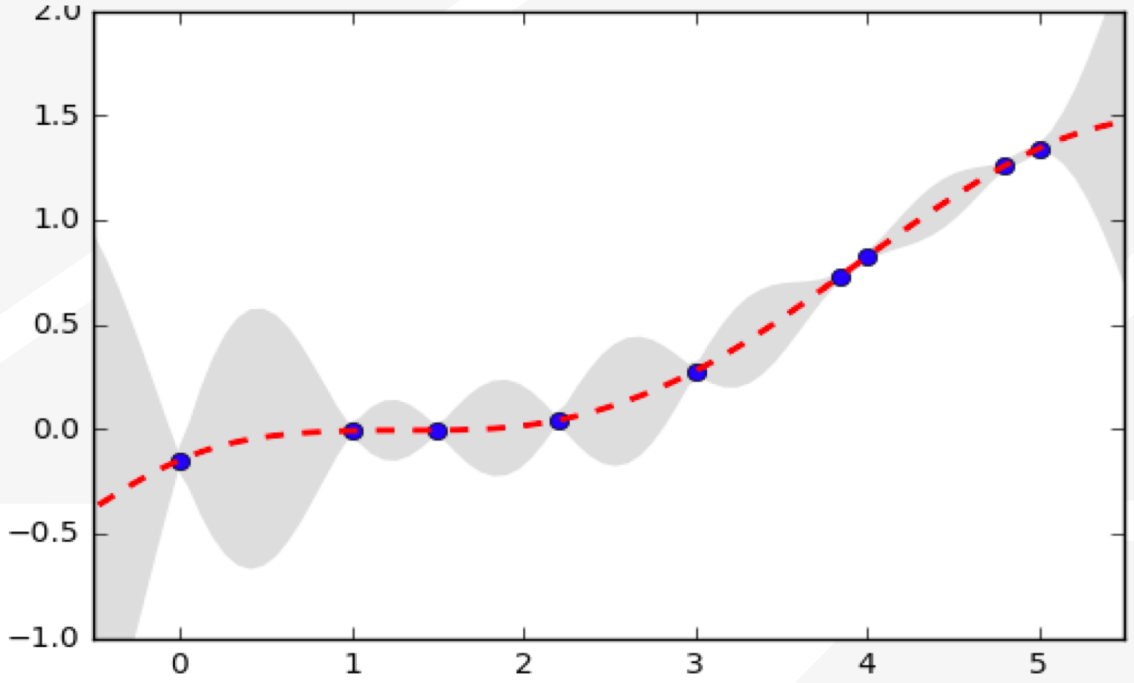
Gaussian Process Regression

- Gaussian Process Regression (GPR) is a non-parametric “machine learning” tool for regression analysis
- GPR does not impose a restriction on the functional form of the output
- Instead, just assume the output is **auto-correlated**: if the inputs are similar, then the output should also be similar
  - This auto-correlation violates the independent errors assumption in OLS linear regression

Two Important Features

- The BPM-V3 model for the California HSR has two features that make it work well with GPR:
  - **Deterministic**: Re-run the model with the same inputs, get the same output
  - **Smooth**: Re-run the model with the infinitesimally different inputs, get the only infinitesimally different output
- Conveniently, many travel demand models share these features
  - Although it makes things simpler, neither is strictly necessary for the use of GPR

GPR Illustration in One Dimension



Gaussian Process Regression $R^2 = 1.00$

2 Standard Deviations

We will come back to this

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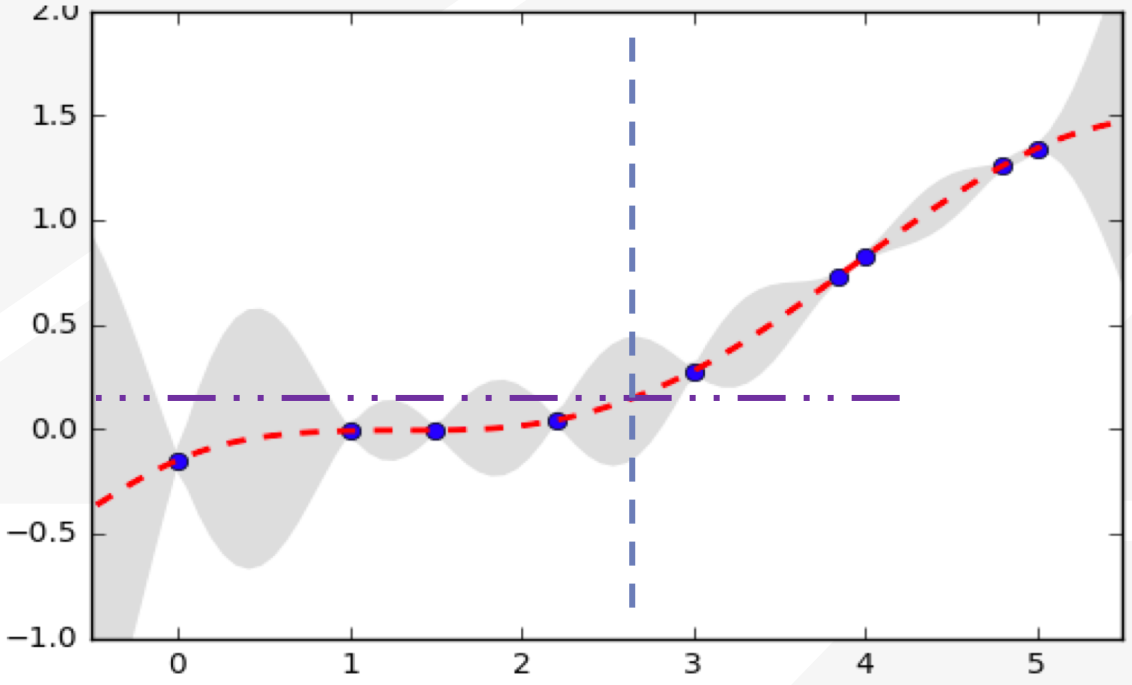
GPR Illustration in One Dimension



Bingo!

Gaussian Process Regression Y=0.21

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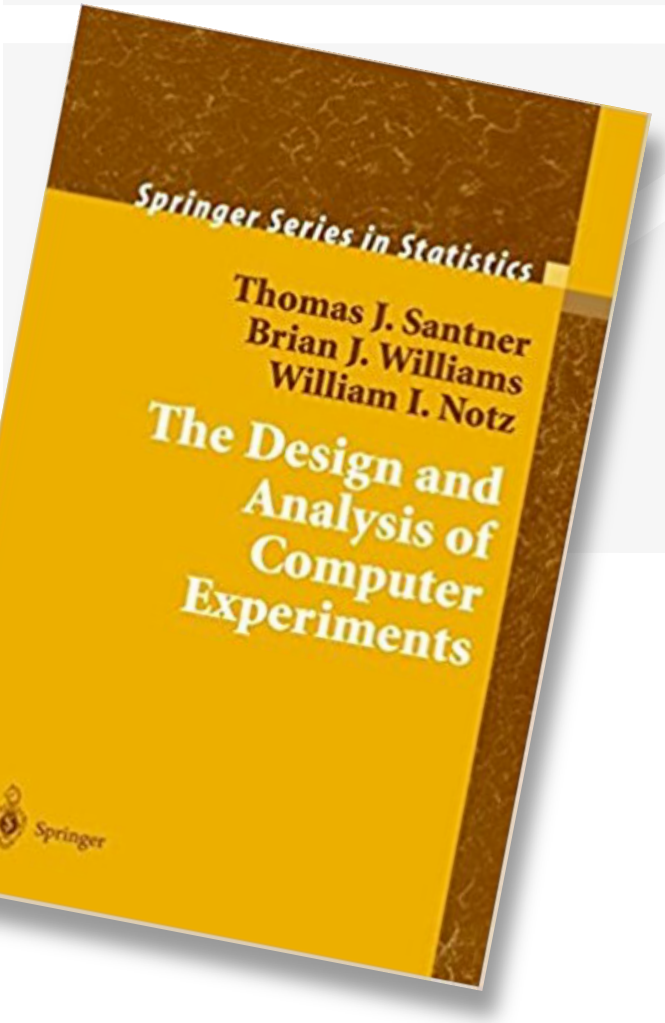
GPR Represents Best Practices



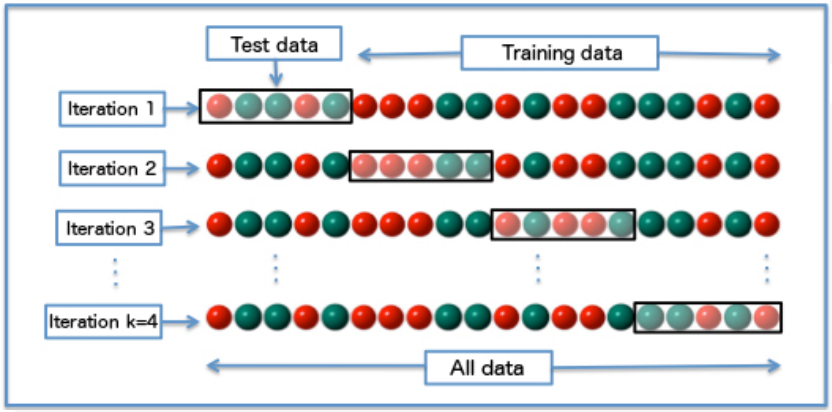
- Although not widely used for transportation planning meta-model applications, it is widely used for computer simulation meta-models in other fields
- Gaussian Process Regression is the textbook approach for modern meta-models of computer experiments
- And this is the textbook:
  Santner, T. J., Williams, B. J., & Notz, W. I. (2013). *"The design and analysis of computer experiments."* Springer Science & Business Media.

About that R^2 of 1.0

- GPR meta-models cannot be evaluated based on traditional “goodness of fit” measures derived from the estimation data, as for deterministic models they by design always fit all the estimation data perfectly
- Instead it is necessary to measure fit on a validation data set that is not used for model estimation
- Since additional data is expensive to collect, it is preferred to use k-fold cross-validation

K-fold Cross-Validation

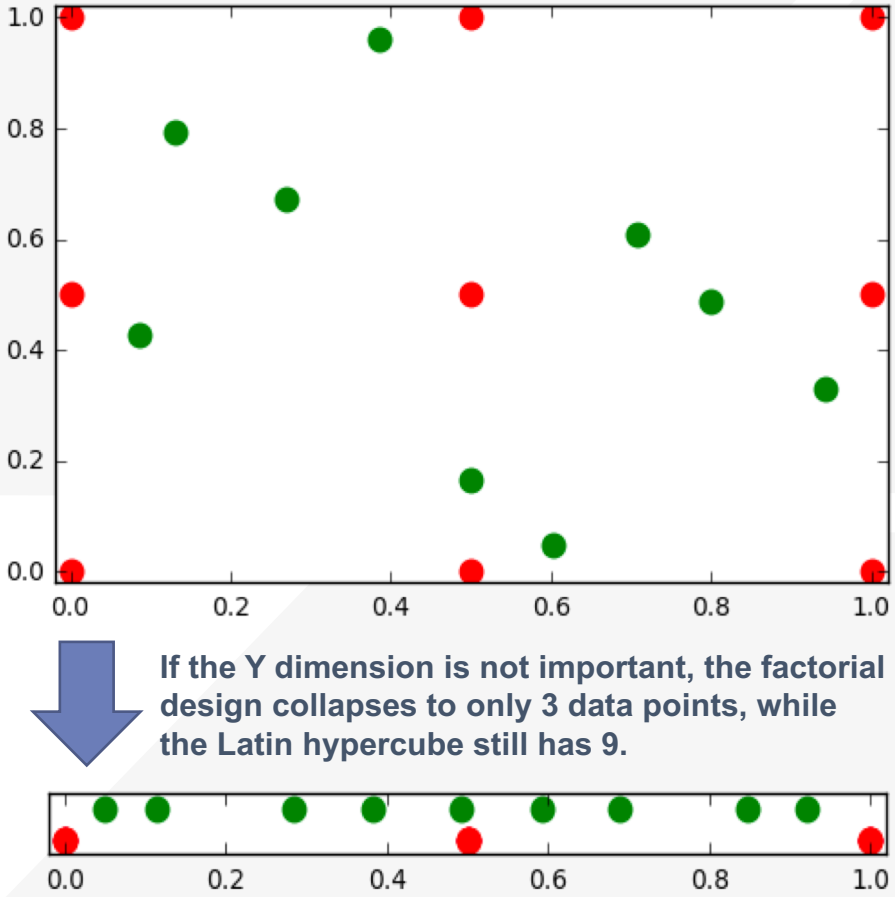
- Data is randomly split into K groups of roughly even size
- The model is fit using only K-1 groups, then evaluated based on the fit of the remaining holdout group
- Process is repeated for each of the K groups and averaged across them to create fit statistics



Source: https://commons.wikimedia.org/wiki/File:K-fold_cross_validation_EN.jpg

Design of Experiments

- Previous transportation planning applications have focused on a **factorial** or fractional factorial design of experiments (example in red)
- GPR instead is better supported by a **Latin Hypercube** design with irregular distances between experiments (example in green)
- If some dimensions are not important, the factorial design partially collapses but the hypercube design still recovers maximum information.



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Application: California High Speed Rail

- GPR was employed to conduct a risk analysis for the ridership and revenue forecasts for the California High Speed Rail Authority 2018 Business Plan
- The Latin Hypercube design was adopted to allow for 13 to 15 risk factors (varies by forecast year) — prior Business plans relied on a fractional factorial designs that limited the analysis to only 10 risk factors.

Risk Factors Included

- High speed rail constants
- Trip frequency constants
- Quality of connecting bus service
- Coefficient on access/egress time by distance
- Coefficient on extremely long access/egress
- Impact of Automated Vehicles
- Automobile operating cost
- Air and High speed rail fares
- High speed rail frequency of service
- High speed rail reliability
- Number and distribution of households throughout the state
- Level of visitor travel
- Level of extra induced ridership

Note: Not all risk factors are relevant for every forecast year

Still a Heavy Computation Load

- A single run of the full BPM-V3 simulation requires about 12 hours of CPU time
- One pass of this risk analysis involved conducting 150 runs for each of 3 forecast years = 5,400 CPU-hours
- We built an *ad hoc* cluster using Python and Dask with on average about 200 CPU cores available to complete the experimental runs in just a few days

Results: Revenue

	2029 – VtoV	2033 – Phase 1	2040 – Phase 1
GPR Cross Validation Score (Improvement over Linear Regression)	0.747	0.987	0.983
RMSE of Cross Validation Predictions (millions of 2017$)	$14.4	$7.1	$9.0
Long Distance HSR Revenue – 2018 Business Plan Base Runs (millions of 2017$)	$823	$2,085	$2,329
RMSE as a percent of Base Run Long Distance HSR Revenue	1.7%	0.3%	0.4%

Results: Ridership

	2029 – VtoV	2033 – Phase 1	2040 – Phase 1
GPR Cross Validation Score (Improvement over Linear Regression)	0.834	0.986	0.983
RMSE of Cross Validation Predictions (millions of annual riders)	0.25	0.16	0.19
Long Distance HSR Revenue – 2018 Business Plan Base Runs (millions of 2017$)	14.4	35.6	39.4
RMSE as a percent of Base Run Long Distance HSR Revenue	1.7%	0.4%	0.5%

http://www.hsr.ca.gov/docs/about/business_plans/2018_Business_Plan_Ridership_Revenue_Risk_Model.pdf

http://www.hsr.ca.gov/docs/about/business_plans/2018_CA_High_Speed_Rail_Business_Plan_Ridership_and_Revenue_Risk_Analysis.pdf

jnewman@camsys.com