Bridging Survey Response and Revealed Preference in a Quantum Cognition Model

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Why Bother Using Quantum Mechanics

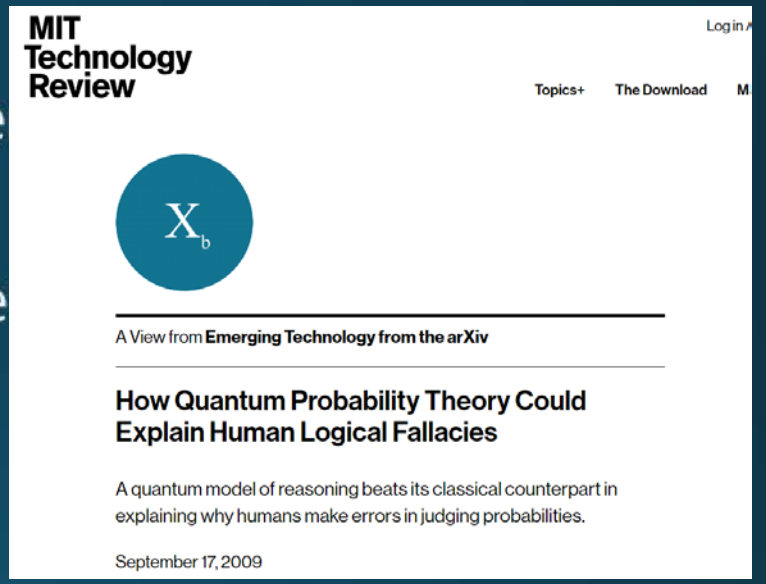
- Precedents of using methods from physics
  - Gravity Model, Equilibrium, Fluid Mechanics, Maximum Entropy…
- Quantum Mechanics was accepted (only) because it works (and classical mechanics doesn't)
- Non-Boolean Quantum Logic. E.g., $(A\ and\ B) \neq (B\ and\ A)$
- An Experiment

What is Quantum Logic?

Intuitive Explanation: **Measurement Alters the System Itself!**

In A *and* B experiement, the measurement **B** changes the syste
$|\updownarrow\rangle$, and **B** measures $|\updownarrow\rangle$, not $|\nearrow\swarrow\rangle$.

In B *and* A experiement, the measurement **A** changes the syste
$|\nearrow\swarrow\rangle$, and **A** measures $|\nearrow\swarrow\rangle$, not $|\updownarrow\rangle$.



Survey Question Alters A Participant's Mental State!

Disclaimer: using quantum principles, not implying quantum nature of cognition/consciousness

- Quantum Cognition (Quantum-like Mind) vs. Quantum

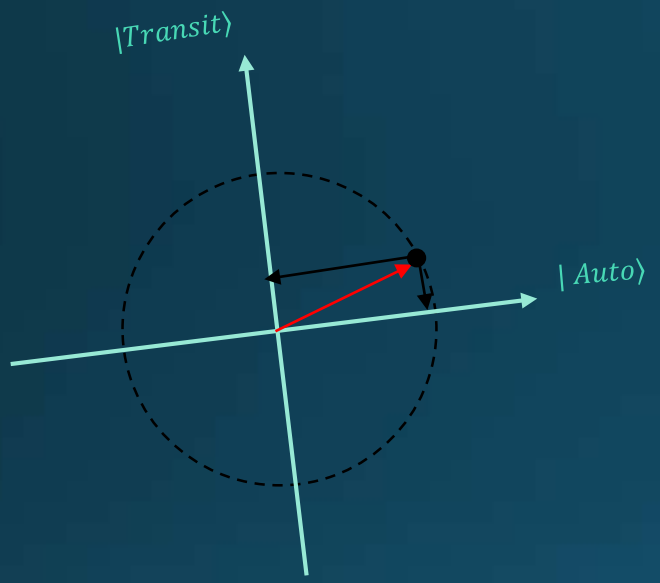
Challenges in quantifying survey bias

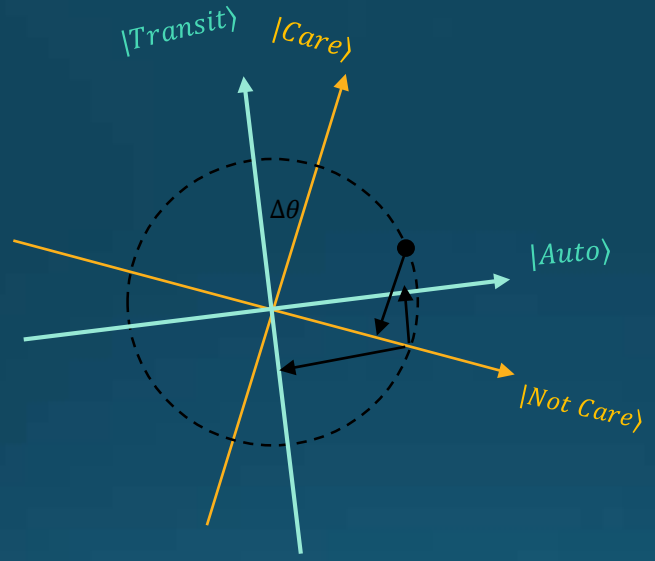
- Important role of stated preference survey when no observation available
- "Framing Effect" and "Mere Measurement Effect"
  - Stopher et al. (07)
    - "25% more trips were over-reported than trips that were under-reported in terms of actual travel time in 2003/2004 Syndey Household Travel Survey.
  - McFadden and Leonard (92)
    - "The results the stated preference (SP) survey change noticeably when response format and the wording of the questions were varied"
  - Morwitz et al. (93)
    - "… merely measuring intent will increases subsequent purchase behavior."
  - Surgery decision (McNeil, 1982; Tversky & Kahneman, 1986)
    - "There's a 90% chance of death without the surgery."
    - "There's a 10% chance of survival without surgery."
  - Water & Electricity Bills (Richard Thaler, 2008)
    - "Please pay $xxx for water and electricity."
    - "Most of your neighbors have paid the bill, but you still have $xxx deficit."
  - Traveler Information
    - "You have 80% chance of arriving on time or early."
    - "You have 20% chance of arriving late."
  - Various Survey Wording, Sequence, Instruments (picture/video/VR), etc.

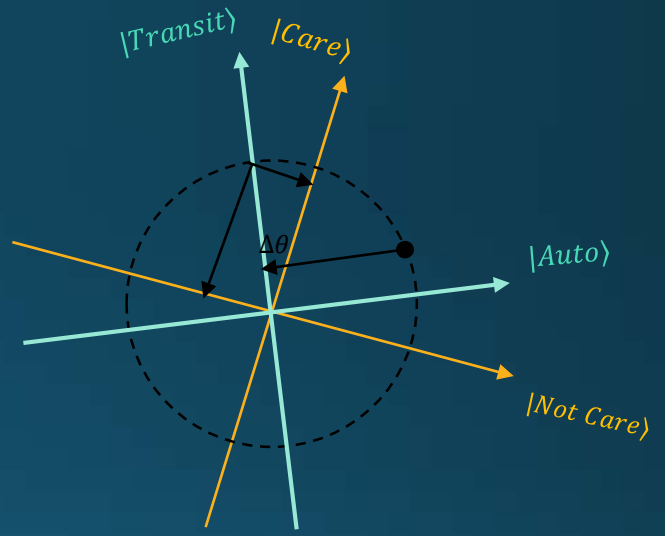
Simple Survey Example

- Survey Format 1
  - Q_A: Would you consider environmental impact when making travel decision?
  - Q_B: Do you prefer transit or auto?
- Survey Format 2
  - Q_B: Do you prefer transit or auto?
  - Q_A: Would you consider environmental impact when making travel decision?

A Geometric Illustration





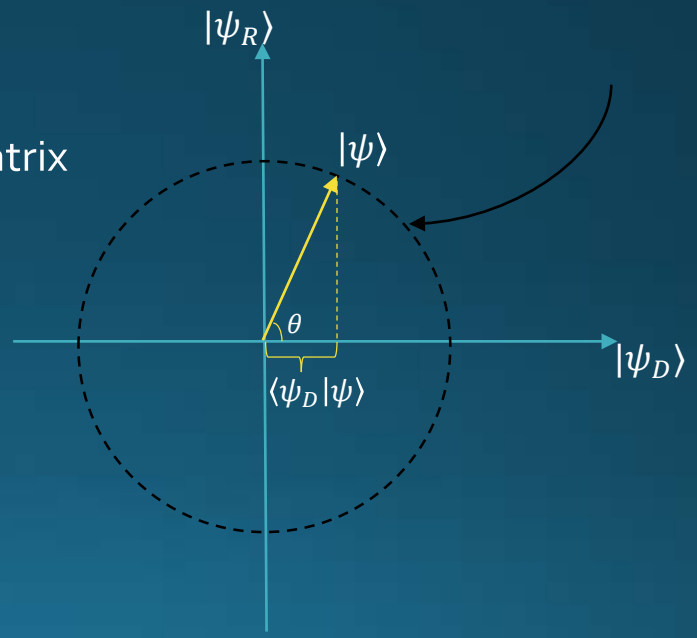


Quick Intro to Some Key Concepts

	Event D	Event R
Probability	0.3	0.7

- Sample Space vs. (Unit-Length) Vector Space
  - Bra vector, say $\langle\phi| = [1 \;\; 0]$. Ket vector, say $|\psi\rangle = \begin{bmatrix}\sqrt{2/3}\\ \sqrt{1/3}\end{bmatrix}$
  - "Bracket": $\langle\phi|\psi\rangle = [1 \;\; 0]\begin{bmatrix}\sqrt{2/3}\\ \sqrt{1/3}\end{bmatrix} = \sqrt{2/3} \approx 0.82$. $\langle\psi|\psi\rangle = \frac{2}{3} + \frac{1}{3} = 1$
- Probability & Probability Amplitude, $|\langle\psi_D|\psi\rangle|^2 = \langle\psi|\psi_D\rangle\langle\psi_D|\psi\rangle \approx 0.67$
- *Matrix representation of an observable (e.g., a survey response)
- Each "coordinate systems" are the set of eigenvectors of a (Hermitian) matrix
- Note that **once we observe, the system "collapses" into one of the eigenvectors**

$$p_D + p_R = \psi_D^*\psi_D + \psi_R^*\psi_R = 1$$



Core Ideas of Applying QM to travel survey study

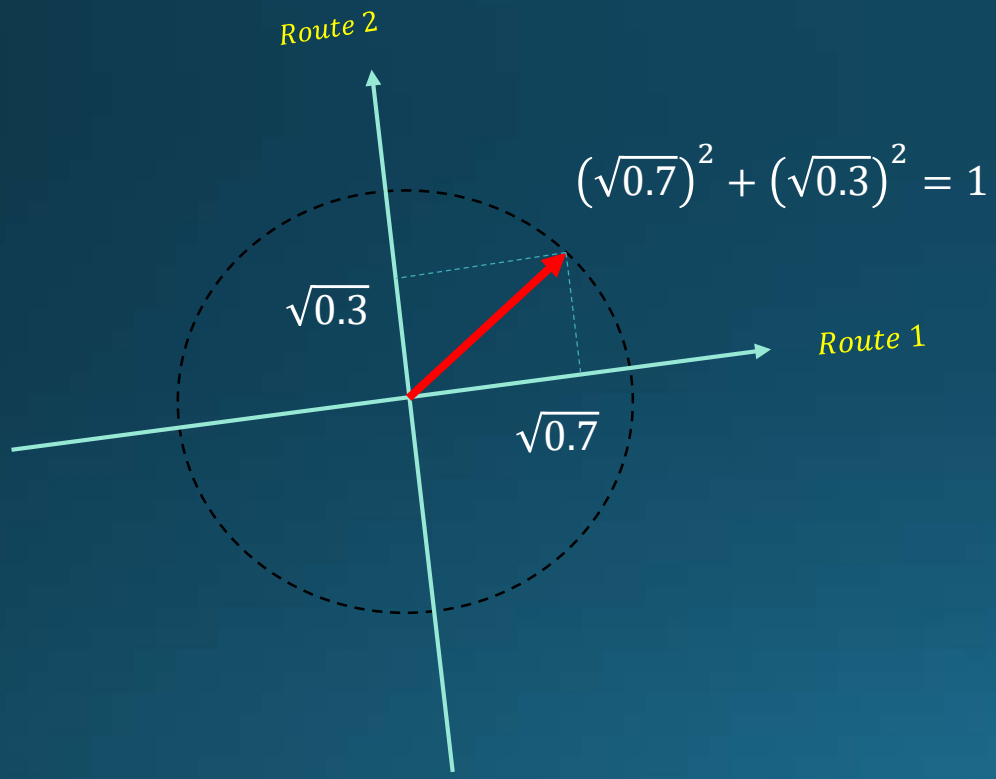
- **The unit-length state vector captures respondent's attributes/characteristics relevant to answering the survey question**
  - Different angles represent different mental states relevant to the survey question
  - The angel is a function of attributes/characteristics of the observer
- **The "coordinate system" captures information relevant to the survey question (measurement)**
  - Different survey instruments, platforms, wordings are represented by different coordinate systems formed by different sets of bases.
  - The state vector BECOMES one of the eigenvectors after the measurement

Presents the survey differently

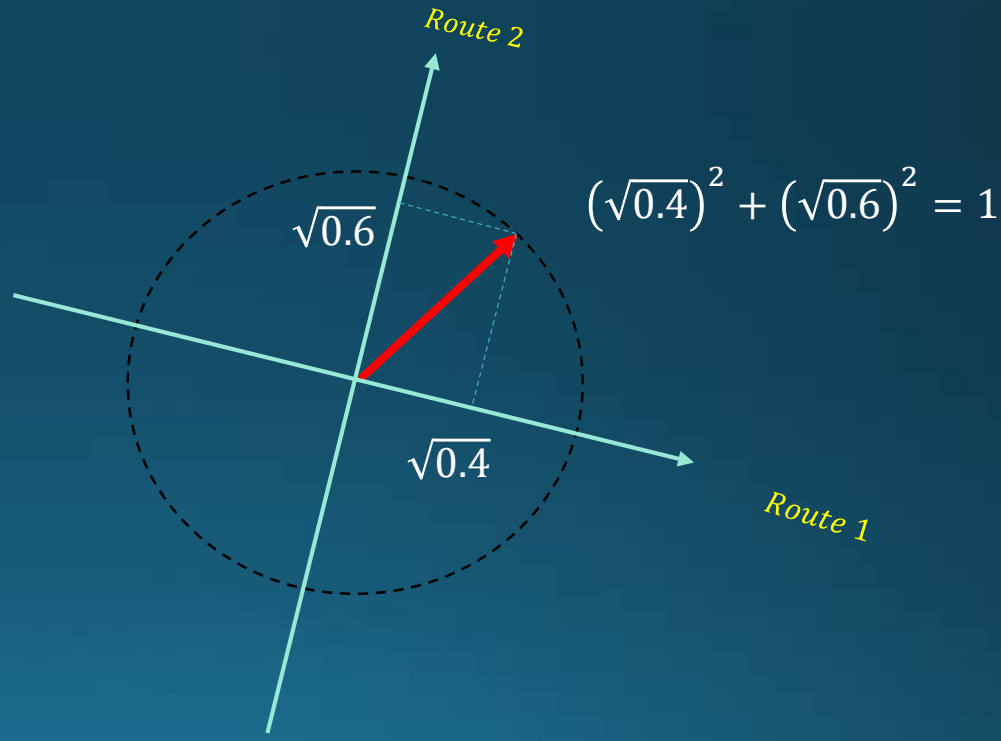
- Q1: Verbal Description about Route 1 and Route 2
- Q2: Photo/VR Description about Route 1 and Route 2

Intuitive (Geometric) Understanding of Quantum Cognition Model

Vector representation of a person's probability to choose between Route 1 and Route 2



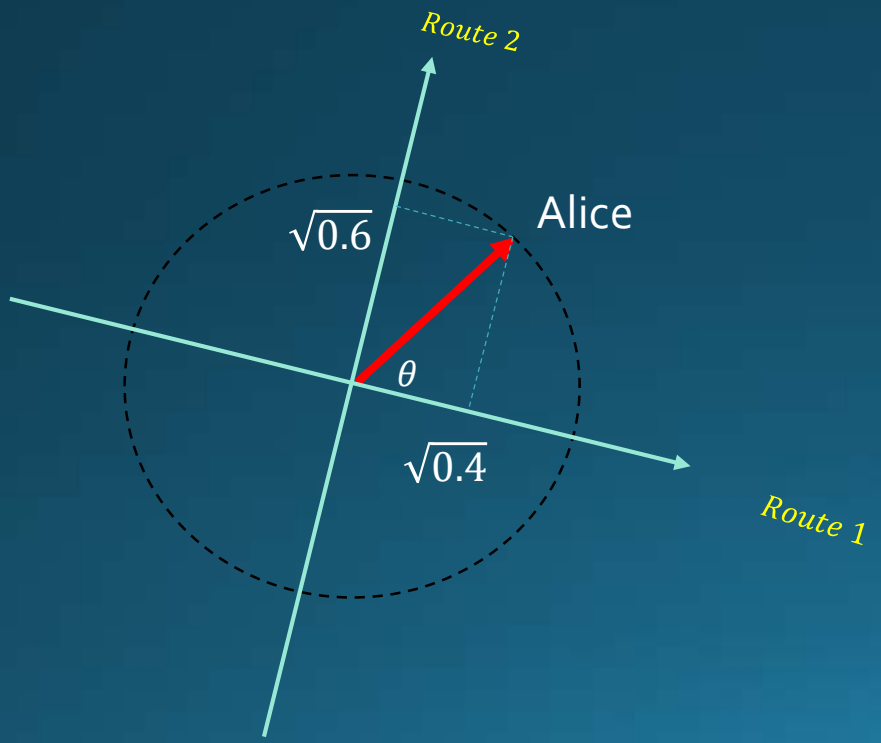
Question 1 biased towards Route 1

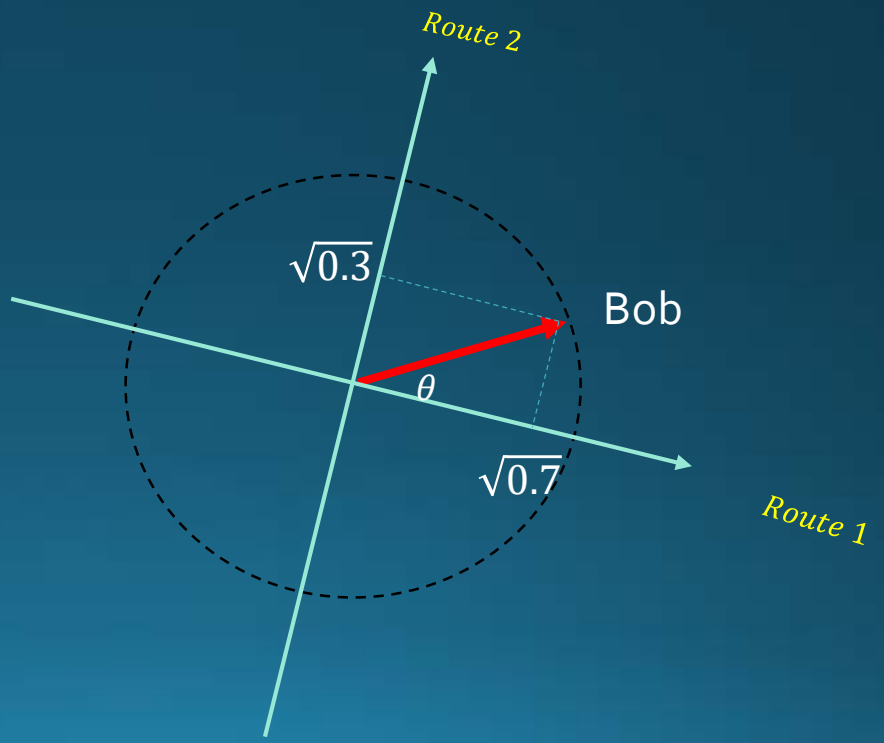


Question 2 biased towards Route 2

Asking different participants the same way

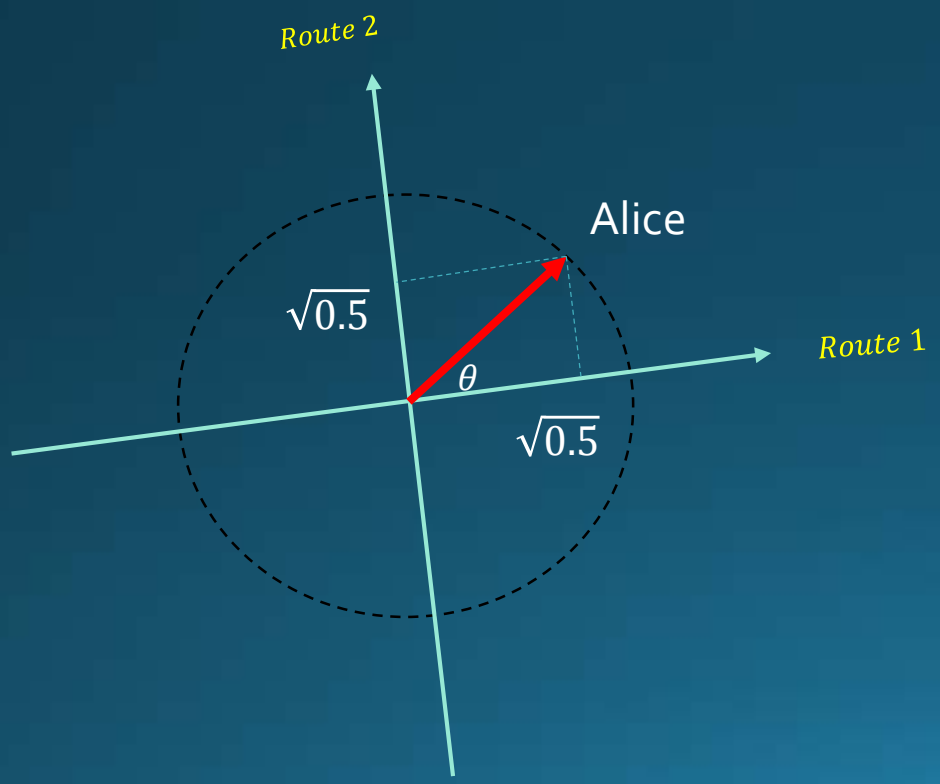
- In heterogenous case, $\theta \in [0,2\pi]$ is a function of an individual's attributes
- Question 1 for Alice and Bob

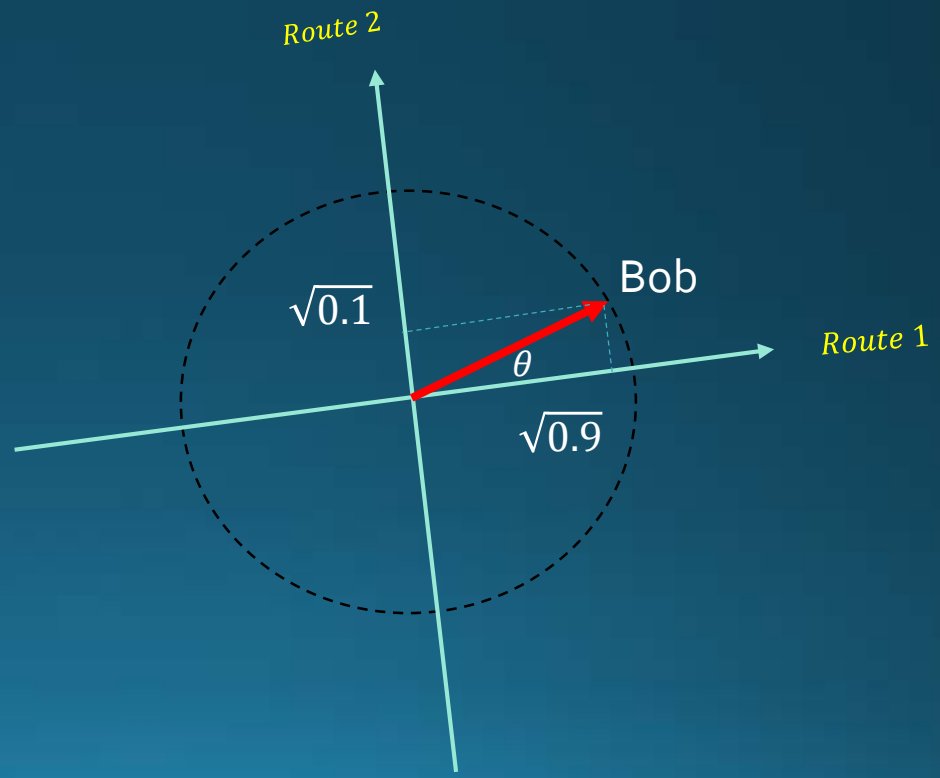




Heterogeneous sample

- In heterogenous case, θ is a function of their attributes, $\theta \in [0,2\pi]$
- Question 2 for Alice and Bob





Example 1: Case study using Fujii & Garling's data

- Kyoto, Japan, 2003
- 903/20000 respondents before-and-after survey (by mails)
  - First wave
    - info about the new service and socio-demographic attributes of the participants
  - Second wave (6 months later)
    - Whether moved since the first wave, and whether "kept their words."

Revealed Choice (%) for Non Transit Users

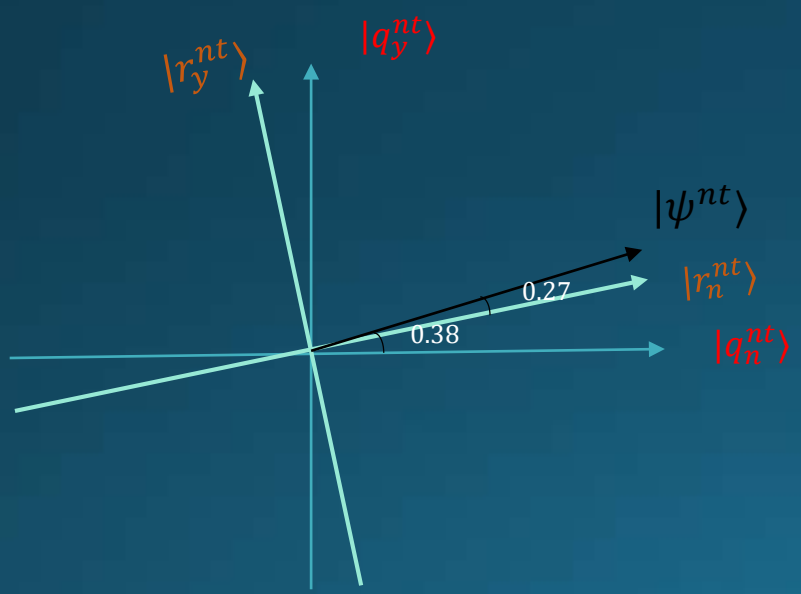
		Non-Transit Users		
		No	Yes	Total
Stated Preference (%)	No	82.90	3.62	86.52
	Yes	9.84	3.64	13.48
	Total	92.74	7.26	

Revealed Choice (%) for Transit Users

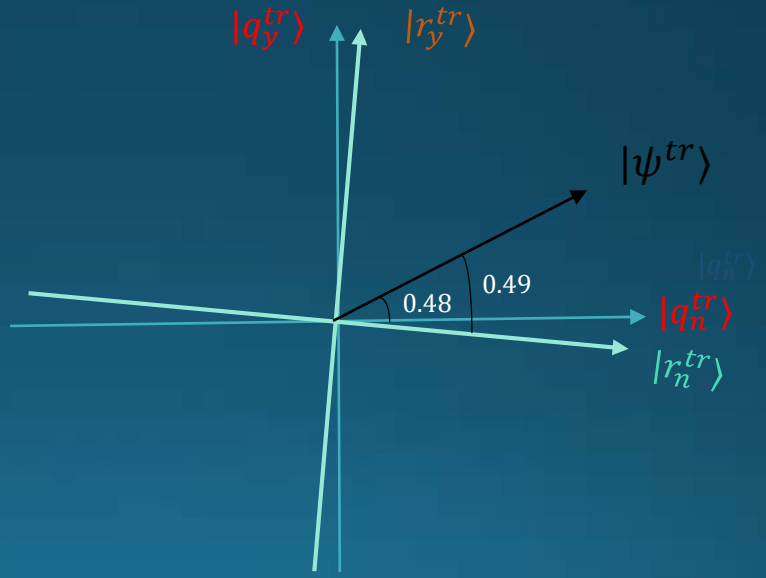
		Transit Users		
		No	Yes	Total
Stated Preference (%)	No	73.13	5.10	78.23
	Yes	6.46	15.31	21.77
	Total	79.58	20.42	

Example 1: Case study using Fujii & Garling's data

- They selected responses only when the participants experienced little changes (e.g., no home/work relocation and no major income change)
- Revealed preference is considered as a "self-imposing question"
- New transit service (e.g., frequency, fare) was verified to be consistent with what was provided in the survey
- Original transit users tend to predict their behavior better; OR, transit users are less likely to be framed by the question



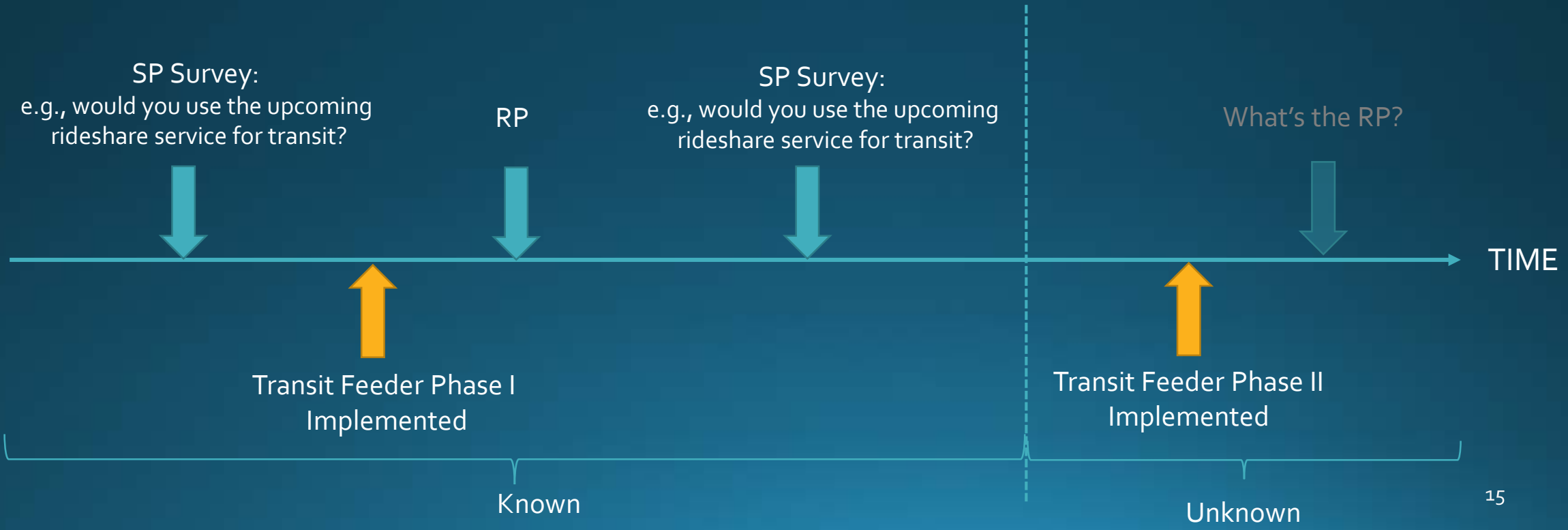
Originally Non-Transit Users



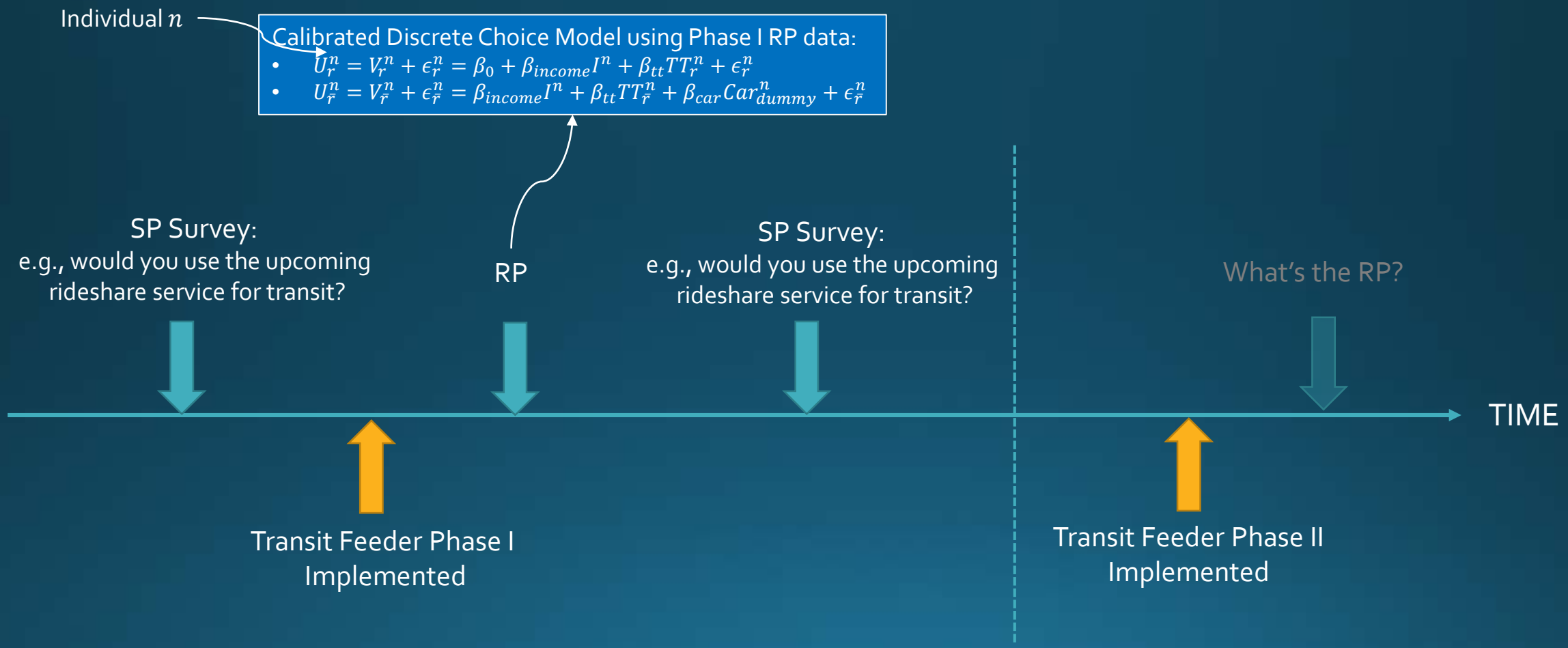
Originally Transit Users

Example 2 (A mixed approach)

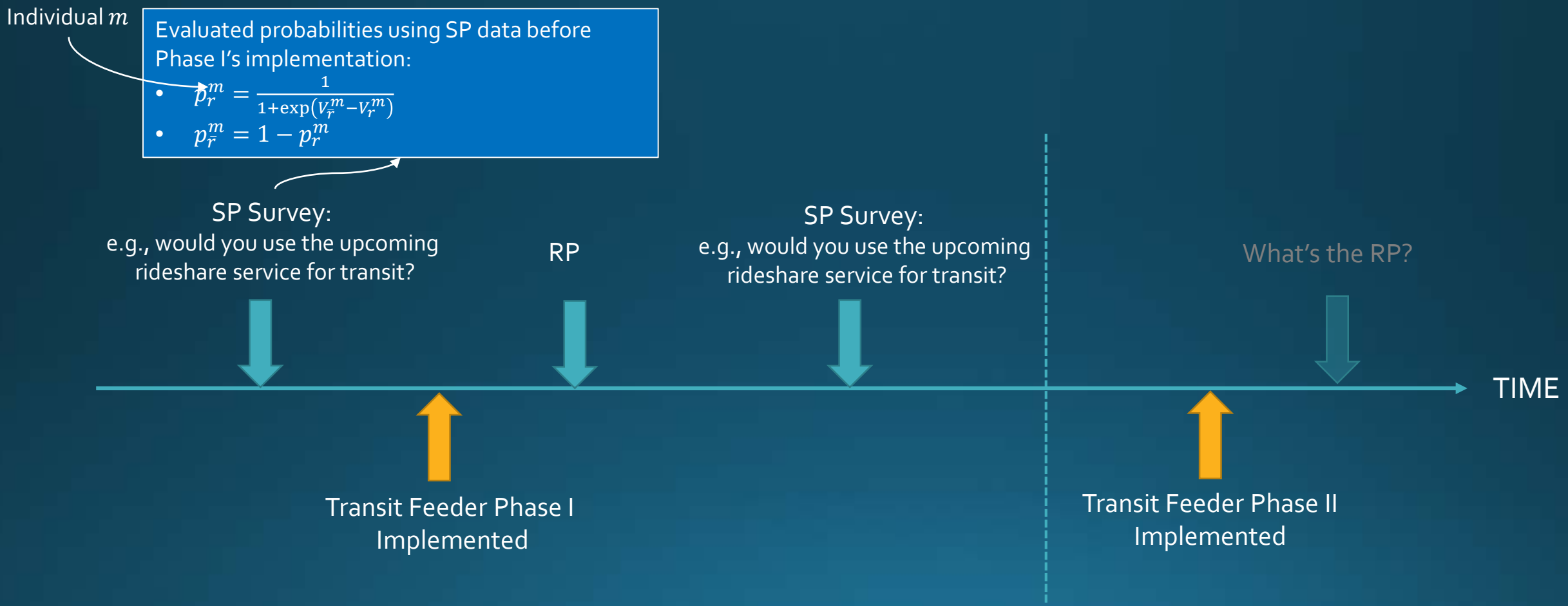
- Suppose we estimated parameters based on RP data of a new transit feeder system with rideshare service
- We can evaluate the SP data using the calibrated model and a "quantum corrector"
- Now, we can then use it to predict new actual behavior based on what we give in SP



Example 2
Step 1: Calibrate Discrete Choice Model Using RP Data after Phase I



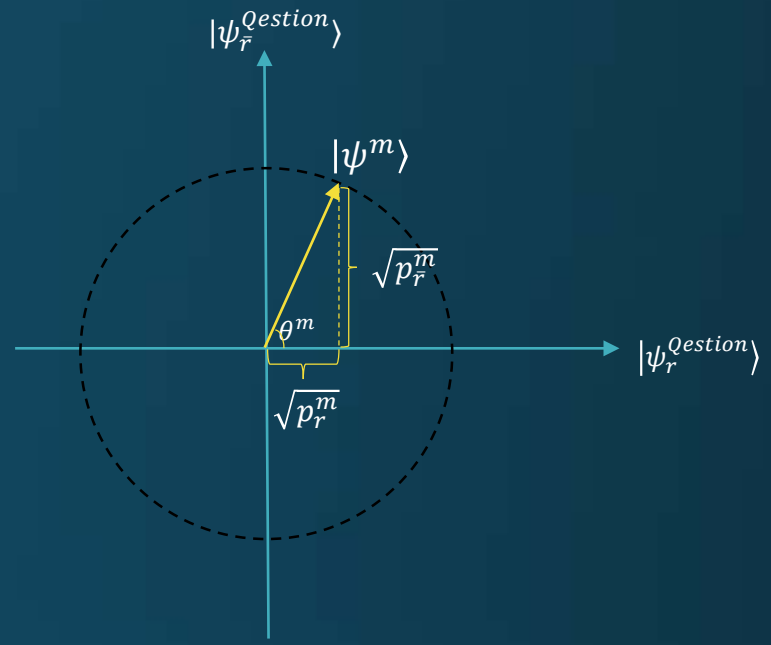
Example 2
Step 2a: Evaluate Discrete Choice Model
Using SP data obtain probabilities



Example 2
Step 2b: Obtain the angel, θ, between the two coordinate systems

Since $\frac{p_{\bar{r}}^m}{p_r^m} = \exp(V_{\bar{r}}^m - V_r^m)$, we have $\sqrt{\frac{p_{\bar{r}}^m}{p_r^m}} = \sqrt{\exp(V_{\bar{r}}^m - V_r^m)} = \tan(\theta^m)$

So, $\theta^m = \arctan\left(\sqrt{\exp(V_{\bar{r}}^m - V_r^m)}\right)$. Or equivalently $V_{\bar{r}}^m - V_r^m = 2\ln(\tan(\theta^m))$



SP Survey:
e.g., would you use the upcoming rideshare service for transit?

RP

SP Survey:
e.g., would you use the upcoming rideshare service for transit?

What's the RP?

TIME

Transit Feeder Phase I Implemented

Transit Feeder Phase II Implemented

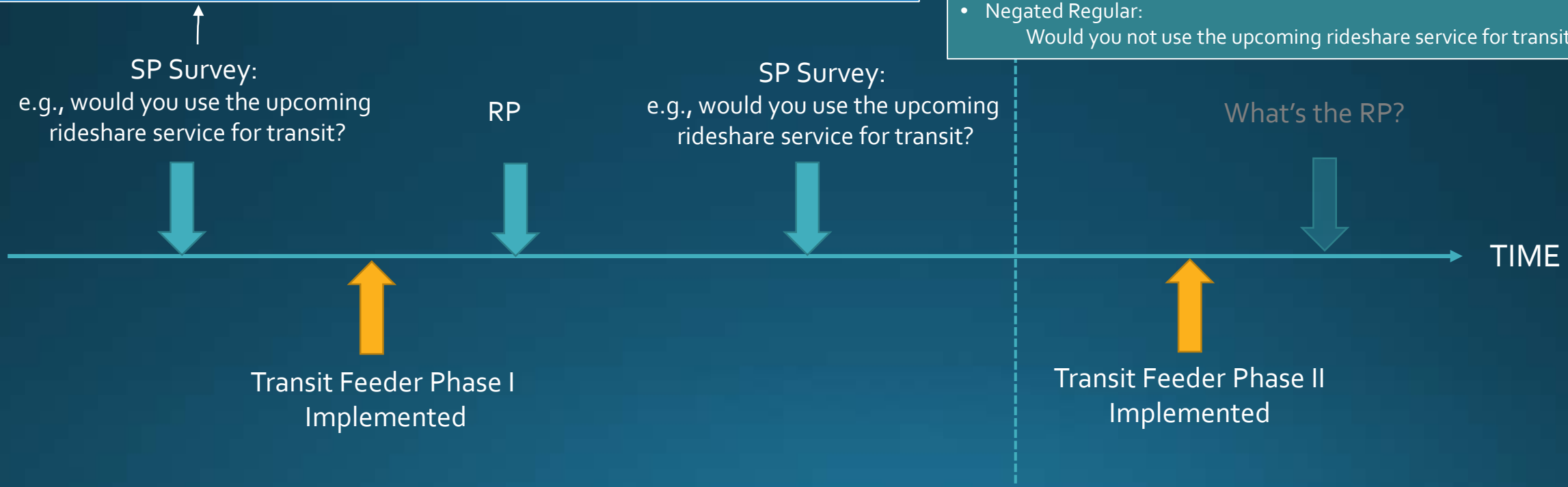
Example 2
Step 3: Obtain $\Delta\theta$ between the two coordinate systems

$\max \sum_m (y_r^m log p_r^m + y_{\bar{r}}^m log p_{\bar{r}}^m)$, where $p_r^m = \frac{1}{1+\exp(2\ln(\tan(\theta^m + \Delta\theta^m)))}$, $p_{\bar{r}}^m = 1 - p_r^m$

Note: the only unknown is $\Delta\theta^m$, which might be a function of m's attributes and question's wording type dummy. The optimal $\Delta\theta^m$ function specification maximizes the likelihood function.

E.g., four wording types suggested by Schriesheim (1995):

- Regular:
  Would you use the upcoming rideshare service for transit?
- Polar Opposite:
  Would you keep using the current mode after the implementation of the rideshare service for transit?
- Negated Polar Opposite:
  Would you not keep using the current after the implementation of the rideshare service for transit?
- Negated Regular:
  Would you not use the upcoming rideshare service for transit?



Example 2
Step 4: Predict based on new SP data using the joint model

Update Discrete Choice Model using the RP and "corrected" SR data (or Bayesian):

$$\max \sum_n \left(y_r^n log p_r^n + y_{\bar{r}}^n log p_{\bar{r}}^n\right) + \sum \left(y_r^k log p_r^k + y_{\bar{r}}^k log p_{\bar{r}}^k\right),$$

where $p_r^k = \frac{1}{1+\exp\left(2\ln\left(\tan\left(\theta^k - \Delta\theta\right)\right)\right)}$, $p_{\bar{r}}^k = 1 - p_r^k$

Individual k

SP Survey:
e.g., would you use the upcoming rideshare service for transit?

RP

SP Survey:
e.g., would you use the upcoming rideshare service for transit?

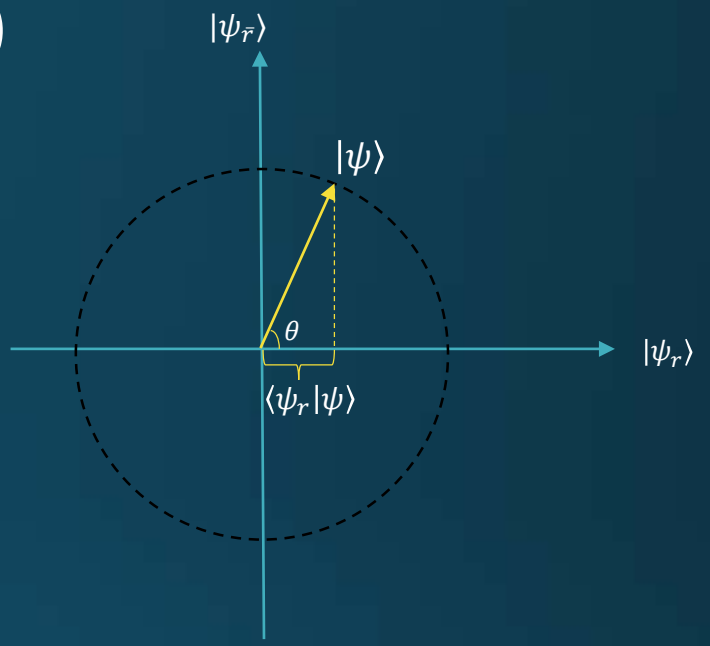
What's the RP?

TIME

Transit Feeder Phase I Implemented

Transit Feeder Phase II Implemented

Greater Flexibility (Through Hilbert Space)



Note that, since the ratio $\sqrt{\frac{p_{\bar{\mathrm{r}}}}{p_{\mathrm{r}}}} = \sqrt{\frac{|\langle\psi_{\bar{\mathrm{r}}}|\psi_{\bar{\mathrm{r}}}\rangle|^2}{|\langle\psi_{\mathrm{r}}|\psi_{\mathrm{r}}\rangle|^2}}$ determines the revealed probabilities, any $\psi_{\bar{\mathrm{r}}} = \sqrt{p_{\bar{r}}}e^{-i\delta_{\bar{r}}}$ and $\psi_r = \sqrt{p_r}e^{-i\delta_r}$ would produce the same ratio.

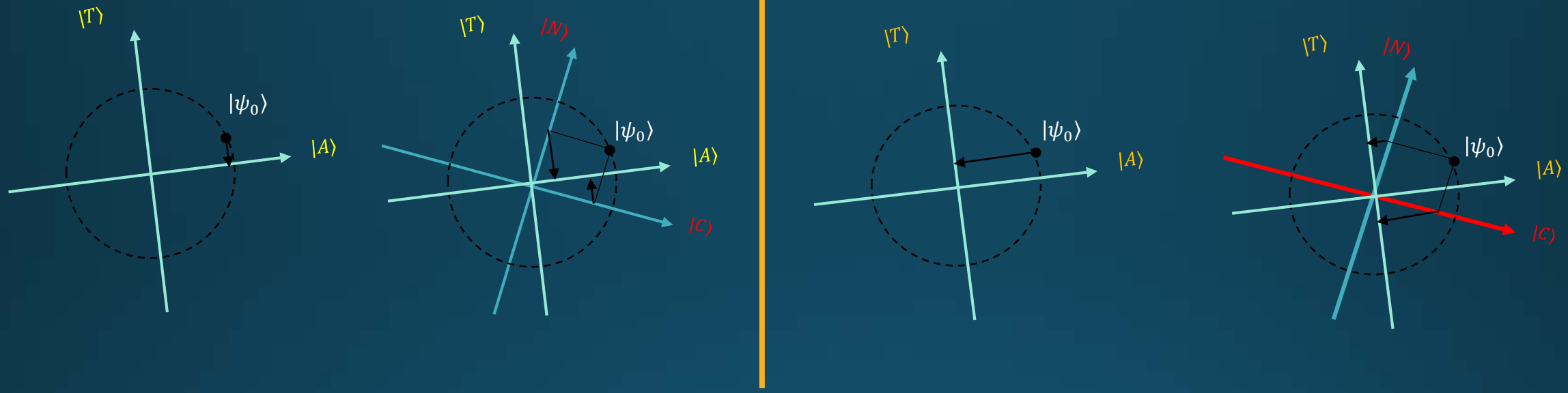
E.g.,

$$\psi_{\bar{\mathrm{r}}}^* \cdot \psi_{\bar{\mathrm{r}}} = \sqrt{p_{\bar{r}}}e^{i\delta_{\bar{r}}} \cdot \sqrt{p_{\bar{r}}}e^{-i\delta_{\bar{r}}} = p_{\bar{\mathrm{r}}}$$

In other words, the same $\sqrt{\frac{p_{\bar{\mathrm{r}}}}{p_{\mathrm{r}}}}$ could come from different combinations of framing effects.

$e^{-i\delta_{\bar{r}}}$ is also related to the time evolution of the system.

Interference Effect of Survey Question



- $\Pr(A) = |\langle A|\psi_0\rangle|^2 = \langle\psi_0|A\rangle\langle A|\psi_0\rangle$
- $\Pr(A|N\ or\ C) = |\langle A|N\rangle\langle N|\psi_0\rangle + \langle A|C\rangle\langle C|\psi_0\rangle|^2$

$$= |\langle A|N\rangle\langle N|\psi_0\rangle|^2 + |\langle A|C\rangle\langle C|\psi_0\rangle|^2 + \langle A|N\rangle\langle N|\psi_0\rangle\langle A|C\rangle\langle C|\psi_0\rangle + \langle A|C\rangle\langle C|\psi_0\rangle\langle A|N\rangle\langle N|\psi_0\rangle$$

$$= |\langle r_D|q_D\rangle\langle q_D|\psi_0\rangle|^2 + |\langle r_D|q_R\rangle\langle q_R|\psi_0\rangle|^2 + 2\cdot\delta_D\cdot Int(\Delta\theta)$$

Summary

- SP-RP relationship could potentially benefit from the quantum cognition approach.
- Travel behavior studies can provide an application field and contribute to the development of this new method
- Just a Beginning!
  - Identifying factors that affect θ and $\Delta\theta$
  - Sequencing Effect of Information Provision (e.g., traveler information, stakeholder investment decisions, etc.)
  - Considering dynamics – time evolution of mental state vectors
  - Modeling impact of format/instrument of traveler information on system performance
  - Modeling impact of various information provision scheme to decision makers

Selected References

- Bruza, P. D., Wang, Z., & Busemeyer, J. R. (2015). *Quantum cognition: a new theoretical approach to psychology*. Trends in cognitive sciences, 19(7), 383-393.
- Busemeyer, J. R., & Bruza, P. D. (2012). *Quantum models of cognition and decision*. Cambridge University Press.
- Franco, R. (2009). The conjunction fallacy and interference effects. *Journal of Mathematical Psychology*, *53*(5), 415-422.
- Fujii, S., & Gärling, T. (2003). Application of attitude theory for improved predictive accuracy of stated preference methods in travel demand analysis. *Transportation Research Part A: Policy and Practice*, *37*(4), 389-402.
- Haven, E., & Khrennikov, A. (2013). *Quantum social science*. Cambridge University Press.
- Railsback, S. F., & Grimm, V. (2011). *Agent-based and individual-based modeling: a practical introduction*. Princeton university press.
- Shannon, C. E. (1948). A mathematical theory of communication. *Bell System Technical Journal*, 30:47-51
- Stopher, P., FitzGerald, C., & Xu, M. (2007). Assessing the accuracy of the Sydney Household Travel with GPS. *Transportation*, *34*(6), 723-741.
- Thaler, R. H. (2015). *Misbehaving: how economics became behavioural*. Allen Lane.
- Thaler, R., Sunstein C. (2008). Nudge: Improving decisions about health, wealth, and happiness.
- Tversky, A., & Kahneman, D. (1983). Extensional versus intuitive reasoning: The conjunction fallacy in probability judgment. *Psychological review*, *90*(4), 293.
- Ben-Akiva, M., Bradley, M., Morikawa, T., Benjamin, J., Novak, T., Oppewal, H., & Rao, V. (1994). Combining revealed and stated preferences data. *Marketing Letters*, *5*(4), 335-349.
- Ben-Elia, E., Erev, I., & Shiftan, Y. (2008). The combined effect of information and experience on drivers' route-choice behavior. *Transportation*, *35*(2), 165-177.

Thank You