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Urban Arterial Speed-Flow Equations For Travel Demand Models

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ABSTRACT

This paper describes the effort to improve the speed-flow relationships for urban arterial streets that are contained in the Southern California Association of Governments metropolitan area travel demand model. Intersection traffic counts and floating car runs were made over 4-hour long periods on one-mile long sections of 8 different arterial streets within the City of Los Angeles.

The field data was then filtered to identify which speed measurements were taken during under-capacity conditions and which measurements were made during congested conditions when demand exceeded the capacity of one or more intersections on the arterial. Since the traditional manual intersection traffic count method that was used to gather volumes did not measure queue buildup, and therefore demand, the speed data points obtained during congested conditions were set aside and not used in the fitting of speed-flow equations.

Several different speed-flow relationships were evaluated against the field data for under-capacity conditions.

The most promising speed-flow equations for under capacity conditions were then evaluated for their ability to predict delays for congested conditions where one or more intersections on the arterial are over-capacity. The theoretical delay due to vehicles waiting their turn to clear the bottleneck intersection on the arterial was computed using classical deterministic queuing theory. Speed-flow equations that under-predicted the delay to clear a congested intersection were rejected.

Of the speed-flow equations tested, the Akcelik equation performed the best for over-capacity situations and performed as well as other possible equations for under-capacity conditions.

KEYWORDS

Travel Demand Models

Speed-Flow Equations

Arterial Streets

INTRODUCTION

The objective of the study was to develop improved field calibrated speed-flow equations for use in travel demand models to predict the mean speed of traffic on signalized urban arterial streets.

FIELD DATA COLLECTION

Intersection turning movement counts and GPS equipped floating cars were used to gather a total of 216 hourly observations of speed and flow were obtained on 54 directional street segments (defined as a one-way link between two signalized intersections) at 8 different sites in the Los Angeles basin. A total of 12.8 directional miles of arterial streets were surveyed. Table 1 shows the salient characteristics of each survey site.

DATA FILTERING

The method used to collect intersection volumes measured intersection discharge rates rather than demand. When the demand is less than the discharge capacity for the intersection approach then discharge rate and demand are identical. When the demand exceeds capacity, then the demand diverges from the counted discharge rate. Consequently it was necessary to identify data points when the counted volume did not equal the demand and drop these points from the data set.

CANDIDATE SPEED-FLOW EQUATIONS

There are several candidate speed-flow equations that might be fitted to the observed data. Table 1. Functional Form Candidates for Speed-Flow Curves, describes several candidates. The first five candidates, linear, logarithmic, exponential, power, and polynomial are standard mathematical functions commonly used in data analysis. The last two equation forms; BPR and Akcelik, are unique to travel time and delay analysis.

The BPR (Bureau of Public Roads) equation [**1**] has been the traditional method for predicting vehicle speed as a function of volume/capacity ratio in travel demand models.

$$S = \frac{S_0}{\left[1 + a(X)^b\right]}$$ **Equation 1**

where

S = average link speed (mph or km/hr)
S_0 = free-flow link speed (mph or km/hr)
X = volume/capacity ratio
$a = 0.15$
$b = 4$

The BPR equation was originally fitted to 1965 Highway Capacity Manual [**2**] freeway speed-flow data. Since then additional research has indicated a less significant effect of volume/capacity ratio on mean speeds until capacity is reached (see Exhibit 13-4 of the Year 2000 Highway Capacity Manual [**3**]).

The Akcelik equation was derived by Akcelik [**4**] from the steady state delay equation for a single channel queuing system. He derived the following time dependent form.

$$S = \frac{L}{L/S_0 + 0.25T\left[(x-1)+\sqrt{(x-1)^2+\frac{8Jx}{cT}}\right]}$$

Equation 2

where

L = Link length (miles)
S = Average link travel time (hrs)
S_0 = Free-flow link travel time (hrs)
x = Volume/capacity ratio
T = Duration of analysis period (hrs)
c = Capacity (vph)
J = Calibration parameter

PRELIMINARY SCREENING OF CANDIDATE SPEED-FLOW EQUATIONS

Speed-flow equations must meet several behavioral requirements in order to permit capacity constrained equilibrium assignment to be performed by travel demand models. The speed-flow equations must be monotonically decreasing and continuous functions of the volume/capacity ratio (v/c) in order for an equilibrium assignment process to arrive at a single unique solution. As a practical matter, the speed-flow equations should also never intersect the *x*-axis (that is, the predicted speed should never reach precisely zero), so that the computer implementing the travel demand model is never confronted with a "divide by zero" problem.

Table 1. Functional Form Candidates for Speed-Flow Curves

Functional Form	Example	Comments
Linear	$s = -a\,x + b$	Not acceptable. Reaches zero speed at high v/c.
Logarithmic	$s = -a \ln x + b$	Not acceptable. Has no value at x = 0 (the logarithm of "x" approaches negative infinity).
Exponential	$s = a\,s_0 \exp(-bx)$	Has all required traits for equilibrium assignment.
Power	$s = a / x^b$	Not acceptable. It goes to infinity at v/c = x = 0.
Polynomial	$s = -ax^2 - bx + c$	Not acceptable. It reaches zero speed at high v/c.
BPR	$s = s_0/(1+a\,(x)^b)$	Has all required traits for equilibrium assignment.
Akcelik	$s = L/[L/s_0+0.25\{(x-1)+ \text{SQRT}\{(x-1)^2+ax\}\}]$	Has all required traits for equilibrium assignment.

s = predicted speed
a,b,c = global parameters for equation.
L = link length
x = volume/capacity ratio
s_0 = Link free-flow speed.

Three of the candidate functional forms meet the equilibrium assignment requirements for a speed flow curve: exponential, BPR, and Akcelik.

MODEL SPEED-FLOW EQUATION CALIBRATION – V/C < 1.00

The exponential, BPR, and Akcelik equations were fitted through a least squared error fitting process to the observed speed-flow data. Figure 2. Speed-Flow Equations Versus Field Data for V/C < 1.00, compares the fit of the Standard BPR and the other fitted curves to the data. As can be seen, the wide scatter of the observed data allows most any speed-flow curve to be drawn through the cloud of data. All three functional forms appear to account for some of the observed variation in speeds.

The Akcelik equation is of interest because it is not a smooth curve in v/c like the others. The Akcelik equation predicted speed is sensitive to the link length in addition to the v/c ratio. The Akcelik equation adds the same delay to a link for a given v/c ratio, regardless of the link length (The assumption being that all the delay occurs at the downstream signal at the end of the link. There is no delay accruing over the length of the link.) The result is that the Akcelik curve shows a bit more scatter (similar to the observed data) than the other curves for which the predicted speeds are not sensitive to link length.

The reader will note that we have calibrated a simplified version of the original Akcelik equation. The constant multiplier of 8 for the "J" calibration parameter has been subsumed within the J calibration parameter itself. We also dropped the variable "capacity" from the Akcelik equation because the simplified equation fit the data better. Note also, that since the length of our analysis period is 1 hour, it is no longer necessary to carry the time period duration variable, "T". The final equation is shown below.

$$S = \frac{L}{L/S_0 + 0.25\left[(x-1) + \sqrt{(x-1)^2 + Jx}\right]}$$

Equation 3

Where all variables are the same as defined before.

A statistical comparison of the equations is presented in Table 3. Quality of Fit to Observed One-Hour Data. This table shows the root mean square error and the bias for each curve when compared against the observed data. The fitted equations (BPR, Exponential, and Akcelik) naturally do better against the field data than the standard BPR equation because they have been fitted to the data. While the standard BPR equation over estimates arterial speeds by an average of 11.5 mph (bias) (18.4 km/hr), the other curves over estimate arterial speeds by less than one-half mile per hour on average. The root mean square (RMS) error for the standard BPR curve is 16 mph (25.6 km/hr), while the other curves have significantly lower RMS errors. The best fitting curve, the Akcelik equation, has about a 40% better RMS error than the standard BPR equation.

MODEL SPEED-FLOW EQUATION CALIBRATION – V/C > 1.00

The field data could not be used to evaluate the speed-flow curve candidates for demands greater than capacity because the standard traffic counting procedure used could only count the served demand, not the unserved demand. Thus a theoretical evaluation was conducted of the speed-flow curves comparing their predicted delays for volumes greater than capacity against the delays predicted by queuing theory.

According to classical queuing theory, when demand is greater than capacity, vehicles must wait their turn in line until the vehicles in front of them have had a chance to pass through the intersection. This theoretical average delay can be graphed and compared to the predictions produced by the candidate speed-flow curves.

Figure 3. Speed-Flow Equations Versus Queuing Theory illustrates this (the chart plots travel time per segment, the inverse of speed, so that the theoretical delay due to queuing can be included in the chart). Points that fall on the horizontal portion of the queuing theory line, represent traffic moving at free-flow speeds with no delay. Points above this horizontal line represent speeds below free-flow speeds, with delay.

The theoretical average delay due to queuing is the thick solid line at the bottom of the chart. The line is flat until the real-world capacity of the link is reached, than the predicted travel time increases rapidly, but linearly with increasing demand.

The ideal speed-flow curve would not cross the theoretical solid line for queue delay.

As can be seen, both the Standard and fitted BPR curves cross the theoretical queuing delay line. Both of these curves under estimate the delay due to queuing when demand exceeds the real-world capacity of an intersection at the end of a link.

The fitted Akcelik curve is consistent with the queue delay line, because the Akcelik curve is derived from classical queuing theory.

CONCLUSIONS

There is a great deal of variation in the observed arterial street segment speeds even when averaged over an hour that cannot be explained solely based on the volume/capacity ratio for the signalized intersection at the terminus of the segment. Volume/capacity ratio appears to explain about 30% of the variation. Other factors besides capacity, such as signal timing offsets, affect the observed mean hourly speed on a segment.

When evaluating data for demands less than the approach capacity, many equations perform equally as well. The fitted BPR, fitted exponential, and the fitted Akcelik equations all performed equally as well. The fitted Akcelik equation performed slightly better because it adds signal delay to the segment free-flow travel time, rather than treating delay as a multiplicative factor of the segment length, as is done in the BPR and exponential equations.

When evaluating the speed-flow equations against theoretical delays for hourly demands greater than hourly capacities, only the Akcelik equation produced the expected delays due to oversaturated conditions at the downstream signal on a street segment. The other equations significantly underestimated delay within the 1.00 to 2.00 v/c range (At significantly higher v/c's the BPR curve eventually catches up to and surpasses the delay estimates produced by queuing theory and the Akcelik equation.

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Robert Dulla and Mr. Thomas Carlson of Sierra Research prepared the sampling plan for the study and the recommended sampling plan for monitoring of regional travel speeds on an on-going basis in the future. Moses Wilson of Wiltech led the floating car and traffic count data collection in the field. Mr. Chris Ferrell and David Reinke of Dowling Associates led the data analysis effort.

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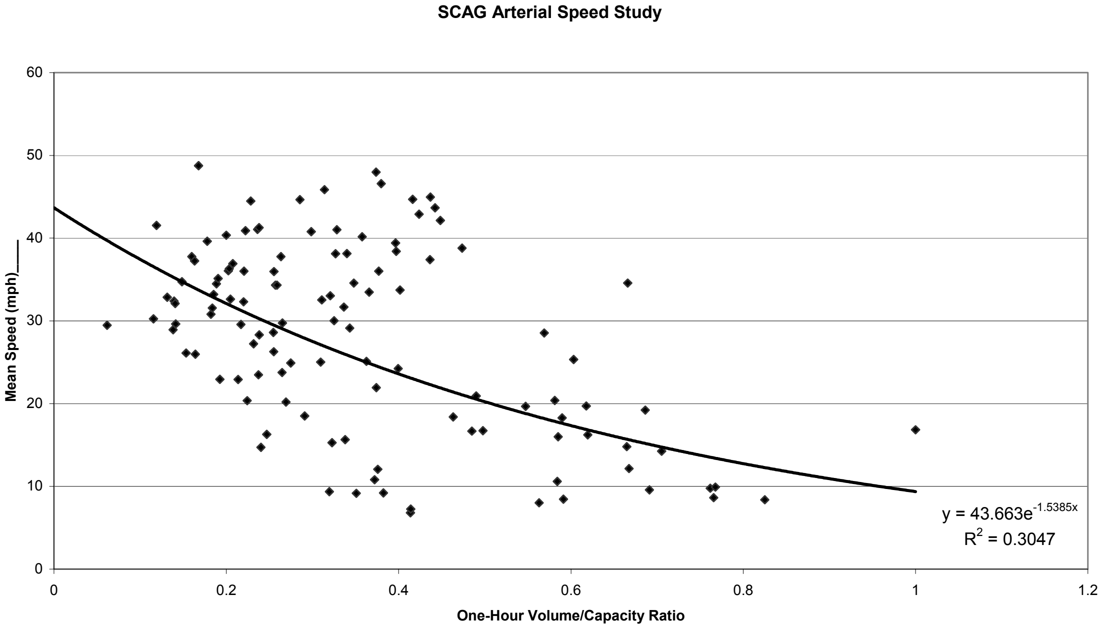

Figure 1. **Hourly Speed-Flow Observations**

Table 2. **Speed Survey Site Characteristics**

Street	From	To	Length (miles)	Facility Type	Area Type	Lanes (both dir.)	Signals (#)	Speed Limit (mph)	ADT (2-way)
1st St	Ford Blvd	Gage Ave	0.90	3	4	4	4	35	19,300
Aviation Blvd	W 120th St	W 135th St	0.99	2	4	4	4	40	33,800
Beverly Blvd	Robertson Blvd	La Cienega Blvd	0.44	2	2	4	4	40	34,700
Lincoln Blvd	Fiji Way	Venice Blvd	1.43	2	3	6	7	35-40	54,600
San Vicente Blvd	Curson Ave	Hauser Blvd	0.64	2	2	6	4	35-40	39,900
Sunset Blvd	N La Brea Ave	N Cherokee Ave	0.51	2	3	6	4	40	33,200
Verdugo Rd	Colorado Blvd	N Shasta Cir	0.83	3	5	4	4	35-40	16,000
Western Ave	W 111th St	W 120th St	0.68	2	4	4-6	4	35	21,300

Facility Type	Area Type
2-= Principal Arterial 3-= Minor Arterial	2-= Central Business District 3-= Urban Business District 4-= Urban 5-= Suburban

Table 3. Quality of Fit to Observed One-Hour Data for V/C < 1.00

Fitted parameters	Standard BPR	Fitted BPR	Fitted Exponential	Fitted Akcelik
S_0 (free-flow speed)	40 mph	40 mph	40 mph	40 mph
A	0.15	2.248	1.0512	0.0019
B	4.00	1.584	-1.185	
Bias (mph)	11.53	0.30	0.04	0.13
RMSE (mph)	16.00	9.83	9.84	9.40

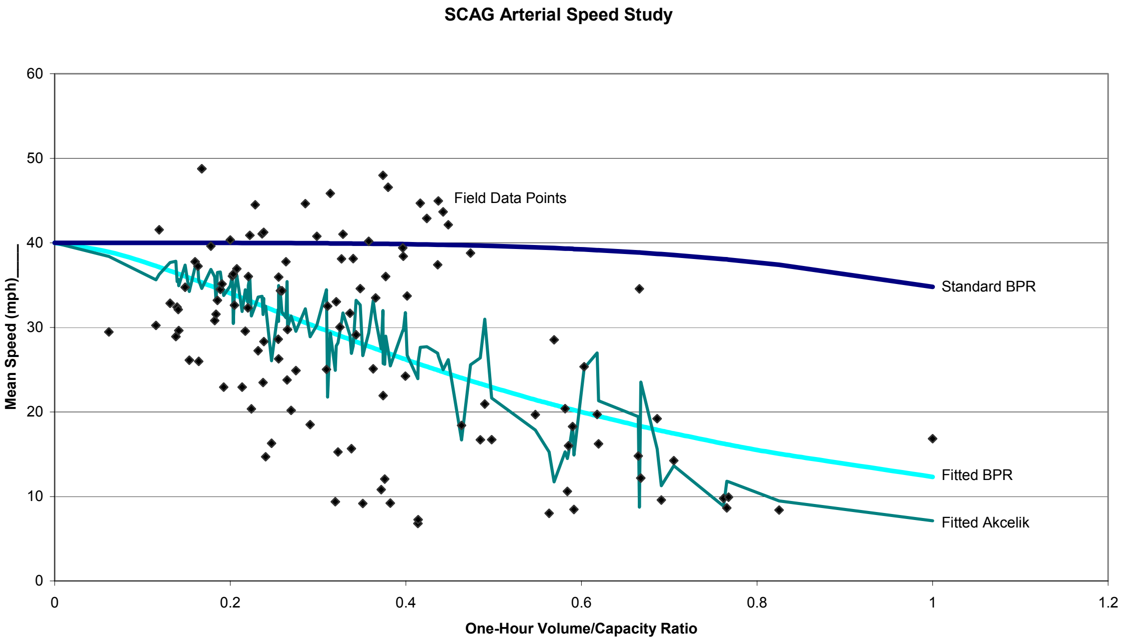

Figure 2. Speed-Flow Equations Versus Field Data for V/C < 1.00

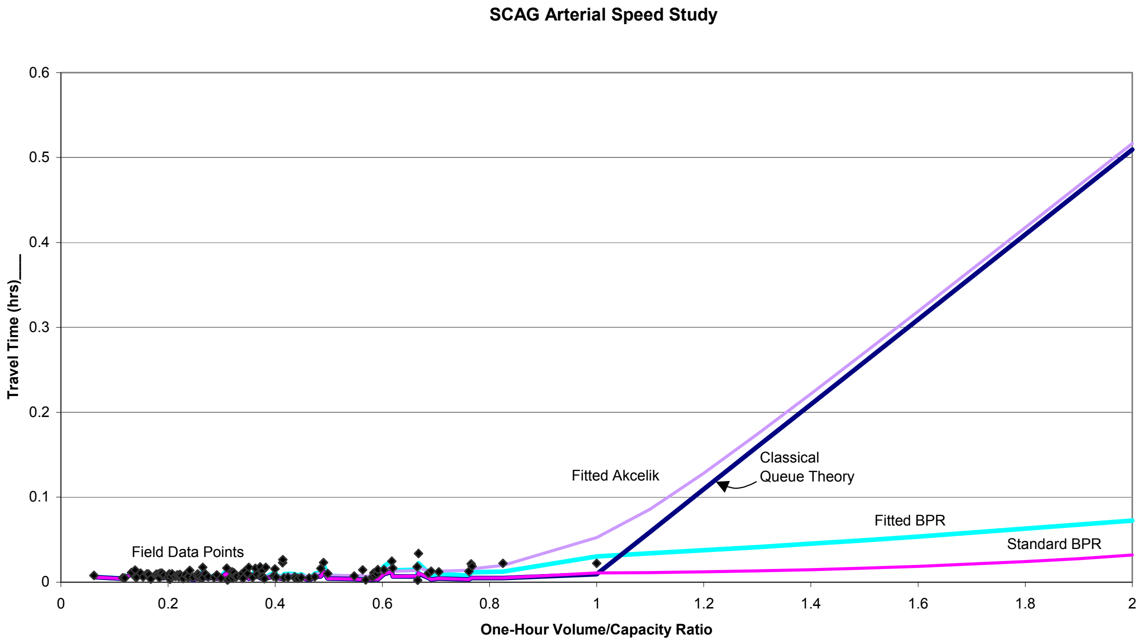

Figure 3. Speed-Flow Equations Versus Queuing Theory