APPENDIX A – MATERIAL PROPERTIES STUDY

A.1 Introduction

The development of high-strength concrete (HSC) has led to more efficient designs of buildings and bridges. HSC allows designers to utilize shallower cross sections and longer spans. Although various national and international specifications include the use of HSC, certain limits were set on the strength of concrete due to the lack of sufficient research data when the specifications were developed. The AASHTO LRFD Bridge Design Specifications (2004) limits the use of HSC with strength no more than 10 ksi (69 MPa). The goal of the National Cooperative Highway Research Program (NCHRP) Research Project 12-64 is to extend the limit of applicability of the LRFD Specifications (2004) to include HSC for flexure and compression members with concrete strength up to 18 ksi (124 MPa).

A.2 Objective and Scope

This appendix summarizes an extensive experimental program to determine the material characteristics of three HSC mixtures with target compressive strengths ranging from 10 to 18 ksi (69 to 124 MPa). The material properties investigated were compressive strength, elastic modulus, Poisson’s ratio, modulus of rupture, creep and shrinkage. More detailed discussions of this experimental program have been presented by Logan (2005) and Mertol (2006).

A.3 Test Program

A.3.1 Test Specimens

A total of 321 specimens of different sizes and shapes were tested to determine the material characteristics of HSC with concrete strengths ranging from 10 to 18 ksi (69 to 124 MPa).
MPa). The variables investigated in this study were the concrete strength, specimen size, curing process, age of loading and sustained stress level. At least three identical specimens were tested for each variable except for the creep study where two replicate specimens were loaded and one companion specimen was used as control specimen without loading. The test matrix for the specimens tested in this program is provided in Table A1 and A2.

Table A1 – Matrix for compressive strength, elastic modulus and modulus of rupture

<table>
<thead>
<tr>
<th>Specimen Type</th>
<th>Size of Specimens</th>
<th>Target Concrete Strength (ksi)</th>
<th>Curing Type</th>
<th>Day of Testing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressive Strength and Elastic Modulus</td>
<td>4×8 in. Cylinders</td>
<td>10</td>
<td>1-Day Heat</td>
<td>1, 7, 14, 28, 56</td>
</tr>
<tr>
<td>Compressive Strength</td>
<td>6×12 in. Cylinders</td>
<td>14, 18</td>
<td>7-Day Moist</td>
<td></td>
</tr>
<tr>
<td>Modulus of Rupture</td>
<td>6×6×20 in. Beams</td>
<td>1-Day Heat</td>
<td>Continual Moist</td>
<td>1, 7, 14, 28, 56</td>
</tr>
</tbody>
</table>

Table A2 – Testing scheme for creep and shrinkage specimens

<table>
<thead>
<tr>
<th>Specimen Type</th>
<th>Size of Specimens</th>
<th>Target Concrete Strength (ksi)</th>
<th>Curing Type</th>
<th>Day of Loading</th>
<th>Loading Stress Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Creep</td>
<td>4×12 in. Cylinders</td>
<td>10</td>
<td>1-Day Heat</td>
<td>1, 7, 14, 28</td>
<td>0.2 (f'_c), 0.4 (f'_c)</td>
</tr>
<tr>
<td>Shrinkage (Cylinder)</td>
<td>4×12 in. Cylinders</td>
<td>14, 18</td>
<td>7-Day Moist</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Shrinkage (Prism)</td>
<td>3×3×11¼ in. Prisms</td>
<td>10</td>
<td>1-Day Heat</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

A.3.2 Material Properties

After numerous trial batches, the three concrete mixtures selected to provide the specified target strengths of 10, 14, and 18 ksi (69, 97, and 124 MPa) are shown in Table A3.

The coarse aggregate was #78M crushed stone with a nominal maximum size of \(\frac{3}{8}\) in. (10 mm), obtained from Carolina Sunrock Corporation. Two types of fine aggregate were used depending on the target compressive strength: (i) natural sand used by the Ready-Mixed Concrete Company in all of their commercial concrete mixtures, and (ii) manufactured sand
known as 2MS Concrete Sand produced by Carolina Sunrock Corporation. The cement used was a Type I/II cement produced by Roanoke Cement Company. The silica fume was supplied by Elkem Materials, Inc, and the fly ash was provided by Boral Material Technologies. Both the high-range water-reducing and the retarding admixtures were manufactured by Degussa Admixtures, Inc. The high-range water-reducing admixture (HRWRA) used was Glenium® 3030 and the retarding admixture was DELVO® Stabilizer.

Table A3 – Three mixture designs for target concrete strength

<table>
<thead>
<tr>
<th>Material</th>
<th>Target Strengths</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10 ksi</td>
</tr>
<tr>
<td>Cement (lbs/yd³)</td>
<td>703</td>
</tr>
<tr>
<td>Silica Fume (lbs/yd³)</td>
<td>75</td>
</tr>
<tr>
<td>Fly Ash (lbs/yd³)</td>
<td>192</td>
</tr>
<tr>
<td>Sand (lbs/yd³)</td>
<td>1055**</td>
</tr>
<tr>
<td>Rock (lbs/yd³)</td>
<td>1830</td>
</tr>
<tr>
<td>Water (lbs/yd³)</td>
<td>292</td>
</tr>
<tr>
<td>High Range Water-Reducing Agent (oz./cwt)*</td>
<td>17</td>
</tr>
<tr>
<td>Retarding Agent (oz./cwt)*</td>
<td>3</td>
</tr>
<tr>
<td>w/cm</td>
<td>0.30</td>
</tr>
<tr>
<td>28-Day Compressive Strength (ksi)</td>
<td>11.5</td>
</tr>
</tbody>
</table>

* Ounces per 100 pounds of cementitious materials, ** Natural Sand, *** Manufactured Sand

A.4 Test Method

The three different curing conditions used in this investigation were: 1-day heat curing, 7-day moist curing and continuous moist curing until the day of testing. The 1-day heat curing was selected to simulate the conditions of precast-prestressed concrete members. The 7-day moist curing was selected to represent typical curing procedures for reinforced concrete members in the field. One-day heat-cured specimens and 7-day moist-cured specimens were subsequently stored in the laboratory, where the temperature was maintained at approximately 72°F (22°C) with 50 percent relative humidity until the day of testing. The 28-day continuously moist-cured specimens were cured according to the ASTM standards which are used for quality control testing by the concrete industry.
Different end treatments for concrete cylinders can significantly affect the measured strength and the variability of the resulting data (ACI 363.2R-98 (1998)). Review of studies related to end treatments showed that grinding of the cylinders provided the highest strength and the lowest coefficient of variation (Zia et al. 1997). Therefore prior to testing, all cylinders were prepared by grinding both ends to remove irregularities in the surfaces and to ensure that the ends were perpendicular to the sides of the cylinders.

A.4.1 Compressive Strength

Compressive strength tests were performed using 4×8 in. (100×200 mm) and 6×12 in. (150×300 mm) cylinders in accordance with AASHTO T 22. The test set-up is shown in Figure A1. The 4×8 in. (100×200 mm) cylinders were tested using a 500-kip (2225 kN) compression machine at concrete ages of 1, 7, 14, 28, and 56 days. The 6×12 in. (150×300 mm) cylinders for the 10 ksi (69 MPa) target concrete compressive strength were tested at 28 and 56 days using the same compression machine. For target concrete strengths of 14 and 18 ksi (97 and 124 MPa), cylinders were tested using a 2000-kip (9000 kN) compression machine at 28 and 56 days. The loading rate used was approximately 35±7 psi/sec (0.25±0.05 MPa/sec) as specified by AASHTO T 22.
A.4.2 Elastic Modulus

ASTM C 469 method was followed to determine the elastic modulus using the 4×8 in. (100×200 mm) concrete cylinders, as shown in Figure A1. One of the three cylinders used for each curing method was tested solely to determine the compressive strength. Subsequently, the remaining two cylinders from each curing method were used to determine the elastic modulus and then tested to failure to determine the compressive strength. Strains were determined using four potentiometers attached to two fixed rings. Four potentiometers were used to measure the axial deformation. In addition, two potentiometers at mid-height were used to measure the lateral dilation of the cylinder. The collected data was used to calculate the elastic modulus and the Poisson’s ratio. The elastic modulus test consisted of three loading cycles. The first loading cycle, which was only intended to seat the gages and the specimen, began at zero applied load and the cylinders were unloaded at 40 percent of the anticipated capacity of the specimen. The second and third loading cycles were applied also up to 40 percent of the anticipated capacity of the specimen. In the third loading cycle, the specimen was loaded ultimately to failure to measure its compressive strength.

A.4.3 Modulus of Rupture

The modulus of rupture tests were carried out using the 6×6×20 in. (150×150×500 mm) beam specimens. The test set-up is illustrated in Figure A2. The specimens were tested under four point loading in accordance to AASHTO T 97. A 90-kip (400 kN) hydraulic jack mounted on a steel frame was used to apply the load, which was measured by a load cell. Below the load cell, there was a spherical head and a plate/roller assembly to distribute the load evenly to the two loading points at the top surface of the specimen. The span length of the specimen was 18 in. (450 mm). The load was applied such that the stress at the extreme bottom fiber of the specimen
increased at a rate of 150 psi/min (1 MPa/min).

Figure A2 – Test set-up for modulus of rupture

A.4.4 Creep

Creep test was performed using 4×12 in. (100×300 mm) cylindrical specimens. The test set-up is shown in Figure A3. Two creep specimens were stacked and loaded in each creep rack using a 120-kip (535 kN) hydraulic jack to induce a stress level of $0.2 f'_c A_g$ or $0.4 f'_c A_g$ where $f'_c$ is the target compressive strength of concrete. The load in each creep rack was monitored using a pressure gage connected to the hydraulic jack at the time of loading and, also by the strain gages attached to the three threaded rods of each rack. Six demec inserts were embedded in each creep specimen and along the height at three 120° angle planes to measure the concrete strain using 8 in. (200 mm) Demec gage. One-day heat-cured specimens were loaded at the end of curing, whereas the 7-day moist-cured specimens were loaded at the 7th, 14th and 28th days. The creep tests were continuously monitored by a datalogger. Disk springs were used in each rack to maintain the necessary load. If the load was reduced by more than 5 percent of the specified load, the load was adjusted to the specified value. Each pair of the creep specimens had a
companion 4×12 in. (100×300 mm) cylinder to measure shrinkage strains, which were then used to adjust the creep strain readings. The two ends of the shrinkage cylinders were sealed with epoxy to achieve the same surface-to-volume ratio of the loaded creep cylinders.

![Test set-up for creep](image)

**Figure A3 – Test set-up for creep**

A.4.5 Shrinkage

Prism specimens of 3×3×11¼ in. (75×75×280 mm) were used to measure shrinkage in accordance with ASTM C 157. The test set-up is presented in Figure A4. Two inserts were embedded at the top and bottom of each specimen to monitor the shrinkage strain using a dial gage. Tests for the 1-day heat-cured specimens were started at the end of the first day, whereas tests for the 7-day moist-cured specimens were started at the 7th day.
A.5 Test Results

A.5.1 Compressive Strength

The effect of curing on the compressive strength of concrete measured by 4×8 in. (100×200 mm) cylinders is illustrated in Figure A5. Numerical values of the tests results of compressive strength for all the specimens are given in Table A4 and Appendix G. Similar behaviors were observed for all the strength levels considered in this study. Cylinders subjected to 7-day moist curing showed the highest compressive strengths at 28 and 56 days. The 1-day heat-cured cylinders typically resulted in the lowest compressive strength at 28 and 56 days, although the major portion of the strength was gained during the first day. Therefore, heat curing to gain early strength as in the precast plant operations may reduce the strength of concrete at later ages. This behavior is attributed to the rapid hydration of cement, which would cause the structure of the cement paste to be more porous than the cement paste subjected to moist curing. The increase in porosity leads to overall reduction of the compressive strength (Neville 1996).
Figure A5 – Effect of curing process on the compressive strength for 16.7 ksi (115 MPa)

Table A4 – Effect of curing process on compressive strength for different ages

<table>
<thead>
<tr>
<th>Target Concrete Strength</th>
<th>Curing Type</th>
<th>Average Concrete Compressive Strength (ksi) at</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1 Day</td>
</tr>
<tr>
<td>10 ksi</td>
<td>1-Day Heat</td>
<td>9.57</td>
</tr>
<tr>
<td></td>
<td>7-Day Moist</td>
<td>5.06</td>
</tr>
<tr>
<td></td>
<td>Continuous Moist</td>
<td>5.06</td>
</tr>
<tr>
<td></td>
<td>7-Day Moist</td>
<td>6.17</td>
</tr>
<tr>
<td></td>
<td>Continuous Moist</td>
<td>6.17</td>
</tr>
<tr>
<td></td>
<td>7-Day Moist</td>
<td>5.99</td>
</tr>
<tr>
<td></td>
<td>Continuous Moist</td>
<td>5.99</td>
</tr>
</tbody>
</table>

The compressive strength of the 28-day moist-cured cylinder was found to be less than the compressive strength of the 7-day moist-cured specimens. This result seemingly contradicts the phenomenon that occurs routinely with normal-strength concrete where extended moist
curing leads to a higher compressive strength. However, the difference is believed to be related to the low permeability of HSC. In the first few days after casting of HSC, the capillary pores within concrete become segmented by the cement gel created in the hydration process. The time required for the capillary pores to get disconnected, decreases significantly as the water to cementitious material ratio decreases. For a w/cm of 0.45, the time that it takes for the capillary pores to become segmented is approximately 7 days (Neville (1996)). Since the concrete used in this study had w/cm less than 0.3, the capillary pores probably became segmented after 1 to 3 days of curing. Once the capillary pores were segmented, they could no longer convey water from the surface of the specimen to the un-hydrated cement inside. Hydration beyond this point could only develop with the water trapped in the pores, not from the surface of the concrete during the remaining moist curing process. Therefore, extended moist curing of HSC beyond 7 days could not generate significantly more cement hydration.

Test data showed that the specimens moist-cured up to the time of testing typically failed at lower strengths than the 7-day moist-cured specimens. This relationship is most likely the result of the testing procedure. At 28 and 56 days, the continuously moist-cured specimens were tested with the inside still in moist condition, while the 7-day moist-cured specimens would have dried out for several weeks. This result is in agreement with the general observation that for compressive strength of concrete, dry specimens exhibit higher strengths than moist specimens. It has been reported that for the 5 ksi (34 MPa) concrete, drying would increase the compressive strength by as much as 10 percent (Neville (1996)).

Figure A6 shows the ratio of the average compressive strength measured by 4×8 in. (100×200 mm) cylinders to the average compressive strength measured by 6×12 in. (150×300 mm) cylinders tested for this research at the ages of 28 and 56 days. Numerical values of the
tests results are given in Appendix G. The results are divided into three categories according to the curing method. By averaging the ratios for each curing method, it can be seen that the strengths of 7-Day moist-cured 4×8 in. (100×200 mm) cylinders were approximately 5 percent greater than that of the strength of the 6×12 in. (150×300 mm) cylinders. For the heat-cured and continuously moist-cured specimens, the compressive strengths of the 4×8 in. (100×200 mm) cylinders were approximately 3 percent greater. It should be noted that testing of the two different cylinders were conducted using two different machines to match their respective capacity. In general, the data from the larger and stiffer testing machine showed greater amount of variation. The average coefficient of variation for the 6×12 in. (150×300 mm) cylinders, when tested on the smaller compression machine was 2.1. When tested in the larger compression machine, the average coefficient of variation for the similar size cylinders was 4.7.

![Concrete Compressive Strength of 4x8 in. / 6x12 in.](image)

Figure A6 – Specimen size effect on compressive strength
A.5.2 Elastic Modulus

The results from the elastic modulus tests for the 16.7 ksi (115 MPa) concrete strength at different ages are shown in Figure A7. The figure also shows the predicted values from the LRFD Specifications (2004) and ACI 318-05 (2005). Numerical values of the tests results of elastic modulus for all the specimens are given in Appendix G. Similar behaviors were observed for all three concrete strengths in this study. After one day of curing, the highest value for the modulus of elasticity was measured in the heat-cured specimens. This was expected due to high early compressive strength achieved by the heat-cured specimens. The elastic modulus of the heat-cured specimens increased slightly by age. The highest increase was approximately 14 percent for the 12.1 ksi (83 MPa) concrete strength.

Figure A7 – Effect of curing process on the elastic modulus for the 16.7 ksi (115 MPa) concrete compressive strength

The elastic moduli for the 7-day moist-cured specimens at testing ages of 7 days or
higher were similar to the values attained from the heat-cured specimens. The 7-day moist-cured specimens showed significant increases in the elastic modulus from the 1<sup>st</sup> day up to the 7<sup>th</sup> day of curing. At the ages of 28 and 56 days, cylinders that were continuously moist-cured, typically resulted in higher values of elastic modulus when compared to the heat-cured and 7-day moist-cured specimens.

The current code equation in the LRFD Specifications (2004) and ACI 318-05 (2005) for estimating the elastic modulus of concrete is:

\[
E_c (\text{psi}) = 33w_c (pcf)^{1.5} \sqrt{f'_{c}(\text{psi})}
\]

Equation A1

\[
E_c (\text{MPa}) = 0.043\left[w_c (\text{kg/m}^3)\right]^{1.5} \sqrt{f'_{c}(\text{MPa})}
\]

where \(w_c\) is the unit weight of the concrete and \(f'_{c}\) is the specified compressive strength. Based on the research by Carasquillo et al. (1981), the following equation was proposed and published in ACI 363R-92 (1997) for estimating the elastic modulus of concrete with strengths ranging from 3.0 to 12.0 ksi (21 to 83 MPa):

\[
E_c (\text{psi}) = \left[40,000 \sqrt{f'_{c}(\text{psi})} + 10^6\right]^{1.5}\left(\frac{w_c (pcf)}{145}\right)
\]

Equation A2

\[
E_c (\text{MPa}) = \left[3,320 \sqrt{f'_{c}(\text{MPa})} + 6,900\right] \left(\frac{w_c (\text{kg/m}^3)}{2,323}\right)^{1.5}
\]

Figure A8 shows test results from this research as well as those from the literature compared with the LRFD Specifications (2004), ACI 318-05 (2005), and ACI 363R-92 (1997). Numerical values of the tests results from other researches are presented in Appendix G. Note that, for the unit weight of the concrete, the average of the unit weights of the three target strength mixtures, 159 pcf (2547 kg/m<sup>3</sup>), was used in these equations. The measured values are generally in good agreement with the ACI 363R-92 (1997) equation regardless of curing method.
or compressive strength. The data also supports the findings of ACI 363R-92 (1997) that the LRFD Specifications (2004) (ACI 318-05 (2005)) equation consistently over-estimates the elastic modulus for HSC.

Over 4000 test results for elastic modulus were collected from Cook (2006), Noguchi Laboratory in Japan and Tadros (2003) as shown in Table A5.

Based on the collected data, the following equation (Equation A3) for the elastic modulus for concrete compressive strength up to 18 ksi (124 MPa) is proposed.

![Figure A8 – Elastic modulus vs. concrete compressive strength](image)

* Sources for this data were Le Roy (1996), Dong and Keru (2001), Chin and Mansur (1997), Carrasquillo et al. (1981), Khan et al. (1995), Iravani (1996), and Cusson and Paultre (1994).

<table>
<thead>
<tr>
<th>Table A5 – Range of the collected data</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Data</td>
</tr>
<tr>
<td>-------------</td>
</tr>
<tr>
<td>4388</td>
</tr>
</tbody>
</table>
\[ E_c(ksi) = 310,000K_1 \left( w_c(kcf) \right)^{2.5} \cdot \left( f'_c(ksi) \right)^{0.33} \]

\[ E_c(MPa) = 0.000035K_1 \left( w_c(kg/m^3) \right)^{2.5} \cdot \left( f'_c(MPa) \right)^{0.33} \]

where \( K_1 \) is the correction factor to account for the source of aggregate which should be taken as 1.0 unless determined by physical test, and as approved by the authority of jurisdiction, \( w_c \) is the unit weight of concrete and \( f'_c \) is the specified compressive strength of concrete.

The collected data including the results from this study are compared to the following equations: LRFD Specifications (2004), ACI 363R-92 (1997), and proposed equation in Figures A9 (a), (b) and (c), respectively.

(a) LRFD Specifications (2004)
Results of the statistical analysis for the ratio of the predicted to the measured elastic modulus are presented in Table A6. The normal distributed data with respect to various equations are shown in Figure A10.
Table A6 – Results of statistical analysis

<table>
<thead>
<tr>
<th></th>
<th>Mean (m)</th>
<th>Standard Deviation (σ)</th>
<th>1/(σ√2π)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LRFD (2004)</td>
<td>1.06</td>
<td>0.18</td>
<td>2.18</td>
</tr>
<tr>
<td>ACI 363R-92 (1997)</td>
<td>0.95</td>
<td>0.15</td>
<td>2.72</td>
</tr>
<tr>
<td>Proposed</td>
<td>0.99</td>
<td>0.16</td>
<td>2.48</td>
</tr>
</tbody>
</table>

It can be clearly seen from Figure A10 that the current LRFD Specifications (2004) over-estimates the elastic modulus. On the other hands, the ratio of the predicted elastic modulus using the ACI 363R-92 (1997) has the lowest standard deviation among the three predictions. However, ACI 363R-92 (1997) provides a slightly conservative prediction. Finally, the proposed equation shows that the mean of the ratio of the predicted to the measured elastic modulus is close to 1, even though the standard deviation is slightly higher than that based on ACI 363R-92 (1997) equation. Therefore, the proposed equation for elastic modulus is recommended for concrete compressive strength up to 18 ksi (124 MPa).

Figure A10 – Normal distribution for the ratio of predicted to measured elastic modulus
A.5.3 Poisson’s Ratio

Poisson’s ratio was determined using the measured lateral and axial strains of the cylinders tested in compression. The range used corresponds to an axial strain of 50 $\mu$ε which corresponds to 40 percent of the measured peak stress. The measured Poisson’s ratios for concrete of different strength showed large variations, as seen in Figure A11. Numerical values of the Poisson’s ratios obtained in this research are given in Appendix G. Test results do not show an apparent correlation between the Poisson’s ratio and the measured compressive strength. In addition, it was observed that curing procedures and age of concrete showed little or no effect on the Poisson’s ratio. The average Poisson’s ratio for all measured cylinders is 0.17 with a standard deviation of 0.07. The generally accepted range for the Poisson’s ratio of normal-strength concrete is between 0.15 and 0.25, while it is generally assumed to be 0.20 for analysis (Nawy (2001)). The test data from this project suggest that it is equally reasonable to use 0.2 as Poisson’s ratio for HSC up to 18 ksi (124 MPa).

![Figure A11 – Poisson’s ratio for various concrete compressive strengths](image_url)
A.5.4 Modulus of Rupture

Test results for the 16.7 ksi (115 MPa) concrete compressive strength with different curing conditions are shown in Figure A12. The results suggest that the modulus of rupture is significantly affected by curing conditions. Numerical values of the tests results of modulus of rupture for all the specimens are given in Appendix G. Similar behavior was observed for all of the three concrete strengths considered in this study. The trend indicates that removal of beam specimen after 7-day from the curing tank causes significant reduction of the modulus of rupture. Similarly, the 1-day heat-cured beam specimens showed low values of the modulus of rupture due to the dryness after removal from the molds which prevented moisture loss during the first day of curing.

![Figure A12 – Effect of curing process on the modulus of rupture for the 16.7 ksi (115 MPa) concrete compressive strength](image-url)
In both cases, the reduced modulus of rupture is believed to be the result of micro-cracks initiated by drying shrinkage. The low permeability of the HSC causes internal differential shrinkage strains due to the fact that the moisture trapped in the interior part of the specimens cannot evaporate as quickly as the surface moisture. This relative shrinkage difference causes micro-cracking of concrete (Neville (1996)). Therefore, the specimens that were moist-cured up to the time of testing showed much higher modulus of rupture than those cured for only 7 days.

Figure A13 shows test data from material study tested in this program along with the data collected from others (Legeron and Paultre (2000), Paultre and Mitchell (2003), Mokhtarzadeh and French (2000), Li (1994) and the Noguchi Laboratory). Two equations for modulus of rupture given in Section 5.4.2.6 of the current LRFD Specifications (2004) are also shown in the figure. Some of the tests results correspond better to the current upper bound of the LRFD Specifications (2004). This is mainly due to the curing condition and moisture content of the specimens. Test results suggest that the current lower bound of the LRFD Specifications (2004) overestimates the modulus of rupture for HSC. A better predictive equation, using the lower bound of the test data, \( f_r = 0.19\sqrt{f'_c (\text{ksi})} \) (\( f_r = 0.5\sqrt{f'_c (\text{MPa})} \)), is proposed for HSC with compressive strengths up to 18 ksi (124 MPa).
A.5.5 Creep

A total of thirty six (36) 4×12 in. (100×300 mm) cylinders were used to evaluate the creep behavior of HSC. Details of the creep test program are given in Table A7.
Table A7 – Details of creep test program

<table>
<thead>
<tr>
<th>Rack No</th>
<th>Target Concrete Compressive Strength (ksi)</th>
<th>Curing Type</th>
<th>Concrete Compressive Strength @ 28 days (ksi)</th>
<th>Day of Loading (days)</th>
<th>Concrete Compressive Strength @ Day of Loading (ksi)</th>
<th>Applied Stress (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10Rack1</td>
<td>10</td>
<td>1-Day Heat</td>
<td>10.4</td>
<td>1</td>
<td>1</td>
<td>9.6</td>
</tr>
<tr>
<td>10Rack2</td>
<td></td>
<td>7-Day Moist</td>
<td>12.1</td>
<td>14</td>
<td>10.8</td>
<td>2</td>
</tr>
<tr>
<td>10Rack3</td>
<td></td>
<td></td>
<td></td>
<td>28</td>
<td>12.1</td>
<td></td>
</tr>
<tr>
<td>10Rack4</td>
<td></td>
<td></td>
<td></td>
<td>8</td>
<td>8.3</td>
<td></td>
</tr>
<tr>
<td>10Rack5</td>
<td></td>
<td></td>
<td></td>
<td>14</td>
<td>10.8</td>
<td></td>
</tr>
<tr>
<td>10Rack6</td>
<td></td>
<td></td>
<td></td>
<td>28</td>
<td>12.1</td>
<td></td>
</tr>
<tr>
<td>14Rack1</td>
<td>14</td>
<td>1-Day Heat</td>
<td>14.3</td>
<td>14</td>
<td>14.5</td>
<td>2.8</td>
</tr>
<tr>
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Measured creep strains were adjusted by subtracting the measured shrinkage strain of an unloaded companion cylinder for each rack. The average specific creep, defined as creep strain per unit stress (ksi or MPa) and the average creep coefficients, defined as the ratios between the creep deformations at time \( t \) and the instantaneous elastic strain, were calculated to evaluate the creep behaviour for HSC, as given in Appendix G. The average creep coefficients of specimens with 18 ksi (124 MPa) target concrete strength are shown in Figure A14. In general, test results of three concrete strengths considered in this investigation indicate that as concrete gets older and stronger, the creep of concrete decreases. The creep behavior of 1-day heat-cured cylinders is less than that of the 7-day moist-cured cylinders. The creep for HSC is proportional to the applied stress provided that the applied stress is less than the proportional limit.
Variations of the temperature and humidity during the test are given in Appendix G and the average creep coefficients were adjusted accordingly. The procedure used to calculate the adjusted average creep coefficients for each rack is given in Appendix G. The adjusted average creep coefficients compared to creep prediction equations by the LRFD Specifications (2004) are shown in Figure A15 and Figure A16.
Figure A15 – Comparison of adjusted creep coefficient of 10Rack2 and 10Rack 5

Figure A16 – Comparison of adjusted creep coefficient of 18Rack3 and 18Rack6
Test results indicate that the equations of the LRFD Specifications (2004) provide adequate predictions of the creep behavior of HSC. However, it was found that the time-development correction factor, as shown below yields negative results in the first few days after loading if concrete compressive strengths were greater than 15 ksi (103 MPa):

\[ k_{td} = \frac{t}{61 - 4f'_{ci}} + t \quad (f'_{ci} \text{ in ksi}) \]

Equation A5

\[ k_{td} = \frac{t}{61 - 0.58f'_{ci}} + t \quad (f'_{ci} \text{ in MPa}) \]

where \( t \) is the age of concrete after loading in days, \( f'_{ci} \) is the specified compressive strength at prestress transfer for prestressed members or 80 percent of the strength at service for non-prestressed members.

The above equation also gives abrupt changes in the slope of the predicted creep in the first few days for concrete compressive strengths greater than 12 ksi (83 MPa). This equation was developed by Tadros et al. (2003) based on research data with concrete strengths up to 12 ksi (83 MPa), but extended to include strengths up to 15 ksi (103 MPa). Although for the pretensioned members, it is unlikely to require a concrete compressive strength of more than 10 ksi (69 MPa) at transfer of prestress, the equation must be suitable also for applications to other cases such as cast-in-place columns and post-tensioned girders where the concrete compressive strength at the time of loading could conceivably be higher than 12 ksi (83 MPa). After a detailed examination of the test results of this study, the following modified time-development correction factor is proposed to extend the applicability of creep relationship to 18 ksi (124 MPa):
\[ k_{td} = \frac{t}{12 \left( \frac{100 - 4f'_{ci}}{f'_{ci} + 20} \right) + t} \quad (f'_{ci} \text{ in ksi}) \]

Equation A6

\[ k_{td} = \frac{t}{12 \left( \frac{100 - 0.58f'_{ci}}{0.145f'_{ci} + 20} \right) + t} \quad (f'_{ci} \text{ in MPa}) \]

In Figures A17 to A24, the proposed time-development correction factor and the current expression (Equation A5) are compared for different concrete compressive strengths up to 18 ksi (124 MPa). In these figures, the dotted and the solid lines represent the current and the proposed time-development correction factors, respectively. It can be seen that for concrete compressive strengths greater than 12 ksi (83 MPa), the proposed time-development correction factor eliminates the erroneous predictions given by the current time-development correction factor.

Figure A17 – \( k_{td} \) for \( f'_{ci} = 4 \) ksi (28 MPa)

Figure A18 – \( k_{td} \) for \( f'_{ci} = 6 \) ksi (41 MPa)

Figure A19 – \( k_{td} \) for \( f'_{ci} = 8 \) ksi (55 MPa)

Figure A20 – \( k_{td} \) for \( f'_{ci} = 10 \) ksi (69 MPa)
A.5.6 Shrinkage

Six (6) 4×12 in. (100×300 mm) cylinders and eighteen 3×3×11¼ in. (75×75×280 mm) prisms were used to monitor the shrinkage behavior of HSC. Details of the cylindrical and prismatic shrinkage specimens are given in Table A8 and A9, respectively.

The measured shrinkage strains of cylindrical and prismatic specimens with 10, 14, and 18 ksi (69, 97, and 124 MPa) target concrete compressive strengths are tabulated in Appendix G. The measured shrinkage strains of cylindrical and prismatic specimens with 18 ksi (124 MPa) target concrete strength are shown in Figure A25 and A26. The results indicate that heat-cured specimens have less shrinkage as compared to the moist-cured cylinders. Furthermore, the data given in the Appendix G also indicates that the differences in the measured shrinkage for concrete specimens with target concrete strengths of 10, 14, and 18 ksi (69, 97, and 124 MPa)
are insignificant.

Table A8 – Details of cylindrical shrinkage specimens

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<th>Curing Type</th>
<th>Concrete Compressive Strength @ 28 days (ksi)</th>
<th>Day of Initial Monitoring</th>
<th>Concrete Compressive Strength @ Day of Initial Monitoring (ksi)</th>
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Table A9 – Details of prismatic shrinkage specimens

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Figure A25 – Shrinkage strain of cylindrical specimens with 18 ksi (124 MPa) concrete compressive strength

Figure A26 – Shrinkage strain of prismatic specimens with 18 ksi (124 MPa) concrete compressive strength
The ambient temperature and humidity in the laboratory for the shrinkage specimens varied during the experimental program, as shown in Appendix G. Although the temperature during the experimental period was fairly constant, the variation in the humidity was significant. To account for the variation of humidity on the shrinkage behavior of HSC, the shrinkage strains were adjusted according to a procedure described fully in Appendix G. The adjusted shrinkage strains of cylindrical and prismatic specimens are compared to the predictions of the LRFD Specifications (2004) in Figures A27 and A28.

![Graph showing comparison of adjusted shrinkage strain of 18SC1](image)

**Figure A27 – Comparison of adjusted shrinkage strain of 18SC1**

Test results indicate that the equation of the LRFD Specifications (2004) is adequate to predict the shrinkage of HSC. As before, for concrete compressive strengths greater than 12 ksi (83 MPa), using the proposed time-development correction factor eliminates the unreasonable predictions given by the current time-development correction factor for the shrinkage predictions.
A.6 Summary and Conclusion

Based on the research findings, the following conclusions can be drawn:

- Of the three different curing methods, cylinders moist-cured for 7 days exhibited the highest compressive strengths at ages of 28 and 56 days. In contrast, 1-day heat curing generally resulted in the lowest strength. Cylinders moist-cured up to the time of testing resulted in strengths slightly lower than the 7-day moist-cured specimens. The reduction in strength may be attributed to the differences in the internal moisture conditions of the concrete at the time of testing.

- Comparisons of the compressive strengths of the 7-day moist-cured and the continuously moist-cured specimens indicated that, for HSC, moist curing beyond 7 days did not result in any significant increase in strength. It is believed to be due to the low permeability of HSC
and the short time required for the capillary pores of HSC to be blocked.

- The effect of specimen size on the compressive strength of HSC is negligible, same as in NSC. The average ratios of compressive strengths of the 4×8 in. (102×203 mm) to the 6×12 in. (152×305 mm) cylinders for the 1-day heat-cured, 7-day moist-cured, and continuously moist-cured specimens were 1.05, 1.03 and 1.03, respectively.

- At ages of 28 and 56 days, the continuously moist-cured specimens were found to have the highest values of elastic modulus. This result may be attributed to the moist surface conditions at the time of testing.

- The elastic moduli of the 1-day heat-cured and 7-day moist-cured specimens were comparable despite the difference in their compressive strengths.

- The equation specified by the LRFD Specifications (2004) over-estimated the elastic modulus for all specimens. Based on the tests results and the collected data in the literature, the following equation for the elastic modulus of concrete with compressive strength up to 18 ksi (124 MPa) is proposed.

\[
E_c(\text{ksi}) = 310,000K_i\left(w_c(kcf)\right)^{2.5}\cdot\left(f_c'(\text{ksi})\right)^{0.33}
\]

\[
E_c(\text{MPa}) = 0.000035K_i\left(w_c(\text{kg/m}^3)\right)^{2.5}\cdot\left(f_c'(\text{MPa})\right)^{0.33}
\]

Equation A3

- Poisson’s ratio of 0.2 specified by the LRFD Specifications (2004) can adequately be used for HSC up to 18 ksi (124 MPa).

- The modulus of rupture was reduced significantly for test specimens removed from their sealed or moist environments and allowed to dry. The continuously moist-cured specimens developed modulus of rupture values, in some cases, twice as much as the values obtained from the 7-day moist-cured specimens.

- The upper bound equation specified by the LRFD Specifications (2004) provided a good
estimate of the modulus of rupture for the continuously moist-cured specimens but overestimated the modulus of rupture for the 1-day heat-cured and 7-day moist-cured specimens. Test results suggest that the current lower bound of the LRFD Specifications (2004) overestimates the modulus of rupture for HSC. A better predictive equation, lower bound of the test data, \( f_r = 0.19\sqrt{f'_{c}(ksi)} \) (\( f_r = 0.5\sqrt{f'_{c}(MPa)} \)), is proposed for HSC up to 18 ksi (124 MPa).

- Creep of 1-day heat-cured cylinders was less than the 7-day moist-cured cylinders for the same concrete strength.
- Creep of HSC is proportional to the applied stress provided that the applied stress is less than the proportional limit.
- Heat-cured specimens have less shrinkage compared to moist-cured specimens.
- There was little difference in shrinkage for HSC specimens higher than 12 ksi (83 MPa).
- The test results indicated that the creep and shrinkage prediction relationships specified by the LRFD Specifications (2004) are sufficiently accurate for HSC with the exception of the expression for the time-development correction factor. The following modified time-development correction factor is proposed as the replacement.

\[
k_{id} = \frac{t}{12\left(\frac{100-4f'_{ci}}{f'_{ci}+20}\right) + t} \quad (f'_{ci} \text{ in ksi})
\]

Equation A6

\[
k_{id} = \frac{t}{12\left(\frac{100-0.58f'_{ci}}{0.145f'_{ci}+20}\right) + t} \quad (f'_{ci} \text{ in MPa})
\]
A.7 References


AASHTO T 22, “Compressive Strength of Cylindrical Concrete Specimens,” American Association of State Highway and Transportation Officials, Washington, DC.

AASHTO T 97, “Standard Method of Test for Flexural Strength of Concrete” American Association of State Highway and Transportation Officials, Washington, DC.


ACI Committee 318, “Building Code Requirements for Structural Concrete (ACI 318-05) and Commentary (318R-05),” American Concrete Institute, Farmington Hills, MI, 2005, 430 p.


Chin, M. S., Mansur, M. A. and Wee, T. H., “Effect of Shape, Size and Casting Direction of


Mertol, H. C., “Characteristics of High Strength Concrete for Combined Flexure and Axial Compression Members,” *Ph.D. Thesis*, Department of Civil, Construction and Environmental


Noguchi Laboratory Data, Department of Architecture, University of Tokyo, Japan, [http://bme.t.u-tokyo.ac.jp/index_e.html](http://bme.t.u-tokyo.ac.jp/index_e.html).


APPENDIX B – DISTRIBUTION OF STRESSES IN THE COMPRESSION ZONE OF FLEXURAL MEMBERS

B.1 Introduction

Flexural failure of reinforced concrete member occurs when the extreme compression fiber within the compression zone reaches the ultimate compressive strain of concrete. The concrete in the compression zone has a stress distribution, referred to as the generalized stress block, similar to the stress-strain relationship of concrete cylinder tested in axial compression. This appendix focuses on the evaluation of the stress block of high-strength concrete (HSC) ranging from 10 to 18 ksi (69 to 124 MPa) in the compression zone of flexural members.

Many researchers have investigated the stress-strain distribution of compression zone of flexural concrete members. Hognestad et al. (1955) developed a test set-up to determine the stress-strain distribution for concrete. Their specimens were mostly referred to as C-shaped specimens or eccentric bracket specimens. In their test set-up, they simulated the compression zone of a flexural member on a rectangular cross-section by varying the axial load and the moment on the section. In the research presented in this appendix, the same method was utilized to obtain the stress-strain distribution in the compression zone of HSC flexural members.

B.2 Objective and Scope

This section presents the research findings from twenty-one (21) unreinforced HSC members with concrete compressive strengths ranging from 10.4 to 16.0 ksi (71 to 110 MPa), tested under combined axial and flexure to evaluate the stress-strain distribution of compression zone of concrete members in flexure. Stress-strain curves and stress block parameters for HSC were obtained, evaluated and compiled with the results available in the literature. The results
serve as the basis for the proposed revisions for the AASHTO LRFD Bridge Design Specifications (2004) to increase the limit on the compressive strength of concrete from 10 to 18 ksi (69 and 124 MPa).

B.3 Test Program

B.3.1 Test Specimens

The test program consisted of 21 concrete specimens with a cross-section of 9×9 in. (225×225 mm) and 40 in. (1 m) in length. A general view of the specimens is shown in Figure B1. The end sections of the eccentric bracket specimens were heavily reinforced, while the test region in the middle of the specimens was plain concrete. The main parameter considered in this study was the concrete compressive strength. Three different HSC mixture designs were used to cast the specimens. The target concrete compressive strengths of these mixtures at 28 days were 10, 14, and 18 ksi (69, 97, and 124 MPa). Three 4×8 in. (100×200 mm) cylinders were cast for each test specimen which were tested on the same testing day as the specimen.

The ends sections of the specimens were reinforced with three #4 U-shaped longitudinal and three #3 transverse reinforcement. Steel reinforcement configuration of the specimens is shown in Figure B2. Furthermore, the ends of the specimens were confined with ½ in. (13 mm) thick 10 in. (250 mm) high rectangular steel tubes with holes on two opposite faces. The combination of the steel tubes and heavy reinforcement ensures proper transfer of the axial load and moment and eliminates possible localized failures at the ends of the specimens. The plain concrete test section of the specimens is the middle section of 16 in. (400 mm) in length.

The specimens and control cylinders cast to determine concrete strength were stripped 24 hours after casting and they were covered with wet burlap and plastic sheets for a week. The
specimens were then stored in the laboratory where the temperature was maintained at approximately 72°F (22°C) with 50 percent relative humidity until the time of testing. The cylinders were prepared by grinding both end surfaces before testing. The mixture designs for the three different concrete target strengths, 10, 14, and 18 ksi (69, 97, and 124 MPa) and the type of materials used were given in Appendix A.

![Neutral Face View](image1)

![Typical Side View](image2)

![Typical Side View](image3)

![Neutral Face View](image4)

**Figure B1 – General view**

**Figure B2 – Steel reinforcement configuration**

B.3.2 Test Method and Test Set-Up

Figure B3 shows a schematic view of the test setup. The two axial loads of $P_1$ and $P_2$ are adjusted during the test to maintain the location of the neutral axis, i.e., zero strain at the exterior edge of the specimen. On the opposite side of the cross-section, the extreme fiber is subjected to a monotonically increasing compressive strain. In each load increment, the main axial load from the test machine, $P_1$, creates a constant axial strain in the section. The secondary load applied by a jack, $P_2$, creates a moment such as to maintain zero strain at one face, and the maximum strain at the other.

Two steel arms are connected to the concrete specimen which is confined with rectangular steel tubes at the ends. The holes in the steel tubes allow the arms to be connected to
the concrete section using threaded rods. Two roller connections eliminate the end restrictions due to the applied axial load from the machine. Each roller connection consists of six 1 in. (25 mm) diameter rollers and two curved plates, tapering through inside and outside, respectively. Details of the test set-up can be found in Mertol (2006).

![Test set-up diagram]

**Figure B3 – Test set-up**

B.3.3 Instrumentation

Each specimen was instrumented with 2.4 in. (60 mm) surface mounted strain gages, model PL-60-3L. A total of 9 strain gages were used for each test specimen. Two of them were applied on the zero strain face. Four of them were mounted on the two sides of the specimen. Three of them were located on the maximum compression side of the specimen, one of which was used to measure the transverse strain of concrete. Three 1 in. (25 mm) linear variable displacement transducers (LVDT) were placed at the top, bottom and mid-section in order to obtain the deflected shape of the specimen and to incorporate the secondary moment effects in later analysis. The location of the instrumentation for the test specimen is illustrated in Figure B4.
B.3.4 Test Procedure

The specimen was first leveled using a thin layer of hydrostone (gypsum cement) placed between the roller connections and the specimen at the top and the bottom. The initial readings from the instrumentation were balanced to zero. As the main axial load was increased incrementally, the secondary load was applied by a hydraulic jack and a hand-pump to maintain the neutral axis on the exterior face. The loading rate was 2 microstrains per second on the opposite compression face of the specimen. The duration of each test was about 25 minutes. The test was terminated when the concrete failed in an explosive manner. For each specimen tested, three concrete cylinders were also tested in accordance with ASTM C 39 on the same day.

B.4 Test Results

Although the three target concrete strengths were 10, 14, and 18 ksi (69, 97, and 124 MPa). However, the actual average cylinder strengths were 11.1, 14.9, and 15.4 ksi (76, 103, and 106 MPa), respectively. The highest cylinder strength achieved in this research was 16.0 ksi (110 MPa). All the test specimens had similar explosive failure mode. No cracks were observed prior to failure. Typical failure mode for the eccentric bracket tests is shown in Figure B5. The
cylinder strength, age at testing, the loading rate and the ultimate compressive strain achieved by the specimens are summarized in Table B2.

![Image of eccentric bracket specimens](image)

**Figure B5 – Typical failure mode for eccentric bracket specimens (18EB6)**

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<th>Spec. ID</th>
<th>$f'_c$ at Testing (ksi)</th>
<th>Age at Testing (days)</th>
<th>Loading Rate ($\mu$ε/sec)</th>
<th>Ultimate Strain ($\mu$ε)</th>
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</table>
B.4.1 Concrete Strain Measurements

The surface strain measurements for different loading stages of Specimen 18EB#2 are shown in Figure B6. Similar behavior was observed in all other specimens. The graph confirms that plane sections remain plane after deformation is valid for HSC.

![Figure B6 – Typical strain distribution on side face of specimen 18EB#2](image)

The ultimate concrete compressive strains measured at failure on the extreme compression face of concrete are shown in Table B2. The comparison of the proposed ultimate concrete compressive strain with the test results of this research as well as other researches reported in the literature is shown in Figure B7. The researches reported in the literature consists of test programs performed by Hognestad (1951), Sargin (1971), Nedderman (1973), Kaar et al. (1978a, 1978b), Swartz et al. (1985), Pastor (1986), Schade (1992), Ibrahim (1994), Tan and Nguyen (2005). The complete tabulated values of the research data is presented in Appendix G.
A total of 188 test results from this research and the literature with concrete compressive strengths up to 20 ksi (138 MPa) under eccentric loading were evaluated using regression analysis technique to develop the relationship between the ultimate concrete strain, $\varepsilon_{cu}$, and concrete compressive strength, $f'_c$. Details of the regression analysis are presented in Mertol (2006). Based on this evaluation, the ultimate concrete compressive strain of 0.003 is proposed to be used for design purposes for concrete compressive strengths up to 18 ksi (124 MPa). When only the test results for concrete compressive strength over 10 ksi (69 MPa) are considered, the 90 percent regression line corresponding to the lower bounds of the 90 percent of the test results for $\varepsilon_{cu}$ becomes very close to the proposed value for HSC.

A sensitivity analysis was also performed to assess how sensitive the ultimate flexural resistance of a reinforced concrete member would be affected by the ultimate concrete strain, $\varepsilon_{cu}$. Details of this sensitivity analysis are also presented in Mertol (2006). In this figure, the ratio of
the ultimate moment capacity is used for comparison purposes. This ratio can be defined as the ultimate moment capacities obtained from various ultimate compressive strain values (0.003 to 0.0025) were divided by the ultimate moment capacity obtained from the ultimate compressive strain value of 0.003. The results of the sensitivity analysis for ultimate concrete strain are shown in Figure B8. The analysis indicates that the change in ultimate concrete strain, $\varepsilon_{cu}$, has no effect on the flexural capacity of an under-reinforced concrete section. However, for an over-reinforced concrete section, a reduction of the ultimate concrete strain, $\varepsilon_{cu}$, by 16.7 percent (0.003 vs. 0.0025) leads to a reduction of the ultimate moment capacity by only 2.6 percent.

Figure B8 - Ratio of ultimate moment capacity versus change in $\varepsilon_{cu}$ from 0.003 to 0.0025

The measurements of the horizontal strain gage on the compression face were used to calculate the Poisson’s ratio ($\nu$) for HSC. The calculated values of Poisson’s ratio for all specimens are shown in Table B2 and Figure B9. The graph is extended for concrete compressive strains up to 1,400 microstrains after which, the effect of micro-cracks in the
concrete matrix leads to higher Poisson’s ratios beyond service loading conditions. The figure indicates that the Poisson’s Ratio ranges between 0.20 and 0.25 for HSC specimens tested in this research. There is no apparent trend for Poisson’s Ratio as concrete compressive strength increases. The comparison of the proposed relationship with the test results of this research and other data from the literature is shown in Figure B10. Test data of other researchers consist of those obtained by Komendant et al. (1978), Perenchio and Kliger (1978), Carrasquillo et al. (1981), Swartz et al. (1985), Jerath and Yamane (1987), Radain et al. (1993), and Iravani (1996). The complete tabulated values of the research data is presented in Appendix G.

A total of 246 test results including results from Appendix A with concrete compressive strengths up to 20 ksi (124 MPa) were evaluated using regression analysis technique to develop the relationship between Poisson’s ratio, \( \nu \), and concrete compressive strength, \( f' c \). Details of the

![Figure B9 – Poisson’s ratio for test specimens](image)

\( (f' c)_{test} = 10.4 \sim 16.0 \text{ ksi} \)
regression analysis are presented in Mertol (2006). Based on the evaluation, Poisson’s ratio of 0.2 was found to be adequate for concrete compressive strengths up to 18 ksi (124 MPa). When only test results of specimens with concrete compressive strengths over 10 ksi (69 MPa) are considered, there is a slight increase in the Poisson’s ratio as concrete compressive strength increases. The proposed value for Poisson’s ratio establishes the lower bound limit of 44 percent of the test results.

![Graph showing Poisson's ratio vs. concrete compressive strength](image)

Figure B10 – Proposed value for Poisson’s ratio

B.4.2 Stress Block Parameters

The approach presented by Hognestad et al. (1955) was used to determine the stress-strain relationship for each specimen. This approach can be used to calculate the concrete stress $f_c$ as a function of measured strain at the most compressed fiber $\varepsilon_c$ and the applied stresses $f_o$ and $m_o$. The following equations were obtained from equilibrium of external and internal forces and moments. Note that secondary moment effects were also considered in the calculation of the
applied moment, $M$.

$$C = P_1 + P_2 = f_o \cdot b_c = \frac{b_c}{\epsilon_c} \int_0^\epsilon \sigma(\epsilon_x) d\epsilon_x$$  \hspace{1cm} \text{Equation B1}$$

$$M = P_1 a_1 + P_2 a_2 = m_o \cdot b_c^2 = \frac{b_c^2}{\epsilon_c^2} \int_0^\epsilon \sigma(\epsilon_x) \epsilon_x d\epsilon_x$$  \hspace{1cm} \text{Equation B2}$$

where, $C$ is the total applied load, $M$ is the total applied moment, $P_1$ is the main axial load, $P_2$ is the secondary load, $a_1$ and $a_2$ are the eccentricities with respect to the neutral surface, $b$ is the width of the section, $c$ is the depth of neutral axis,

$$f_o = \frac{P_1 + P_2}{b_c}$$  \hspace{1cm} \text{Equation B3}$$

and

$$m_o = \frac{P_1 a_1 + P_2 a_2}{b_c^2}$$  \hspace{1cm} \text{Equation B4}$$

are the applied stresses. Some of these definitions are illustrated in Figure B11.

![Figure B11 – Eccentric bracket specimen](image-url)
Differentiating the last terms of the equations for $C$ and $M$ with respect to $\varepsilon_c$ yields the following equations.

$$\sigma_c = \varepsilon_c \frac{df_o}{d\varepsilon_c} + f_o \quad \text{Equation B5}$$

$$\sigma_c = \varepsilon_c \frac{dm_o}{d\varepsilon_c} + 2m_o \quad \text{Equation B6}$$

Using these equations, two similar stress-strain relationships were obtained for each eccentric bracket specimen and the average of these two was used to determine the stress-strain relationship of the specimen. A typical stress-strain distribution for HSC is shown in Figure B12. The numerical values of the simplified stress-strain relationships for all the specimens are given in Appendix G. These stress-strain relationships were used to calculate the stress block parameters for HSC.

![Figure B12 – Typical stress-strain distribution for eccentric bracket specimens (18EB9)](image-url)
In general, the stress block in the compression zone of a flexure member can be defined by three parameters, \( k_1 \), \( k_2 \) and \( k_3 \). The parameter \( k_1 \) is defined as the ratio of the average compressive stress to the maximum compressive stress in the compression zone, \( k_3 f'_c \). The parameter \( k_2 \) is the ratio of the depth of the resultant compressive force, \( C \), to the depth of the compression zone, \( c \). The parameter \( k_3 \) is the ratio of the maximum compressive stress in the compression zone to the compressive strength measured by concrete cylinder, \( f'_c \). The design values of the stress block parameters are determined when the strains at the extreme fibers reach the ultimate strain of the concrete, \( \varepsilon_{eu} \). The three generalized parameters of a stress block can be reduced into two parameters to establish equivalent rectangular stress block using \( \alpha_1 \) and \( \beta_1 \), which ensure the location of the compressive stress resultant to remain at the same location. These parameters are shown in Figure B13. The stress block parameters for each specimen are also given in Table B2.

\[
\alpha_1 = \frac{k_1 k_3}{2k_2} \quad \beta_1 = 2k_2
\]

Figure B13 – Stress block parameters for rectangular sections

The stress-strain distribution of NSC can be generalized by the curved shape shown in Figure B14. For this type of stress distribution, \( k_1 \) and \( k_2 \) are equal to 0.85 and 0.425, respectively. When converted to a rectangular distribution, \( \alpha_1 \) and \( \beta_1 \) correspond to \( k_3 \) and 0.85, respectively. If the stress-strain distribution of HSC is assumed to be a triangular distribution, \( k_1 \) and \( k_2 \) would be equal to 0.50 and 0.333, respectively. The rectangular stress block parameters,
\( \alpha_1 \) and \( \beta_1 \), for triangular distribution would be \( 0.75k_3 \) and 0.667, respectively. These parameters are shown in Figure B14.

\[
\begin{align*}
k_1 &= 0.85 \\
k_2 &= 0.425 \\
\alpha_1 &= k_3 \\
\beta_1 &= 0.85
\end{align*}
\]

Normal-Strength Concrete Stress Distribution

\[
\begin{align*}
k_1 &= 0.50 \\
k_2 &= 0.333 \\
\alpha_1 &= 0.50 \\
\beta_1 &= 0.667
\end{align*}
\]

Triangular Stress Distribution

\[
k_1 = \frac{Shaded Area}{Area \ of \ Dotted \ Rectangle}
\]

Figure B14 – Stress block parameters for different stress distributions

Test results of this research and other researchers in the literature indicate that the generalized stress block parameter \( k_1 \) is rarely less than 0.58 when concrete compressive strengths is between 10 and 18 ksi (69 and 124 MPa) as shown in Figure B15. Therefore, the lower bound of \( k_1 = 0.58 \) was proposed for concrete compressive strengths beyond 15 ksi (103 MPa). The collected data for stress block parameters from other researchers consist of test results obtained by Hognessad et al. (1955), Nedderman (1973), Kaar et al. (1978a, 1978b), Swartz et al. (1985), Pastor (1986), Schade (1992), Ibrahim (1994), and Tan and Nguyen (2005). The tabulated values of the research data is presented in Appendix G.
The $k_2$ parameter in the LRFD Specifications (2004) is already set to 0.33 for concrete compressive strengths beyond 8 ksi (55 MPa), since the assumed $\beta_1$ parameter used in design is equal to 0.65. The test results of this research and other researchers in the literature indicate that the stress block parameter $k_2$ for HSC between 8 and 18 ksi (55 and 124 MPa) can be assumed to be 0.33 as shown in Figure B16.

The test results of this research and other researchers in the literature indicate that the stress block parameter $k_3$ for HSC is similar to NSC as shown in Figure B17. Hence, using the same value of $k_3$ parameter, 0.85, for concrete compressive strengths up to 18 ksi (124 MPa) is completely appropriate for design purposes.

![Proposed value for the stress block parameter $k_1$](image)

Figure B15 – Proposed value for the stress block parameter $k_1$
Figure B16 – Proposed value for the stress block parameter $k_2$

Figure B17 – Proposed value for the stress block parameter $k_3$
Using the values proposed for the generalized stress block parameters, the lower bound relationships for rectangular stress block parameters $\alpha_i$ and $\beta_i$ can be obtained as follows:

$$\alpha_i = \frac{k_2 k_3}{2k_2} = \frac{0.58 \times 0.85}{2 \times 0.33} = 0.75$$

Equation B7

$$\beta_i = 2k_2 = 2 \times 0.33 \approx 0.65$$

Equation B8

In light of the above discussions, the following relationship is proposed for the rectangular stress block parameters, $\alpha_i$ and $\beta_i$, for concrete compressive strengths up to 18 ksi (124 MPa) using the results of this research and other researchers in the literature.

$$\alpha_i = \begin{cases} 0.85 & \text{for } f'_c \leq 10 \text{ ksi} \\ 0.85 - 0.02(f'_c - 10) & \text{for } f'_c > 10 \text{ ksi} \end{cases}$$

where $f'_c$ in ksi

Equation B9

$$\alpha_i = \begin{cases} 0.85 & \text{for } f'_c \leq 69 \text{ MPa} \\ 0.85 - 0.003(f'_c - 69) & \text{for } f'_c > 69 \text{ MPa} \end{cases}$$

where $f'_c$ in MPa

Equation B10

$$\beta_i = \begin{cases} 0.85 & \text{for } f'_c \leq 4 \text{ ksi} \\ 0.85 - 0.05(f'_c - 4) & \text{for } f'_c > 4 \text{ ksi} \end{cases}$$

where $f'_c$ in ksi

Equation B10

$$\beta_i = \begin{cases} 0.85 & \text{for } f'_c \leq 28 \text{ MPa} \\ 0.85 - 0.0073(f'_c - 28) & \text{for } f'_c > 28 \text{ MPa} \end{cases}$$

where $f'_c$ in MPa

at

$\varepsilon_{cu} = 0.003$.

Equation B11

The comparisons of the proposed relationships and the product of the relationships to the test results of this research and other researchers in the literature are shown in Figures B18 to B20.

A total of 159 eccentric bracket specimen test results from this research and the literature with concrete compressive strengths up to 20 ksi (124 MPa) were evaluated using regression analysis technique to develop the relationship between the rectangular stress block parameters,
\( \alpha_i \) and \( \beta_i \) and concrete compressive strength, \( f'_{c} \). Details of the regression analysis are presented in Mertol (2006). It was observed that the standard deviations of the rectangular stress block parameter, \( \alpha_i \), for all three concrete compressive strength ranges - below 10 ksi (69 MPa), over 10 ksi (69 MPa) and up to 20 ksi (138 MPa) - were very close to each other which indicated that the variability of \( \alpha_i \) was same for all three strength ranges. When the test results for concrete compressive strength over 10 ksi (69 MPa) was considered, the 90 percent regression line for \( \beta_i \) became almost identical with the proposed equation for HSC.

A sensitivity analysis was performed to evaluate how sensitive the ultimate moment capacity of a reinforced concrete member would be affected by the rectangular stress block parameters, \( \alpha_i \) and \( \beta_i \). Details of the sensitivity analysis are presented in Mertol (2006). The results of the analysis are shown in Figure B21 and B22. In these figures, the ratio of the ultimate moment capacity is used for comparison purposes. This ratio represents the ultimate moment capacities obtained for various \( \alpha_i \) and \( \beta_i \) values (\( \alpha_i = 0.85 \) vs. 0.75, \( \beta_i = 0.85 \) vs. 0.65) were divided by the ultimate moment capacity obtained from \( \alpha_i \) and \( \beta_i \) values of 0.85. Figure B21 indicates that, for under-reinforced concrete sections, a reduction in the rectangular stress block parameter \( \alpha_i \) by 11.8 percent (0.85 vs. 0.75) leads to a reduction of the ultimate moment capacity only by 1.9 percent. However, for an over-reinforced concrete section, a reduction in \( \alpha_i \) by 11.8 percent (0.85 vs. 0.75) leads to a reduction of the ultimate moment capacity by 10.3 percent. Figure B22 indicates that, for under-reinforced concrete sections, a reduction in the rectangular stress block parameter \( \beta_i \) by 23.5 percent (0.85 vs. 0.65) had no effect on the ultimate moment capacity. However, for an over-reinforced concrete section, a reduction in \( \alpha_i \) by 23.5 percent (0.85 vs. 0.65) leads to a reduction of the ultimate moment capacity by 12.2 percent.
Figure B18 – Proposed relationship for the rectangular stress block parameters $\alpha_i$

Figure B19 – Proposed relationship for the rectangular stress block parameters $\beta_i$
Figure B20 – Proposed relationship for the product of rectangular stress block parameters $\alpha_1\beta_1$

Figure B21 – Ratio of ultimate moment capacity versus change in $\alpha_1$ from 0.85 to 0.75
Figure B22 – Ratio of ultimate moment capacity versus change in $\beta_1$ from 0.85 to 0.65

B.5 Conclusion

Based on the research findings, the following conclusions can be drawn:

- The assumption that plane sections remain plane after deformation is valid for HSC up to 18 ksi (124 MPa).

- The ultimate concrete compressive strain value of 0.003 specified by the LRFD Specifications (2004) is acceptable for HSC up to 18 ksi (124 MPa).

- Poisson’s ratio of 0.2 specified by the LRFD Specifications (2004) can adequately be used for HSC up to 18 ksi (124 MPa).

- The test results, confirmed by other data in the literature, indicate that the stress block parameter $\alpha_i$ of 0.85 should be reduced where the compressive strength of concrete increases beyond 10 ksi (69 MPa). The recommended value for the parameter $\alpha_i$ is:
\[
\alpha_i = \begin{cases}
0.85 & \text{for } f'_{c} \leq 10 \text{ ksi} \\
0.85 - 0.02(f'_{c} - 10) & \text{for } f'_{c} > 10 \text{ ksi}
\end{cases}
\]
\text{where } f'_{c} \text{ in ksi}

\[
\alpha_i = \begin{cases}
0.85 & \text{for } f'_{c} \leq 69 \text{ MPa} \\
0.85 - 0.0029(f'_{c} - 69) & \text{for } f'_{c} > 69 \text{ MPa}
\end{cases}
\]
\text{where } f'_{c} \text{ in MPa}

- The current value of \( \beta_i \), 0.65 for \( f'_{c} > 8 \) ksi (55 MPa), specified by AASHTO LRFD Bridge Design Specifications is appropriate for HSC up to 18 ksi (124 MPa).

### B.6 References


ACI Committee 318, “Building Code Requirements for Structural Concrete (ACI 318-05) and Commentary (318R-05),” American Concrete Institute, Farmington Hills, MI, 2005, 430 p.


Schade, J. E., “Flexural Concrete Stress in High Strength Concrete Columns,” *M.S. Thesis*, Civil Engineering Department, the University of Calgary, Calgary, Alberta, Canada, Sept. 1992, 156 p.


APPENDIX C – BEAMS UNDER FLEXURAL AND AXIAL-FLEXURAL LOADINGS

C.1 Objective and Scope

The objectives of this study were to:

- Examine the usable ultimate strain of unconfined HSC for flexural members, which is a general assumption of the LRFD Specifications (2004) 5.7.2.1;
- Investigate the accuracy of the current stress block parameters ($\alpha_1$, $\beta_1$) and their proposed relationships for determining the flexural resistance of beams;
- Validate the methods of predicting the cracking moment and crack widths;
- Examine the accuracy of the equations in the current LRFD Specifications (2004) equations for predicting deflection at the service level.

The scope of this investigation comprised of both experimental and analytical work. The experimental program included pure flexure tests, axial-flexural tests and material property tests for three levels of HSC with target strengths of 10, 14, and 18 ksi (69, 97, and 124 MPa). The analytical program was conducted to develop recommendations for modifications to the LRFD Specifications (2004) in order to extend its applicability to concrete strength up to 18 ksi (124 MPa).

C.2 Test Program

The test program consisted of 14 pure flexure specimens and five axial-flexural specimens with concrete strength, reinforcement ratio, size and shape of the specimen, and level of applied axial load as main parameters. Mixture designs for three different concrete target
strengths, 10, 14, and 18 ksi (69, 97, and 124 MPa) and the type of materials used are given in Appendix A.

C.2.1 Specimens and Material Properties

Table C1 shows the overall test matrix. The specimens were cast in five batches of concrete. The first two characters of the specimen number identify the target concrete strength, the last two or three digits indicate the reinforcement ratio, the character “B” or “BA” represents beam or beam-column specimen. The last character “R” stands for “replicate specimen”. The compressive strength ($f'_c$) of each specimen shown in this table is based on the average compressive strength of three 4×8 in. (100×200 mm) concrete cylinders tested on the same day as the specimen.

The cross-sections of pure flexure specimens and the test set-up are shown in Figure C1. The pure flexure specimens of Batch 1 and 2 had 9×12 in. (225×300 mm) rectangular sections, and were designed for a test span of 10 ft. (3 m). The pure flexure specimens in Batch 3, 4 and 5 had inverted-T cross-section to accommodate the higher amount of reinforcement so that the specimen would fail in concrete crushing before yielding of the longitudinal reinforcement. To reduce the shear demand, the span for these specimens was increased from 10 ft. (3 m) to 13 ft. (3.9 m). For all pure flexure specimens, no stirrups were used in the constant-moment region.

The cross-sections of the axial-flexural specimens and the test set-up are shown in Figure C2. The axial-flexural specimens of Batch 1, 2 and 5 had 9×12 in. (225×300 mm) rectangular sections with the same reinforcement design as the columns tested in this project (see Appendix D). These specimens were tested with a flexural span of 10 ft. (3 m).

The cross-section of the axial-flexural specimens of Batch 3 and 4 was reduced to 7×9 in. (175×225 mm) to be consistent with the size of the column specimens (see Appendix D). The
size was reduced for the column specimens because of the capacity limitation of the testing machine. The span length of these axial-flexural specimens was therefore reduced from 10 ft. (3 m) to 9 ft. (2.7 m).

In testing of the first axial-flexural specimen 10BA4, the axial load increased during the test up to 20 percent upon crushing of concrete. Therefore, it was decided to repeat this test. A pressure-relieve valve was used to enable the manual release of the axial load. For the rest of the specimens, axial load was maintained within ±3 percent of its predetermined value.

Table C1 – Overall test matrix

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<th>Batch No.</th>
<th>Specimen No.</th>
<th>$f'_{c}$ (ksi)</th>
<th>$f_{r}$ (psi)</th>
<th>$\rho$ (%)</th>
<th>Steel Reinforcement</th>
<th>d (in.)</th>
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C-3
Figure C1 – Test set-up and details for pure flexure tests

Figure C2 – Test Setup and Details for Axial-flexural Test
All the reinforcement used in this project was of Grade 60. Their material properties are shown in Table C2. The strength of the reinforcement coupons was determined using the MTS tension machine.

Table C2 – Summary of the Measured Yield Strength and Elastic Modulus of Steel Rebars

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<th>No.8</th>
<th>No.9</th>
<th>No.10</th>
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<th>No.14</th>
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C.2.2 Preparation and Instrumentation of Specimens

All specimens were cast in plywood forms using ready-mixed concrete delivered from a local plant. After casting and finishing, the top surfaces were covered with polystyrene sheets to maintain a moist environment for 2 hours and then were covered with wet burlap. Specimens were demolded one day after casting and then moist-cured for 7 days. Afterwards, specimens were moved outside the laboratory and cured under the natural environment for at least 28 days before testing.

All specimens were painted white using latex material before testing. Grids were marked to facilitate tracking and mapping crack patterns. The primary interest in the instrumentation for the pure flexure test was the strain distribution in the constant moment region as well as the deflection at mid-span. Pi-gages were used to measure the strain within the constant moment zone. They were attached to the specimens using Demec points. Two types of instruments were used to measure mid-span or support deflections: cable-extension transducers (wire potentiometers) or linear potentiometers. The instrumentation was calibrated before each test. For axial-flexural specimens, axial load was measured using a load cell, which is commercially calibrated each year.
C.2.3 Test Procedure

Both pure flexure and axial-flexural specimens were tested using an MTS hydraulic actuator with a maximum capacity of 220 kips (1000 kN). The applied load was measured by a built-in load cell of the actuator.

MTS hydraulic actuator was operated in the displacement-control mode during the test. An initial deflection rate of 0.02 in./min (0.5 mm/min) was chosen before the first cracking of the beam. After the first tensile crack was observed, a constant rate of 0.04 in./min (1 mm/min) was used, which resulted in testing time for each specimen less than an hour to minimize the potential creep effect. Test was paused for 5 to 10 seconds after each load increment of 10 kips (45 kN) to allow checking of stability of the loading system. Visual inspection of the cracks was carried out throughout the test, and cracks were mapped until the applied load reached 80 percent of the predicted failure load. Tests were terminated after crushing of concrete within the constant moment region.

Testing of axial-flexural specimen required a hydraulic jack and a hand-pump. Specimen was axially loaded to the predetermined load level before transverse load was applied. The procedure for testing of the axial-flexural specimens is nearly the same as testing of the pure flexure specimens, except that the level of axial load needed to be monitored throughout the test. Once the axial load rose by more than 2 percent of the expected value, the pressure was manually released. The axial load is kept within ±3 percent of the desired value. After the peak load was reached, the axial load was released. For the safety of test setup and instrumentation, tests were terminated when the deflection at mid-span reached 3 in. (75 mm).
C.2.4 Failure Modes

A typical failure mode of a pure flexure specimen (10B5.7) is shown in Figure C3. This figure shows the size of 2 triangular shape zones where concrete crushed. Initially, concrete crushed locally within the smaller triangular zone (4 in. (100 mm) deep). This was followed immediately by the crushing of the entire test zone, as there was no stirrup to restrain crack propagation. In this particular specimen, a secondary crack expended diagonally, until it reached the top layer of tension reinforcement. Similar failure mode was observed for all the pure flexure specimens; the depth of the crushing zone depended on the reinforcement ratio.

![Figure C3 – Typical failure of a pure flexure specimen (10B5.7)](image_url)

A typical failure mode of an axial-flexural specimen (10BA4) is shown in Figure C4. Due to the presence of stirrups and compression reinforcement within the constant moment zone, failure was initiated by crushing of the concrete cover at the top fiber of the compression zone, accompanied by a sudden drop of the load. After the peak load, gradual spalling of the concrete...
cover within the compression zone was observed with an increase of deflection due to the
confinement provided by the stirrups in the compression zone. It is noted that even though all
tests were terminated when the mid-span deflection became greater than 3 in. (75 mm), the
specimens were still capable of sustaining additional deflection without significant loss of
resistance.

Figure C4 – Typical failure mode of the axial-flexural specimen (10BA4)

C.3 Test Results

C.3.1 Ultimate Compressive Strain of Concrete:

The compressive strain of concrete measured at the time of concrete crushing is shown in
Figure C5. The reported $f'_c$ is the average of compressive strength of three cylinders tested on the
same day as the test specimen. For this project, there were at least 3 pi-gages attached to the top
surface of each specimen to monitor the compressive strain of concrete. The magnitude of
measured strain values depended on the relative location of the gage to where concrete crushing
occurred. Only the largest measured value for each specimen is reported. It should also be noted that since the pi-gages are mounted approximately ½ in (13 mm) above the concrete surface; their original readings were adjusted based on the measured strain profile, neutral axis depth and geometric relationship.

Figure C5 – Ultimate compressive strain of concrete for various concrete strengths

All specimens had concrete strain reading greater than 0.003 at the time of crushing of concrete. It can also be seen that the magnitude of ultimate strain of concrete is more or less independent of the compressive strength of the concrete.

C.3.2 Load-Deflection Response:

The load-deflection curves at mid-span for all tested specimens are shown in Figure C6 and C7. The load was based on the reading of the load cell of the actuator.
Figure C6 – Load vs. deflection at mid-span for pure-flexure specimens with rectangular section

Figure C7 – Load vs. deflection at mid-span for pure flexure specimens with inverted-T section
Load-deflection curves for the six pure flexure specimens with 9×12 in. (225×300 mm) rectangular sections are shown in Figure C6. As some specimens cracked before testing, not all cracking moments can be clearly identified from the curves. The behavior of the HSC members is generally the same as NSC members. For an under-reinforced beam, as the reinforcement ratio increases both the capacity and the stiffness of the section increase, but the ultimate deflection decreases. For an over-reinforced specimen, this is not always true as the resistance is mainly controlled by the compressive strength of concrete. The load-deflection curves generally remain linear after cracking. The over-reinforced specimens failed upon reaching the peak load; while the under-reinforced specimens show a more ductile failure.

Load-deflection curves for the seven pure flexure specimens with inverted-T sections are shown in Figure C7. Due to the low concrete strength achieved, none of the specimens showed a clear ductile response. Specimens with target strength of 18 ksi (124 MPa) had higher cylinder strength but did not achieve as high a flexural resistance. This is possibly due to the inadequate curing of the specimens since they were cured outside during the winter and exposed to low temperatures at night.

The shape of the load-deflection curves of the axial-flexural specimens as shown in Figures C8 and C9 is different from those of the pure flexure specimens, mainly due to the effect of applied axial load. The cracking load, identified as the first change in the slope, can clearly be seen except for the “re-test” curve for specimen 18BA4, which had cracked due to shrinkage prior to testing. The second change in the slope reflects the yielding of the bottom layer of the tensile reinforcement. As the reinforcement was placed in three layers, the middle layer of the longitudinal reinforcement remained elastic and continued to increase its contribution to the capacity of the section until the crushing of concrete. A sudden drop of load was observed for all
specimens upon crushing of concrete. Unlike the pure flexure specimens, concrete in the constant moment region was confined by the stirrups. For specimens 10BA4, 10BA4R and 14BA4, which had 9×12 in. (225×300 mm) cross-sections, about 20 percent of the resistance was lost upon the crushing of concrete cover at the top compression zone. After that, all specimens showed some ductility and some limited gain of strength due to the confinement effect. For specimens 14BA4R and 18BA4, which had 7×9 in. (175×225 mm) sections, the loss in the resistance due to the crushing of top cover was more significant. This is because the concrete cover constituted a greater portion of the smaller cross-section.

Figure C8 – Load vs. deflection at mid-span for axial-flexural specimens with 9×12 in. (225×300 mm) rectangular section
C.3.3 Moment-Curvature Response

Typical mid-span moment-curvature curves for the pure flexure and axial-flexural specimens are shown in Figures C10 and C11.

Curvature at mid-span was derived based on linear regression analysis of measured strains from a set of 4 to 5 pi-gages attached to the top, bottom and side of the specimens. Determination of mid-span moment was slightly different between two types of specimens:

- For pure flexure specimens, moment was determined by multiplying half of the applied load by the shear span.
- For axial-flexural specimens, the “primary moment” and “secondary moment” were added together, with primary moment being applied by the actuator, and secondary moment being the product of axial load and mid-span deflection.
Figure C10 – Moment-curvature response for pure flexure specimens with rectangular cross-section and target strength of 10 ksi (69 MPa)

Figure C11 – Moment-curvature response for axial-flexural specimens with 9×12 in. (225×300 mm) rectangular section
As the shape of moment-curvature curve is almost the same as that observed for load-deflection curve, only typical curves are presented in this section.

C.3.4 Neutral Axis Depth

Typical curves for the change in the depth of neutral axis at mid-span for the pure flexure and axial-flexural specimens are shown in Figure C12 and C13, respectively. Neutral axis depth is calculated based on a regression analysis of the readings of same group of pi-gages, which are used for the determination of moment-curvature relationship.

Figure C12 – Neutral axis depth vs. load for pure flexure specimens with inverted-T section and target strength of 18 ksi (124 MPa)

For pure flexure specimens, the neutral axis depth increases as the reinforcement ratio increases. The initial part of the curves reflects the noise in the data, which is mainly due to the error from seating of the instrumentation as well as the effect of initial cracking. Because the stress-strain curve of HSC is more or less linear until reaching the peak stress, the cracked
section at mid-span behaves elastically and the neutral axis depth remains almost constant after cracking. Neutral axis shifts upward after yielding of the longitudinal reinforcement for under-reinforced beams. For over-reinforced beams, specimens fail before the reinforcement yield. If the stress of concrete reduces, neutral axis may shift downwards. In this project, most of over-reinforced specimens failed suddenly without much change in the neutral axis depth.

Figure C13 – Neutral axis depth vs. load for axial-flexural specimens with 9×12 in. (225×300 mm) rectangular cross-section

For axial-flexural specimens, the shape of the curves is quite different due to the applied axial load. Since the axial load was applied at the section concentrically, the theoretical neutral axis depth is infinite at the beginning of the test. The neutral axis shifted upward as the lateral load was increased. The yielding of the bottom layer of the longitudinal reinforcement created a discontinuity in the curve and the shifting of neutral axis become somehow irregular. Neutral axis quickly moved upward after the peak resistance was reached due to crushing of concrete.
Some of the post-peak behavior of the cross section can also be observed from these curves.

C.4 Discussion of Results

C.4.1 Ultimate Compressive Strain of Concrete

In the LRFD Specifications (2004), “maximum usable strain of concrete” refers to the strain at which concrete crushes. This strain is also used for determining the strain profile and ductility of a section. For over-reinforced specimens, the strain profile is critical in determining the nominal resistance of the beam. For this reason, the data at the peak load and at the time of failure when concrete crushed are reported separately.

The strain values at the peak load for various concrete compressive strengths are shown in Figure C14. The collected data consists of test results obtained by Alca and MacGregor (1997), Mansur and Chin (1997), Weiss and Shah (2001), Kaminska (2002), as well as the data from the present study. The measured strain values from the present study shown in the figure have an average value of 0.0038 and standard deviation of 0.0004. All measured values exceed a value of 0.003 except one of the pure flexure specimens with concrete strength of 13.1ksi (90 MPa).

The maximum strains of concrete measured at failure are shown in Figure C15. The measured maximum values from this project have an average of 0.0040 and a standard deviation of 0.0005. The measured strains from the present study exceeded the value of 0.003. It should be noted that for the axial-flexural specimens, the recorded maximum strain was measured immediately before the loss of the instrumentation due to the crushing of concrete cover. The measured value of the present study confirms the finding of others, and suggests that using an ultimate concrete stain of 0.003 is valid for HSC up to 18 ksi (124 MPa).
Figure C14 – Concrete strain at peak load

Figure C15 – Concrete strain at crushing
C.4.2 Resistance of Flexural Members

In this section, the measured flexural resistance $M_{\text{Exp}}$ is compared to the predicted value $M_{\text{LRFD}}$ in Figure C16 for all tested beams of the present study as well as data from literature with concrete strengths over 10 ksi (69 MPa). For under-reinforced pure flexure specimens, the predicted value is based on the LRFD Specifications (2004) Equation 5.7.3.2.2.1, using the current value of 0.85 for $\alpha_i$, 0.65 for $\beta_i$, and the measured material properties. For over-reinforced specimens or axial-flexural specimens, resistance is determined by solving two equations based on force equilibrium and strain compatibility. An ultimate strain value of 0.003 is assumed in the calculations, and plane sections are presumed to remain plane.

![Figure C16 – Comparison of the experimental and predicted values of flexural resistance using the current LRFD Specifications (2004)](image)

The flexural resistance is also determined using the proposed stress-block factors (see Appendix B), with a reduced value for $\alpha_i$. The predicted value $M_{\text{Prop}}$ is compared with the
measured value $M_{Exp}$ in Figure C17.

Figure C17 – Comparison of the experimental and predicted values of flexural resistance using the proposed stress block factors

Comparisons of the results presented in Figures C16 and C17 indicate that using the proposed smaller value of $\alpha_1$ leads to a more conservative prediction of the nominal flexural resistance for all the specimens tested in this project. Table C3 shows the statistical information of the predicted values of $M_{LRFD}$ and $M_{Prop}$ versus the measured values of $M_{Exp}$ for a total of 141 specimens in the database (including tested in the present study). It can be seen that the prediction using proposed $\alpha_1$ value is more conservative, especially for the over-reinforced beams.
Table C3 – Statistical data for over-reinforced and under-reinforced HSC beams

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<tr>
<th>Type</th>
<th>Total Number of Specimens</th>
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<th>$M_{Exp}/M_{prop}$</th>
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<td></td>
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<td>Std. Dev.</td>
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<td>Under-Reinf.</td>
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<td>16 0.82 1.55 1.12 0.15</td>
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C.4.3 Cracking Load

The measured modulus of rupture values as well as data from the literature is shown in Figure C18. Predicted values using the current LRFD Specifications (2004) equation is also shown in the figure. Two equations for modulus of rupture given in Section 5.4.2.6 of the current LRFD Specifications (I) are also shown in the figure. Some of the tests results correspond better to the current upper bound of the LRFD Specifications (I). This is mainly due to the curing condition and moisture content of the specimens. Test results suggest that the current lower bound of the LRFD Specifications (I) overestimates the modulus of rupture for HSC. A better predictive equation, $f_r = 0.19 \sqrt{f_{c}'} (ksi)$ ($f_r = 0.5 \sqrt{f_{c}'} (MPa)$), is proposed for HSC up to 18 ksi (124 MPa).

The observed and calculated cracking loads are summarized and compared in Table C4. The observed cracking load data are obtained in two ways: observed during test and identified from load-deflection curves. The cracking load of a specimen was noted during the test when the first vertical crack was observed. These loads are listed under “@ testing”. Some of the specimens had hairline cracks on the surface before loading, possibly due to drying shrinkage. In these cases, data is marked as NA. The cracking load is also estimated from the measured load deflection curve at mid-span. Theoretically, upon the first cracking, the stiffness of the cross-section is significantly reduced. These loads are reported under “From P-$\Delta$”.

C-21
Figure C18 – Modulus of Rupture vs. Concrete Compressive Strength

Table C4 – Summary of cracking load data

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>$f'_c$ @ Testing Day (ksi)</th>
<th>$f_r$ (psi)</th>
<th>AASHTO LRFD Measured</th>
<th>Observed @ Testing From P-∆</th>
<th>Predicted Code $f_r$ Using Measured $f_c$</th>
<th>Cracking Load (kips)</th>
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</table>

Proposed Modulus of Rupture

\[ f_r = 0.19 \sqrt{f'_c} \text{ (ksi)} = 6 \sqrt{f'_c} \text{ (psi)} \]
The cracking load for each specimen is also calculated using equation (5.7.3.6.2-2). Two expression of modulus of rupture values are used: the current LRFD Specifications (2004) value as well as measured \( f_c \) value. It should be noted that the LRFD Specifications (2004) provide two modulus rupture values: one for calculating deflection and the other for the determining the minimum reinforcement ratio. The smaller value of the two, equation for calculating the deflection and camber is used here, which is:

\[
f_r = 0.24 \sqrt{f'_c} \quad (f'_c \text{ in ksi})
\]

\[
f_r = 0.62 \sqrt{f'_c} \quad (f'_c \text{ in MPa})
\]

Overall, the observed value and the predicted value do not compare very well. This is attributed to inaccuracies from both observed and predicted data. The inaccuracy from observed data can be from the following sources:

1. The first cracking could not be recognized until it propagated to the surface of the specimen. Also, the crack might be so small that it would take some time to be noticed. Therefore, it is believed the reported value is usually slightly over-estimated.

2. The cracking load from load-deflection curve is not very accurate either. This is because the development of micro-crack in the mortar is gradual. As a result, the change in the slope of the load-deflection curve is not sharp and distinctive.

### C.4.4 Crack Width

In the present study, crack width was measured using pi-gages installed on the bottom surface of the specimens. Readings were adjusted based on the neutral axis depth to reflect the crack width at the extreme tension fiber of concrete. The reported values are the readings at 45 percent of the measured peak load, which is considered to represent the average level of service load.
The measured crack widths for the tested pure flexure members are shown in Figure C19. The crack width of 0.017 in. (0.425 mm), representing the crack width specified for Class 1 exposure condition, is also shown in the same figure. Since all the specimens tested in this project utilized mild steel reinforcement with spacing closer than the maximum allowed by Eq.5.7.3.4.1, all of the measured crack widths are much less than 0.017 in (0.425 mm), as would be expected.

![Figure C19 – Measured crack width vs. concrete compressive strength at service load](image)

The current equation in the LRFD Specifications (2004) for control of cracking is based on a physical crack model (Frosch 2001) rather than the statistically-based model used in previous editions of the specifications. In Figure C20, the measured crack widths are compared with the predicted value using Frosch's model (Frosch 2000). In calculating the crack width, the measured steel strains were used. It can be seen that Frosch’s model over-predicts the crack widths for the majority of the beams tested in this project. For three of the over-reinforced
beams, the model provides un-conservative predictions since the steel stress at service load was much less than 0.6 $f_y$.

Figure C20 – Measured vs. predicted crack width using Frosch’s Model (Frosch 2000)

C.4.5 Deflection and Camber

In the present study, all the pure flexure specimens were tested under 4-point bending. Therefore, Equation C2 is used for the prediction of the deflection at mid-span.

$$\Delta_c = \frac{Pa}{48EI_e}(3L^2 - 4a^2)$$  \hspace{1cm} \text{Equation C2}

where $\Delta_c$ is the mid-span deflection; $P$ is the applied load; $L$ is the span of the beam; $a$ is the shear span; $E$ is the elastic modulus of concrete and $I_e$ is the effective moment of inertia, given by the LRFD Specifications (2004) equation (5.7.3.6.2.1) as:

$$I_e = (1 - \left(\frac{M_{cr}}{M_a}\right)^3)I_{cr} + \left(\frac{M_{cr}}{M_a}\right)^3I_g$$  \hspace{1cm} \text{Equation C3}

where $M_{cr}$ is the cracking moment, $M_a$ is the applied moment, $I_{cr}$ is the moment of inertia of the
cracked section, and $I_g$ is the gross moment of inertia of the un-cracked section.

Table C5 shows the ratio of the applied moment ($M_a$) to cracking moment ($M_{cr}$) at service load, and demonstrates how this ratio affects the deflection when using Equation C3 to determine the effective moment of inertia ($I_{ef}$). Service load is taken as 45 percent of the measured peak load. It can be seen that for all pure flexure members tested in this project, except one, have the ratio of $M_a/M_{cr}$ greater than three. As a result, the difference between the $I_e$ and $I_{cr}$ in Equation C3 is almost negligible. Therefore, it seems reasonable to neglect the last term of Equation C3 for HSC members, and simplifies the computations by using $I_{cr}$ alone.

**Table C5 – Effect of $M_a/M_{cr}$ ratio on effective moment of inertia**

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>$M_a/M_{cr}$</th>
<th>$I_{cr}/I_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10B2.1</td>
<td>4.18</td>
<td>0.98</td>
</tr>
<tr>
<td>10B4.3</td>
<td>4.80</td>
<td>0.99</td>
</tr>
<tr>
<td>10B5.7</td>
<td>5.04</td>
<td>0.99</td>
</tr>
<tr>
<td>10B10.2</td>
<td>3.06</td>
<td>0.96</td>
</tr>
<tr>
<td>14B3.3</td>
<td>5.51</td>
<td>0.99</td>
</tr>
<tr>
<td>14B7.7</td>
<td>7.85</td>
<td>1.00</td>
</tr>
<tr>
<td>14B12.4</td>
<td>8.60</td>
<td>1.00</td>
</tr>
<tr>
<td>14B7.6</td>
<td>3.29</td>
<td>0.95</td>
</tr>
<tr>
<td>14B12.7</td>
<td>3.72</td>
<td>0.98</td>
</tr>
<tr>
<td>14B17.7</td>
<td>3.67</td>
<td>0.99</td>
</tr>
<tr>
<td>18B5.9</td>
<td>2.55</td>
<td>0.88</td>
</tr>
<tr>
<td>18B12.7</td>
<td>3.24</td>
<td>0.97</td>
</tr>
<tr>
<td>18B17.7</td>
<td>3.20</td>
<td>0.98</td>
</tr>
</tbody>
</table>

*Calculated based on 45 percent of the peak load

The mid-span deflections of the 13 beams tested in this project are compared with the predicted values from the LRFD Specifications (2004) equations in Table C6. In calculating the deflections in Table C6, previously proposed new expression for the elastic modulus of HSC, together with the value using the current LRFD Specifications (2004) equation and the measured elastic modulus were used. It should be noted that the moment of inertia of the cracked section is affected by the elastic modulus of concrete. Therefore, it needs to be re-calculated each time the elastic modulus is changed.
Table C6 – Comparison of measured and predicted mid-span deflections at service load for the pure flexure specimens

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>( \frac{\Delta_{\text{predicted}}}{\Delta_{\text{measured}}} ) using E from AASHTO LRFD Proposed Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>10B2.1</td>
<td>0.84 0.85 0.91</td>
</tr>
<tr>
<td>10B4.3</td>
<td>0.81 0.83 0.90</td>
</tr>
<tr>
<td>10B5.7</td>
<td>0.89 0.91 0.98</td>
</tr>
<tr>
<td>10B10.2</td>
<td>0.75 0.76 0.93</td>
</tr>
<tr>
<td>14B3.3</td>
<td>0.77 0.78 0.82</td>
</tr>
<tr>
<td>14B7.7</td>
<td>0.81 0.81 0.88</td>
</tr>
<tr>
<td>14B12.4</td>
<td>0.79 0.80 0.88</td>
</tr>
<tr>
<td>14B7.6</td>
<td>0.81 0.83 0.88</td>
</tr>
<tr>
<td>14B12.7</td>
<td>0.75 0.78 0.82</td>
</tr>
<tr>
<td>14B17.7</td>
<td>0.77 0.80 0.85</td>
</tr>
<tr>
<td>18B5.9</td>
<td>0.69 0.71 0.84</td>
</tr>
<tr>
<td>18B12.7</td>
<td>0.66 0.68 0.83</td>
</tr>
<tr>
<td>18B17.7</td>
<td>0.78 0.80 0.99</td>
</tr>
<tr>
<td>Average</td>
<td>0.78 0.80 0.89</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.060 0.059 0.056</td>
</tr>
</tbody>
</table>

It can be seen from Table C6 that the current LRFD Specifications (2004) underestimates the deflection for all tested specimens in this project. The measured elastic modulus from cylinder tests provides the closest prediction. Using the proposed expression of the elastic modulus improves the prediction of deflection only slightly, as compared the current LRFD Specifications (2004) equation.

Similar results have been reported in the literature as summarized by Rashid and Mansur (2005) in Table C7, which is updated to include test results of this project for comparison.

It should be noted that Eq. 5.7.3.6.2-1 of the LRFD Specifications (2004) was proposed by Branson in 1963 based on test results of normal strength concrete members, and it was adopted by the ACI code in 1971. A statistical study of short-term deflection of simply-supported beams was conducted by ACI Committee 435 in 1972. It was reported that under controlled laboratory conditions, there is a 90 percent chance that deflection of a beam would fall
within -20 to +30 percent of the calculated value.

Table C7 – Statistical information on service load deflection from literature

<table>
<thead>
<tr>
<th>Researchers</th>
<th>No. of Beams</th>
<th>$f'_c$ (ksi)</th>
<th>$\Delta$ (%)</th>
<th>Measured/Predicted Average</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rashid and Mansur (2005)</td>
<td>16</td>
<td>6.2 – 18.3</td>
<td>1.3 – 5.3</td>
<td>1.26</td>
<td>0.08</td>
</tr>
<tr>
<td>Ashour (2000)</td>
<td>9</td>
<td>7.1 – 14.8</td>
<td>1.2 – 2.4</td>
<td>1.17</td>
<td>0.07</td>
</tr>
<tr>
<td>Lin (1992)</td>
<td>9</td>
<td>3.9 – 10.0</td>
<td>2.3 – 3.7</td>
<td>1.27</td>
<td>0.12</td>
</tr>
<tr>
<td>Lambotte and Taerwe (1990)</td>
<td>5</td>
<td>4.9 – 11.7</td>
<td>0.5 – 1.5</td>
<td>1.17</td>
<td>0.12</td>
</tr>
<tr>
<td>Paulson et al. (1991)</td>
<td>9</td>
<td>5.4 – 13.2</td>
<td>1.5</td>
<td>1.37</td>
<td>0.14</td>
</tr>
<tr>
<td>Shin et al. (1989)</td>
<td>23</td>
<td>3.9 – 14.5</td>
<td>0.4 – 3.6</td>
<td>1.56</td>
<td>0.27</td>
</tr>
<tr>
<td>Pastor et al. (1984)</td>
<td>12</td>
<td>3.8 – 9.3</td>
<td>1.1 – 5.3</td>
<td>1.09</td>
<td>0.08</td>
</tr>
<tr>
<td>This Research</td>
<td>13</td>
<td>11.4-16.1</td>
<td>2.1-17.7</td>
<td>1.29</td>
<td>0.10</td>
</tr>
</tbody>
</table>

The current LRFD Specifications (2004) tend to underestimate the measured deflection of reinforced HSC beams. However, the discrepancy is within the commonly acknowledged limit. Since data on deflection of reinforced HSC beams are very limited, it is proposed not to change the current LRFD Specifications (2004) equations for deflection calculation.

C.5 Conclusions

Based on the research findings, the following conclusions can be drawn:

- Ultimate concrete strain of 0.003 is still valid for HSC up to 18 ksi (124 MPa).
- Prediction of nominal flexural resistance using the current LRFD Specifications (2004) is less conservative and less accurate. Using proposed stress block factor $\alpha_l$ (Appendix B), would enhance the conservativeness of the prediction for HSC beams and beam-columns.
- The observed cracking moment and the predicted value did not always compare very well in this study, which is attributed to the inaccuracies inherent in the methods of determining the values.
- All the measured crack widths were much less than 0.017 in. (0.425 mm), which represents the crack width specified for Class 1 exposure condition in the LRFD Specifications (2004).
For the majority of the beams tested in this study, Frosch’s model (Frosch 2000) provided conservative predictions of the crack widths.

- The current LRFD Specifications (2004) underestimated the deflection at service load for all tested specimens in this project. However, the discrepancy is within the commonly acknowledged limit.

C.6 References


Frosch, R.J., “Flexural Crack Control in Reinforcement Concrete,” *ACI International SP 204*, “Design and Construction Practices to Mitigate Cracking,” Concrete International, 2001, pp. 135-153.

Hognestad, E., Hanson, N.W. and McHenry, D., “Concrete Stress Distribution in Ultimate


Lambotte, H and Taerwe, L.R. “Deflection and Cracking of High-strength Concrete Beams and Slabs,” SP121, High-Strength Concrete, Second International Symposium, American Concrete Institute, 1990, pp. 109-128.


Wiegrink, K, Marikunte, S., and Shah, S. P., “Shrinkage Cracking of High-Strength Concrete,”

APPENDIX D – COLUMNS UNDER CONCENTRIC AND ECCENTRIC LOADINGS

D.1  Introduction

Many tests have been conducted since the early 1900s on normal-strength concrete (NSC) columns under axial load. In the early 1930s, ACI Committee 105 reported the results of 564 column tests, primarily carried out at Lehigh University and the University of Illinois. Thereafter, many researchers have also investigated axial resistance of columns as well as the confinement and ductility produced by transverse reinforcement.

Several researches on columns with HSC indicated that their behavior is quite different from that of columns with NSC. The current LRFD Specifications (2004) are based on tests conducted using columns with NSC. Therefore, it is necessary to examine the behavior of HSC columns in order to extend the use of HSC with compressive strength up to 18 ksi (124 MPa).

D.2  Objective and Scope

The main objective of this study was to develop recommended revisions to extend the current compression design provisions of the LRFD Specifications (2004) to include concrete compressive strengths up to 18 ksi (124 MPa).

The study included testing of thirty-two (32) rectangular and twenty-four (24) circular columns with target strengths of 10, 14, and 18 ksi (69, 97, and 124 MPa), subjected to concentric and eccentric axial compression. The axial resistance of HSC columns and the effect of transverse reinforcement on the axial resistance were investigated. The test results together with extensive data reported in the literature were also analyzed.
D.3 Experimental Program

D.3.1 Test Specimens

A total of thirty-two (32) rectangular and twenty-four (24) circular reinforced concrete columns with concrete compressive strengths ranging from 7.9 ksi (55 MPa) to 16.5 ksi (114 MPa) were tested under monotonic concentric and eccentric loadings. The parameters considered for the concentrically loaded columns were concrete compressive strength, size and shape of the cross section, and the longitudinal and transverse reinforcement ratios. For eccentrically loaded columns, the parameters were concrete compressive strength, column size and eccentricity of the applied load. The clear concrete cover used for all tested columns was approximately ½ in. (13 mm) to the surface of the ties or spirals. All columns were reinforced with six longitudinal steel bars. The transverse reinforcement consisted of #4 ties for the rectangular columns and #3 or #4 spirals for the circular columns. The longitudinal bars were cut to match the column height and were placed flush with the ends of the columns. Two different spacing of ties were used, one being the maximum spacing according to the minimum transverse reinforcement requirement of the LRFD Specifications (2004), and the other being half of the required spacing. The two ends of the test specimens were reinforced heavily with closely spaced ties and also confined with external steel tubes, as shown in Figure D1, to avoid premature localized failure of the test specimens. All columns were cast vertically to simulate typical column construction practice as shown in Figure D2. Details of the concentric and eccentric columns are given in Tables D1 through D3. Geometric overview of the columns and their instrumentation are shown in Figure D3.
Figure D1 – Typical column specimens

Figure D2 – Casting of the columns

Table D1 – Details of concentrically loaded rectangular columns*

<table>
<thead>
<tr>
<th>Column ID</th>
<th>Size</th>
<th>Longitudinal Reinforcement</th>
<th>Transverse Reinforcement</th>
<th>Measured Concrete Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>b×h×L (in)</td>
<td>No. &amp; Size</td>
<td>ρ (%)</td>
<td>fy (ksi)</td>
</tr>
<tr>
<td>10CR9-p1</td>
<td>9×12×40</td>
<td>6 #4</td>
<td>1.11</td>
<td>59</td>
</tr>
<tr>
<td>10CR4½-p1</td>
<td>9×12×40</td>
<td>6 #4</td>
<td>1.11</td>
<td>59</td>
</tr>
<tr>
<td>10CR9-p2.5</td>
<td>9×12×40</td>
<td>6 #6</td>
<td>2.44</td>
<td>63</td>
</tr>
<tr>
<td>10CR4½-p2.5</td>
<td>9×12×40</td>
<td>6 #6</td>
<td>2.44</td>
<td>63</td>
</tr>
<tr>
<td>10CR9-p4</td>
<td>2 #7 + 4 #8</td>
<td>4.04</td>
<td>61, 60</td>
<td></td>
</tr>
<tr>
<td>10CR4½-p4</td>
<td>2 #7 + 4 #8</td>
<td>4.04</td>
<td>61, 60</td>
<td></td>
</tr>
<tr>
<td>A10CR9-p1</td>
<td>9 × 12 × 40</td>
<td>6 #4</td>
<td>1.11</td>
<td>67</td>
</tr>
<tr>
<td>A10CR9-p2.5</td>
<td>9 × 12 × 40</td>
<td>6 #6</td>
<td>2.44</td>
<td>63</td>
</tr>
<tr>
<td>A10CR9-p4</td>
<td>2 #7 + 4 #8</td>
<td>4.04</td>
<td>62, 61</td>
<td></td>
</tr>
<tr>
<td>14CR9-p1</td>
<td>9×12×40</td>
<td>6 #4</td>
<td>1.11</td>
<td>58</td>
</tr>
<tr>
<td>14CR4½-p1</td>
<td>9×12×40</td>
<td>6 #4</td>
<td>1.11</td>
<td>58</td>
</tr>
<tr>
<td>14CR9-p2.5</td>
<td>9×12×40</td>
<td>6 #6</td>
<td>2.44</td>
<td>63</td>
</tr>
<tr>
<td>14CR4½-p2.5</td>
<td>9×12×40</td>
<td>6 #6</td>
<td>2.44</td>
<td>63</td>
</tr>
<tr>
<td>14CR9-p4</td>
<td>2 #7 + 4 #8</td>
<td>4.04</td>
<td>62, 61</td>
<td></td>
</tr>
<tr>
<td>14CR4½-p4</td>
<td>2 #7 + 4 #8</td>
<td>4.04</td>
<td>62, 61</td>
<td></td>
</tr>
<tr>
<td>A18CR7-p2</td>
<td>7×9×36</td>
<td>6 #4</td>
<td>1.9</td>
<td>67</td>
</tr>
<tr>
<td>A18CR7-p3</td>
<td>7×9×36</td>
<td>6 #5</td>
<td>2.95</td>
<td>61</td>
</tr>
<tr>
<td>A18CR7-p4</td>
<td>7×9×36</td>
<td>6 #6</td>
<td>4.19</td>
<td>63</td>
</tr>
<tr>
<td>18CR7-p2</td>
<td>7×9×36</td>
<td>6 #4</td>
<td>1.9</td>
<td>58</td>
</tr>
<tr>
<td>18CR3½-p2</td>
<td>7×9×36</td>
<td>6 #4</td>
<td>1.9</td>
<td>58</td>
</tr>
<tr>
<td>18CR7-p3</td>
<td>7×9×36</td>
<td>6 #5</td>
<td>2.95</td>
<td>63</td>
</tr>
<tr>
<td>18CR3½-p3</td>
<td>7×9×36</td>
<td>6 #5</td>
<td>2.95</td>
<td>63</td>
</tr>
<tr>
<td>18CR7-p4</td>
<td>7×9×36</td>
<td>6 #6</td>
<td>4.19</td>
<td>63</td>
</tr>
<tr>
<td>18CR3½-p4</td>
<td>7×9×36</td>
<td>6 #6</td>
<td>4.19</td>
<td>63</td>
</tr>
</tbody>
</table>
* $\rho$ is the longitudinal reinforcement ratio.

* $f_y$ is the yield strength of longitudinal reinforcement.

* $\rho_s$ is the volumetric ratio of transverse reinforcement.

* $f_{ys}$ is the yield strength of transverse reinforcement.

### Table D2 – Details of concentrically loaded circular columns

<table>
<thead>
<tr>
<th>Column ID</th>
<th>Size</th>
<th>Longitudinal Reinforcement</th>
<th>Spiral Reinforcement</th>
<th>Measured Concrete Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>dxL (in)</td>
<td>No. &amp; Size</td>
<td>$\rho$ (%)</td>
<td>$f_y$ (ksi)</td>
</tr>
<tr>
<td>10CC2¾-p1</td>
<td>12×40</td>
<td>6 #4</td>
<td>1.0</td>
<td>59</td>
</tr>
<tr>
<td>10CC1¾-p1</td>
<td>6 #4</td>
<td>1.0</td>
<td>59</td>
<td>2¾</td>
</tr>
<tr>
<td>10CC2¼-p2.5</td>
<td>6 #6</td>
<td>2.19</td>
<td>63</td>
<td>2¾</td>
</tr>
<tr>
<td>10CC1¾-p2.5</td>
<td>6 #6</td>
<td>2.19</td>
<td>63</td>
<td>1½</td>
</tr>
<tr>
<td>10CC2¾-p4</td>
<td>6 #8</td>
<td>3.94</td>
<td>60</td>
<td>2¾</td>
</tr>
<tr>
<td>10CC1¾-p4</td>
<td>6 #8</td>
<td>3.94</td>
<td>60</td>
<td>1½</td>
</tr>
<tr>
<td>A10CC2¾-p1</td>
<td>6 #4</td>
<td>1.0</td>
<td>67</td>
<td>2¾</td>
</tr>
<tr>
<td>A10CC2¾-p2.5</td>
<td>6 #8</td>
<td>2.19</td>
<td>63</td>
<td>2¾</td>
</tr>
<tr>
<td>A10CC2¾-p4</td>
<td>6 #6</td>
<td>2.19</td>
<td>63</td>
<td>2¾</td>
</tr>
<tr>
<td>14CC2-p1</td>
<td>6 #4</td>
<td>1.0</td>
<td>58</td>
<td>2</td>
</tr>
<tr>
<td>14CC1-p1</td>
<td>6 #4</td>
<td>1.0</td>
<td>58</td>
<td>2</td>
</tr>
<tr>
<td>14CC2-p2.5</td>
<td>6 #6</td>
<td>2.19</td>
<td>63</td>
<td>2</td>
</tr>
<tr>
<td>14CC1-p2.5</td>
<td>6 #6</td>
<td>2.19</td>
<td>63</td>
<td>1</td>
</tr>
<tr>
<td>14CC2-p4</td>
<td>6 #8</td>
<td>3.94</td>
<td>61</td>
<td>2</td>
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<td>14CC1-p4</td>
<td>6 #8</td>
<td>3.94</td>
<td>61</td>
<td>1</td>
</tr>
<tr>
<td>18CC1½-p2</td>
<td>9×36</td>
<td>6 #4</td>
<td>1.89</td>
<td>67</td>
</tr>
<tr>
<td>18CC1½-p3</td>
<td>6 #5</td>
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<td>61</td>
<td>1½</td>
</tr>
<tr>
<td>18CC1½-p4</td>
<td>6 #6</td>
<td>4.15</td>
<td>63</td>
<td>1½</td>
</tr>
<tr>
<td>18CC2¾-p2</td>
<td>6 #4</td>
<td>1.89</td>
<td>58</td>
<td>2¾</td>
</tr>
<tr>
<td>18CC1¾-p2</td>
<td>6 #4</td>
<td>1.89</td>
<td>58</td>
<td>1½</td>
</tr>
<tr>
<td>18CC2¾-p3</td>
<td>6 #5</td>
<td>2.92</td>
<td>63</td>
<td>2¾</td>
</tr>
<tr>
<td>18CC1¾-p3</td>
<td>6 #5</td>
<td>2.92</td>
<td>63</td>
<td>1½</td>
</tr>
<tr>
<td>18CC2¾-p4</td>
<td>6 #6</td>
<td>4.15</td>
<td>63</td>
<td>2¾</td>
</tr>
<tr>
<td>18CC1¾-p4</td>
<td>6 #6</td>
<td>4.15</td>
<td>63</td>
<td>1½</td>
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Table D3 – Details of eccentrically loaded rectangular columns

<table>
<thead>
<tr>
<th>Column ID</th>
<th>Size</th>
<th>$e^*$ (in.)</th>
<th>Longitudinal Reinforcement</th>
<th>Transverse Reinforcement</th>
<th>Measured Concrete Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$b \times h \times L$ (in.)</td>
<td>No. &amp; Size</td>
<td>$\rho$ (%)</td>
<td>$f_y$ (ksi)</td>
<td>Size</td>
</tr>
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<td>10CE1</td>
<td>9×12×40</td>
<td>1.22</td>
<td>2 #7 + 4 #8</td>
<td>4.04</td>
<td>61, 60</td>
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<td>9</td>
<td>72</td>
<td>72</td>
<td>10.9</td>
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<td>62, 61</td>
<td>62, 61</td>
<td>62, 61</td>
<td>16.4</td>
</tr>
<tr>
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<td>72</td>
<td>72</td>
<td>72</td>
<td>16.5</td>
</tr>
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<td>62</td>
<td>62</td>
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<td>66</td>
<td>66</td>
<td>66</td>
<td>15.6</td>
</tr>
</tbody>
</table>

*Initial eccentricity of the applied load and the lateral deflection at the mid-height of the column prior to failure.

Figure D3 – Geometric overview and instrumentation

The column specimens were identified by target strength (10, 14, or 18 ksi [69, 97, or 124 MPa]), type of specimen (Column), shape of cross section (Rectangular or Circular), spacing of transverse reinforcement (9, 4½, 2¾, 1½ in. [225, 113, 69, 34 mm], etc.), and longitudinal
reinforcement ratio (ρ1, ρ2.5, etc.). For example, specimen 10CC2\(\frac{3}{4}\)-ρ1 is a 10 ksi (69 MPa) concrete target strength, circular column, with 2 \(\frac{3}{4}\) in. (69 mm) pitch of spirals, and one percent longitudinal reinforcement ratio. The eccentrically loaded columns were labeled in a similar way, and the eccentricity, e, with respect to the depth of the cross-section, h, was added as the last number to the label. For example, specimen 10CE2 is eccentrically loaded column (CE) with 10 ksi (69 MPa) target concrete strength and eccentricity (e/h) of 20 percent. If a column test was replicated, the character ‘A’ would be used to identify the repeated tests (see Tables D1 to D3).

D.3.2 Material Properties

The three target concrete strengths considered in this study were developed after laboratory and plant trial batches (Logan 2005). Mixture designs for the three concrete target strengths, 10, 14, and 18 ksi (69, 97, and 124 MPa) and the type of materials used are given in Appendix A. Three 4×8 in. (100×200 mm) cylinders were cast for each test specimen to determine the strength at the time of column testing.

Both the longitudinal and transverse reinforcement used for the test specimens were Grade 60 steel. A 220 kip capacity MTS testing machine was used to determine the properties of the longitudinal and transverse reinforcement. The yield stress of longitudinal reinforcement ranged from 58 ksi (400 MPa) to 67 ksi (462 MPa). The transverse reinforcement (#4 tie) exhibited non-linear behavior within the yielding range, without a well-defined yield point. Therefore, the 0.2 percent offset method was used to determine the yield strength. The yield strength of the spiral reinforcement was obtained from the mill tests provided by the steel supplier. Typical stress-strain relationships of longitudinal and transverse reinforcements are shown in Figure D4.
D.3.3 Instrumentation and Test Set-Up

The axial shortening of the columns was measured using four 4 in. (100 mm) pi-gages, located at the mid region of the test specimens. Two pi-gages in longitudinal direction were attached to threaded rods embedded in the core concrete while the other two pi-gages were mounted on the concrete surface. Two additional 4 in. (100 mm) pi-gages were used to measure the transverse deformations at the mid region for the rectangular columns. The strains in the longitudinal and transverse reinforcement were also measured using electrical resistance strain gages. A total of four and eight strain gages were used for the concentrically and eccentrically loaded columns, respectively, as shown in Figure D3. Three linear variable displacement transducers (LVDT) were used to measure the lateral deflections of the eccentrically loaded specimens.

Readings from the pi-gages, strain gages, applied load and stroke of the testing machine were recorded using a Vishay Data Acquisition System during testing. A 2,000 kip (8,896 kN) capacity compression testing machine was used to apply the compression load monotonically at an average rate of 0.014 in./min (0.35 mm/min) for column specimens. Eight columns (two
rectangular columns, 14CR9-ρ4, 14CR4½-ρ4; and six circular columns of 14CC series), of which the load carrying capacity was estimated to be too close to the capacity of the testing machine, were tested using a 5,000 kip (22,240 kN) capacity Baldwin testing machine by special arrangement with Lehigh University.

Thin layers of hydrostone were used at the top and the bottom ends of each column for leveling and to ensure uniform distribution of the applied load across the section. For eccentric tests, the load was applied with specific eccentricities through specially designed curved plates and roller bearing assembly. Figure D5 shows typical test set-up for the concentrically and eccentrically loaded columns. Each test was continued until a significant loss of load-resistance of the columns occurred.

![Figure D5 – Test set-up for columns](image)

(a) Concentric test  
(b) Eccentric test
D.4 Test Results and Discussion

D.4.1 Concentrically Loaded Rectangular Columns

D.4.1.1 Observed Behavior

Typical axial load–axial shortening curves of concentrically loaded columns are shown in Figure D6. No cracks were observed up to the measured peak load in most of the columns, except in some specimens which were subjected to small unintentional eccentricities during testing. The maximum axial resistance of columns was affected by the longitudinal reinforcement ratio as shown in Figure D6.

![Graph showing load-axial shortening curves for concentrically loaded rectangular columns with tie reinforcement.](image)

Figure D6 – Load-axial shortening graphs for concentrically loaded rectangular columns with tie reinforcement

At the peak load, the concrete cover spalled off suddenly and explosively at the mid region of the column for all the columns with larger tie spacing as shown in Figure D7(a). Spalling of the concrete cover for the columns with closer tie spacing occurred relatively less...
explosively (Figure D7 (b)). Spalling of the concrete cover was also accompanied by some loss of core concrete and resulted in a sudden drop in load carrying capacity of the columns as shown in Figure D6. This was more pronounced for columns with higher concrete strength. Relatively higher residual resistance was noted for columns with closer tie spacing. This behavior suggests that the remaining resistance of the column is highly dependent on the local buckling resistance of the individual longitudinal reinforcement.

In general, the measured longitudinal reinforcement strains at failure exceeded the yield strain of the reinforcement. At this stage, the transverse reinforcement remained well below the respective yield strain. For some of the columns with closer tie spacing, the transverse reinforcement yielded at a later stage of loading. The average measured axial concrete strains corresponding to the peak load ranged from 0.0022 to 0.0029.

(a) Column with larger tie spacing  
(b) Column with closer tie spacing

Figure D7 – Typical failure shapes of concentrically loaded rectangular columns
D.4.1.2 Nominal Axial Resistance at Zero Eccentricity

The nominal axial resistance of a column at zero eccentricity, \( P_o \), can be determined using the equation as follows:

\[
P_o = k_c f'_c (A_g - A_s) + f_y A_s
\]

Equation D1

where the parameter \( k_c \) is the ratio of the in-place concrete strength to the compressive strength of control cylinder, \( f'_c \); \( A_g \) is the gross area of the column; \( f_y \) is the yield strength of the longitudinal reinforcement and \( A_s \) is the area of the longitudinal reinforcement.

The parameter \( k_c \) accounts for the size effect and the concrete casting process of column versus standard concrete cylinder. Currently the value for \( k_c \) specified by the LRFD Specifications (2004) for concentrically loaded column is 0.85 for NSC.

The ratio of the measured maximum capacity \( P_{\text{max}} \) to the predicted capacity \( P_o \), using \( k_c = 0.85 \) in the above equation from the test data of this study, along with test data found in the literature by Sheik and Uzumeri (1980), Cusson and Paultre (1994), Saatcioglu and Razvi (1998), and Sharma et al. (2005) are shown in Figure D8. These data suggest that using a value of 0.85 for \( k_c \) could overestimate the column capacity for HSC.
Figure D8 – Ratio of $P_{\text{max}} / P_o$ (based on $k_c = 0.85$) with respect to concrete compressive strength

The parameter $k_c$ was further analyzed with data reported in the literature for tied columns. The magnitude of $k_c$ based on test results of the concentrically loaded columns with tie reinforcement tested in this study as well as based on the reported data by others are shown in Figure D9. The magnitude of $k_c$ was calculated by substituting the measured maximum capacity, $P_{\text{max}}$ for $P_o$ in the equation above. It should be noted that the data shown in this figure also includes columns with closer tie spacing than that required by the LRFD Specifications (2004). The figure clearly shows a trend that, for concrete higher than 10 ksi (69 MPa), the value of $k_c$ decreases with increasing concrete compressive strength.
Figure D9 – Comparison of $k_c$ parameters of concentrically loaded columns with tie reinforcement

For columns with concrete strength around 15 ksi (103 MPa) tested in this study, the magnitude of $k_c$ for smaller size columns is generally less than that of larger size columns. This behavior could be attributed to the size of the cross section, which becomes more sensitive when subjected to the same unintended eccentricity.

Test results for this study which included concrete strengths ranging from 7.9 to 16.1 ksi (54 to 111 MPa) show the same trend reported by other researchers.

A regression analysis of the collected data, shown in Figure D9, indicates that 80 percent of the $k_c$ values are higher than 0.75 for concrete strength greater than 10 ksi (69 MPa). Using it as the lower bound, the following expression for the parameter $k_c$ is proposed for concrete strength up to 18 ksi (124 MPa).
The proposed expression for $k_c$ matches the current LRFD Specifications (2004) for NSC and extends its use to include HSC up to 18 ksi (124 MPa).

**D.4.1.3 Tie Spacing**

The primary purpose of tie reinforcement in columns is to prevent concrete splitting and to provide lateral support for the longitudinal reinforcement to avoid buckling below yield strength. The current LRFD Specifications (2004) require that the ties should have a spacing equal to or less than the least lateral dimension of the column.

The typical load vs. average longitudinal reinforcement strain and average transverse reinforcement strain of concentrically loaded rectangular columns up to and slightly beyond the maximum load are shown in Figure D10. As seen in the figures, the measured strains in the longitudinal reinforcement exceeded the yield strain at the peak load, which occurred in all tested columns with tie reinforcement. The measured strains in the ties of the columns with larger tie spacing were much lower than the yield strain of the ties. These results suggest that ties in the columns spaced according to the spacing allowed by the LRFD Specifications (2004), are sufficient to provide adequate lateral support to prevent buckling of longitudinal reinforcement below its yield strength. However, the behavior of the tested columns with such large tie spacing showed no confinement effect to the concrete core, as reported also by other researchers. Strength enhancement of the column core with smaller tie spacing, one half of the specification requirement, was negligible. The test results show that using smaller spacing of ties merely
improved the residual strength of the columns after peak load as shown in Figure D6.

![Graph showing residual strength improvement](image1.png)

(a) Average longitudinal steel strain  
(b) Average transverse steel (Tie) strain

Figure D10 – Strains of longitudinal and transverse reinforcement (14CR9-p1)

In addition, inclined shear sliding was observed at later stages of testing in most of the columns as shown in Figure D11, much like the failure of a concrete cylinder.

![Images of inclined shear failure plane](image2.png)

(a) 14CR9-p2.5  
(b) 18CR7-p2

Figure D11 – Inclined shear failure plane of rectangular columns with tie reinforcement
D.4.2 Concentrically Loaded Circular Columns

**D.4.2.1 Observed Behavior**

As shown in Figure D12, the initial behavior of the circular columns was almost linear up to the initiation of the longitudinal crack. For the majority of the columns, the initial longitudinal crack occurred at an average measured concrete strain ranging from 0.0015 to 0.0022. These cracks led to the spalling of cover concrete (Figure D13 (a)), as evidenced by separation of large pieces of cover from core concrete. The spalling of cover concrete caused small drop of the load carrying capacity, which was subsequently recovered due to the confinement action by the spiral. Beyond this stage, the applied load was resisted mainly by the confined concrete core and the longitudinal reinforcement.

![Behavior of concentrically loaded circular columns with spiral reinforcement](image)

**Figure D12 – Behavior* of concentrically loaded circular columns with spiral reinforcement**

* $P_{CR}$: measured load at the initiation of spalling of concrete cover
* $P_{max}$: measured maximum load resistance of column
(a) Longitudinal crack of concrete cover
(b) Local buckling of longitudinal reinforcement, rupture of spiral, and crushing of core concrete

Figure D13 – Typical failure modes of concentrically loaded circular columns

The measured strains in the spiral reinforcement increased rapidly after spalling of concrete cover and were higher than the yield strain at the maximum measured axial load in all tested columns. These results suggest that the concrete core was confined by the spiral reinforcements, which affected significantly the overall behavior of the circular column. The effect of the confinement is clearly recognizable by comparing the behaviors of the columns with closely spaced spiral versus widely spaced spiral, as shown in Figure D12.

After peak load, the load carrying capacity of the columns with widely spaced spiral decreased relatively more than the columns with closely spaced spiral. The load carrying capacity was gradually reduced until rupture of spiral occurred.

The initial loss of the load carrying capacity was due to spalling of the concrete cover, cracking of the core concrete, and the local buckling of the longitudinal reinforcement. Afterwards, initiation of the shear sliding within the cracked core concrete was observed.
second significant loss of the load carrying capacity of the columns was due to rupture of the spiral reinforcement, followed by crushing of the core concrete as shown in Figure D13 (b).

D.4.2.2 Nominal Axial Resistance at Zero Eccentricity

The measured load at the initiation of spalling of concrete cover, $P_{CR}$, was used for $P_o$ in Equation D1 to determine the parameter $k_c$ for circular columns with spiral reinforcement. The parameters $k_c$ obtained from the concentrically loaded circular columns, from this study and other reported tests are shown in Figure D14. The figure shows the same trend of parameter $k_c$ as observed for tied rectangular columns.

![Figure D14 – Comparison of $k_c$ parameters of concentrically loaded columns with spiral reinforcement](image)

It should be noted that the longitudinal reinforcement for some of the circular columns did not yield before spalling of the concrete cover as evidenced by the strain measurements shown in Figure D15(b). In these cases, the measured strain in the longitudinal reinforcement
was used instead of the yield strength for the prediction of the nominal axial resistance. The measured strain in the spiral reinforcement was considerably less than the yield strain at $P_{CR}$, (Figure D15(c)), which means that the concrete core was not confined by the spiral reinforcement at the level of spalling load, $P_{CR}$. The measured concrete strain in the columns indicated that spalling of concrete cover occurred prior to the development of the ultimate compressive strain of the concrete as shown in Figure D15(a). This early spalling of the concrete cover is due to the closely spaced spiral separating the concrete cover from the inner core of the circular column and resulted in lower values of $k_c$. This phenomenon is more pronounced for columns with HSC.

(a) Avg. axial concrete strain  (b) Avg. longitudinal steel strain  (c) Avg. spiral steel strain

Figure D15 – Strains of concrete and reinforcements (18CC2½ -ρ3)

Although the load carried by the circular column corresponding to the initiation of spalling of concrete cover was used to determine the values of $k_c$, the load-carrying capacity of the circular columns increased after the spalling of the concrete cover due to the confinement effect produced by the spiral reinforcement. As shown in Figure D12, the behavior clearly reflected the significant increase of the maximum load-carrying capacity $P_{max}$ with respect to the
load at the initiation of spalling of the concrete cover, \( P_{CR} \), due to confinement effect produced by spiral reinforcement. Accordingly, the same expression for \( k_c \) proposed for columns with tied reinforcement can be equally used for columns with spiral reinforcement.

**D.4.2.3 Pitch of Spirals**

Richart et al. (1928 and 1929) studied confinement effects under lateral fluid pressure and in spirally reinforced concrete columns, and proposed the compressive strength of confined concrete by spiral reinforcement \( f'_{cc} \), in term of the unconfined strength \( f'_{co} \) and the lateral confinement stress \( f_i \) as follows:

\[
 f'_{cc} = f'_{co} + 4.1 f_i \quad \text{Equation D3}
\]

For circular column with spiral reinforcement, \( f_i \) can be determined in term of the area of the spiral reinforcement, \( A_{sp} \) and the spacing of the spiral \( s \), as follows:

\[
 f_i = 2A_{sp} f_{sp} / d_c s \quad \text{Equation D4}
\]

where \( f_{sp} \) is the stress in the spiral at maximum column load, \( d_c \) is the outside diameter of the spiral.

Richart’s equation has been the basis to determine the required minimum spiral steel ratio for columns with spiral reinforcement in the current LRFD Specifications (2004).

However, studies have shown that confinement effectiveness is less for HSC. For passive confinement such as in columns confined by lateral steel, confining pressure is dependent on the lateral dilation of concrete under axial load. Since lateral dilation of HSC is less than that of NSC, the effectiveness of confinement becomes less for the columns with HSC. Setunge et al. (1993) reported that in their HSC specimens, failure occurred through the aggregate particles as well as the mortar, unlike NSC in which the failure occurs mainly through the mortar and
aggregate interfaces. Failure through the aggregates leads to a lower shear resistance as well as less lateral dilation and thus lower confinement effectiveness.

The confined concrete strength $f_{cc}'$ with the lateral pressure $f_l$, normalized by $f_{co}'$, for the tested columns as well as other reported data are shown in Figure D16. The comparisons indicate that the confinement factor, represented by the slope of trend line, is reduced for concrete strength greater than 10 ksi (69 MPa).

\[
(f'_{cc} / f'_c) = 1 + 4.1 \left( f_l / f'_co \right)
\]

(a) $f_c' \leq 10$ ksi (69 MPa) \hspace{2cm} (b) $f_c' > 10$ ksi (69 MPa)

Figure D16 – Confined concrete strength with lateral confinement stress

The minimum amount of spiral reinforcement required by the LRFD Specifications (2004) is selected to ensure that the second maximum load carried by the column core and longitudinal reinforcement would be roughly equal to the initial maximum load carried by the column before spalling of the concrete cover. The first and the second peak loads ($P_1$ and $P_2$) of the tested columns with different volumetric ratios of spiral reinforcement, $\rho_s$, are summarized in Table D4. The load-axial shortening relationships of the selected 12 and 9 in. (300 and 225 mm) circular columns reinforced with spirals are shown in Figures D17 and D18, respectively. It can
be seen that for the larger 12 in. (300 mm) columns, there was virtually no load reduction after
the first peak load as opposed to the smaller 9 in. (225 mm) columns, because the concrete cover
of the larger column represents only a smaller portion of the overall section of the column.

Table D4 – Test results of circular columns with different volumetric ratios of spiral
reinforcement

<table>
<thead>
<tr>
<th>Column ID</th>
<th>Section Size (in.)</th>
<th>Concrete $f'_c$ (ksi)</th>
<th>$\rho_s$</th>
<th>$\rho_s/\rho_s$ code</th>
<th>$P_1$ (kips)</th>
<th>$P_2$ (kips)</th>
<th>$P_{cl1}$ (kips)</th>
<th>$P_{cl2}$ (kips)</th>
<th>$P_{cl1}/P_{cl2}$</th>
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<td>1003</td>
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<td>817</td>
<td>991</td>
<td>1.21</td>
<td></td>
</tr>
</tbody>
</table>

Note: $\rho_s$ code : minimum required volumetric ratio of spiral specified by the LRFD Specifications (2004)

$P_{cl1} = P_1 - A_f f_y$ (axial load carried by concrete at the first peak load)

$P_{cl2} = P_2 - A_f f_y$ (axial load carried by concrete at the second peak load)

In general, the second peak loads were larger than the first maximum loads in most
columns with volumetric ratio of spiral close to the code requirement, which is a favorable
behavior satisfying the premise of the code.
Figure D17 – Load-deflection response of concentrically loaded circular columns with spiral reinforcement

Figure D18 – Load-deflection response of concentrically loaded circular columns with spiral reinforcement
Based on the above data, it appears that the current minimum spiral steel requirement of the LRFD Specifications (2004) is also applicable to HSC columns in non-seismic zones. It should be noted that the minimum spiral requirement, $\rho$, is a function of concrete strength. For HSC, the minimum spiral requirement is significantly increased. However, increased spiral amount which compensates for the lower confinement effectiveness leads to smaller pitches of spiral, if normal grade steel is used for the spiral. Unduly small pitch may result in reinforcement congestion that will hinder concrete placement. A remedy would be to use high strength steel for the spiral reinforcement.

D.4.3 Eccentrically Loaded Rectangular Columns

D.4.3.1 Observed Behavior

Typical axial load shortening responses of eccentrically loaded columns with eccentricity to depth ratios, $e/h$, of 10 and 20 percent are shown in Figure D19.
Figure D19 also includes a concentrically loaded column to emphasize the effect of load eccentricity on the behavior of the column. In most of the columns, no cracks were observed on the compressive side of the column up to the peak load. In some cases, the cracking sound was heard at loads slightly less than the peak load. At the peak load, spalling of the concrete cover and buckling of the longitudinal reinforcement were observed simultaneously at the extreme compression face. When the peak load was reached, inclined flexural cracks propagated rapidly through the tension side, as shown in Figure D20(b). The load carrying capacity of eccentrically loaded columns was reduced due to the presence of the secondary moment resulting from the applied load with eccentricity.

(a) Spalling of cover concrete and local buckling of longitudinal steel
(Compression side)

(b) Inclined and flexural crack
(Tension side)

Figure D20 – Typical failure shapes of eccentrically loaded columns
D.4.3.2 Rectangular Stress Block in Compression Zone of HSC

Prediction of the load carrying capacity is based on an equivalent rectangular stress block representing the stress distribution of concrete in the compression zone for flexural members at ultimate strength. The values of $\alpha_l$ and $\beta_l$ were determined based on specially designed bracket specimens as described in Appendix B. The parameters $\alpha_l$ and $\beta_l$ specified by the current LRFD Specifications (2004) and proposed by this research program are given in Table D5.

Table D5 – Rectangular stress block parameters

<table>
<thead>
<tr>
<th>RSB</th>
<th>$\alpha_l$</th>
<th>$\beta_l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LRFD</td>
<td>0.85</td>
<td>$\begin{align*} 0.85 &amp; \quad \text{for } f'_c \leq 4 \text{ksi} \ 0.85 - 0.05(f'_c - 4) &amp; \geq 0.65 \quad \text{for } f'_c &gt; 4 \text{ksi} \end{align*}$</td>
</tr>
<tr>
<td>Proposed</td>
<td>$\begin{align*} 0.85 &amp; \quad \text{for } f'_c \leq 10 \text{ksi} \ 0.85 - 0.02(f'_c - 10) &amp; \geq 0.75 \quad \text{for } f'_c &gt; 10 \text{ksi} \end{align*}$</td>
<td>Same above</td>
</tr>
</tbody>
</table>

$f'_c$ is in ksi

Data obtained from 20 reinforced concrete columns with concrete compressive strengths higher than 10 ksi (69 MPa), tested in this study, along with the reported data by others under combined axial and flexural loading were examined to verify the applicability of the proposed $\alpha_l$ for the rectangular stress block in the compression zone.

The proposed $\alpha_l$ was used to construct the interaction diagram for each of the tested columns. The predicted values ($M_{\text{pred}}$, $P_{\text{pred}}$) are compared to the experimental values ($M_{\text{exp}}$, $P_{\text{exp}}$) for each tested column using the same eccentricities as illustrated in Figure D21. The ratio ($P_{\text{exp}} / P_{\text{pred}}$) using the modified parameter $\alpha_l$ proposed by this research program as defined in Table D5 is given in Table D6. For comparison, the ratio ($P_{\text{exp}} / P_{\text{pred}}$) using the current value of $\alpha_l = 0.85$ is also given in Table D6.

Although the overall average of the ratio ($P_{\text{exp}} / P_{\text{pred}}$) using the current LRFD Specifications (2004) for $\alpha_l$ was slightly greater than one, the ratio was less than one for...
approximately half of the columns examined, which means that the predictions overestimated the strength of the columns.

![Interaction diagram for rectangular reinforced concrete columns](image)

**Figure D21** Interaction diagram for rectangular reinforced concrete columns

**Table D6** – Comparison between experimental and predicted load using proposed rectangular stress block for eccentrically loaded columns

<table>
<thead>
<tr>
<th>Reference</th>
<th>$f'_c$ (ksi)</th>
<th>e (in.)</th>
<th>(\frac{P_{\text{exp}}}{P_{\text{pred}}})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LRFD</td>
<td>Proposed</td>
<td></td>
</tr>
<tr>
<td>This Research</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16.5</td>
<td>0.10h</td>
<td>0.97</td>
<td>1.08</td>
</tr>
<tr>
<td>16.6</td>
<td>0.21h</td>
<td>0.97</td>
<td>1.08</td>
</tr>
<tr>
<td>15.6</td>
<td>0.10h</td>
<td>0.98</td>
<td>1.09</td>
</tr>
<tr>
<td>15.6</td>
<td>0.20h</td>
<td>1.01</td>
<td>1.12</td>
</tr>
<tr>
<td>14.0</td>
<td>0.09h</td>
<td>1.07</td>
<td>1.16</td>
</tr>
<tr>
<td>10.9</td>
<td>0.09h</td>
<td>1.00</td>
<td>1.01</td>
</tr>
<tr>
<td>Lee and Son (2000)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13.5</td>
<td>0.23h</td>
<td>0.94</td>
<td>1.01</td>
</tr>
<tr>
<td>13.5</td>
<td>0.40h</td>
<td>0.98</td>
<td>1.04</td>
</tr>
<tr>
<td>13.5</td>
<td>0.23h</td>
<td>1.03</td>
<td>1.11</td>
</tr>
<tr>
<td>13.5</td>
<td>0.40h</td>
<td>1.00</td>
<td>1.07</td>
</tr>
<tr>
<td>Foster and Attard (1997)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13.1</td>
<td>0.08h</td>
<td>0.96</td>
<td>1.03</td>
</tr>
<tr>
<td>13.1</td>
<td>0.08h</td>
<td>1.02</td>
<td>1.09</td>
</tr>
<tr>
<td>13.1</td>
<td>0.18h</td>
<td>0.96</td>
<td>1.02</td>
</tr>
<tr>
<td>13.1</td>
<td>0.17h</td>
<td>0.97</td>
<td>1.03</td>
</tr>
<tr>
<td>13.1</td>
<td>0.17h</td>
<td>1.15</td>
<td>1.23</td>
</tr>
<tr>
<td>13.1</td>
<td>0.40h</td>
<td>1.21</td>
<td>1.29</td>
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<tr>
<td>13.1</td>
<td>0.40h</td>
<td>1.12</td>
<td>1.18</td>
</tr>
<tr>
<td>13.1</td>
<td>0.39h</td>
<td>1.34</td>
<td>1.42</td>
</tr>
<tr>
<td>Tan and Nguyen (2005)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.0</td>
<td>0.11h</td>
<td>0.93</td>
<td>0.95</td>
</tr>
<tr>
<td>11.0</td>
<td>0.22h</td>
<td>0.94</td>
<td>0.96</td>
</tr>
<tr>
<td>11.0</td>
<td>0.32h</td>
<td>0.92</td>
<td>0.94</td>
</tr>
<tr>
<td>Average ((\mu))</td>
<td>1.02</td>
<td>1.09</td>
<td></td>
</tr>
<tr>
<td>Standard deviation ((\sigma))</td>
<td>0.10</td>
<td>0.11</td>
<td></td>
</tr>
</tbody>
</table>
The average of the ratio using the proposed $\alpha_i$ was 1.09, and the strength of all the columns was greater than the predictions except for the columns tested by Tan et al. (2005). Accordingly, using the modified $\alpha_i$ parameter for concrete strength higher than 10 ksi (69 MPa) would produce improved comparison between the predictions and the test results.

The interaction diagrams for the columns with concrete strengths of 10.9 to 16.5 ksi (75 to 114 MPa) are shown in Figure D22. They illustrate graphically the applicability of the modified $\alpha_i$ parameter for concrete strength higher than 10 ksi (69 MPa). It should be noted that the proposed $\alpha_i$ parameter is the same as the proposed $k_c$ parameter for concentrically loaded columns, which is a convenience in design applications.

![Interaction diagrams](image)

Figure D22 – Interaction diagrams based on the current LRFD Specifications (2004) and modified parameters $\alpha_i$, $\beta_i$ and $k_c$

### D.4.3.2 Nominal Axial Resistance of Column

As mentioned before, the nominal axial load carrying capacity of a column at zero eccentricity ($P_o$) can be determined by using $k_c$ parameter introduced by this research program. However, a truly concentrically loaded column is rare. Unintentional eccentricities should be expected due to end conditions, inaccuracy of construction, and normal variation in material
properties. Hence in design, a minimum eccentricity of 10 percent of the thickness of the column in the direction perpendicular to its axis of bending is normally considered for columns with ties and 5 percent for spirally reinforced columns. To reduce the calculations necessary for analysis and design for minimum eccentricity, the LRFD Specifications (2004) prescribes a reduction of 20 percent in the axial load for tied columns and a 15 percent reduction for spiral columns. Using these factors, the nominal axial resistance of columns can be determined as follows:

\[ P_{n\text{(max)}} = 0.8 \left[k_e f'_c (A_g - A_t) + f_y A_t \right] \text{ for tied reinforced columns} \quad \text{Equation D5} \]

\[ P_{n\text{(max)}} = 0.85 \left[k_e f'_c (A_g - A_t) + f_y A_t \right] \text{ for spirally reinforced columns} \quad \text{Equation D6} \]

In Table D7, the measured maximum load \( P_{\text{max}} \) of the columns with an eccentricity of 0.09h or 0.1h is compared with 80 percent of the predicted nominal strength (0.8\( P_o \)) of the tied columns using Equation D5. The results indicate that measured maximum axial load of the columns, \( P_{\text{max}} \) is consistently greater than the predicted nominal strength. The difference ranged between 6 and 21 percent with an average of 12 percent. Therefore, for design purpose, the 20 percent reduction in the axial load capacity of the column as specified by the LRFD Specifications (2004) to account for unintentional eccentricity for tied columns with HSC is on the conservative side.

Table D7 – Comparison between maximum measured load and 80 percent of predicted load of tied columns with eccentricity of 0.1h

<table>
<thead>
<tr>
<th>Column ID</th>
<th>( e )</th>
<th>( f'_c ) (ksi)</th>
<th>Measured ( P_{\text{max}} ) (kips)</th>
<th>0.8( P_o ) (kips)</th>
<th>Measured ( P_{\text{max}} ) of 0.8( P_o )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10CE1</td>
<td>0.1h</td>
<td>7.9</td>
<td>893</td>
<td>770</td>
<td>1.16</td>
</tr>
<tr>
<td>A10CE1</td>
<td>0.09h</td>
<td>10.9</td>
<td>1023</td>
<td>966</td>
<td>1.06</td>
</tr>
<tr>
<td>14CE1</td>
<td>0.1h</td>
<td>16.4</td>
<td>1358</td>
<td>1234</td>
<td>1.10</td>
</tr>
<tr>
<td>18CE1</td>
<td>0.1h</td>
<td>15.6</td>
<td>767</td>
<td>698</td>
<td>1.10</td>
</tr>
<tr>
<td>A18CE1</td>
<td>0.09h</td>
<td>14.0</td>
<td>789</td>
<td>654</td>
<td>1.21</td>
</tr>
<tr>
<td>Average (( \mu ))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
D.4.3.3 Ultimate Compressive Strain of Concrete

The measured ultimate compressive strains of concrete from eccentrically loaded columns ranged from 0.0025 to 0.0046. The average value of the strains was 0.0033 for the columns with concrete strength greater than 10 ksi (69 MPa), and was 0.0034 for all tested columns. Figure D23 shows the measured ultimate compressive strains with respect to concrete strength. No definite trend is observed. If the single exceptional value of 0.0046 is excluded, the average value for the columns with concrete strength larger than 10 ksi (69 MPa) approaches to a lower value of 0.003. Therefore, the value of 0.003 for the ultimate compressive strain specified by the current specification seems appropriate as a conservative lower bound.

![Figure D23 – Ultimate concrete strain from eccentrically loaded columns](image)

D.4.4 Reinforcement Limits

The current LRFD Specifications (2004) have two relationships to limit the maximum reinforcement and one criterion to limit the minimum reinforcement for compression members.
The maximum area of prestressed and non-prestressed longitudinal reinforcement for non-composite compression components according to the LRFD Specifications (2004) is limited by the two following equations:

\[
\frac{A_s}{A_g} + \frac{A_{ps} f_{pu}}{A_g f_y} \leq 0.08 \quad \text{LRFD Equation 5.7.4.2-1} \quad \text{Equation D7}
\]

and

\[
\frac{A_{ps} f_{pe}}{A_g f'_c} \leq 0.30 \quad \text{LRFD Equation 5.7.4.2-2} \quad \text{Equation D8}
\]

where \(f'_c\) is the concrete compressive strength, \(A_{ps}\) is the area of prestressing steel, \(f_{pu}\) is the specified tensile strength of prestressing steel, \(f_{pe}\) is the effective prestress after losses, \(A_s\) and \(f_y\) are the area and yield strength of mild tension steel, respectively, and \(A_g\) is the gross area of the section.

The minimum area of prestressed and non-prestressed longitudinal reinforcement for non-composite compression components according to the LRFD Specifications (2004) is:

\[
\frac{A_s f_y}{A_g f'_c} + \frac{A_{ps} f_{pu}}{A_g f'_c} \geq 0.135 \quad \text{Equation D9}
\]

The upper limits were initially established based on practical considerations of concrete placement and have since been maintained for all ranges of concrete compressive strengths. Accordingly, there is no need to change the LRFD Specifications (2004) for the maximum reinforcement ratio for compression members.

However, the current LRFD Specifications (2004) indicate a requirement of 4.05 percent as the minimum reinforcement ratio for 18 ksi (124 MPa) concrete compressive strength and Grade 60 steel, in the absence of any prestressing steel in the section as shown in Figure D24. Such high level of minimum reinforcement ratio is quite unusual and should be examined for
HSC. In order to evaluate this reinforcement limit, it is necessary to review the basis and historical development of the current requirement of the minimum reinforcement.

Figure D24 – Reinforcement limits for compression members with only mild steel according to the current LRFD Specifications (2004)

For non-prestressed sections, the minimum limit for longitudinal reinforcement in compression members originated from the early column tests by Richart et al. (1931a, 1931b, 1931c and 1932) at the University of Illinois. When a column is tested under sustained service loads, the stress distribution between steel and concrete changes over time due to creep and shrinkage of concrete. With creep and shrinkage increasing progressively, concrete relieves itself from its initial share of the axial load. As a result, longitudinal steel reinforcement gradually carries a larger portion of the sustained load over time. Therefore, it is theoretically possible that in columns with small amounts of longitudinal reinforcement, the reinforcing steel could yield, resulting in creep rupture of the column. Tests by Richart et al. (1931a, 1931b, 1931c and 1932)
showed the increase of stress in the steel reinforcement is inversely proportional to the percentage of the longitudinal steel. Results from their tests carried out with concrete strengths between 2 and 8 ksi (14 and 55 MPa), suggested a minimum reinforcement ratio of 1 percent. The application of this limit was later extended by the LRFD Specifications (2004) for concrete compressive strengths up to 10 ksi (69 MPa) without any further tests or analysis, and certainly without any consideration for HSC above 10 ksi (69 MPa).

Three types of strain are normally developed in the longitudinal reinforcement under the effect of sustained loading: initial elastic strain, strain developed due to shrinkage of concrete and strain developed due to creep of concrete.

When a sustained load is applied to a reinforced concrete column, initial elastic strain, $\varepsilon_1$, is observed immediately as shown in Figure D25.

![Figure D25 – Initial elastic strain due to applied sustained load](image)

At this stage the applied load is resisted by both concrete and steel as follows:

$$ P = E_c \varepsilon_1 (1 - \rho_l) A_g + E_s \varepsilon_1 \rho_l A_g $$

Equation D10

where $P$ is the applied axial load, $E_c$ is the modulus of elasticity of concrete, $\rho_l$ is the longitudinal reinforcement ratio, $A_g$ is the gross area of concrete and $E_s$ is the modulus of elasticity of steel. The initial elastic strain of concrete and steel can be obtained from the above equilibrium
Following the initial elastic deformation, the time dependent deformations will occur in the concrete due to creep and shrinkage. Since the column contains reinforcement, the shrinkage strain, $\varepsilon_{sh}$, will be restrained by the longitudinal reinforcement of the column causing an increase in the load carried by the reinforcement and a decrease in the load carried by the concrete. The same behavior holds true for creep strain of concrete, $\varepsilon_{cr}$. The behavior of reinforced concrete column due to shrinkage and creep is presented in Figure D26.

![Diagram](image)

a) Due to Shrinkage  

b) Due to Creep

Figure D26 – Shortening of reinforced concrete column due to shrinkage and creep

From equilibrium of forces due to shrinkage,

$$E_s \varepsilon_2 \rho_s A_g = E_c (\varepsilon_{sh} - \varepsilon_2)(1 - \rho_i)A_g$$  

Equation D12

where $\varepsilon_2$ is the strain in the reinforcement due to shrinkage of concrete. Thus the strain developed in the longitudinal reinforcement due to shrinkage of concrete can be determined as:

$$\varepsilon_2 = \frac{(1 - \rho_i)\varepsilon_{sh}E_c}{\rho_sE_g + (1 - \rho_i)E_c}.$$  

Equation D13

Similarly, from equilibrium of forces due to creep,
\[ E_s \varepsilon_3 \rho_s A_s = E_c (\varepsilon_{cr} - \varepsilon_3)(1 - \rho_t)A_g \quad \text{Equation D14} \]

where \( \varepsilon_3 \) is the strain in the reinforcement due to creep of concrete. Then the strain developed in the longitudinal reinforcement due to creep of concrete can be determined as:

\[ \varepsilon_3 = \frac{(1 - \rho_t)\varepsilon_{cr}E_c}{(\rho_s E_s + (1 - \rho_t)E_c)}. \quad \text{Equation D15} \]

To prevent yielding of the longitudinal reinforcement, the summation of the initial elastic strain and the strains due to shrinkage and creep should not reach the yield strain of the longitudinal reinforcement. Thus,

\[ \varepsilon_{total} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \leq \text{yield strain of longitudinal reinforcement} \quad \text{Equation D16} \]

Note that for Grade 60 steel reinforcement, the yield strain is assumed to be 0.002.

The procedure used to calculate the minimum longitudinal reinforcement ratio for compression members was an iterative procedure which was modeled using Microsoft Excel. The amount of reinforcement was determined for a reinforced concrete column under sustained load which would lead to a total strain of 0.002 after specified period of time. The procedure used was as follows:

1. The range of concrete compressive strength used in this study varies between 6 and 18 ksi (41 and 124 MPa).
2. The modulus of elasticity of steel was taken as 30,000 ksi (200,000 MPa). For modulus of elasticity of concrete, the relationships proposed by this research program as well as the current LRFD Specifications (2004) were used for HSC. Most critical conditions were established using the one proposed by this research program. The unit weight of concrete \( (w_c) \) used in the analysis was 0.150 kcf (2400 kg/m³) since HSC is more compact and denser than NSC. The equation for modulus of elasticity \( (E_c) \) proposed by this research program is
as follows:

\[ E_c = 310,000K_1w_c^{2.5}(f'_c)^{0.33} \quad \text{Equation D17} \]

where \( K_1 \) is the correction factor for source of aggregate (taken as 1.0) and \( f'_c \) is the concrete compressive strength.

The analysis was repeated using the current equation specified by the LRFD Specifications (2004):

\[ E_c = 33,000K_1w_c^{1.5}\sqrt{f'_c} \quad \text{LRFD Equation 5.4.2.4-1} \quad \text{Equation D18} \]

3. The shrinkage strain (\( \varepsilon_{sh} \)) and creep coefficient (\( \psi \)) relationship specified by the LRFD Specifications (2004) were used to calculate the shrinkage and creep behavior of concrete.

4. The relative humidity used in the calculation of the creep and shrinkage was 10 percent, since lower relative humidity would produce more critical results.

5. The volume to surface ratio used in the calculation of creep and shrinkage was 3. Note that, the volume to surface ratio for a circular column with 12 in. (300 mm) diameter is 3. It is the same for a 12×12 in. (300×300 mm) square column.

6. The time considered in the calculation of the creep and shrinkage was 10 years which is equal to 3650 days.

7. The age of loading in the calculation of the creep coefficient was 28 days.

8. The sustained load level on the reinforced concrete column considered in this investigation was 50 percent. \( (P/f'_cA_g = 0.5) \). The unfactored permanent load on columns do not exceed \( 0.5A_gf'_c \), which is typically the case encountered in design.

9. The effects associated with stress relief for both creep and shrinkage due to creep of concrete in tension are neglected in the formulation of the equilibrium conditions. By neglecting such effects, the results are more conservative.
10. The initial value for the longitudinal reinforcement ratio ($\rho_l$) for a reinforced concrete column was established. The initial elastic strain and strains due to creep and shrinkage were calculated based on the previous discussions in this section. The sum of all three strain values, the total strain ($\varepsilon_{\text{total}}$), was calculated and compared to the yield strain of steel reinforcement. By changing the initial value of the longitudinal reinforcement ratio, the reinforcement ratio for which the total strain was equal to the yield strain of steel was determined. This reinforcement ratio was used as the minimum amount of longitudinal reinforcement ratio for compression members to prevent creep rupture.

11. Step 10 was performed for all the concrete compressive strengths in the range between 6 and 18 ksi (41 and 124 MPa).

The most critical conditions were evaluated in the calculation of minimum longitudinal reinforcement ratio for compression members. Based on the analysis using the proposed equation for $E_c$ and the current relationship specified by the LRFD Specifications (2004), a new relationship is proposed for minimum reinforcement ratio for compression members as follows:

$$\frac{A_x}{A_g} + \frac{A_{ps} f_{ps}}{A_g f_y} \geq 0.135 \frac{f'_c}{f_y} \quad \text{but not greater than 0.0225.}$$

Equation D19

For concrete compressive strengths up to 10 ksi (69 MPa), the proposed relationship for minimum longitudinal reinforcement ratio requires the same amount as that of the LRFD Specifications (2004). For concrete compressive strengths greater than 10 ksi (69 MPa), the proposed equation requires the same amount of 0.0225 for concrete compressive strengths up to 18 ksi (124 MPa). Furthermore, the proposed minimum reinforcement limitation is similar in format with the maximum reinforcement limitation specified by the LRFD Specifications (2004).

The minimum longitudinal reinforcement ratio for the stress level $P/f'_c A_g = 0.5$ as required by the current LRFD Specifications (2004), by the proposed provision, and based on the
above procedure considering the effects of creep and shrinkage are tabulated in Table D8 and shown in Figure D27. The figure clearly indicates that for concrete strength greater than 10 ksi (69 MPa), the required minimum longitudinal reinforcement ratio by the proposed equation is greatly reduced from that called for by the current LRFD Specifications (2004), but the proposed equation still provides substantial margin against what is needed to prevent creep rupture.

Table D8 – Comparison of the $A_s/A_g$ ratio for $P/f'_c A_g = 0.5$

<table>
<thead>
<tr>
<th>$f'_c$ (ksi)</th>
<th>$P/f'_c A_g = 0.5$</th>
<th>$A_s/A_g$ (LRFD)</th>
<th>$A_s/A_g$ (Proposed)</th>
<th>$A_s/A_g$ (Calculated)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0.0135</td>
<td>0.0135</td>
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<td></td>
</tr>
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<td>8</td>
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</tr>
<tr>
<td>16</td>
<td>0.036</td>
<td>0.0225</td>
<td>0.01672</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>0.03825</td>
<td>0.0225</td>
<td>0.01952</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>0.0405</td>
<td>0.0225</td>
<td>0.02259</td>
<td></td>
</tr>
</tbody>
</table>

Figure D27 – Comparison of the $A_s/A_g$ ratio for $P/f'_c A_g = 0.5$
The calculated values for minimum reinforcement ratio for compression members based on prevention of creep rupture for $P/f'_c A_g = 0.5$ are tabulated in Table D9. Note that the summation of the initial elastic, shrinkage and creep strains are equal to the yield strain of Grade 60 steel reinforcement, 0.002. It is clear from the table that the creep and shrinkage strains of concrete decreases as concrete compressive strength increases. However, the initial elastic strain also increases as concrete compressive strength increases since the same stress level was applied on each column with different concrete compressive strengths. When columns with 6 and 18 ksi (41 and 124 MPa) concrete compressive strengths are compared under $P/f'_c A_g = 0.5$, the load applied on the column with 18 ksi (124 MPa) concrete compressive strength is 3 times that of applied on the column with 6 ksi (41 MPa) concrete compressive strength. However, the modulus of elasticity of the column with 18 ksi (124 MPa) concrete compressive strength is only 1.44 times that of the column with 6 ksi (41 MPa) concrete compressive strength. Therefore, the minimum reinforcement ratio for compression members can not be reduced for HSC compared to NSC, although HSC creeps and shrinks less.

Table D9 – Calculated values for $P/f'_c A_g = 0.5$

<table>
<thead>
<tr>
<th>$f'_c$ (ksi)</th>
<th>$\rho$ (%)</th>
<th>$E_c$ (ksi)</th>
<th>Initial Elastic Strain ($\varepsilon_1$)</th>
<th>Shrinkage Strain ($\varepsilon_2$)</th>
<th>Creep Strain ($\varepsilon_3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>1.109</td>
<td>4880</td>
<td>0.000582</td>
<td>0.000633</td>
<td>0.000785</td>
</tr>
<tr>
<td>7</td>
<td>0.892</td>
<td>5134</td>
<td>0.000653</td>
<td>0.000563</td>
<td>0.000784</td>
</tr>
<tr>
<td>8</td>
<td>0.764</td>
<td>5365</td>
<td>0.000720</td>
<td>0.000505</td>
<td>0.000775</td>
</tr>
<tr>
<td>9</td>
<td>0.707</td>
<td>5578</td>
<td>0.000782</td>
<td>0.000457</td>
<td>0.000761</td>
</tr>
<tr>
<td>10</td>
<td>0.712</td>
<td>5776</td>
<td>0.000841</td>
<td>0.000415</td>
<td>0.000744</td>
</tr>
<tr>
<td>11</td>
<td>0.770</td>
<td>5960</td>
<td>0.000895</td>
<td>0.000380</td>
<td>0.000725</td>
</tr>
<tr>
<td>12</td>
<td>0.874</td>
<td>6134</td>
<td>0.000946</td>
<td>0.000350</td>
<td>0.000704</td>
</tr>
<tr>
<td>13</td>
<td>1.020</td>
<td>6298</td>
<td>0.000994</td>
<td>0.000323</td>
<td>0.000683</td>
</tr>
<tr>
<td>14</td>
<td>1.204</td>
<td>6454</td>
<td>0.001039</td>
<td>0.000299</td>
<td>0.000662</td>
</tr>
<tr>
<td>15</td>
<td>1.422</td>
<td>6602</td>
<td>0.001081</td>
<td>0.000278</td>
<td>0.000641</td>
</tr>
<tr>
<td>16</td>
<td>1.672</td>
<td>6744</td>
<td>0.001122</td>
<td>0.000259</td>
<td>0.000619</td>
</tr>
<tr>
<td>17</td>
<td>1.952</td>
<td>6881</td>
<td>0.001159</td>
<td>0.000242</td>
<td>0.000599</td>
</tr>
<tr>
<td>18</td>
<td>2.259</td>
<td>7012</td>
<td>0.001195</td>
<td>0.000227</td>
<td>0.000578</td>
</tr>
</tbody>
</table>
D.5 Conclusions

Based on the test results, the following conclusions can be drawn:

- The factor 0.85 $f'_c$ for the concrete contribution to the factored axial resistance of concrete compressive components in the LRFD Specifications (1) Equations 5.7.4.4-2, 5.7.4.4-3, and 5.7.4.5-2 should be replaced by $k_c f'_c$ where $k_c$ is defined as follows:

  $$k_c = \begin{cases} 
  0.85 & \text{for } f'_c \leq 10 \text{ ksi} \\
  0.85 - 0.02(f'_c - 10) \geq 0.75 & \text{for } f'_c > 10 \text{ ksi}
  \end{cases} \quad (f'_c \text{ in ksi})$$

  Equation D2

  $$k_c = \begin{cases} 
  0.85 & \text{for } f'_c \leq 69 \text{ MPa} \\
  0.85 - 0.003(f'_c - 69) \geq 0.75 & \text{for } f'_c > 69 \text{ MPa}
  \end{cases} \quad (f'_c \text{ in MPa})$$

  where $k_c$ is the ratio of in-place concrete compressive strength to the specified compressive strength of concrete, $f'_c$.

- For concrete compressive strengths exceeding 10 ksi (69 MPa), the modified interaction diagrams according to the proposed parameters $\alpha_i$, $\beta_i$ (Appendix B) and $k_c$ are more conservative than those based on the LRFD Specifications (2004), especially for compression members subjected to small eccentricity.

- The maximum tie spacing and minimum volumetric ratio of spiral required by the LRFD Specifications (2004) are applicable for reinforced concrete columns with compressive strengths up to 18 ksi (124 MPa).

- For design purposes, setting the maximum limit of 80 percent of the axial load capacity for tied columns with HSC to account for the unintentional eccentricity seems to be reasonable and conservative.

- The ultimate compressive strain of 0.003 specified by the current LRFD Specifications (2004) is appropriate for analysis of reinforced HSC columns up to 18 ksi (124 MPa).
• The minimum area of prestressed and non-prestressed longitudinal reinforcement for non-composite compression components required by the LRFD Specifications (Equation 5.7.4.2-3) should be replaced by the following equation:

\[
\frac{A_g}{A_g} + \frac{A_{ps}f_{pu}}{A_g f_y} \geq 0.135 \frac{f'_{c}}{f_y}
\]

but not greater than 0.0225. Equation D19

D.6 References


Issa, M. A., and Tobaa, H. “Strength and Ductility Enhancement in High-Strength Confined


Richart, F. E., Brandtzaeg, A., and Brown, R. L., “The Failure of Plain and Spirally Reinforced


Yong, Y., Nour, M. G., and Nawy, E. G., “Behavior of Laterally Confined High Strength
APPENDIX E – PRESTRESSED GIRDERS

E.1 Introduction

This appendix presents a summary of an investigation on the flexural behavior of prestressed girders cast with high-strength concrete (HSC) up to 18 ksi (124 MPa). The investigation included an experimental program and analytical study. Detailed discussions on this investigation can be found in Choi (2006).

E.2 Objective and Scope

The main goal of this research was to evaluate the flexural behavior of full-size prestressed HSC girders with or without a cast-in-place deck of normal-strength concrete (NSC). The specific objectives were:

- To assess the modulus of rupture and elastic modulus of the HSC used in girders.
- To evaluate the losses of prestress and cracking moment of prestressed girders cast with HSC.
- To examine the current procedures of AASHTO LRFD Bridge Design Specifications (2004) including the ultimate flexural resistance of prestressed HSC girders with or without a NSC deck slab.

The experimental program consisted of nine (9) AASHTO girders tested in flexure under static loading up to failure. The first three of the nine HSC girders with three different target concrete strengths and a cast-in-place NSC deck, were designed to have the compression zone to be located within the NSC deck slab. The second three prestressed HSC girders with a narrower cast-in-place NSC deck were designed to have the compression zone to occur in both HSC and
NSC. The third group of three prestressed HSC girders without a deck slab was designed to have the entire compression zone to occur in HSC.

E.3 Test Program

E.3.1 Specimens and Material Properties

The nine prestressed HSC girders were designed for three different target concrete strengths of 10, 14, and 18 ksi (69, 97, and 124 MPa). All girders were designed to avoid premature failure due to shear and bond slippage before flexural failure. The design was based on the LRFD Specifications (2004). Each girder used No. 4 stirrups at a spacing of 3 in. (75 mm) near the end regions and at 6 in. (150 mm) of spacing for the remainder of the span.

The girders were all AASHTO Type II with a span of 40 ft. (12 m). They were tested under static loading with four-point bending. The concrete mixtures used for the girders were designed for target compressive strengths of 10, 14, and 18 ksi (69, 97, and 124 MPa). Details of the concrete mixtures are shown in Table E1.

<table>
<thead>
<tr>
<th>Target Strength</th>
<th>10 ksi</th>
<th>14 ksi</th>
<th>18 ksi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cement (lbs)</td>
<td>670</td>
<td>703</td>
<td>890</td>
</tr>
<tr>
<td>Fly Ash (lbs)</td>
<td>150</td>
<td>192</td>
<td>180</td>
</tr>
<tr>
<td>Silica Fume (lbs)</td>
<td>50</td>
<td>75</td>
<td>75</td>
</tr>
<tr>
<td>#67 Granite (lbs)</td>
<td>1727</td>
<td>1700</td>
<td>1700</td>
</tr>
<tr>
<td>Concrete Sand (River) (lbs)</td>
<td>1100</td>
<td>1098</td>
<td>917</td>
</tr>
<tr>
<td>Water (lbs)</td>
<td>280</td>
<td>250</td>
<td>265</td>
</tr>
<tr>
<td>Recover (Hydration stabilizer) (oz.)</td>
<td>26</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>ADVA 170 (Water reducer) (oz.)</td>
<td>98</td>
<td>125</td>
<td>135</td>
</tr>
<tr>
<td>w/cm</td>
<td>0.32</td>
<td>0.26</td>
<td>0.23</td>
</tr>
</tbody>
</table>

The girders were fabricated by Standard Concrete Products in Savannah, GA. Each $\frac{1}{2}$ in. (13mm) diameter, 7-wire, 270K Grade prestressing strand was tensioned to 75 percent of
ultimate strength for a total load of 31 kips. Casting and curing of the girders followed the typical procedures used by the producer. During the fabrication process, the applied prestressing force, strain of prestressing strands, their elongation and end slippage were monitored. The compressive strength and elastic modulus of concrete were measured using 4×8 cylinders. The modulus of rupture was determined using 6×6×20 in. (150×150×500 mm) beam specimens.

Three identical girders were cast for each of the three target concrete strengths. Prior to testing, one of the three girders received a 5 ft. (1.5 m) wide cast-in-place deck slab, the second received a 1 ft. (0.3 m) wide cast-in-place deck slab, and the third was tested without a composite deck slab.

The cast-in-place deck slab was 8 in. (200 mm) thick, using ready-mixed concrete supplied by a local concrete producer. The average compressive strength of the concrete used for the 5 ft. (1.5 m) wide deck slab was 4.07 ksi (28 MPa) and that for the 1 ft. (0.3 m) wide deck slab was 5.56 ksi (38 MPa).

E.3.2 Instrumentation and Test Procedure

The prestressing force in each girder was monitored by load cells and checked by the elongation of selected strands at the time of jacking. It was checked again by load cells at the time of casting, during curing, and just before transfer of prestress.

Two weldable strain gages were installed on two strands at the bottom row for each girder to measure the strains in the prestressing strands. These strains were used to determine the elastic shortening, prestress losses, and the strain changes in the prestressing strands during testing up to failure.

A schematic view of the test set-up and locations of the instrumentations are shown in Figure E1. Potentiometers were used to measure deflections along the girder and at the two
supports. Both electrical resistance strain gages and pi-gages, mounted at the top surface of girder, were used to measure the strain of concrete within the constant moment region.

![Test set-up and locations of the instrumentations](image)

Figure E1 – Test set-up and locations of the instrumentations

The load was applied in displacement control at a rate 0.1 in./min (2.5 mm/min) in order to observe crack initiation in the girder and at a rate 0.25 in./min (6.3 mm/min) after the yielding of prestressing strands and up to failure. Visual inspection of the cracks was performed throughout the tests, and cracks were mapped. Tests were terminated after crushing of concrete occurred in the constant moment zone.

**E.4 Test Results and Discussions**

**E.4.1 Material Properties**

The concrete properties determined on the test day for each girder and deck slab are shown in Table E2. Each specimen is identified first by two digits representing the target concrete strength followed by the characters “PS” which stands for prestressed concrete girder. The final two characters such as “5S” represents 5 ft. (1.5m) wide deck slab, and the letter “N” means without deck slab.
The test cylinders were air-cured inside the laboratory, same as the girders. All the tests regarding material properties were conducted in accordance with ASTM Specifications. All specimens achieved their target concrete compressive strengths except for girder 18PS-1S.

Table E2 – Material properties for each test specimens

<table>
<thead>
<tr>
<th>Specimens</th>
<th>Age (days)</th>
<th>$f_{c\test}$ (ksi)</th>
<th>E (ksi)</th>
<th>$f_r$ (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10PS-5S</td>
<td>Girder</td>
<td>120</td>
<td>11.49</td>
<td>5360</td>
</tr>
<tr>
<td></td>
<td>Deck</td>
<td>29</td>
<td>3.78</td>
<td>2690</td>
</tr>
<tr>
<td>14PS-5S</td>
<td>Girder</td>
<td>143</td>
<td>16.16</td>
<td>5560</td>
</tr>
<tr>
<td></td>
<td>Deck</td>
<td>43</td>
<td>5.34</td>
<td>3300</td>
</tr>
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<td>18PS-5S</td>
<td>Girder</td>
<td>175</td>
<td>18.06</td>
<td>5970</td>
</tr>
<tr>
<td></td>
<td>Deck</td>
<td>67</td>
<td>3.99</td>
<td>2660</td>
</tr>
<tr>
<td>10PS-1S</td>
<td>Girder</td>
<td>189</td>
<td>13.19</td>
<td>5630</td>
</tr>
<tr>
<td></td>
<td>Deck</td>
<td>77</td>
<td>5.04</td>
<td>2770</td>
</tr>
<tr>
<td>14PS-1S</td>
<td>Girder</td>
<td>184</td>
<td>15.53</td>
<td>5440</td>
</tr>
<tr>
<td></td>
<td>Deck</td>
<td>70</td>
<td>5.04</td>
<td>2770</td>
</tr>
<tr>
<td>18PS-1S</td>
<td>Girder</td>
<td>199</td>
<td>14.49</td>
<td>5150</td>
</tr>
<tr>
<td></td>
<td>Deck</td>
<td>84</td>
<td>5.04</td>
<td>2770</td>
</tr>
<tr>
<td>10PS-N</td>
<td>Girder</td>
<td>222</td>
<td>11.81</td>
<td>5540</td>
</tr>
<tr>
<td>14PS-N</td>
<td>Girder</td>
<td>228</td>
<td>15.66</td>
<td>5330</td>
</tr>
<tr>
<td>18PS-N</td>
<td>Girder</td>
<td>232</td>
<td>18.11</td>
<td>6020</td>
</tr>
</tbody>
</table>

E.4.2 Load-Deflection Responses

The load-deflection responses at the mid-span of the three composite girders each having 5 ft. (1.5 m) deck slab are presented in Figure E2. The response indicates that prior to cracking, the initial flexural stiffness of the composite girders was practically the same and not affected by the compressive strength of concrete, since there were only small differences in the elastic modulus of the three different concretes.

The load-deflection curves of the three composite girders with 1 ft. (0.3 m) wide deck slab are shown in Figure E3. Similar trend was observed prior to the initiation of cracks. The response reflects a small drop in load carrying capacity near failure due to complete crushing of the entire depth of the NSC concrete deck slab followed by crushing of a portion of the HSC flange of the AASHTO girder.
Figure E2 – Load-deflection responses for the composite girders each having 5 ft. (1.5 m) deck

Figure E3 – Load-deflection responses for the composite girders each having 1 ft. (0.3 m) deck
The load-deflection curves of the three girders without a deck slab are shown in Figure E4. Similar behavior was observed except that the failure modes for the three HSC girders were more brittle relative to the girders with NSC deck.

![Graph showing load-deflection responses for the girders without deck](image)

**Figure E4 – Load-deflection responses for the girders without deck**

Based on the load-deflection response for each specimen, the observed cracking and ultimate moments are given in Table E3.

**Table E3 – Test results for cracking moment and ultimate moment**

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Observed Cracking Moment (kip-ft.)</th>
<th>Observed Ultimate Moment (kip-ft.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10PS-5S</td>
<td>1097</td>
<td>2123</td>
</tr>
<tr>
<td>14PS-5S</td>
<td>1267</td>
<td>2349</td>
</tr>
<tr>
<td>18PS-5S</td>
<td>1377</td>
<td>2543</td>
</tr>
<tr>
<td>10PS-1S</td>
<td>935</td>
<td>1752</td>
</tr>
<tr>
<td>14PS-1S</td>
<td>1054</td>
<td>1941</td>
</tr>
<tr>
<td>18PS-1S</td>
<td>1131</td>
<td>2083</td>
</tr>
<tr>
<td>10PS-N</td>
<td>799</td>
<td>1465</td>
</tr>
<tr>
<td>14PS-N</td>
<td>867</td>
<td>1688</td>
</tr>
<tr>
<td>18PS-N</td>
<td>918</td>
<td>1808</td>
</tr>
</tbody>
</table>
E.4.3 Failure Modes

Failure of the girders with 5 ft. (1.5 m) deck slab occurred gradually and was due to crushing of the concrete within the NSC deck slab as shown in Figure E5 (a). On the other hand, failure of the girders with 1 ft. (0.3 m) deck slab occurred suddenly after crushing of the deck slab followed by crushing of the top flange of the HSC girder. Buckling of the longitudinal reinforcement and prestressing strand in the compression zone was also observed, as shown in Figure E5 (b).

![Figure E5](image)

a) AASHTO girders each having 5 ft. (1.5 m) deck slab

b) AASHTO girders with 1 ft. (0.3 m) deck slab

c) AASHTO girders without any deck slab

Figure E5 – Typical failure modes

For the girders without a deck slab, failure also occurred suddenly followed by the buckling of prestressing strands in the compression zone, as shown in Figure E5 (c). In the latter
two cases, the sudden crushing of the compression zone also led to immediate crushing of concrete in the web area.

E.4.4 Ultimate Compressive Strain of Concrete

The maximum strain of concrete measured at ultimate load is shown in Figure E6. The strains were based on average readings of 5 strain gages installed at the top surface of the concrete in the compression zone. The results indicate that the ultimate strains of girders with NSC deck slab exceeded 0.003 by a substantial margin since the concrete strengths was only in the range of 4 to 6 ksi (28 to 41 MPa). The ultimate strains of the girders without deck slab were close to 0.003, which indicates that the usual limiting strain of 0.003 for design purposes is valid for HSC up to 18 ksi (124 MPa).

![Figure E6 – Ultimate strain for each specimen at failure](image)

Figure E6 – Ultimate strain for each specimen at failure
E.4.5 Elastic Modulus

The elastic modulus obtained from the control cylinders of the nine prestressed AASHTO girders tested in this project is shown in Figure E7. The predicted values using the LRFD Specifications (2004) as well as the prediction according to the proposed equation in Appendix A are shown in the same figure, using a unit weight of 149pcf (2,387 kg/m$^3$) for concrete. The results indicate that the predictive equation in the LRFD Specifications (2004) overestimates the elastic modulus, while the proposed equation provides a closer prediction.

![Figure E7 – Compressive strength vs. elastic modulus](image)

E 4.6 Modulus of Rupture

The modulus of rupture obtained from the control specimens for the nine tested girders are shown in Figure E8. Two expressions for modulus of rupture as given by the current LRFD Specifications (2004) are also plotted in this figure. It should be noted that the LRFD Specifications (2004) give two modulus rupture values; one ($f_r = 0.24\sqrt{f'_c} (ksi)$) used for
computing cracking moment under service limit load combination, the other \( f_r = 0.37\sqrt{f'_c \text{ (ksi)}} \), used for determining minimum reinforcement.

![Modulus of rupture vs. compressive strength](image)

**Figure E8 – Modulus of rupture vs. compressive strength**

Test results suggest that the current lower bound of the LRFD Specifications (2004) overestimates the modulus of rupture for HSC. A better predictive equation, \( f_r = 0.19\sqrt{f'_c \text{ (ksi)}} \) (\( f_r = 0.5\sqrt{f'_c \text{ (MPa)}} \)), is proposed for HSC up to 18 ksi (124 MPa).

**E.4.7 Transfer Length**

To determine the transfer length, the end slippages of six pre-selected strands were measured using tape measurement before and after transfer. The following equation (Oh and Kim 2000) was used to determine the transfer length from end slippage measurements:
\[ l_t = \frac{2E_p \delta}{f_{pi}} \]  

Equation E1

where \( l_t \) = transfer length, \( E_p \) = modulus of elasticity of strand, \( \delta \) = strand end slippage, and \( f_{pi} \) = initial prestress of the strand just before de-tensioning.

The calculated transfer lengths of the tested girders are given in Table E4. The range of the measured transfer length varied from 21 to 34 in. (525 to 850 mm) depending on the compressive strength of concrete. These transfer length values indicate that the predicted value of 30 in. (750 mm) for ½ in. (13 mm) diameter strand by the LRFD Specifications (2004) is reasonable for the purpose of design. Accordingly, the current LRFD Specifications (2004) is considered to be appropriate for concrete strengths up to 18 ksi (124 MPa).

<table>
<thead>
<tr>
<th>Specimen</th>
<th>( \delta ) (in.)</th>
<th>( l_t ) (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>18PS-1S</td>
<td>0.10</td>
<td>29</td>
</tr>
<tr>
<td>18PS-5S</td>
<td>0.08</td>
<td>23</td>
</tr>
<tr>
<td>18PS-N</td>
<td>0.04</td>
<td>12</td>
</tr>
<tr>
<td>Average</td>
<td>0.07</td>
<td>21.3</td>
</tr>
<tr>
<td>14PS-1S</td>
<td>0.08</td>
<td>22</td>
</tr>
<tr>
<td>14PS-5S</td>
<td>0.10</td>
<td>29</td>
</tr>
<tr>
<td>14PS-N</td>
<td>0.13</td>
<td>36</td>
</tr>
<tr>
<td>Average</td>
<td>0.10</td>
<td>29</td>
</tr>
<tr>
<td>10PS-1S</td>
<td>0.10</td>
<td>30</td>
</tr>
<tr>
<td>10PS-5S</td>
<td>0.20</td>
<td>58</td>
</tr>
<tr>
<td>10PS-N</td>
<td>0.05</td>
<td>15</td>
</tr>
<tr>
<td>Average</td>
<td>0.12</td>
<td>34.3</td>
</tr>
</tbody>
</table>

E.4.8 Prestress Losses

The measured and calculated loss due to elastic shortening according to the LRFD Specifications (2004) are summarized in Table E5. The calculated values were based on two different equations for elastic modulus; the equation specified by the current LRFD Specifications (2004) and the proposed equation in Appendix A. Table E5 indicates that the
average loss due to elastic shortening in the bottom level strands was 7.6 percent, which is very close to the predicted values, using the current LRFD Specifications (2004) as well as the proposed equation.

Table E5 – Losses due to elastic shortening in the bottom level strands

<table>
<thead>
<tr>
<th>Identification</th>
<th>Measured Losses (%)</th>
<th>Calculated Losses</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>LRFD Eqn. (%)</td>
<td>Proposed Eqn. (%)</td>
</tr>
<tr>
<td>18PS-1S</td>
<td>9.5</td>
<td>7.9</td>
<td>7.9</td>
</tr>
<tr>
<td>18PS-5S</td>
<td>5.0</td>
<td>7.8</td>
<td>7.9</td>
</tr>
<tr>
<td>18PS-N</td>
<td>7.0</td>
<td>7.6</td>
<td>7.7</td>
</tr>
<tr>
<td>14PS-1S</td>
<td>8.0</td>
<td>7.7</td>
<td>7.5</td>
</tr>
<tr>
<td>14PS-5S</td>
<td>7.0</td>
<td>8.2</td>
<td>7.9</td>
</tr>
<tr>
<td>14PS-N</td>
<td>8.5</td>
<td>8.2</td>
<td>7.9</td>
</tr>
<tr>
<td>10PS-1S</td>
<td>8.5</td>
<td>6.8</td>
<td>6.6</td>
</tr>
<tr>
<td>10PS-5S</td>
<td>8.0</td>
<td>6.8</td>
<td>6.6</td>
</tr>
<tr>
<td>10PS-N</td>
<td>7.0</td>
<td>6.7</td>
<td>6.6</td>
</tr>
<tr>
<td>Average</td>
<td>7.61</td>
<td>7.5</td>
<td>7.4</td>
</tr>
</tbody>
</table>

All girders were initially loaded up to cracking and unloaded. Each girder was loaded again to the load level that caused the cracks to re-open. Once this cracking load was established, then the effective prestressing force \( P_e \), was determined by using the following equation:

\[
0 = -\frac{P_e}{A} - \frac{P_e \cdot e \cdot y_b}{I_g} + \frac{M_{\text{dead}} \cdot y_b}{I_g} - f_y + \frac{M_{\text{cr}} \cdot y_{bc}}{I_c}
\]

Equation E2

By subtracting the effective prestressing force \( P_e \) from the initial prestressing force the total prestress loss was determined for each of the tested girders. These prestress losses, at the time of testing are summarized and compared to the calculated values in Table E6. It can be seen that the predicted losses using both the LRFD Specifications (2004) and proposed equations are slightly higher than the actual losses, but on the conservative side. Therefore, the current LRFD Specifications (2004) equation for predicting the prestress losses can be considered acceptable for HSC up to 18 ksi (124 MPa).
Table E6 – Summary of prestress losses

<table>
<thead>
<tr>
<th>Identification</th>
<th>Actual Prestress Losses (Using measured $M_{cr}$ and $f_r$) (%)</th>
<th>Computed Total Prestress Losses</th>
<th>LRFD Eqn. (%)</th>
<th>Proposed Eqn. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10PS-5S</td>
<td>12.9</td>
<td>13.9</td>
<td>13.7</td>
<td></td>
</tr>
<tr>
<td>14PS-5S</td>
<td>11.2</td>
<td>15.3</td>
<td>15.0</td>
<td></td>
</tr>
<tr>
<td>18PS-5S</td>
<td>12.9</td>
<td>14.2</td>
<td>14.2</td>
<td></td>
</tr>
<tr>
<td>10PS-1S</td>
<td>13.9</td>
<td>15.1</td>
<td>14.8</td>
<td></td>
</tr>
<tr>
<td>14PS-1S</td>
<td>10.8</td>
<td>15.8</td>
<td>15.6</td>
<td></td>
</tr>
<tr>
<td>18PS-1S</td>
<td>11.3</td>
<td>15.0</td>
<td>15.1</td>
<td></td>
</tr>
<tr>
<td>10PS-N</td>
<td>8.3</td>
<td>14.9</td>
<td>14.7</td>
<td></td>
</tr>
<tr>
<td>14PS-N</td>
<td>7.3</td>
<td>17.6</td>
<td>17.2</td>
<td></td>
</tr>
<tr>
<td>18PS-N</td>
<td>10.1</td>
<td>14.1</td>
<td>14.3</td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>11.0</td>
<td>15.1</td>
<td>14.9</td>
<td></td>
</tr>
<tr>
<td>Standard Dev.</td>
<td>0.022</td>
<td>0.01</td>
<td>0.01</td>
<td></td>
</tr>
</tbody>
</table>

E.4.9 Cracking Moment

In Table E7, the measured cracking moment of the nine girders tested in this project are compared to the calculated values using the following the LRFD Specifications (2004) equation:

$$M_{cr} = S_{bc} (f_r + f_{ce} - f_{d/nc})$$

Equation E3

where $S_{bc}$ is composite section modulus, $f_r$ is modulus of rupture, $f_{ce}$ is compressive stress due to effective prestress only at the bottom fibers, and $f_{d/nc}$ is stress due to non-composite dead loads at the same load level.

Two different values were used for $f_r$ in predicting the cracking moment, are being specified by the current LRFD Specifications (2004) and the other being proposed in Appendix A. It should be noted that the predicted cracking moment is highly dependent on the value of modulus of rupture. The results shown in the table indicate that for all the tested girders, the predicted cracking moment using the proposed modulus of rupture produced conservative results. Therefore, the proposed modulus of rupture is more appropriate for use in determining cracking moment of prestressed HSC girders.
Table E7 – Summary of measured and predicted cracking moments

<table>
<thead>
<tr>
<th>I.D.</th>
<th>Measured (kip-ft.)</th>
<th>Predicted Cracking Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>f_r (LRFD)</td>
</tr>
<tr>
<td>10PS-5S</td>
<td>1097</td>
<td>1123</td>
</tr>
<tr>
<td>14PS-5S</td>
<td>1267</td>
<td>1314</td>
</tr>
<tr>
<td>18PS-5S</td>
<td>1377</td>
<td>1436</td>
</tr>
<tr>
<td>10PS-1S</td>
<td>935</td>
<td>974</td>
</tr>
<tr>
<td>14PS-1S</td>
<td>1054</td>
<td>1084</td>
</tr>
<tr>
<td>18PS-1S</td>
<td>1131</td>
<td>1183</td>
</tr>
<tr>
<td>10PS-N</td>
<td>799</td>
<td>751</td>
</tr>
<tr>
<td>14PS-N</td>
<td>867</td>
<td>843</td>
</tr>
<tr>
<td>18PS-N</td>
<td>918</td>
<td>964</td>
</tr>
</tbody>
</table>

E.4.10 Ultimate Moment

The ultimate moments of the nine tested girders were calculated in three different approaches. In the first approach, the LRFD Specifications (2004) Equation 5.7.3.2.2.1 was followed. Modeling of the concrete in the compression zone was based on the current values of α_i and β_i. In the second approach, the modeling of the concrete in the compression zone was based on the proposed relationships for α_i and β_i (Appendix B). In the third approach, the more exact method based on the strain compatibility and force equilibrium was used along with the concrete compressive stress distribution obtained from tests of control cylinders.

The stress distribution for the composite girder with a 5 ft. (1.5 m) wide deck is shown in Figure E9. The flexural resistance of the composite girder with flanged sections depends on whether the neutral axis is located in the flange or in the girder. Since the neutral axis depth, c, is located in the deck concrete, the composite girder would behave as a rectangular section. The stress block parameters for computing the flexural strength of the composite AASHTO girders can be determined by using the current LRFD Specifications (2004).
Table E8 shows the comparisons between the measured ultimate moments of the three girders each having 5 ft. (1.5 m) deck slab and the predicted values using the three approaches mentioned above. The comparisons indicate that the current LRFD Specifications (2004) can be used to predict the ultimate moment capacity when the compression zone is in the NSC deck slab.

Table E8 – Ultimate moment capacity of girders each having 5 ft. (1.5 m) deck slab

<table>
<thead>
<tr>
<th>I.D.</th>
<th>Experiment</th>
<th>Ultimate moment computed with</th>
<th>Strain compatibility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\alpha_1$ and $\beta_1$ (LRFD)</td>
<td>$\alpha_1$ and $\beta_1$ (Recommendation)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(kip-ft.)</td>
<td>Exp./Pre.</td>
</tr>
<tr>
<td>10PS-5S</td>
<td>2123</td>
<td>1904</td>
<td>1.12</td>
</tr>
<tr>
<td>14PS-5S</td>
<td>2349</td>
<td>2181</td>
<td>1.08</td>
</tr>
<tr>
<td>18PS-5S</td>
<td>2543</td>
<td>2344</td>
<td>1.08</td>
</tr>
</tbody>
</table>

The current LRFD Specifications (2004) do not provide clear recommendations on how to determine the flexural strength of a section where its compression zone includes two different
concrete compressive strengths. Since the neutral axis is located below the deck, as shown in Figure E10 (b), the compression zone require two different concrete stress-strain distributions as shown in Figure E10 (c). However, for simplicity, the stress distribution in the compression zone may be assumed conservatively by using the stress-strain relationship of NSC as shown in Figure E10 (d). The equivalent rectangular stress block is shown in Figure E10 (e).

![Figure E10](image)

Figure E10 – Design approach: (a) cross-section; (b) strain compatibility; (c) measured stress-strain distribution in the compression zone; (d) modified stress-strain distribution; (e) equivalent rectangular stress block

The computed flexural strength using the recommended method are approximately 12 to 14 percent less than the measured flexural resistance as shown in Table E9. These results confirm that the recommended method to determine the nominal flexural resistance, $M_n$, is reasonably conservative, yet still accurate. Similarly, the predicted nominal flexural resistance based on the strain compatibility with the measured material properties shows more accurate results within a ± 1 percent difference of the measured flexural resistance.
Table E9 – Ultimate moment capacity for three girders each having 1 ft. (0.3 m) deck slab

<table>
<thead>
<tr>
<th>I.D.</th>
<th>Experiment (kip-ft.)</th>
<th>Ultimate flexural moment computed with Recommendation (kip-ft.)</th>
<th>Strain compatibility Exp./Pre.</th>
</tr>
</thead>
<tbody>
<tr>
<td>10PS-1S</td>
<td>1752</td>
<td>1735</td>
<td>1.12</td>
</tr>
<tr>
<td>14PS-1S</td>
<td>1941</td>
<td>1928</td>
<td>1.14</td>
</tr>
<tr>
<td>18PS-1S</td>
<td>2083</td>
<td>2107</td>
<td>1.14</td>
</tr>
</tbody>
</table>

Since the current LRFD Specifications (2004) limits the use of the stress block parameter $\alpha_1$, to concrete strength of 10 ksi (69 MPa), the proposed equivalent rectangular stress block parameter for HSC up to 18 ksi (given in Appendix B), was used to determine the flexural strength for HSC girders without deck slabs as shown in Figure E11. The measured and the predicted ultimate flexural strength, the recommended $\alpha_l$ and $\beta_l$ and the measured stress-strain in the compression zone are given in Table E10, respectively. This table indicates that the proposed parameters $\alpha_l$ and $\beta_l$ can be used to predict the flexural strength of prestress girders with concrete strength up to 18 ksi (124 MPa).

Figure E11 – Design approach: (a) cross-section; (b) strain compatibility; (c) measured stress-strain distribution in the compression zone; (d) equivalent rectangular stress block
Table E10 – Ultimate moment capacity for three girders without deck slab

<table>
<thead>
<tr>
<th>I.D.</th>
<th>Experiment</th>
<th>Ultimate flexural moment computed with</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Recommendation</td>
</tr>
<tr>
<td></td>
<td>(kip-ft.)</td>
<td>(kip-ft.)</td>
</tr>
<tr>
<td>10PS-N</td>
<td>1465</td>
<td>1324</td>
</tr>
<tr>
<td>14PS-N</td>
<td>1688</td>
<td>1519</td>
</tr>
<tr>
<td>18PS-N</td>
<td>1808</td>
<td>1692</td>
</tr>
</tbody>
</table>

E.5 Conclusions

Based on the research findings, the following conclusions can be drawn:

- The current LRFD Specifications (2004) equation to evaluate the elastic modulus of concrete overestimated the measured values. The proposed equation in Appendix A provides better agreement with the measured values.

- The proposed modulus of rupture can be used to predict the cracking moment of prestressed HSC girders for concrete strengths up to 18 ksi (124 MPa).

- Based on the test results of prestressed concrete girders, the LRFD Specifications (1) may be used to determine transfer length of prestressed HSC girders with concrete compressive strength up to 18 ksi (124 MPa).

- For composite girder section in which the neutral axis is located below the deck and within the prestressed high-strength concrete girder, the nominal flexural resistance may be determined based on the concrete compressive strength of the deck (and, the $\alpha_l$ and $\beta_l$ of the deck concrete).

- For prestressed girder section with HSC, the nominal flexural strength can be determined using the LRFD Specifications (2004) procedure and the proposed relationships in Appendix B for $\alpha_l$ and $\beta_l$ for concrete strengths up to 18 ksi (124 MPa).
E.7 References


ACI Committee 318, “Building Code Requirements for Structural Concrete (ACI 318-02) and Commentary (318R-02),” American Concrete Institute, Farmington Hills, MI, 2002, 443 p.


Noguchi Laboratory Data, Department of Architecture, University of Tokyo, Japan, [http://bme.t.u-tokyo.ac.jp/index_e.html](http://bme.t.u-tokyo.ac.jp/index_e.html).


APPENDIX F – PROPOSED REVISIONS

5.1 SCOPE

The provisions in this section apply to the design of bridge and retaining wall components constructed of normal weight or lightweight concrete and reinforced with steel bars, welded wire reinforcement, and/or prestressing strands, bars, or wires. The provisions are based on concrete specified compressive strengths varying from 2.4 ksi to 10.0 ksi for both normal weight and lightweight concrete, except where higher strengths are allowed for normal weight concrete.

The provisions of this section combine and unify the requirements for reinforced, prestressed, and partially prestressed concrete. Provisions for seismic design, analysis by the strut-and-tie model, and design of segmentally constructed concrete bridges and bridges made from precast concrete elements have been added.

A brief outline for the design of some routine concrete components is contained in Appendix A.

5.3 NOTATION

\[ k_c = \frac{\text{ratio of the in-place concrete compressive strength to the specified compressive strength of concrete}}{} \]  
\[ \alpha_1 = \frac{\text{ratio of equivalent rectangular concrete compressive stress block intensity to the specified compressive strength of concrete}}{} \]

5.4 Material Properties

5.4.2.1 Compressive Strength

For each component, the specified compressive strength, \( f'_c \), or the class of concrete shall be shown in the contract documents.

Design concrete strengths above 10.0 ksi for normal weight concrete shall only be used only when allowed by specific articles or when physical tests are made to establish the relationships between the concrete strength and other properties. Specified concrete with strengths below 2.4 ksi should not be used in structural applications.

The specified compressive strength for prestressed concrete and decks shall not be less than 4.0 ksi.

For lightweight structural concrete, air dry unit weight, strength and any other properties required for the evaluation of the strength of the concrete used in the work should be based on test cylinders produced, tested, and evaluated in accordance with Section 8 of the AASHTO LRFD Bridge Construction Specifications.

It is common practice that the specified strength be attained 28 days after placement. Other maturity ages may be assumed for design and specified for components that will receive loads at times appreciably different than 28 days after placement.

It is recommended that the classes of concrete shown in Table C1 and their corresponding specified strengths be used whenever appropriate. The classes of concrete indicated in Table C1 have been developed for general applications.
application shall be specified in the contract documents. use and are included in *AASHTO LRFD Bridge Construction Specifications*, Section 8, “Concrete Structures,” from which Table C1 was taken.

These classes are intended for use as follows:

- **Class A** concrete is generally used for all elements of structures, except when another class is more appropriate, and specifically for concrete exposed to saltwater.

- **Class B** concrete is used in footings, pedestals, massive pier shafts, and gravity walls.

- **Class C** concrete is used in thin sections, such as reinforced railings less than 4.0 in. thick, for filler in steel grid floors, etc.

- **Class P** concrete is used when strengths in excess of 4.0 ksi are required. For prestressed concrete, consideration should be given to limiting the nominal aggregate size to 0.75 in.

- **Class S** concrete is used for concrete deposited underwater in cofferdams to seal out water.

Strengths above 5.0 ksi should be used only when the availability of materials for such concrete in the locale is verified.

Lightweight concrete is generally used only under conditions where weight is critical.

In the evaluation of existing structures, it may be appropriate to modify the $f'_c$ and other attendant structural properties specified for the original construction to recognize the strength gain or any strength loss due to age or deterioration after 28 days. Such modified $f'_c$ should be determined by core samples of sufficient number and size to represent the concrete in the work, tested in accordance with AASHTO T 24 (ASTM C 42).
For concrete Classes A, A(AE), and P used in or over saltwater, the W/C ratio shall be specified not to exceed 0.45.

The sum of Portland cement and other cementitious materials shall be specified not to exceed 800 pcy, except for Class P (HPC) concrete where the sum of Portland cement and other cementitious materials shall be specified not to exceed 1000 pcy.

Air-entrained concrete, designated “AE” in Table C1, shall be specified where the concrete will be subject to alternate freezing and thawing and exposure to deicing salts, saltwater, or other potentially damaging environments.

There is considerable evidence that the durability of reinforced concrete exposed to saltwater, deicing salts, or sulfates is appreciably improved if, as recommended by ACI 318, either or both the cover over the reinforcing steel is increased or the W/C ratio is limited to 0.40. If materials, with reasonable use of admixtures, will produce a workable concrete at W/C ratios lower than those listed in Table C1, the contract documents should alter the recommendations in Table C1 appropriately.

The specified strengths shown in Table C1 are generally consistent with the W/C ratios shown. However, it is possible to satisfy one without the other. Both are specified because W/C ratio is a dominant factor contributing to both durability and strength; simply obtaining the strength needed to satisfy the design assumptions may not ensure adequate durability.

Table C5.4.2.1-1 Concrete Mix Characteristics By Class.

<table>
<thead>
<tr>
<th>Class of Concrete</th>
<th>Minimum Cement Content pcy</th>
<th>Maximum W/C Ratio</th>
<th>Air Content Range</th>
<th>Coarse Aggregate Per AASHTO M 43 (ASTM D 448)</th>
<th>28-Day Compressive Strength ksi</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>611</td>
<td>0.49</td>
<td>—</td>
<td>1.0 to No. 4</td>
<td>4.0</td>
</tr>
<tr>
<td>A(AE)</td>
<td>611</td>
<td>0.45</td>
<td>6.0 ± 1.5</td>
<td>1.0 to No. 4</td>
<td>4.0</td>
</tr>
<tr>
<td>B</td>
<td>517</td>
<td>0.58</td>
<td>—</td>
<td>2.0 to No. 3 and No. 3 to No. 4</td>
<td>2.4</td>
</tr>
<tr>
<td>B(AE)</td>
<td>517</td>
<td>0.55</td>
<td>5.0 ± 1.5</td>
<td>2.0 to No. 3 and No. 3 to No. 4</td>
<td>2.4</td>
</tr>
<tr>
<td>C</td>
<td>658</td>
<td>0.49</td>
<td>—</td>
<td>0.5 to No. 4</td>
<td>4.0</td>
</tr>
<tr>
<td>C(AE)</td>
<td>658</td>
<td>0.45</td>
<td>7.0 ± 1.5</td>
<td>0.5 to No. 4</td>
<td>4.0</td>
</tr>
<tr>
<td>P</td>
<td>564</td>
<td>0.49</td>
<td>As specified elsewhere</td>
<td>1.0 to No. 4 or 0.75 to No. 4</td>
<td>As specified elsewhere</td>
</tr>
<tr>
<td>P(HPC)</td>
<td>564</td>
<td>0.58</td>
<td>—</td>
<td>1.0 to No. 4</td>
<td>—</td>
</tr>
<tr>
<td>Lightweight</td>
<td>658</td>
<td>As specified in the contract documents</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5.4.2 Normal and Structural Lightweight Concrete

5.4.2.3 Shrinkage and Creep

5.4.2.3.1 General

Values of shrinkage and creep, specified herein and in Articles 5.9.5.3 and 5.9.5.4, shall be used to determine the effects of shrinkage and creep on the loss of prestressing force in bridges other than segmentally...

Creep and shrinkage of concrete are variable properties that depend on a number of factors, some of which may not be known at the time of design.
constructed ones. These values in conjunction with the moment of inertia, as specified in Article 5.7.3.6.2, may be used to determine the effects of shrinkage and creep on deflections.

These provisions shall be applicable for normal-weight concrete with specified concrete compressive strengths up to 45.0 ksi. In the absence of more accurate data, the shrinkage coefficients may be assumed to be 0.0002 after 28 days and 0.0005 after one year of drying.

When mix-specific data are not available, estimates of shrinkage and creep may be made using the provisions of:

- Articles 5.4.2.3.2 and 5.4.2.3.3,
- The CEB-FIP model code, or
- ACI 209.

For segmentally constructed bridges, a more precise estimate shall be made, including the effect of:

- Specific materials,
- Structural dimensions,
- Site conditions, and
- Construction methods, and
- Concrete age at various stages of erection.

5.4.2.3.2 Creep

The creep coefficient may be taken as:

\[ \psi(t, t_{i}) = 1.9k_{cvi} k_{h} k_{i} f_{ci}^{-0.118} \]  
(5.4.2.3.2-1)

in which:

\[ k_{cvi} = 1.45 - 0.13(V/S) \geq 0.0 \]  
(5.4.2.3.2-2)

\[ k_{h} = 1.56 - 0.008H \]  
(5.4.2.3.2-3)

\[ k_{i} = \frac{5}{1 + \frac{f_{ci}}{f_{ci}'}} \]  
(5.4.2.3.2-4)

\[ k_{ed} = \frac{t}{61 - 4f_{ci}' + t} \]

Without specific physical tests or prior experience with the materials, the use of the empirical methods referenced in these Specifications cannot be expected to yield results with errors less than ±50 percent.
\[ k_{sd} = \frac{t}{12 \left( \frac{100 - 4 f'_{ci}}{f'_{ci} + 20} \right) + t} \quad (5.4.2.3.2-5) \]

where:

- **H** = relative humidity (%). In the absence of better information, **H** may be taken from Figure 5.4.2.3.3-1.
- **k_{vs}** = factor for the effect of the volume-to-surface ratio of the component
- **k_{f}** = factor for the effect of concrete strength
- **k_{hc}** = humidity factor for creep
- **k_{td}** = time development factor
- **t** = maturity of concrete (day), defined as age of concrete between time of loading for creep calculations, or end of curing for shrinkage calculations, and time being considered for analysis of creep or shrinkage effects
- **t_{i}** = age of concrete when load is initially applied (day)
- **V/S** = volume-to-surface ratio (in.)
- **f'_{ci}** = specified compressive strength of concrete at time of prestressing for pretensioned members and at time of initial loading for nonprestressed members. If concrete age at time of initial loading is unknown at design time, \( f'_{ci} \) may be taken as 0.80 \( f'_{c} \) (ksi).

- Maturity of the concrete at the time of loading, and
- Temperature of concrete.

Creep shortening of concrete under permanent loads is generally in the range of 0.5 to 4.0 times the initial elastic shortening, depending primarily on concrete maturity at the time of loading.

The time development of shrinkage, given by Eq. 5, is proposed to be used for both precast concrete and cast-in-place concrete components of a bridge member, and for both accelerated curing and moist curing conditions. This simplification is based on a parametric study documented in Tadros (2003) on prestress losses in high strength concrete. It was found that various time development prediction methods have virtually no impact on the final creep and shrinkage coefficients, prestress losses, or member deflections. It was also observed in that study that use of modern concrete mixtures with relatively low water/cement ratios and with high range water reducing admixtures, has caused time development of both creep and shrinkage to have similar patterns. They have a relatively rapid initial development in the first several weeks after concrete placement and a slow further growth thereafter. For calculation of intermediate values of prestress losses and deflections in cast-in-place segmental bridges constructed with the balanced cantilever method, it may be warranted to use actual test results for creep and shrinkage time development using local conditions. Final losses and deflections would be substantially unaffected whether Eq. 5 or another time-development formula is used.

It should be noted that the previous versions of the equation for \( k_{sd} \) give negative values if the specified compressive strength at the time of loading is greater than 15 ksi (Rizkalla et al. 2007).

In determining the maturity of concrete at initial loading, \( t_{i} \), one day of accelerated curing by steam or radiant heat may be taken as equal to seven days of normal curing.

The surface area used in determining the volume-to-surface ratio should include only the area that is exposed to atmospheric drying. For poorly ventilated enclosed cells, only 50 percent of the interior perimeter should be used in calculating the surface area. For pretensioned stemmed members (I-beams, T-beams, and box beams), with an average web thickness of 6.0 to 8.0 in., the value of \( k_{vs} \) may be taken as 1.00.

The factors for the effects of volume-to-surface ratio are an approximation of the following formulas:

For creep:
\( k_v = \left[ \frac{t}{26e^{0.36(V/S)} + t} \right] \left[ \frac{1.80 + 1.77e^{-0.54(V/S)}}{2.587} \right] \) (C5.4.2.3.2-1)

For shrinkage:

\( k_s = \left[ \frac{t}{26e^{0.36(V/S)} + t} \right] \left[ \frac{1064 - 94(V/S)}{923} \right] \) (C5.4.2.3.2-2)

The maximum V/S ratio considered in the development of Eqs. C1 and C2 was 6.0 in.

Ultimate creep and shrinkage are less sensitive to surface exposure than intermediate values at an early age of concrete. For accurately estimating intermediate deformations of such specialized structures as segmentally constructed balanced cantilever box girders, it may be necessary to resort to experimental data or use the more detailed Eqs. C1 and C2.

### 5.4.2.4 Modulus of Elasticity

In the absence of measured data, the modulus of elasticity, \( E_c \), for concretes with unit weights between 0.090 and 0.155 kcf and normal-weight concrete with specified compressive strengths up to 15.0 ksi, 18.0 ksi may be taken as:

\[
E_c = 33,000K_1w_c^{1.5}\sqrt{f'_c}
\]

\[
E_c = 310,000K_1w_c^{2.5}(f'_c)^{0.33}
\] (5.4.2.4-1)

where:

- \( K_1 \) = correction factor for source of aggregate to be taken as 1.0 unless determined by physical test, and as approved by the authority of jurisdiction
- \( w_c \) = unit weight of concrete (kcf); refer to Table 3.5.1-1 or Article C5.4.2.4
- \( f'_c \) = specified compressive strength of concrete (ksi)

Equation 5.4.2.4-1 may be used for normal weight concrete with specified compressive strengths up to 18.0 ksi.

### C5.4.2.4

See commentary for specified strength in Article 5.4.2.1.

For normal weight concrete with \( w_c = 0.145 \) kcf, \( E_c \) may be taken as:

\[
E_c = 1,820\sqrt{f'_c}
\]

\[
E_c = 2,480(f'_c)^{0.33}
\] (C5.4.2.4-1)

Test data show that the modulus of elasticity of concrete is influenced by the stiffness of the aggregate. The factor \( K_1 \) is included to allow the calculated modulus to be adjusted for different types of aggregate and local materials. Unless a value has been determined by physical tests, \( K_1 \) should be taken as 1.0. Use of a measured \( K_1 \) factor permits a more accurate prediction of modulus of elasticity and other values that utilize it.

This equation is based on the study by Rizkalla et al. (2007). In that study, \( K_1 \) was assumed to be equal to 1.
5.4.2.5 Poisson’s Ratio

Unless determined by physical tests, Poisson’s ratio may be assumed as 0.2 for light weight concrete with specified compressive strengths up to 10 ksi and for normal weight concrete with specified compressive strengths up to 18.0 ksi. For components expected to be subject to cracking, the effect of Poisson’s ratio may be neglected.

C5.4.2.5

This is a ratio between the lateral and axial strains of an axially and/or flexurally loaded structural element.

5.4.2.6 Modulus of Rupture

Unless determined by physical tests, the modulus of rupture, \( f' \), in ksi, for specified concrete strengths up to 15.0 ksi, may be taken as:

- For normal-weight concrete with specified compressive strengths up to 18.0 ksi:
  - When used to calculate the cracking moment of a member in Articles 5.7.3.4 and 5.7.3.6.2: \( 0.24 \sqrt{f'_c} \)
  - When used to calculate the cracking moment of a member in Article 5.7.3.3.2: \( 0.37 \sqrt{f'_c} \)

- For lightweight concrete:
  - For sand-lightweight concrete: \( 0.20 \sqrt{f'_c} \)
  - For all-lightweight concrete: \( 0.17 \sqrt{f'_c} \)

When physical tests are used to determine modulus of rupture, the tests shall be performed in accordance with AASHTO T 97 and shall be performed on concrete using the same proportions and materials as specified for the structure.

C5.4.2.6

Data show that most modulus of rupture values are between 0.24\( \sqrt{f'_c} \) and 0.37\( \sqrt{f'_c} \) (ACI 1992; Walker and Bloem 1960; Khan, Cook, and Mitchell 1996). It is appropriate to use the lower bound value when considering service load cracking. The purpose of the minimum reinforcement in Article 5.7.3.3.2 is to assure that the nominal moment capacity of the member is at least 20 percent greater than the cracking moment. Since the actual modulus of rupture could be as much as 50 percent greater than 0.24\( \sqrt{f'_c} \), the 20 percent margin of safety could be lost. Using an upper bound is more appropriate in this situation.

The properties of higher strength concretes are particularly sensitive to the constitutive materials. If test results are to be used in design, it is imperative that tests be made using concrete with not only the same mix proportions, but also the same materials as the concrete used in the structure. The given values may be unconservative for tensile cracking caused by restrained shrinkage, anchor zone splitting, and other such tensile forces caused by effects other than flexure. The direct tensile strength stress should be used for these cases.

5.7 Design of Flexural and Axial Force Effects

5.7.2 Assumptions for Strength and Extreme Event Limit States

The following assumptions may be used for normal weight concrete with specified compressive strengths up to 18.0 ksi.
5.7.2.1 General

Factored resistance of concrete components shall be based on the conditions of equilibrium and strain compatibility, the resistance factors as specified in Article 5.5.4.2, and the following assumptions:

- In components with fully bonded reinforcement or prestressing, or in the bonded length of locally debonded or shielded strands, strain is directly proportional to the distance from the neutral axis, except for deep members that shall satisfy the requirements of Article 5.13.2, and for other disturbed regions.

- In components with fully unbonded or partially unbonded prestressing tendons, i.e., not locally debonded or shielded strands, the difference in strain between the tendons and the concrete section and the effect of deflections on tendon geometry are included in the determination of the stress in the tendons.

- If the concrete is unconfined, the maximum usable strain at the extreme concrete compression fiber is not greater than 0.003.

- If the concrete is confined, a maximum usable strain exceeding 0.003 in the confined core may be utilized if verified. Calculation of the factored resistance shall consider that the concrete cover may be lost at strains compatible with those in the confined concrete core.

- Except for the strut-and-tie model, the stress in the reinforcement is based on a stress-strain curve representative of the steel or on an approved mathematical representation, including development of reinforcing and prestressing elements and transfer of pretensioning.

- The tensile strength of the concrete is neglected.

- The concrete compressive stress-strain distribution is assumed to be rectangular, parabolic, or any other shape that results in a prediction of strength in substantial agreement with the test results.

- The development of reinforcing and prestressing elements and transfer of pretensioning are considered.

C5.7.2.1

The first paragraph of C5.7.1 applies.

The results of Rizkalla et al. (2007) have shown that the maximum usable strain at the extreme concrete compression fiber of 0.003 is valid for flexural members with specified compressive strengths up to 18 ksi for normal weight concrete.

Research by Bae and Bayrak (2003) has shown that, for well-confined High Strength Concrete (HSC) columns, the concrete cover may be lost at maximum usable strains at the extreme concrete compression fiber as low as 0.0022. The heavy confinement steel causes a weak plane between the concrete core and cover, causing high shear stresses and the resulting early loss of concrete cover.

Test results from Rizkalla et al. (2007) confirm the above findings.
• Balanced strain conditions exist at a cross-section when tension reinforcement reaches the strain corresponding to its specified yield strength $f_y$ just as the concrete in compression reaches its assumed ultimate strain of 0.003.

• Sections are compression-controlled when the net tensile strain in the extreme tension steel is equal to or less than the compression-controlled strain limit at the time the concrete in compression reaches its assumed strain limit of 0.003. The compression-controlled strain limit is the net tensile strain in the reinforcement at balanced strain conditions. For Grade 60 reinforcement, and for all prestressed reinforcement, the compression-controlled strain limit may be set equal to 0.002. The nominal flexural strength of a member is reached when the strain in the extreme compression fiber reaches the assumed strain limit of 0.003. The net tensile strain $\varepsilon_t$ is the tensile strain in the extreme tension steel at nominal strength, exclusive of strains due to prestress, creep, shrinkage, and temperature. The net tensile strain in the extreme tension steel is determined from a linear strain distribution at nominal strength, as shown in Figure C5.7.2.1-1, using similar triangles.

![Figure C5.7.2.1-1 Strain Distribution and Net Tensile Strain.](image)

• Sections are tension-controlled when the net tensile strain in the extreme tension steel is equal to or greater than 0.005 just as the concrete in compression reaches its assumed strain limit of 0.003. Sections with net tensile strain in the extreme tension steel between the compression-controlled strain limit and 0.005 constitute a transition region between compression-controlled and tension-controlled sections.

• The use of compression reinforcement in conjunction with additional tension reinforcement is permitted to increase the strength of flexural members.

When the net tensile strain in the extreme tension steel is sufficiently large (equal to or greater than 0.005), the section is defined as tension-controlled where ample warning of failure with excessive deflection and cracking may be expected. When the net tensile strain in the extreme tension steel is small (less than or equal to the compression-controlled strain limit), a brittle failure condition may be expected, with little warning of impending failure. Flexural members are usually tension-controlled, while compression members are usually compression-controlled. Some sections, such as those with small axial load and large bending moment, will have net tensile strain in the extreme tension steel between the above limits. These sections are in a transition region between compression- and tension-controlled sections. Article 5.5.4.2.1 specifies the appropriate resistance factors for tension-controlled and compression-controlled sections, and for intermediate cases in the transition region.

Before the development of these provisions, the limiting tensile strain for flexural members was not stated, but was implicit in the maximum reinforcement limit that was given as $c/d_e \leq 0.42$, which corresponded to a net tensile strain at the centroid of the tension reinforcement of 0.00414. The net tensile strain limit of 0.005 for tension-controlled sections was chosen to be a single value that applies to all types of steel ( prestressed and nonprestressed) permitted by this Specification.

Unless unusual amounts of ductility are required,
the 0.005 limit will provide ductile behavior for most
designs. One condition where greater ductile behavior
is required is in design for redistribution of moments in
continuous members and frames. Article 5.7.3.5
permits redistribution of negative moments. Since
moment redistribution is dependent on adequate
ductility in hinge regions, moment redistribution is
limited to sections that have a net tensile strain of at
least 0.0075.

For beams with compression reinforcement, or T-
beams, the effects of compression reinforcement and
flanges are automatically accounted for in the
computation of net tensile strain $\varepsilon_t$.

Additional limitations on the maximum usable extreme
cement compressive strain in hollow rectangular
compression members shall be investigated as specified
in Article 5.7.4.7.

5.7.2.2  Rectangular Stress Distribution

The natural relationship between concrete stress and
strain may be considered satisfied by an equivalent
rectangular concrete compressive stress block of $0.85 \alpha f'_c$
over a zone bounded by the edges of the cross-section
and a straight line located parallel to the neutral axis at
the distance $a = \beta_1 c$ from the extreme compression
fiber. The distance $c$ shall be measured perpendicular to
the neutral axis. The factor $\alpha_1$ shall be taken as 0.85 for
specified compressive strengths not exceeding 10.0 ksi.
For specified compressive strengths exceeding 10.0 ksi,
$\alpha_1$ shall be reduced at a rate of 0.02 for each 1.0 ksi of
strength in excess of 10.0 ksi, except that $\alpha_1$ shall not be
taken to be less than 0.75. The factor $\beta_1$ shall be taken as
0.85 for concrete specified compressive strengths not
exceeding 4.0 ksi. For concrete specified compressive
strengths exceeding 4.0 ksi, $\beta_1$ shall be reduced at a rate
of 0.05 for each 1.0 ksi of strength in excess of 4.0 ksi,
except that $\beta_1$ shall not be taken to be less than 0.65.

Additional limitations on the use of the rectangular
stress block when applied to hollow rectangular
compression members shall be investigated as specified
in Article 5.7.4.7.

C5.7.2.2

For practical design, the rectangular compressive stress
distribution defined in this article may be used in lieu
of a more exact concrete stress distribution. This
rectangular stress distribution does not represent the
actual stress distribution in the compression zone at
ultimate, but in many practical cases it does provide
essentially the same results as those obtained in tests.
All strength equations presented in Article 5.7.3 are
based on the rectangular stress block.

Rizkalla et al. (2007) determined that $\alpha_1$ gradually
decreases for specified compressive strengths in excess
of 10 ksi.

The factor $\beta_1$ is basically related to rectangular
sections; however, for flanged sections in which the
neutral axis is in the web, $\beta_1$ has experimentally been
found to be an adequate approximation.

For sections that consist of a beam with a composite
slab of different concrete strength, and the
compression block includes both types of concrete, it is
conservative to assume the composite beam to be of
uniform strength at the lower of the concrete strengths
in the flange and web. If a more refined estimate of
flexural capacity is warranted, a more rigorous analysis
method should be used. Examples of such analytical
techniques are presented in Weigel, Seguirant, Brice,
and Khaleghi (2003) and Seguirant, Brice, and
For specified compressive strengths between 10 and 15 ksi, the value of $\alpha_1$ may be determined by linear interpolation, as shown in Figure C5.7.2.2-1.

![Figure C5.7.2.2-1 – Variation of $\alpha_1$ with specified compressive strength](image)

5.7.3 Flexural Members

The following assumptions may be used for normal weight concrete with specified compressive strengths up to 18.0 ksi.

5.7.3.1 Stress in Prestressing Steel at Nominal Flexural Resistance

5.7.3.1.1 Components with Bonded Tendons

For rectangular or flanged sections subjected to flexure about one axis where the approximate stress distribution specified in Article 5.7.2.2 is used and for which $f_{pe}$ is not less than 0.5 $f_{pu}$, the average stress in prestressing steel, $f_{ps}$, may be taken as:

$$f_{ps} = f_{pu} \left(1 - k \frac{c}{d_p}\right)$$  \hspace{1cm} (5.7.3.1.1-1)

Equations in this article and subsequent equations for flexural resistance are based on the assumption that the distribution of steel is such that it is reasonable to consider all of the tensile reinforcement to be lumped at the location defined by $d_t$ and all of the prestressing steel can be considered to be lumped at the location defined by $d_p$. Therefore, in the case where a significant number of prestressing elements are on the compression side of the neutral axis, it is more
in which:

$$k = 2 \left( 1.04 - \frac{f_{py}}{f_{pu}} \right)$$ \hspace{1cm} (5.7.3.1.1-2)

for T-section behavior:

$$c = \frac{A_p f_{pu} + A_s f_y - A'_s f'_y - 0.85 \alpha_1 f'_y (b - b_w) h_f}{0.85 \alpha_1 f'_y b_w + k A_p \frac{f_{pu}}{d_p}}$$ \hspace{1cm} (5.7.3.1.1-3)

for rectangular section behavior:

$$c = \frac{A_p f_{pu} + A_s f_y - A'_s f'_y}{0.85 \alpha_1 f'_y b_w + k A_p \frac{f_{pu}}{d_p}}$$ \hspace{1cm} (5.7.3.1.1-4)

appropriate to use a method based on the conditions of equilibrium and strain compatibility as indicated in Article 5.7.2.1.

The background and basis for Eqs. 1 and 5.7.3.1.2-1 can be found in Naaman (1985), Loov (1988), Naaman (1989), and Naaman (1990-1992).

Values of $f_{py}/f_{pu}$ are defined in Table C1. Therefore, the values of $k$ from Eq. 2 depend only on the type of tendon used.

<table>
<thead>
<tr>
<th>Type of Tendon</th>
<th>$f_{py}/f_{pu}$</th>
<th>Value of $k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low relaxation strand</td>
<td>0.90</td>
<td>0.28</td>
</tr>
<tr>
<td>Stress-relieved strand and Type 1 high-strength bar</td>
<td>0.85</td>
<td>0.38</td>
</tr>
<tr>
<td>Type 2 high-strength bar</td>
<td>0.80</td>
<td>0.48</td>
</tr>
</tbody>
</table>

where:

- $A_p$ = area of prestressing steel (in.$^2$)
- $f_{pu}$ = specified tensile strength of prestressing steel (ksi)
- $f_{py}$ = yield strength of prestressing steel (ksi)
- $A_s$ = area of mild steel tension reinforcement (in.$^2$)
- $A'_s$ = area of compression reinforcement (in.$^2$)
- $f_y$ = yield strength of tension reinforcement (ksi)
- $f'_y$ = yield strength of compression reinforcement (ksi)
- $b$ = width of compression flange (in.)
- $b_w$ = width of web (in.)
- $h_f$ = depth of compression flange (in.)
- $d_p$ = distance from extreme compression fiber to the centroid of the prestressing tendons (in.)
- $c$ = distance between the neutral axis and the compressive face (in.)
The stress level in the compressive reinforcement shall be investigated, and if the compressive reinforcement has not yielded, the actual stress shall be used in Eq. 3 instead of $f_y'$.

### 5.7.3.1.2 Components with Unbonded Tendons

For rectangular or flanged sections subjected to flexure about one axis and for biaxial flexure with axial load as specified in Article 5.7.4.5, where the approximate stress distribution specified in Article 5.7.2.2 is used, the average stress in unbonded prestressing steel may be taken as:

$$f_{ps} = f_{pe} + 900 \left( \frac{d_p - c}{\ell_e} \right) \leq f_{py} \quad (5.7.3.1.2-1)$$

in which:

$$\ell_e = \left( \frac{2 \ell_{ce}}{2 + N_s} \right) \quad (5.7.3.1.2-2)$$

for T-section behavior:

$$c = \frac{A_{ps} f_{ps} + A_{c} f_{y} - A'_{c} f_y' - 0.85 \alpha_i f_y' (b - b_e) h_c}{0.85 \alpha_i f_y' \beta_i b_e} \quad (5.7.3.1.2-3)$$

for rectangular section behavior:

$$c = \frac{A_{ps} f_{ps} + A_{c} f_{y} - A'_{c} f_y'}{0.85 \alpha_i f_y' \beta_i b_e} \quad (5.7.3.1.2-4)$$

where:

- $c$ = distance from extreme compression fiber to the neutral axis assuming the tendon prestressing steel has yielded, given by Eqs. 3 and 4 for T-section behavior and rectangular section behavior, respectively (in.)
- $\ell_e$ = effective tendon length (in.)

A first estimate of the average stress in unbonded prestressing steel may be made as:

$$f_{ps} = f_{pe} + 15.0 \text{ (ksi)} \quad (C5.7.3.1.2-1)$$

In order to solve for the value of $f_{ps}$ in Eq. 1, the equation of force equilibrium at ultimate is needed. Thus, two equations with two unknowns ($f_{ps}$ and $c$) need to be solved simultaneously to achieve a closed-form solution.
\[ \ell_i = \text{length of tendon between anchorages (in.)} \]
\[ N_s = \text{number of support hinges crossed by the tendon between anchorages or discretely bonded points} \]
\[ f_{py} = \text{yield strength of prestressing steel (ksi)} \]
\[ f_{pe} = \text{effective stress in prestressing steel at section under consideration after all losses (ksi)} \]

The stress level in the compressive reinforcement shall be investigated, and if the compressive reinforcement has not yielded, the actual stress shall be used in Eq. 3 instead of \( f' \).

### 5.7.3.2 Flexural Resistance

#### 5.7.3.2.1 Factored Flexural Resistance

The factored resistance \( M_r \) shall be taken as:

\[
M_r = \phi M_n \tag{5.7.3.2.1-1}
\]

where:

\[ M_n = \text{nominal resistance (kip-in.)} \]
\[ \phi = \text{resistance factor as specified in Article 5.5.4.2} \]

#### 5.7.3.2.2 Flanged Sections

For flanged sections subjected to flexure about one axis and for biaxial flexure with axial load as specified in Article 5.7.4.5, where the approximate stress distribution specified in Article 5.7.2.2 is used and where the compression flange depth is less than \( a = \beta_1 c \), as determined in accordance with Eqs. 5.7.3.1.1-3, 5.7.3.1.1-4, 5.7.3.1.2-3, or 5.7.3.1.2-4, the nominal flexural resistance may be taken as:

\[
M_r = A_{ps} f_{pe} \left( d - \frac{a}{2} \right) + A f_y \left( d - \frac{a}{2} \right) - A' f_y' \left( d' - \frac{a}{2} \right) + 0.85 \alpha f_y' \left( b - b_t \right) h_t \left( \frac{a}{2} - \frac{h_t}{2} \right) \tag{5.7.3.2.2-1}
\]

where:

\[ A_{ps} = \text{area of prestressing steel (in.}^2) \]

Moment at the face of the support may be used for design. Where fillets making an angle of 45° or more with the axis of a continuous or restrained member are built monolithic with the member and support, the face of support should be considered at a section where the combined depth of the member and fillet is at least one and one-half times the thickness of the member. No portion of a fillet should be considered as adding to the effective depth when determining the nominal resistance.

In previous editions and interims of the LRFD Specifications, the factor \( \beta_1 \) was applied to the flange overhang term of Eqs. 1, 5.7.3.1.1-3, and 5.7.3.1.2-3. This was not consistent with the original derivation of the equivalent rectangular stress block as it applies to flanged sections (Mattock, Kriz, and Hognestad 1961). For the current LRFD Specifications, the \( \beta_1 \) factor has been removed from the flange overhang term of these equations. See also Seguirant (2002), Girgis, Sun, and Tadros (2002), Naaman (2002), Weigel, Seguirant, Brice, and Khaleghi (2003), Baran, Schultz, and French (2004), and Seguirant, Brice, and Khaleghi (2004).
\[ f_{ps} = \text{average stress in prestressing steel at nominal bending resistance specified in Eq. 5.7.3.1.1-1 (ksi)} \]

\[ d_p = \text{distance from extreme compression fiber to the centroid of prestressing tendons (in.)} \]

\[ A_s = \text{area of nonprestressed tension reinforcement (in.}^2\text{)} \]

\[ f_y = \text{specified yield strength of reinforcing bars (ksi)} \]

\[ d_s = \text{distance from extreme compression fiber to the centroid of nonprestressed tensile reinforcement (in.)} \]

\[ A'_s = \text{area of compression reinforcement (in.}^2\text{)} \]

\[ f'_y = \text{specified yield strength of compression reinforcement (ksi)} \]

\[ d'_s = \text{distance from extreme compression fiber to the centroid of compression reinforcement (in.)} \]

\[ f'_c = \text{specified compressive strength of concrete at 28 days, unless another age is specified (ksi)} \]

\[ b = \text{width of the compression face of the member (in.)} \]

\[ b_w = \text{web width or diameter of a circular section (in.)} \]

\[ \alpha_{f} = \text{stress block factor specified in Article 5.7.2.2} \]

\[ \beta_{l} = \text{stress block factor specified in Article 5.7.2.2} \]

\[ h_f = \text{compression flange depth of an I or T member (in.)} \]

\[ a = \text{depth of the equivalent stress block (in.)} \]

### 5.7.3.2.3 Rectangular Sections

For rectangular sections subjected to flexure about one axis and for biaxial flexure with axial load as specified in Article 5.7.4.5, where the approximate stress distribution specified in Article 5.7.2.2 is used and where the compression flange depth is not less than \( a = \beta_{l} c \) as determined in accordance with Eqs. 5.7.3.1.1-4 or 5.7.3.1.2-4, the nominal flexural resistance \( M_n \) may be determined by using Eqs. 5.7.3.1.1-1 through 5.7.3.2.2-1, in which case \( b_w \) shall be taken as \( b \).
5.7.3.2.4 Other Cross-Sections

For cross-sections other than flanged or essentially rectangular sections with vertical axis of symmetry or for sections subjected to biaxial flexure without axial load, the nominal flexural resistance, $M_n$, shall be determined by an analysis based on the assumptions specified in Article 5.7.2. The requirements of Article 5.7.3.3 shall apply.

5.7.3.2.5 Strain Compatibility Approach

Alternatively, the strain compatibility approach may be used if more precise calculations are required. The appropriate provisions of Article 5.7.2.1 shall apply.

The stress and corresponding strain in any given layer of reinforcement may be taken from any representative stress-strain formula or graph for mild reinforcement and prestressing strands.

5.7.3.2.6 Composite Girder Section

For composite girder section in which the neutral axis is located below the deck and within the prestressed high-strength concrete girder, the nominal flexural resistance, $M_n$, may be determined by Eqs. 5.7.3.2.2-1, based on the concrete compressive strength of the deck.

Test results from Rizkalla et al. (2007) show that the use of lower concrete compressive strength of the deck provides sufficiently accurate yet conservative estimate of the nominal flexural resistance, in lieu of detailed analysis with two different specified compressive strengths in the compression zone.
5.7.3.3 Limits for Reinforcement

5.7.3.3.1 Maximum Reinforcement

In editions of and interims to the LRFD Specifications prior to 2005, Article 5.7.3.3.1 limited the tension reinforcement quantity to a maximum amount such that the ratio $c/d_e$ did not exceed 0.42. Sections with $c/d_e > 0.42$ were considered over-reinforced. Over-reinforced nonprestressed members were not allowed, whereas prestressed and partially prestressed members with PPR greater than 50 percent were if “it is shown by analysis and experimentation that sufficient ductility of the structure can be achieved.” No guidance was given for what “sufficient ductility” should be, and it was not clear what value of $\phi$ should be used for such over-reinforced members. The current provisions of LRFD eliminate this limit and unify the design of prestressed and nonprestressed tension- and compression-controlled members. The background and basis for these provisions are given in Mast (1992). Below a net tensile strain in the extreme tension steel of 0.005, as the tension reinforcement quantity increases, the factored resistance of prestressed and nonprestressed sections is reduced in accordance with Article 5.5.4.2.1. This reduction compensates for decreasing ductility with increasing overstrength. Only the addition of compression reinforcement in conjunction with additional tension reinforcement can result in an increase in the factored flexural resistance of the section.

5.7.3.3.2 Minimum Reinforcement

Unless otherwise specified, at any section of a flexural component, the amount of prestressed and nonprestressed tensile reinforcement shall be adequate to develop a factored flexural resistance, $M_r$, at least equal to the lesser of:
• 1.2 times the cracking moment, $M_{cr}$, determined on the basis of elastic stress distribution and the modulus of rupture, $f_r$, of the concrete as specified in Article 5.4.2.6, where $M_{cr}$ may be taken as:

$$M_{cr} = S_c (f_r + f_{cpe}) - M_{dnc} \left( \frac{S_c}{S_{nc}} - 1 \right) \geq S_c f_r$$

where:

$f_{cpe}$ = compressive stress in concrete due to effective prestress forces only (after allowance for all prestress losses) at extreme fiber of section where tensile stress is caused by externally applied loads (ksi)

$M_{dnc}$ = total unfactored dead load moment acting on the monolithic or noncomposite section (kip-ft.)

$S_c$ = section modulus for the extreme fiber of the composite section where tensile stress is caused by externally applied loads (in.$^3$)

$S_{nc}$ = section modulus for the extreme fiber of the monolithic or noncomposite section where tensile stress is caused by externally applied loads (in.$^3$)

Appropriate values for $M_{dnc}$ and $S_{nc}$ shall be used for any intermediate composite sections. Where the beams are designed for the monolithic or noncomposite section to resist all loads, substitute $S_{nc}$ for $S_c$ in the above equation for the calculation of $M_{cr}$.

• 1.33 times the factored moment required by the applicable strength load combinations specified in Table 3.4.1-1.

The provisions of Article 5.10.8 shall apply.
5.7.3.4 Control of Cracking by Distribution of Reinforcement

The provisions specified herein shall apply to the reinforcement of all concrete components, except that of deck slabs designed in accordance with Article 9.7.2, in which tension in the cross-section exceeds 80 percent of the modulus of rupture, specified in Article 5.4.2.6, at applicable service limit state load combination specified in Table 3.4.1-1.

All reinforced concrete members are subject to cracking under any load condition, including thermal effects and restraint of deformations, which produces tension in the gross section in excess of the cracking strength of the concrete. Locations particularly vulnerable to cracking include those where there is an abrupt change in section and intermediate post-tensioning anchorage zones.

Provisions specified, herein, are used for the distribution of tension reinforcement to control flexural cracking.

Crack width is inherently subject to wide scatter, even in careful laboratory work, and is influenced by shrinkage and other time-dependent effects. Steps should be taken in detailing of the reinforcement to control cracking. From the standpoint of appearance, many fine cracks are preferable to a few wide cracks. Improved crack control is obtained when the steel reinforcement is well distributed over the zone of maximum concrete tension. Several bars at moderate spacing are more effective in controlling cracking than one or two larger bars of equivalent area.

The spacing $s$ of mild steel reinforcement in the layer closest to the tension face shall satisfy the following:

$$s \leq \frac{700\gamma_e}{\beta_s f_s} - 2d_c$$

(5.7.3.4-1)

in which:

$$\beta_s = 1 + \frac{d_c}{0.7(h - d_c)}$$

where:

- $\gamma_e = $ exposure factor
  - 1.00 for Class 1 exposure condition
  - 0.75 for Class 2 exposure condition
- $d_c = $ thickness of concrete cover measured from extreme tension fiber to center of the flexural reinforcement located closest thereto (in.)
- $f_s = $ tensile stress in steel reinforcement at the service limit state (ksi)
- $h = $ overall thickness or depth of the component (in.)

Extensive laboratory work involving deformed reinforcing bars has confirmed that the crack width at the service limit state is proportional to steel stress. However, the significant variables reflecting steel detailing were found to be the thickness of concrete cover and spacing of the reinforcement.

Eq. 1 is expected to provide a distribution of reinforcement that will control flexural cracking. The equation is based on a physical crack model (Frosch 2001) rather than the statistically-based model used in previous editions of the specifications. It is written in a form emphasizing reinforcement details, i.e., limiting bar spacing, rather than crack width. Furthermore, the physical crack model has been shown to provide a more realistic estimate of crack widths for larger concrete covers compared to the previous equation (Destefano 2003).

Eq. 1 with Class 1 exposure condition is based on an assumed crack width of 0.017 in. Previous research indicates that there appears to be little or no correlation between crack width and corrosion, however, the different classes of exposure conditions have been so defined in order to provide flexibility in the application of these provisions to meet the needs of the Authority having jurisdiction. Class 1 exposure condition could be thought of as an upper bound in regards to crack width for appearance and corrosion. Areas that the Authority having jurisdiction may consider for Class 2...
exposure condition would include decks and substructures exposed to water. The crack width is directly proportional to the \( \gamma_e \) exposure factor, therefore, if the individual Authority with jurisdiction desires an alternate crack width, the \( \gamma_e \) factor can be adjusted directly. For example a \( \gamma_e \) factor of 0.5 will result in an approximate crack width of 0.0085 in.

Where members are exposed to aggressive exposure or corrosive environments, additional protection beyond that provided by satisfying Eq. 1 may be provided by decreasing the permeability of the concrete and/or waterproofing the exposed surface.

Cracks in segmental concrete box girders may result from stresses due to handling and storing segments for precast construction and to stripping forms and supports from cast-in-place construction before attainment of the nominal \( f'_c \).

Class 1 exposure condition applies when cracks can be tolerated due to reduced concerns of appearance and/or corrosion. Class 2 exposure condition applies to transverse design of segmental concrete box girders for any loads applied prior to attaining full nominal concrete strength and when there is increased concern of appearance and/or corrosion.

In the computation of \( d_c \), the actual concrete cover thickness is to be used.

When computing the actual stress in the steel reinforcement, axial tension effects shall be considered, while axial compression effects may be considered.

The minimum and maximum spacing of reinforcement shall also comply with the provisions of Articles 5.10.3.1 and 5.10.3.2, respectively.

The effects of bonded prestressing steel may be considered, in which case the value of \( f_s \) used in Eq. 1, for the bonded prestressing steel, shall be the stress that develops beyond the decompression state calculated on the basis of a cracked section or strain compatibility analysis.

Where flanges of reinforced concrete T-girders and box girders are in tension at the service limit state, the flexural tension reinforcement shall be distributed over the lesser of:

- The effective flange width, specified in Article 4.6.2.6, or
- A width equal to 1/10 of the average of adjacent spans between bearings.

If the effective flange width exceeds 1/10 the span, additional longitudinal reinforcement, with area not less than 0.4 percent of the excess slab area, shall be provided in the outer portions of the flange.

If the effective depth, \( d_e \), of nonprestressed or partially prestressed concrete members exceeds 3.0 ft., longitudinal skin reinforcement shall be uniformly distributed along both side faces of the component for a distance \( d_e /2 \) nearest the flexural tension reinforcement. The area of skin reinforcement \( A_{sk} \) in in.\(^2\)/ft. of height on each side face shall satisfy:

- Wide spacing of the reinforcement across the full effective width of flange may cause some wide cracks to form in the slab near the web.
- Close spacing near the web leaves the outer regions of the flange unprotected.

The 1/10 of the span limitation is to guard against an excessive spacing of bars, with additional reinforcement required to protect the outer portions of the flange.

The requirements for skin reinforcement are based upon ACI 318. For relatively deep flexural members, some reinforcement should be placed near the vertical faces in the tension zone to control cracking in the web.

Without such auxiliary steel, the width of the cracks in the web may greatly exceed the crack widths at the level of the flexural tension reinforcement.
\[ A_{\text{di}} \geq 0.012 (d_e - 30) \leq \frac{A_s + A_{ps}}{4} \]  (5.7.3.4-2)

where:

\[ A_{ps} = \text{area of prestressing steel (in.}^2) \]

\[ A_s = \text{area of tensile reinforcement (in.}^2) \]

however, the total area of longitudinal skin reinforcement (per face) need not exceed one-fourth of the required flexural tensile reinforcement \( A_s + A_{ps} \).

The maximum spacing of the skin reinforcement shall not exceed either \( d_e/6 \) or 12.0 in.

Such reinforcement may be included in strength computations if a strain compatibility analysis is made to determine stresses in the individual bars or wires.

5.7.3.5 Moment Redistribution

In lieu of more refined analysis, where bonded reinforcement that satisfies the provisions of Article 5.11 is provided at the internal supports of continuous reinforced concrete beams, negative moments determined by elastic theory at strength limit states may be increased or decreased by not more than 1000 \( \varepsilon_t \) percent, with a maximum of 20 percent. Redistribution of negative moments shall be made only when \( \varepsilon_t \) is equal to or greater than 0.0075 at the section at which moment is reduced.

Positive moments shall be adjusted to account for the changes in negative moments to maintain equilibrium of loads and force effects.

5.7.3.6.2 Deflection and Camber

Deflection and camber calculations shall consider dead load, live load, prestressing, erection loads, concrete creep and shrinkage, and steel relaxation.

For determining deflection and camber, the provisions of Articles 4.5.2.1, 4.5.2.2, and 5.9.5.5 shall

In editions and interims to the LRFD Specifications prior to 2005, Article 5.7.3.5 specified the permissible redistribution percentage in terms of the \( c/d_e \) ratio. The current specification specifies the permissible redistribution percentage in terms of net tensile strain \( \varepsilon_t \). The background and basis for these provisions are given in Mast (1992).

For structures such as segmentally constructed bridges, camber calculations should be based on the modulus of elasticity and the maturity of the concrete when each increment of load is added or removed, as specified in Articles 5.4.2.3 and 5.14.2.3.6.
apply.
In the absence of a more comprehensive analysis, instantaneous deflections may be computed using the modulus of elasticity for concrete as specified in Article 5.4.2.4 and taking the moment of inertia as either the gross moment of inertia, \( I_g \), or an effective moment of inertia, \( I_e \), given by Eq. 1:

\[
I_e = \left( \frac{M_{cr}}{M_a} \right)^3 I_g + \left[ 1 - \left( \frac{M_{cr}}{M_a} \right)^3 \right] I_{cr} \leq I_g \tag{5.7.3.6.2-1}
\]

in which:

\[
M_{cr} = f_y \frac{I_e}{y_t} \tag{5.7.3.6.2-2}
\]

where:

- \( M_{cr} \) = cracking moment (kip-in.)
- \( f_y \) = modulus of rupture of concrete as specified in Article 5.4.2.6 (ksi)
- \( y_t \) = distance from the neutral axis to the extreme tension fiber (in.)
- \( M_a \) = maximum moment in a component at the stage for which deformation is computed (kip-in.)

For prismatic members, effective moment of inertia may be taken as the value obtained from Eq. 1 at midspan for simple or continuous spans, and at support for cantilevers. For continuous nonprismatic members, the effective moment of inertia may be taken as the average of the values obtained from Eq. 1 for the critical positive and negative moment sections.

Unless a more exact determination is made, the long-time deflection may be taken as the instantaneous deflection multiplied by the following factor:

- If the instantaneous deflection is based on \( I_g \): 4.0
- If the instantaneous deflection is based on \( I_e \): 3.0–1.2\( (A'_{s}/A_s) \geq 1.6 \)

where:

\( A'_{s} \) = area of compression reinforcement (in.²)
\( A_s \) = area of nonprestressed tension reinforcement

In prestressed concrete, the long-term deflection is usually based on mix-specific data, possibly in combination with the calculation procedures in Article 5.4.2.3. Other methods of calculating deflections which consider the different types of loads and the sections to which they are applied, such as that found in (PCI 1992), may also be used.
The contract documents shall require that deflections of segmentally constructed bridges shall be calculated prior to casting of segments based on the anticipated casting and erection schedules and that they shall be used as a guide against which actual deflection measurements are checked.

5.7.4 Compression Members

5.7.4.2 Limits for Reinforcement

The following reinforcement limits may be used for normal weight concrete with specified compressive strengths up to 18.0 ksi.

Additional limits on reinforcement for compression members in Seismic Zones 3 and 4 shall be considered as specified in Article 5.10.11.4.1a.

The maximum area of prestressed and nonprestressed longitudinal reinforcement for noncomposite compression components shall be such that:

\[
\frac{A_{ps} f_{ps}}{A_y f_y} \leq 0.08 \quad (5.7.4.2-1)
\]

and

\[
\frac{A_{pu} f_{pu}}{A_y f_y} \leq 0.30 \quad (5.7.4.2-2)
\]

The minimum area of prestressed and nonprestressed longitudinal reinforcement for noncomposite compression components shall be such that:

\[
\frac{A_{ps} f_{ps}}{A_y f_y} + \frac{A_{nu} f_{nu}}{A_y f_y} \geq 0.135 \quad (5.7.4.2-3)
\]

\[
\frac{A_{pu} f_{pu}}{A_y f_y} + \frac{A_{nu} f_{nu}}{A_y f_y} \geq 0.135 \frac{f'}{f_y} \quad (5.7.4.2-3)
\]

but not greater than 0.0225.

where:

\[
A_s = \text{area of nonprestressed tension steel (in.}^2)\]

\[
A_g = \text{gross area of section (in.}^2)\]

According to current ACI codes, the area of longitudinal reinforcement for nonprestressed noncomposite compression components should be not less than 0.01 \(A_y\). Because the dimensioning of columns is primarily controlled by bending, this limitation does not account for the influence of the concrete compressive strength. To account for the compressive strength of concrete, the minimum reinforcement in flexural members is shown to be proportional to \(f'/f_y\) in Article 5.7.3.3.2. This approach is also reflected in the first term of Eq. 3. For fully prestressed members, current codes specify a minimum average prestress of 0.225 ksi. Here also the influence of compressive strength is not accounted for. A compressive strength of 5.0 ksi has been used as a basis for these provisions, and a weighted averaging procedure was used to arrive at the equation.
Analyses by Rizkalla et al. (2007) showed that the reinforcement ratio calculated by Eq. 3 need not be greater than 0.0225 when the unfactored permanent loads do not exceed 0.4 \( A_p f_{pc} \), which is typically the case encountered in design.

The minimum number of longitudinal reinforcing bars in the body of a column shall be six in a circular arrangement and four in a rectangular arrangement. The minimum size of bar shall be No. 5.

For bridges in Seismic Zones 1 and 2, a reduced effective area may be used when the cross-section is larger than that required to resist the applied loading. The minimum percentage of total (prestressed and nonprestressed) longitudinal reinforcement of the reduced effective area is to be the greater of 1 percent or the value obtained from Eq. 3. Both the reduced effective area and the gross area must be capable of resisting all applicable load combinations from Table 3.4.1-1.

5.7.4.4 Factored Axial Resistance

The following assumptions may be used for normal weight concrete with specified compressive strengths up to 18.0 ksi.

The factored axial resistance of concrete compressive components, symmetrical about both principal axes, shall be taken as:

\[ P_e = \phi P_{u} \] (5.7.4.4-1)

in which:

- For members with spiral reinforcement:

\[ P_{u} = 0.85 \left[ \frac{0.85 \phi f' \left( A_p - A_{ps} - A_{pc} \right)}{f_r A_p - A_{pc} \left( f_{pc} - E_p \varepsilon_{cu} \right)} \right] \] (5.7.4.4-2)

Where columns are pinned to their foundations, a small number of central bars have sometimes been used as a connection between footing and column.

For low risk seismic zones, the 1 percent reduced effective area rule, which has been used successfully since 1957 in the Standard Specifications, is implemented, but modified to account for the dependency of the minimum reinforcement on the ratio of \( f'_c / f_y \).

For columns subjected to high, permanent axial compressive stresses where significant concrete creep is likely, using an amount of longitudinal reinforcement less than that given by Eq. 3 is not recommended because of the potential for significant transfer of load from the concrete to the reinforcement as discussed in the report of ACI Committee 105.

C5.7.4.4

The values of 0.85 and 0.80 in Eqs. 2 and 3 place upper limits on the usable resistance of compression members to allow for unintended eccentricity.

In the absence of concurrent bending due to external loads or eccentric application of prestress, the ultimate strain on a compression member is constant across the entire cross-section. Prestressing causes compressive stresses in the concrete, which reduces the resistance of compression members to externally applied axial loads. The term, \( E_p \varepsilon_{cu} \), accounts for the fact that a column or pile also shortens under externally applied loads, which serves to reduce the level of compression due to prestress. Assuming a concrete compressive strain at ultimate, \( \varepsilon_{cu} = 0.003 \),
• For members with tie reinforcement:

\[
P_{n} = 0.80 \left[ \frac{0.85 k_c f'_c (A_g - A_{st} - A_{ps})}{f_y A_{st} - A_{ps} f_{pe} E_p} \right] + f_y A_{st} - A_{ps} \left( f_{pe} E_p \right)
\]

(5.7.4.4-3)

The factor \( k_c \) shall be taken as 0.85 for specified compressive strengths not exceeding 10.0 ksi. For specified compressive strengths exceeding 10.0 ksi, \( k_c \) shall be reduced at a rate of 0.02 for each 1.0 ksi of strength in excess of 10.0 ksi, except that \( k_c \) shall not be less than 0.75.

where:

- \( P_r \) = factored axial resistance, with or without flexure (kip)
- \( P_n \) = nominal axial resistance, with or without flexure (kip)
- \( f'_c \) = specified strength of concrete at 28 days, unless another age is specified (ksi)
- \( A_g \) = gross area of section (in.\(^2\))
- \( A_{st} \) = total area of longitudinal reinforcement (in.\(^2\))
- \( f_y \) = specified yield strength of reinforcement (ksi)
- \( \phi \) = resistance factor specified in Article 5.5.4.2
- \( A_{ps} \) = area of prestressing steel (in.\(^2\))
- \( E_p \) = modulus of elasticity of prestressing tendons (ksi)
- \( f_{pe} \) = effective stress in prestressing steel after losses (ksi)
- \( \varepsilon_{cu} \) = failure strain of concrete in compression (in./in.)
- \( k_c \) = ratio of the maximum compressive stress to the specified compressive strength of concrete

and a prestressing steel modulus, \( E_p = 28,500 \) ksi, gives a relatively constant value of 85.0 ksi for the amount of this reduction. Therefore, it is acceptable to reduce the effective prestressing by this amount. Conservatively, this reduction can be ignored.

For specified compressive strengths between 10 and 15 ksi, the value of \( k_c \) may be determined by linear interpolation, as shown in Figure C5.7.4.4-1.
5.7.4.5 Biaxial Flexure

The following assumptions may be used for normal weight concrete with specified compressive strengths up to 18.0 ksi.

In lieu of an analysis based on equilibrium and strain compatibility for biaxial flexure, noncircular members subjected to biaxial flexure and compression may be proportioned using the following approximate expressions:

- If the factored axial load is not less than 0.10 $\phi f'_c A_c$:

\[
\frac{1}{P_{\text{cr}}} = \frac{1}{P_{xx}} + \frac{1}{P_{yy}} - \frac{1}{\phi P_a}\]

(Eq. 5.7.4.5-1)

in which:

\[
P_a = \left[ 0.85 k_c f'_c (A_y - A_{c,y} - A_{u,y}) \left( f_{p,y} - E_y \varepsilon_{p,y} \right) + f_y A_y - A_{u,y} \left( f_{p,y} - E_y \varepsilon_{p,y} \right) \right]

(Eq. 5.7.4.5-2)
• If the factored axial load is less than 0.10 $\phi f_c' A_g$:

$$\frac{M_{ux}}{M_{rx}} + \frac{M_{uy}}{M_{ry}} \leq 1.0 \quad (5.7.4.5-3)$$

where:

- $\phi$ = resistance factor for members in axial compression
- $P_{rx}$ = factored axial resistance in biaxial flexure (kip)
- $P_{ry}$ = factored axial resistance determined on the basis that only eccentricity $e_x$ is present (kip)
- $P_{rx}$ = factored axial resistance determined on the basis that only eccentricity $e_y$ is present (kip)
- $P_a$ = factored applied axial force (kip)
- $M_{ux}$ = factored applied moment about the X-axis (kip-in.)
- $M_{uy}$ = factored applied moment about the Y-axis (kip-in.)
- $e_x$ = eccentricity of the applied factored axial force in the X direction, i.e., $e_x = M_{uy}/P_a$ (in.)
- $e_y$ = eccentricity of the applied factored axial force in the Y direction, i.e., $e_y = M_{ux}/P_a$ (in.)
- $P_o$ = nominal axial resistance of a section at 0.0 eccentricity

The factored axial resistance $P_{rx}$ and $P_{ry}$ shall not be taken to be greater than the product of the resistance factor, $\phi$, and the maximum nominal compressive resistance given by either Eqs. 5.7.4.4-2 or 5.7.4.4-3, as appropriate.

### 5.7.4.6 Spirals and Ties

The following assumptions may be used for normal weight concrete with specified compressive strengths up to 18.0 ksi.

The area of steel for spirals and ties in bridges in Seismic Zones 2, 3, or 4 shall comply with the requirements specified in Article 5.10.11.

Where the area of spiral and tie reinforcement is not controlled by:
• Seismic requirements,
• Shear or torsion as specified in Article 5.8, or
• Minimum requirements as specified in Article 5.10.6,

the ratio of spiral reinforcement to total volume of concrete core, measured out-to-out of spirals, shall satisfy:

\[
\rho_s \geq 0.45 \left( \frac{A_g}{A_c} - 1 \right) \frac{f_c'}{f_{yh}} \quad (5.7.4.6-1)
\]

where:

\(A_g\) = gross area of concrete section (in.\(^2\))

\(A_c\) = area of core measured to the outside diameter of the spiral (in.\(^2\))

\(f_c'\) = specified strength of concrete at 28 days, unless another age is specified (ksi)

\(f_{yh}\) = specified yield strength of spiral reinforcement (ksi)

Other details of spiral and tie reinforcement shall conform to the provisions of Articles 5.10.6 and 5.10.11.
5.10 Details of Reinforcement

5.10.6 Transverse Reinforcement for Compression Members

5.10.6.3 Ties

The following requirements for transverse reinforcement, may be used for normal weight concrete with specified compressive strengths up to 18.0 ksi.

In tied compression members, all longitudinal bars shall be enclosed by lateral ties that shall be equivalent to:

- No. 3 bars for No. 10 or smaller bars,
- No. 4 bars for No. 11 or larger bars, and
- No. 4 bars for bundled bars.

The spacing of ties along the longitudinal axis of the compression member shall not exceed the least dimension of the compression member or 12.0 in. Where two or more bars larger than No. 10 are bundled together, the spacing shall not exceed half the least dimension of the member or 6.0 in.

Deformed wire or welded wire fabric of equivalent area may be used instead of bars.

Figure C5.10.6.3-1 Acceptable Tie Arrangements.
No longitudinal bar shall be more than 24.0 in., measured along the tie, from a restrained bar. A restrained bar is one which has lateral support provided by the corner of a tie having an included angle of not more than 135°. Where the column design is based on plastic hinging capability, no longitudinal bar shall be farther than 6.0 in. clear on each side along the tie from such a laterally supported bar and the tie reinforcement shall meet the requirements of Articles 5.10.11.4.1d through 5.10.11.4.1f. Where the bars are located around the periphery of a circle, a complete circular tie may be used if the splices in the ties are staggered.

Ties shall be located vertically not more than half a tie spacing above the footing or other support and not more than half a tie spacing below the lowest horizontal reinforcement in the supported member.

Columns in Seismic Zones 2, 3, and 4 are designed for plastic hinging. The plastic hinge zone is defined in Article 5.10.11.4.1c. Additional requirements for transverse reinforcement for bridges in Seismic Zones 3 and 4 are specified in Article 5.10.11.4.1. Plastic hinging may be used as a design strategy for other extreme events, such as ship collision.
APPENDIX G – COLLECTION OF EXPERIMENTAL DATA

See enclosed CD for all the collected experimental data from the literature review and the experimental program.