



IDEA

**Innovations Deserving
Exploratory Analysis Programs**

High-Speed Rail IDEA Program

**Low-Cost, Precise Railroad GPS Vehicle Location
System**

Final Report for High-Speed Rail IDEA Project 52

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ABSTRACT

If the precise location of both train and track maintenance vehicles and equipment could be determined automatically at central dispatch, it could significantly improve the safety and efficiency of train movements through track maintenance areas. This could be achieved using an automatic precise location system on the train and on the maintenance vehicles that automatically transmits their respective locations to central dispatch. This study focuses on the feasibility of automatically and precisely determining the location of maintenance vehicles and equipment using a GPS Vehicle Location System (GVLS).

GVLS uses a GPS receiver augmented with dead reckoning sensors, in order to provide the vehicle location when GPS coverage is interrupted. The GVLS achieves the required position accuracy by using a sophisticated carrier differential GPS (CDGPS) position algorithm. This algorithm compares the GPS carrier phase measurements from the GVLS receiver with GPS carrier phase measurements from a nearby reference receiver at a known (surveyed) location. Using these measurements, the algorithm estimates and removes line-of-sight slant range errors, such as the unknown carrier (integer) biases, and determines the relative position between the two receivers. The reference receiver measurements are obtained from a High Accuracy – National Differential GPS (HA-NDGPS) reference station network, which is currently under development as an upgrade to the existing NDGPS reference station network.

Under this feasibility study, a candidate CDGPS algorithm was selected based on a review of the technical literature. This algorithm was coded in Matlab and tested with archived GPS data. This data was obtained from the Continuously Operating Reference Station (CORS) Network that is administered by the National Geodetic Survey, NOAA.

KEY WORDS:

Train Location Systems, Track Maintenance Equipment Location Systems, Carrier Differential GPS (CDGPS) algorithms, High Accuracy National Differential GPS (HA-NDGPS) network, Communications Based Train Control (CBTC), Positive Train Control (PTC).

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1 EXECUTIVE SUMMARY

This High-Speed Rail IDEA project investigated the feasibility of a low-cost, precise GPS vehicle location system (GVLS). The requirement is to automatically determine the precise location of on-track maintenance equipment, off-track equipment such as construction equipment, and the location of small track maintenance gangs. It sends this location information to Central Dispatch via a digital communications link as illustrated in Figure 1. This information enables

dispatchers to more efficiently and safely manage train traffic through track maintenance locations

GVLS uses a GPS receiver augmented with dead reckoning sensors, in order to provide the vehicle location when GPS coverage is interrupted. In addition, it uses broadcast measurements from a High Accuracy – National Differential GPS (HA-NDGPS) reference station network, as illustrated in Figure 2.

GVLS achieves the required position accuracy by using a sophisticated carrier differential GPS (CDGPS) position algorithm. This algorithm compares the GPS carrier phase measurements from the GVLS receiver with GPS carrier phase

measurements from a nearby reference receiver at a known (surveyed) location. Using these measurements, the algorithm estimates and removes line-of-sight slant range errors and determines the relative position between the two receivers. The reference receivers are part of the National Differential GPS (NDGPS) network that is being upgraded to broadcast real-time carrier phase measurements under the HA-NDGPS upgrade.

With this carrier phase positioning accuracy, the absolute position of the maintenance vehicle is determined anywhere on the rail network or off of it with a confidence level of 0.99999 (0.9₅) when GPS coverage is available. When multiple measurements over a one minute period are combined, a confidence level of 0.9999999 (0.9₇) is achieved. Hence, measuring the movement through switches is not required to determine on which of several parallel tracks the maintenance vehicle is located.

When the GPS measurements are temporarily interrupted, a low-cost longitudinal accelerometer and a low-cost heading rate sensor provide a dead reckoning capability. The accuracy of the accelerometer and rate sensor will be selected to allow the location system to dead reckon precisely for up to several minutes when GPS coverage is temporarily masked while the vehicle is moving. For stationary vehicles located in an area without GPS coverage, additional software logic

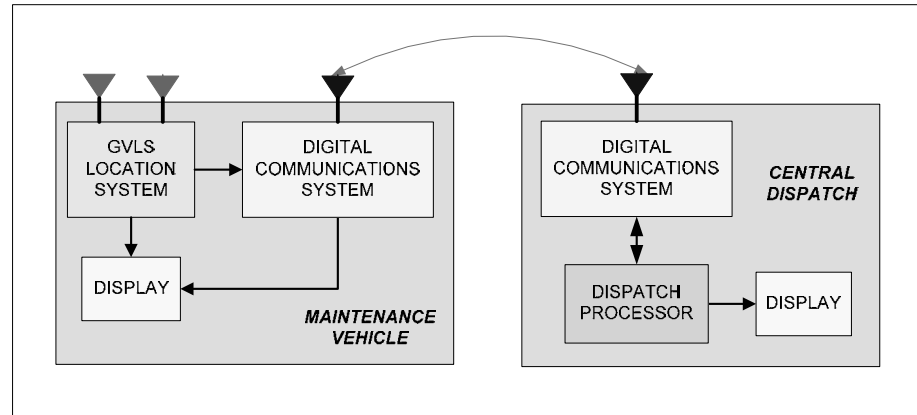


FIGURE 1 Automatic Maintenance Vehicle Location Architecture

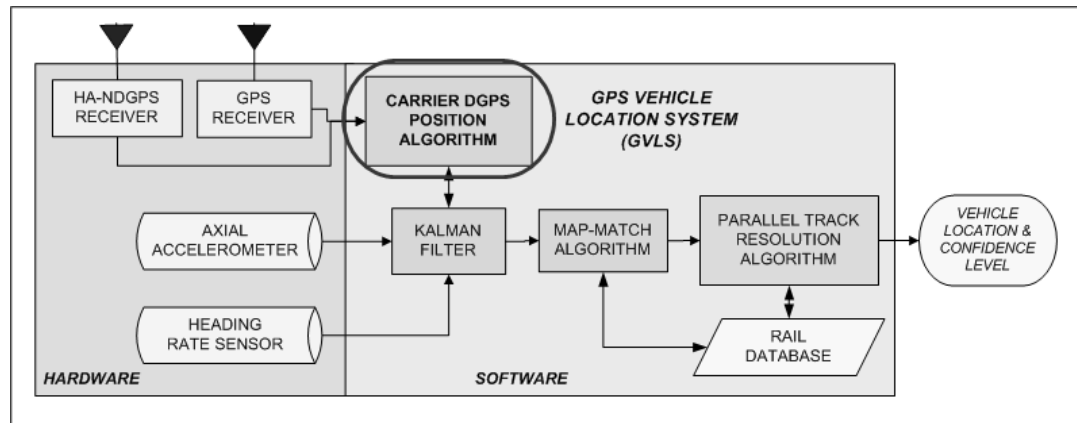


FIGURE 2 GVLS Railroad Vehicle Architecture

can be used to extend the dead reckoning period beyond several minutes. In addition, these dead reckoning sensors will be used to re-initialize the carrier position solution once coverage is restored.

The parallel track resolution algorithm compares the current lateral position of the vehicle to parallel track lateral positions in a digital rail database. Then by considering the vehicle lateral position measurement error, the probability that the vehicle is on one or another track is computed. From these probabilities the most likely track location of the vehicle is determined. The algorithm also establishes whether the maintenance vehicle has left the tracks – although it may still be close to them.

Originally, the GVLS architecture was postulated on the use of a standard single frequency (L1) GPS receiver due to its low cost. A review of the literature identified two studies performed by the National Geodetic Survey (NGS), one of which showed that the required GVLS position accuracy could not be achieved for separation (baseline) distances of greater than 30 km between the single frequency GVLS receiver and the nearest HA-NDPGS reference station. Since these baseline distances may be as large as 150 km, another approach is required.

The second NGS study determined that the required accuracy could be achieved over baseline distances of greater than 150 km with dual-frequency (L1/L2) GPS receivers. With these receivers, the ionospheric delay error, one of the largest error sources, can be measured directly and eliminated within the accuracy of the dual-frequency measurements.

One of the limitations of the current dual-frequency receivers is the fact that they are considerably more expensive than the single frequency receivers. This is primarily due to the use of proprietary algorithms that tap into the current second military (L2) frequency. It is anticipated that future dual-frequency receivers will be considerably cheaper since a next-generation of GPS satellites, with the first launch in September 2005, will also have a civilian L2 frequency. Hence, proprietary algorithms to obtain the L2 frequency will no longer be required. A full constellation of these satellites is anticipated to be available in the 2010-2014 time frame, with a sizable number (50%) available by 2009.

A limitation in using a CDGPS algorithm is that it can require an initialization time of up to several hours depending on the required position accuracy, based on the NGS study. The GVLS CDGPS algorithm that was selected is precise and uses a 2-step procedure that can be initialized with only a few minutes of data. This 2-step CDGPS algorithm was derived from an Ohio University doctoral thesis that defined and evaluated a 3-step CDGPS algorithm [1]. The thesis evaluated the 3-step algorithm with NDGPS reference receiver data for baselines up to 300 km and demonstrated the precise accuracy required by GVLS.

The GVLS CDGPS 2-step algorithm was evaluated with archived GPS data collected by the Continuously Operating Reference Station (CORS) network. Data from 3 receiver sites were obtained with separation (baseline) distances of 52 km, 156 km, and 206 km. In evaluating the performance of the GVLS CDGPS algorithm, it was realized that multipath errors could not be ignored. As a result a separate Kalman Filter was added to each of the algorithms to estimate the desired carrier (integer) bias and the unwanted code multipath errors. With these two filters the multipath errors were eliminated.

In evaluating the performance of the GVLS CDGPS 2-step algorithm, the data analysis under this study was unable to demonstrate the required position accuracies for the 156 km and 206 km distances. A linear error analysis of the algorithm, however, supports the feasibility for the GVLS CDGPS algorithm to meet the required accuracy for the long baseline distances. The error analysis incorporated fundamental error statistics such as the receiver code and carrier measurement noise statistics.

A key limitation for the successful performance of the GVLS CDGPS algorithm is the ionospheric delay estimate, a major error source. This estimate is computed from the dual-frequency code measurements. However this estimate is corrupted by unwanted multipath errors as well as receiver code measurement noise errors. Hence, filtering the ionospheric delay estimates to remove the multipath errors and reduce the measurement noise errors is recommended. This would be accomplished with an initial (third) Kalman Filter that estimates both the ionospheric delay and the unwanted multipath errors before the ionospheric delay estimates are used to correct the code and carrier measurements.

2 IDEA PRODUCT

2.1 GVLS DESCRIPTION

The low-cost, precise GPS vehicle location system (GVLS) automatically determines the location of a railroad vehicle, whether on or off the rail network. It uses a GPS receiver augmented with dead reckoning sensors, in order to provide the vehicle location when GPS coverage is interrupted. In addition, it uses broadcast measurements from a High Accuracy – National Differential GPS (HA-NDGPS) reference station network, as illustrated in Figures 2 and 3.

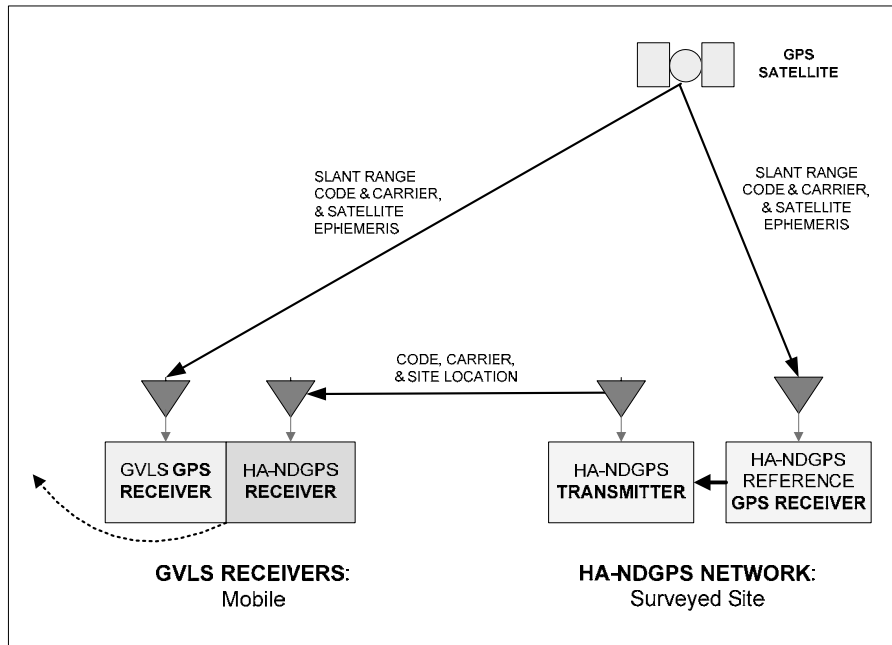


FIGURE 3 Railroad GPS Vehicle Location System (GVLS) System Architecture

The GVLS achieves the required position accuracy by using a sophisticated carrier differential GPS (CDGPS) position algorithm. This algorithm compares the GPS carrier phase measurements from the GVLS receiver with GPS carrier phase measurements from a nearby reference receiver at a known (surveyed) location.

Using these measurements, the algorithm determines and eliminates slant range errors, such as the unknown integer ambiguities, and determines the relative position between the two receivers.

The reference receivers are part of the National Differential GPS (NDGPS) network. While currently providing differential GPS (DGPS) code corrections to

mobile GPS receivers, this network is being upgraded to also broadcast real-time code and carrier phase measurements under the HA-NDGPS upgrade.

With carrier phase positioning accuracy, the absolute position of the maintenance vehicle is determined anywhere on the rail network or off of it with a confidence level of 0.99999 (0.9₅) when GPS coverage is available. By combining measurements over a one minute time interval, the confidence level can be extended to 0.9999999 (0.9₇). Hence, measuring the vehicle movement through switches is not required to determine on which of several parallel tracks the maintenance vehicle is located.

When the GPS measurements are temporarily interrupted, a low-cost longitudinal accelerometer and a low-cost heading rate sensor provide a dead reckoning capability. The accuracy of the accelerometer and rate sensor is selected to allow the location system to dead reckon precisely for up to several minutes when GPS coverage is temporarily masked. In addition, these dead reckoning sensors can be used to re-initialize the carrier position solution.

For vehicles that may be on the rail network, the parallel track resolution algorithm accesses the local digital rail database using the current estimated along-track position. If the maintenance vehicle does not operate on the tracks, the parallel track resolution algorithm and rail database are not required. When the vehicle may be on the tracks, the estimated position is compared to the rail database position of the track segments that are close to the vehicle along-track position estimates. By considering the measurement error statistics, the parallel track resolution algorithm determines the most likely location of the vehicle on one of several parallel tracks and computes the confidence level of that location. It also establishes whether the maintenance vehicle has left the tracks – although it may be still close to them.

2.2 HIGH ACCURACY NATIONAL DIFFERENTIAL GPS NETWORK (HA-NDGPS)

Currently, the Nationwide Differential GPS (NDGPS) network provides 1-3 meter positioning accuracy to receivers capable of receiving the differential correction signals. Stationary receivers with antennae in accurately surveyed locations compute the DGPS code corrections and broadcast them at frequencies around 300 kHz. The NDGPS network

is an expansion of the U.S. Coast Guard's Maritime DGPS network. New stations are using decommissioned U.S. Air Force Ground Wave Emergency Network (GWEN) sites.

Coverage maps for the sites are shown in Figure 4. Current sites (as of August 2005) are shown as solid triangles while open triangles are planned future sites. Areas where a user can receive broadcasts from at least two current sites are shown in light gray. The areas where a user can receive broadcasts from at least one current site are shown in gray.

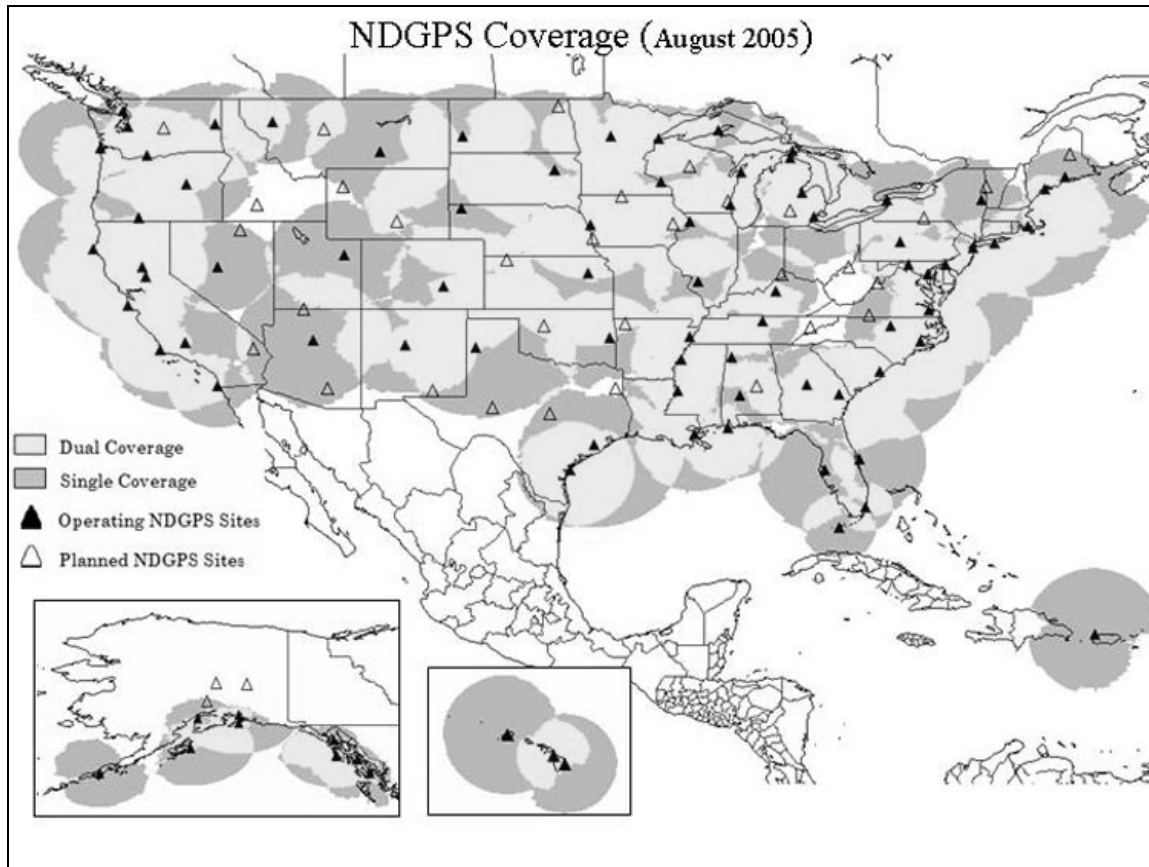


FIGURE 4 National Differential GPS (NDGPS) Sites and Coverage (August 2005)

3 CONCEPT AND INNOVATION

3.1 GVLS CONCEPT

Railroad maintenance-of-way vehicles provide an invaluable service by inspecting and maintaining the track network for



FIGURE 5 Maintenance Vehicles on Mainline Track

trains. These vehicles include sport utility vehicles that can travel on tracks using iron wheels or on roads using their standard tires. More exotic maintenance vehicles run the gamut from small ‘golf cart-sized’ motorized vehicles to large, railroad car-sized tamping vehicles. Due to their slow speeds, limited crash protection, and ability to enter/exit the rail network, their precise location must be known, to avoid conflicts with revenue bearing trains.

To prevent these conflicts, Class 1 railroads, such as the Union Pacific Railroad, will shut down a stretch of parallel tracks if work is required on some of the tracks as shown in Figure 5. While this conservative approach avoids possible accidents with the maintenance vehicles, it is not a cost effective solution when some of the tracks may be usable by revenue bearing trains.

Other railroads require extensive voice radio communications to coordinate the safe passage of a revenue bearing train around a work crew on an adjoining track. This voice coordination takes time and hence is not very efficient.

A new low-cost, precise GPS vehicle location system (GVLS) is proposed for use to automatically track the location of railroad maintenance vehicles. The GVLS uses a GPS receiver, broadcast measurements from a GPS reference station network, and dead reckoning sensors when GPS coverage is interrupted. It is designed to establish the location of the maintenance vehicle on one of several parallel tracks, separated by as little as 11.5 ft center-to-center, with a confidence level of 0.99999 (0.9₅). As a result, the GVLS will have a position accuracy of 40 cm (1.35 ft), one sigma. When measurements over a period of one minute

are combined, the confidence level is increased to 0.9999999 (0.9₇).

This location system is then combined with digital communications to Dispatch as was illustrated in Figure 1. If a similar precise location system is used on the locomotives of revenue bearing trains and connected to Dispatch via digital communications, the coordination required to safely pass through a work area is greatly and safely expedited.

The GVLS will achieve the precise position accuracy by using the GPS carrier phase measurements from a GVLS receiver together with GPS carrier phase measurements from a nearby reference receiver at a known (surveyed) location. The reference receivers are part of the HA-NDGPS network. With GVLS and reference receiver measurements, the absolute position of the maintenance vehicle is determined anywhere on the rail network, or off of it, with a confidence level of 0.99999 (0.9₅) and increased to 0.9999999 (0.9₇) by combining position estimates over a one minute period.

3.2 GVLS INNOVATION

Under the GVLS concept, maintenance vehicles will be tracked precisely and automatically whether they are on a multiple track network or off it. The position will be determined absolutely, requiring no external position initialization, and with a high level of confidence. The concept will provide an affordable location solution that will allow maintenance vehicles to operate on Positive Train Control (PTC) track networks together with PTC-equipped trains.

The GVLS concept also exploits two current developments that facilitate this approach. The first is the availability of free raw GPS code and carrier real-time measurements via the HA-NDGPS network. This network is expected to be complete with dual station coverage throughout the continental US by 2010. Single station coverage is expected to be available as early as 2008.

The second development is the launch of the next generation of GPS satellites that started in September 2005 and will provide a full constellation by 2010-2014. A considerable number (50%) of the satellites are expected to be available by

the 2009 time frame. The advantage of these next generation satellites is that they provide both a civilian L1 and L2 signal transmission. With both signal transmissions readily available, it will significantly reduce the current price of dual-frequency (L1/L2) GPS receivers. In turn, with these dual frequency GPS receivers it is possible to directly measure and eliminate one of the key GPS signal error sources – the ionospheric delay error to within the code measurement noise accuracy of the GPS receivers.

3.3 OTHER PRECISE VEHICLE LOCATION TECHNOLOGIES

3.3.1 NAJPTC System

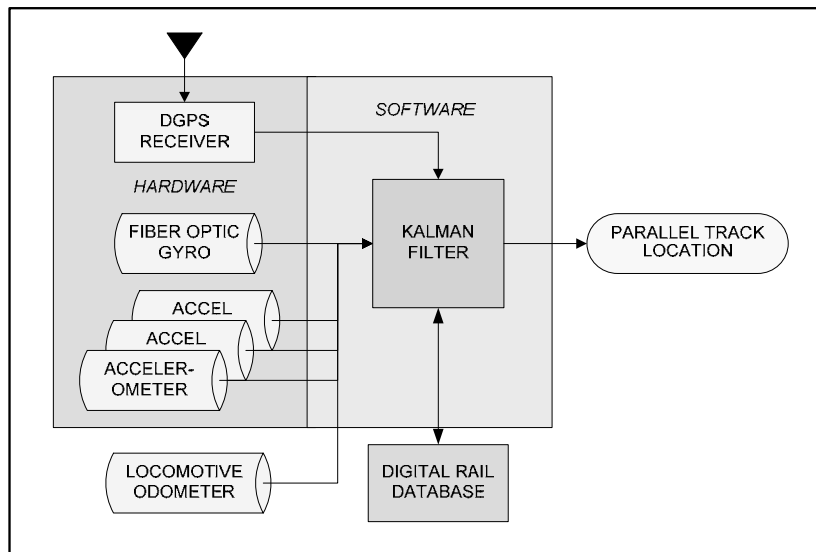


FIGURE 6 NAJPTC Location Determination System (LDS)

Under the North American Joint Positive Train Control Project (NAJPTC), a locomotive location determination system (LDS) is being tested in Illinois. These tests are being performed by a team lead by Lockheed Martin in cooperation with the Federal Railroad Administration, the Association of American Railroads, the Union Pacific Railroad, and the Illinois Department of Transportation. The location system architecture, based on [2], is presented in Figure 6.

This location system uses a precise fiber optic gyro (FOG) developed by KVH. With this gyro, it measures the change in heading rate when the locomotive moves onto a siding or parallel mainline track. In addition, it incorporates 3 accelerometers, the locomotive odometer, and DGPS corrections together with rail database

map matching.

3.3.2 Kayser-Threde

Kayser-Threde GmbH, Munich, Germany, has developed a number of locomotive and separate rail vehicle location systems under funding from the German Space Agency (DLR) since the mid-1990's. Two of the high precision and high integrity systems are the RadioCompass and the INTEGRAIL systems [3, 4]. INTEGRAIL, however, is most directly relevant as a potential maintenance vehicle location system.

3.3.2.1 INTEGRAIL

INTEGRAIL is a lower-cost version of the RadioCompass. It is intended for safety critical applications where the system is integrated with a train control system that is subject to the European Train Control Standard (ETCS), similar to the FRA Positive Train Control (PTC) standard.

As illustrated in Figure 7, the key components of this system are use of a GPS/EGNOS (Novatel Allstar) 12 channel L1 receiver, odometer, along-track accelerometer (Crossbow CXL02LF1), and heading FOG (KVH E-Core 1100 or 2030). It also incorporates a digital track database. Hence, its

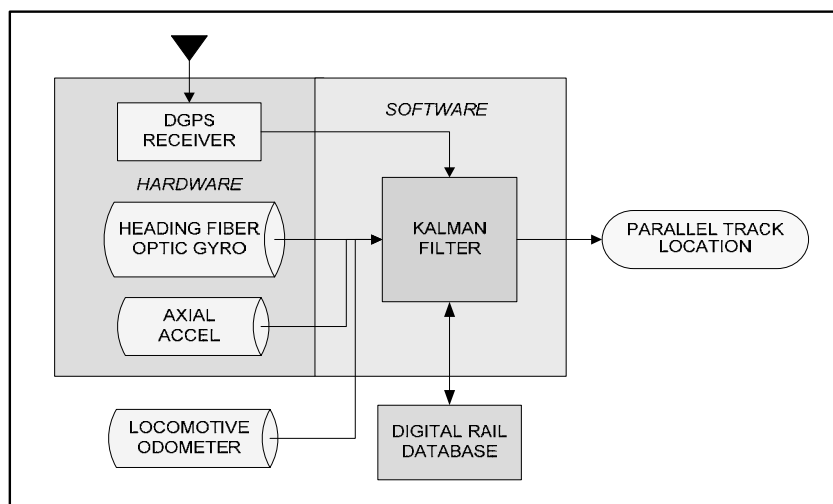


FIGURE 7 Kayser-Threde INTEGRAIL PTC Location System

design is very similar to the NAJPTC system (including the use of a KVH FOG), except that it uses only one accelerometer.

The INTEGRAIL specified accuracy is 5 m (1σ) along track and 0.5 m (1σ) cross-track position. The system availability requirement is 0.99990 (0.9_4) while the positioning availability requirement is 0.99999 (0.9_5) per 2 km of traveled distance or 20 sec of traveled time.

3.3.3 Sensis GLLS

The Seagull Technology Center of Sensis Corp. has developed a prototype GPS locomotive location system (GLLS) that is illustrated in Figure 8 [5 - 11].

GLLS determines the precise location of a locomotive on parallel tracks by measuring the change in heading when the locomotive passes over a switch either into or past a siding or transition track to another track. To achieve the precise location of the locomotive, a multi-GPS receiver antenna system is used to measure the differential carrier phase heading of the locomotive in addition to the GPS (coarse) position and velocity. In order to achieve the required heading accuracy with a 0.99999 (0.9_5) level of confidence, a 2.7 m (8.8 ft) baseline antenna array is mounted on the flat part of the locomotive cab roof. In addition, track database map matching algorithms are used to correct for along track GPS position errors.

Initial funding for GLLS was provided under a HSR IDEA feasibility concept contract (HSR-22) [5]. Under a follow-on HSR IDEA contract (HSR-35) [6], GLLS was subsequently developed into an R&D prototype and field tested. Since then, the Federal Railroad Administration provided additional funding under its Broad Area Announcement (BAA) contract vehicle to further develop GLLS into an operational prototype [7].

In general, all three location systems are limited as maintenance vehicle location system. The principal limitation is that they cannot determine whether a maintenance vehicle is close to but not on the track network. Hence they are not feasible for tracking maintenance vehicles, such as hi-railers, that can be either on or off the track network.

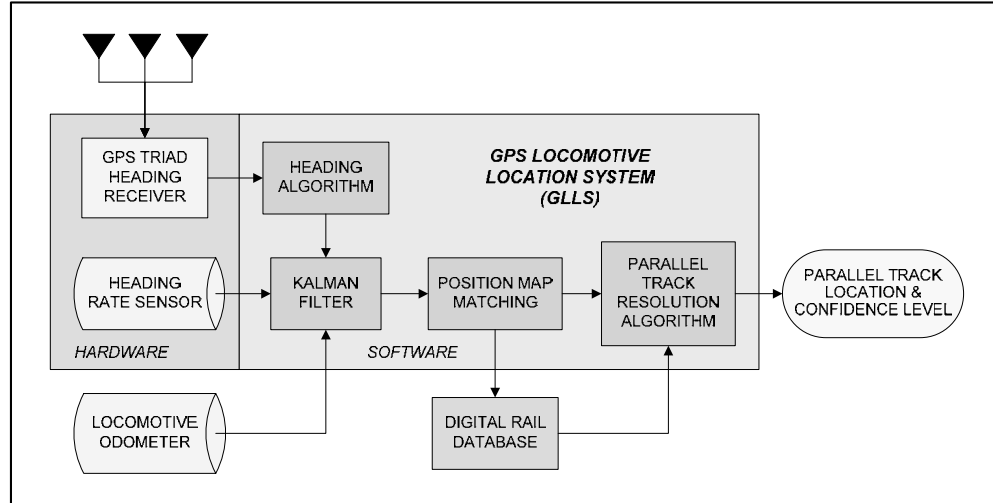


FIGURE 8 Seagull GPS Locomotive Location System (GLLS) Architecture

TABLE 1 GPS Positioning (L1) Error Sources (1σ , m)

ERROR	GPS [12]	DGPS* [13]	CDGPS*+ [13]
Ephemeris	2.1	0	0
Satellite clock	2.1	0.7	0
Ionosphere	4.0	0.5	0.41
Troposphere	0.7	0.5	0.41
Multipath	1.4	1.4	0.014
User Receiver Measurement	0.5	0.2	0.002
Reference Receiver Measurement		0.4	0.002
User Equivalent Range Error (UERE, rms)	5.3	1.8	0.58
Filtered UERE (rms)	5.1	1.1	0.17
Horizontal Position (HDOP = 2.0)	10.2	2.2	0.34
Vertical Position (VDOP = 2.5)	12.8	2.8	0.42

* Reference receiver is located no more than 50 km from user.
 + Assumes that carrier phase integer ambiguity is resolved with single difference (common satellite) solution using L1 frequency. Also, assumes that code ionospheric and tropospheric delays are dominated by quantization errors due to code measurement noise at user and reference receiver. Carrier phase multipath statistics tend to scale by a factor of 0.01 times the code phase multipath statistics, based on measurement data shown in [13, p. 557].

3.3.4 GPS Position Determination Approaches

For civilian applications, the GPS satellites broadcast a coded (Civilian Access C/A) signal on the L1 frequency (1575.42 MHz). This signal is used together with the broadcast orbit ephemeris to determine the location of a user receiver. As illustrated in Table 1, the accuracy of standard GPS leads to accuracies on the order of around 10 m.

Differential GPS (DGPS) allows the user receiver to incorporate GPS code corrections provided by a GPS reference station, which is at a known (surveyed) location. With DGPS, an accuracy of around 2 m is achieved.

To achieve accuracies of < 1 m, the GPS carrier, which carries the code signal, is used, providing CDGPS positioning. For dynamic applications, CDGPS and Kinematic Carrier Phase Positioning are considered to be synonymous. Hence, both will be used under this study.

The problem with using the carrier signal is that only the fractional carrier wavelength (19 cm at L1) is determined automatically by the user receiver hardware. However, the integer number of wavelengths between the user receiver and the GPS satellite is not measured. Hence, when a receiver is turned on, it has an unknown integer bias (integer ambiguity) in the carrier phase measurements that is different for each satellite and that remains fixed until a cycle slip occurs due to an interruption of the satellite signal received by the GPS receiver. After turn on, the receiver keeps track of the change in carrier phase relative to the initial unknown integer ambiguity.

In CDGPS positioning, measurements from a reference station are also used. These measurements, however, are the code and carrier phase measurements at the reference receiver that is at a known (surveyed) location. Hence, by using these code and carrier phase measurements at the user receiver, the problem becomes one of determining the relative position (baseline) of the user to the reference receiver, rather than to the GPS satellite.

The reason that CDGPS positioning works at all, given the large integer ambiguity is that when multiple satellites are tracked by the same user receiver, usually only one unique solution for each integer ambiguity to each satellite is possible. Of course, the code range solution is used to reduce the search space for the carrier phase integer ambiguities. This process of determining the integer ambiguities is a software process rather than a receiver tracking loop hardware process.

3.3.5 Long Baseline CDGPS Positioning

CDGPS positioning uses carrier-phase measurements to improve the 2-sigma accuracy to on the order of 20 cm (one wavelength). However, the carrier phase position estimates are for one GPS antenna relative to another GPS antenna, rather than an absolute position. The reasons for this are that each carrier wave does not carry with it a time of transmission. In addition, each receiver locks on to a satellite carrier at different times with different parts of the transmitted waveforms.

Figure 9 is a schematic representation of CDGPS positioning in 2 dimensions. It shows 2 receiver antennae, a single GPS satellite, the baseline vector between the 2 antennae and the line of sight vector from receiver 1 to the GPS satellite. The projection of the baseline vector onto the line-of-sight vector can be measured using differences between the phase measurements of the two receivers to the same satellite.

The difference in carrier-phase measurements, the single difference, from two receivers to the same satellite eliminates satellite clock and line bias errors. Differencing the single differences to two different satellites, the double difference, eliminates receiver clock and line biases. Hence, double-differences are used extensively in survey or

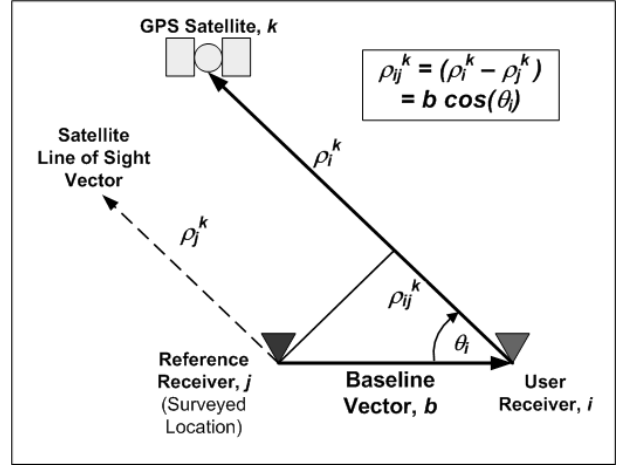


FIGURE 9 Schematic Representation of Differential Carrier Phase GPS Positioning

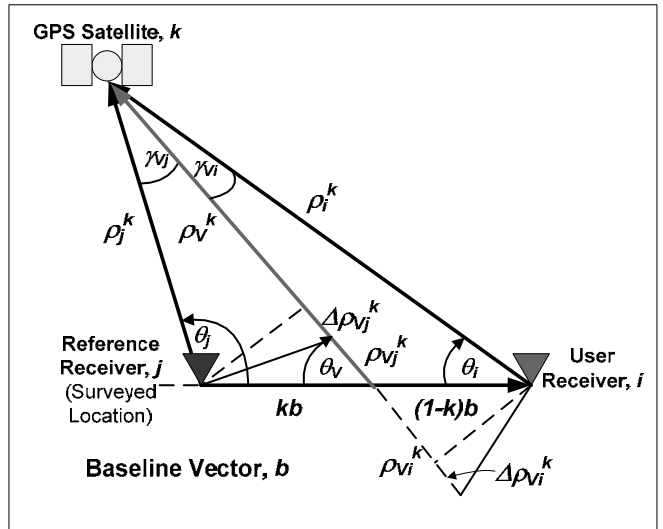


FIGURE 10 Long Baseline Line-of-Sight Geometry (with Virtual Receiver Location)

precision location applications that use GPS carrier phase measurements because the unknown satellite and receiver clock and line bias errors can be eliminated.

In Table 2 are summarized the long baseline error mechanisms that must be considered in addition to the individual error sources that were presented in Table 1. Also summarized are the error mitigation approaches. In the GVLS CDGPS algorithm, which will be described in later sections, all but the antenna phase windup error mitigation approaches are used.

For typical short baseline applications (<40 km), a number of assumptions are typically used to simplify the CDGPS positioning problem. Hence, with respect to Figure 9, one of the assumptions is that the line-of-sight from the vehicle to the satellite is parallel to the line-of-sight from the reference receiver to that satellite. While this is reasonable for the large slant range (20,000 km) to the GPS satellite, line-of-sight corrections have to be applied for long baseline applications.

For longer baselines, the geometry of Figure 10 has to be considered. In this figure, a virtual receiver line-of-sight is introduced to improve the long baseline estimation accuracy [1]

3.3.6 Candidate Integer Ambiguity Fixing Approaches

Determining the integer ambiguity provides a challenging problem. One solution is to impose the operational constraint that the vehicle record GPS phase measurements for 20 to 30 minutes while stationary before each maintenance work assignment. This permits the use of single frequency GPS receivers and the use of a least-squares estimation algorithm to estimate the initial position baseline vector between a GPS reference station and the vehicle GPS antenna. This initial position baseline vector is used to calculate integers and provides an initial condition for position baseline vector estimation when the vehicle begins to move.

Survey and geodetic level GPS receivers use both the L1 and L2 GPS signals and proprietary codeless L2 methods to determine integer ambiguity quickly and reliably. These receivers tend to cost more than the single frequency receivers.

3.3.6.1 Least-squares Ambiguity Decorrelation Algorithm (LAMBDA) Approach

The Least-squares Ambiguity Decorrelation Adjustment (LAMBDA) method was introduced by Teunissen and implemented by de Jong and Tiberius [14-20]. The method uses an integer-valued, linear transformation on the ambiguity float solutions before integer estimation. A discrete search over an ellipsoidal region, the ambiguity search ellipsoid, of candidate integers yields the least-squares solution.

The covariance matrix of the ambiguities governs the shape and orientation of the ellipsoid. The transformation produces an ellipsoid that is much closer to being a sphere. This less eccentric ellipsoid can be searched through very efficiently for candidate integer values.

3.3.6.2 Fast Ambiguity Search Filter (FASF)

The Fast Ambiguity Search Filter (FASF) was introduced by Chen and Lachapelle [21, 22]. The method incorporates a Kalman filter together with a search algorithm that searches for the integers at every epoch until fixed. In addition, an index is used to terminate the search process if integer fixing is unsuccessful.

The Kalman filter state vector includes the float solutions for the integers until they are fixed. The search method uses the satellite geometric information and the effects of the other candidate integers. The method determines the search range for each integer successively by recomputing the search ranges for the unfixed integers each time the fixed search

TABLE 2 Long Baseline Error Mechanism and Mitigation Approaches

Error	Mechanism	Mitigation
Line of Sight (LOS)	User & Ref Receiver LOS to same satellite not parallel	Use analytic geometric correction
Antenna Phase Windup	User & Ref Receiver's LOS to same satellite does not pass through same part of receiver antenna	Approximate analytic approach is available
Ionosphere LOS	User & Ref Receiver's LOS to same satellite does not pass through same part of ionosphere	Use ionospheric delay models or dual-frequency corrections
Troposphere LOS	User & Ref Receiver's LOS to same satellite does not pass through same part of troposphere	Use tropospheric delay models
Orbit LOS	User & Ref Receiver LOS to same satellite do not see same satellite orbit error components	Use separate LOS for each receiver and orbit vector error model
Satellite Transmission Time	User & Reference Receiver slant range differences to same satellite result in different satellite transmission times for common receiver reception time	Use analytic correction

candidates change.

3.4 LOW COST GPS RECEIVERS

A limited survey of lower-cost single and dual-frequency GPS receivers and receiver boards are presented respectively in Tables 3 and 4. All the receivers include antennas. The key metrics are the code and carrier measurement noise accuracy and the cost of the receivers.

TABLE 3 Survey of Single Frequency (L1) GPS Receivers and Receiver Boards with Antennas

MODEL	MANU-FACT.	OUT-PUT (hz)	CHAN-NELS	WARM REAQ (sec)	COLD AQU. (sec)	CODE NOISE (cm)	CAR-RIER NOISE (cm)	DOP-PLER NOISE (cm/s)	COST (\$)
GPS 15H/L Board	Garmin	1	12	15	45		2		183
GPS 25 Board	Garmin	1	12	15	45	80	2		255
Superstar II-5-1CPT (WAAS) Board	Novatel	5	12	45	120	75	1	5	440
FlexPak-SS11 Receiver	Novatel	5	12	45	120	75	1	5	790
OEM4-G2-3151R Board	Novatel	20	24	40	50	6	0.08	3	4,090
JNS 100 Board (WAAS)	Javad	1	50	10	60	10	0.01		3,800

TABLE 4 Survey of Dual-Frequency (L1/L2) GPS Receivers with Antennas

MODEL	MANU-FACT.	OUT-PUT (hz)	CHAN-NELS	WARM REAQ (sec)	COLD AQU. (sec)	CODE NOISE (cm)	CAR-RIER NOISE (cm)	DOP-PLER NOISE (cm/s)	COST (\$)
Lexon-GD Receiver	Javad	1-20	20	10	60	10	0.01		7,000
FlexPak-G2L Receiver	NovAtel	20	24	40	50	6 (L1) 25 (L2)	0.08 (L1) 0.2 (L2)	3	8,940
PolaRx2 Receiver	Septentrio	1-10	48	20	90	15 or 30 (w MP)	0.02 (L1) 0.01 (L2)	0.05 (L1/L2)	9,575

The principal advantage of selecting a dual-frequency receiver is that it can directly measure and remove the ionospheric delay error to within the receiver code measurement noise accuracy. This is currently one of the single largest GPS errors after the integer ambiguity error. However, the cost of these receivers is still considerably higher than the single frequency receivers.

In September 2005, a new generation of GPS satellite was launched. This satellite has both a civilian L1 and L2 signal. With this capability, future GPS receivers will be able measure the ionospheric delay directly without requiring the proprietary codeless techniques that are currently used by the dual-frequency receivers in Table 4. As a result, the slow reduction in dual-frequency receiver costs will significantly accelerate when a full constellation of these new satellites becomes available in 2010-2014.

The National Geodetic Survey, which manages the Continuously Operating Reference Station (CORS) network, has performed research that provides an indication of how accurately one can expect to estimate a baseline vector using

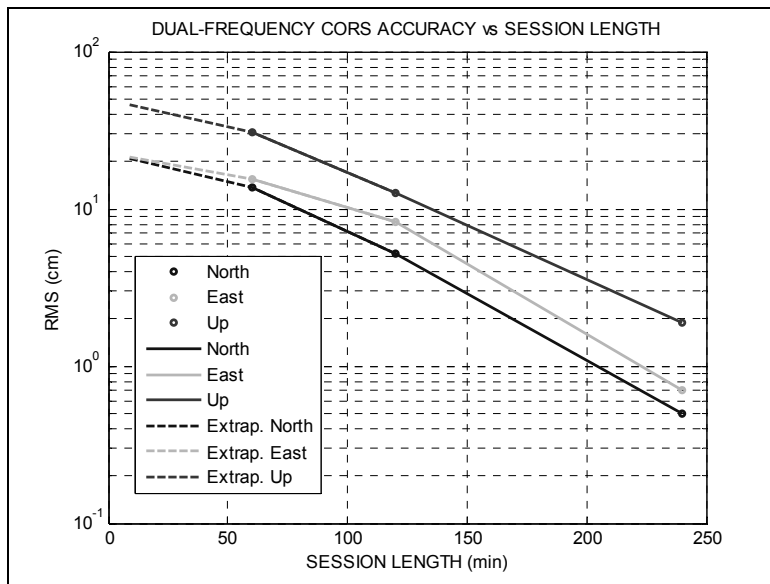
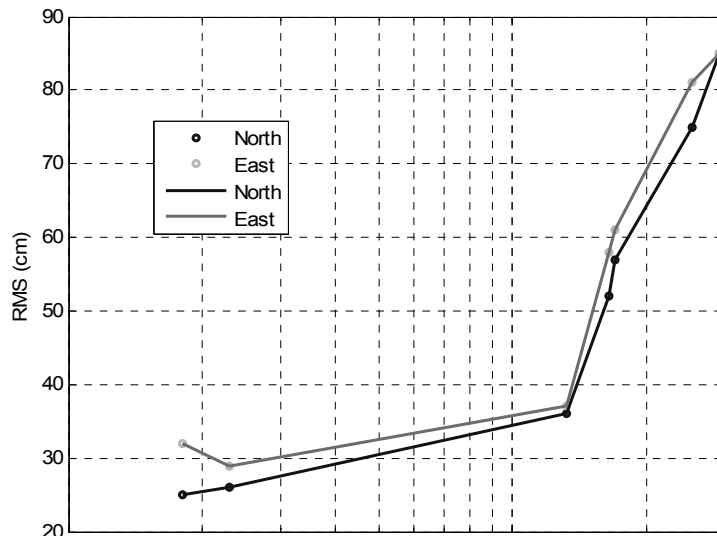


FIGURE 12 RMS Dual-Frequency Receiver Survey Accuracy vs. Initialization Time

accuracy of the fixed double-difference integers. The NGS dual-frequency results for stationary periods less than 4 hours conservatively did not attempt to fix the integer – used the float solution. The North and East accuracy for a 1-hour observation time span is around 15 cm.

3.5 LOW COST ACCELEROMETERS

The specifications for three low cost accelerometers and one inclinometer are presented in Table 5.

stationary single-frequency GPS receiver data [23]. Figure 11 shows how the accuracy of a single-frequency baseline vector estimate depends on the distance from a CORS reference station.

Combining the north and south position accuracy in this figure in a root-sum-square sense, leads to a horizontal position accuracy that ranges from 40 cm to 50 cm for baseline lengths between 15 km and 130 km. The figure also shows that there is a ‘knee’ in the curve at a baseline distance of 132 km (82 miles) after which the accuracy degrades significantly.

Since ionospheric delay is one of the key error sources and since it varies with baseline distance, the baseline-dependent accuracy in Figure 11 is probably due to incorrect knowledge of the ionospheric delay at the single frequency user site. The magnitude of the errors is probably due to the short 1-minute observation time spans. Hence, better accuracies would be expected for 15-30 minute observation time spans

The use of dual frequency user receivers is justified by the research performed by the NGS. This research established horizontal position accuracies of 20 cm using dual-frequency receivers initialized with 1 hour of stationary data as illustrated in Figure 12 [23-25].

It was also found that the dual-frequency carrier-phase user receiver accuracy using dual-frequency CORS reference receiver data was independent of baseline length for baselines ranging from 26 km to 300 km.

The observation time span for these stationary receivers, however, was found to be an important influence in determining the user receiver accuracy for both single and dual-frequency GPS user receivers. This initialization time dependence is due to the

TABLE 5 Solid State Accelerometer Specifications

Company	Crossbow Technology Accelerometer	Jewell Instruments Accelerometer	VTI Hamlin Accelerometer	VTI Hamlin Inclinometer
Model	CXL01LF	LCA-100	SCA110	SCA111T
Type	1 Axis	1 Axis	1 Axis	1 Axis?
Input Range	± 1 g	± 0.5 g	± 1.2 g	± 0.5 g
Temperature Range	-40° to 85° C ⁰	-55° to 85° C ⁰	-40° to 125° C ⁰	-40° to 85° C ⁰
Bandwidth	DC -50 Hz	60 Hz	400 Hz	8-28 Hz
Bias Error	15 mg	10 \square g	± 50 mg	± 3.5 mg
Noise Error	0.07mg/ $\sqrt{\text{Hz}}$	0.5 mg (rms)	0.02 – 0.04mg/ $\sqrt{\text{Hz}}$	0.02 – 0.04mg/ $\sqrt{\text{Hz}}$
Scale Factor Error	$\pm 3\%$ FS	0.05%	$\pm 2\%$ FS	-0.8 to +0.3% (0 to 70° C)
Input Axis Misalign Error	± 2 deg	1 deg (max)	± 1 deg (typical)	1.7 deg (max)
Temperature Sensitivity		0.1 mg/ $^{\circ}$ C (max)	0.5mg/ $^{\circ}$ C	0.1mg/ $^{\circ}$ C (typical)
Cost (1)	\$249	\$495	\$145	\$100
(10)	\$202	\$450	\$110	\$85
(100)	\$163	\$410	\$70	\$63

An inclinometer can be constructed by using a low-g accelerometer that is stationary and whose non-horizontal orientation measures the reaction force of gravitational acceleration. The focus on low-g accelerometers as a dead reckoning sensor for GVLS is motivated by the fact that rail vehicles typically do not accelerate or decelerate that fast. The key metric is the noise error.

3.6 LOW COST RATE SENSORS

Table 6 summarizes a set of low-cost solid state heading rate sensors, as well as a more expensive fiber optic gyro (FOG).

TABLE 6 Low-cost Rate Sensor Survey

Company	BEI Systron Donner	Kionix	Silicon Sensing Systems	KVH
Model	HZ1-90-100A	KX210-075	CRS03	E-Core 2000
Type	Quartz	MEMS	Quartz	FOG
Input Range	± 90 deg/sec	± 75 deg/sec	± 100 deg/sec	± 30 deg/sec
Temperature Range	-25° to $+70^{\circ}$ C	-40° to $+125^{\circ}$ C	-40° to $+85^{\circ}$ C	-40° to $+75^{\circ}$ C
Bandwidth	> 18 Hz	50 Hz	> 10 Hz	100 Hz
Bias Stability	< 0.05 deg/sec		± 1 deg/sec	0.0006 deg/sec
Noise Error	$< 0.025^{\circ}/\text{sec}/\sqrt{\text{Hz}}$	$0.14^{\circ}/\text{sec}/\sqrt{\text{Hz}}$	$\sim 0.005^{\circ}/\text{sec}/\sqrt{\text{Hz}}$	$0.0013^{\circ}/\text{sec}/\sqrt{\text{Hz}}$
Scale Factor Error	$< 0.05\%$ FR	$\pm 1\%$ FS	$\pm 1\%$	0.2% (rms)
Temperature Sensitivity	< 4.5 deg/sec	± 3 deg/sec	± 3 deg/sec	2% (rms)
Cost (1)	\$325		\$500	\$2895
(10)	\$270		\$250	\$2895
(100)	\$175		\$200	\$2495

The key metric is the noise error in addition to the cost. The best heading rate sensor for the GVLS application will depend on the required performance of this sensor. This performance will be established in the next section.

3.7 SYSTEM ARCHITECTURE

3.7.1 System Performance Requirements

Based on an action item received during the Stage 1 review, a more detailed examination was made of the GVLS system performance requirements. The GVLS system performance requirements are derived from the Positive Train Control (PTC) requirements. As stated in [26],

"The single most stressing requirement for the location determination system to support the PTC system is the ability to determine which of two tracks a given train is occupying with a high degree of assurance (an assurance that must be greater than 0.99999 or (09₅)). The minimum center-to-center spacing of parallel tracks is 11.5 feet..."

"The assurance of a navigation system is the probability over both time and area the services will be sufficiently robust to meet the requirements of the user. This differs from availability in that it goes beyond time and beyond a single navigation system..."

Based on comments received from the HSR-52 Review Panel, the 0.9₅ assurance requirement in [25] applies to a PTC overlay system, but not a full PTC system. In a PTC overlay system, existing centralized train control (CTC) technology is augmented with some of the PTC technology while under a full PTC system, the PTC technology is stand-alone. The Panel recommended that for the safety-critical requirements imposed onto PTC, that 0.9₇ or 0.9₈ assurance requirements are more appropriate.

Based on this requirement, the GVLS must be able to determine on which of several parallel tracks a maintenance vehicle is located when the tracks are as little as 11.5 feet (3.5 m) apart. The GVLS must also establish whether the maintenance vehicle is on a track or near the tracks. The location must be established with an assurance level of 99.99999% (0.9₇) or 99.999999% (0.9₈) over a period of time and a region of the rail network. Also, this assurance level applies not only to the performance of the GVLS but also to the robust operation of the hardware and software.

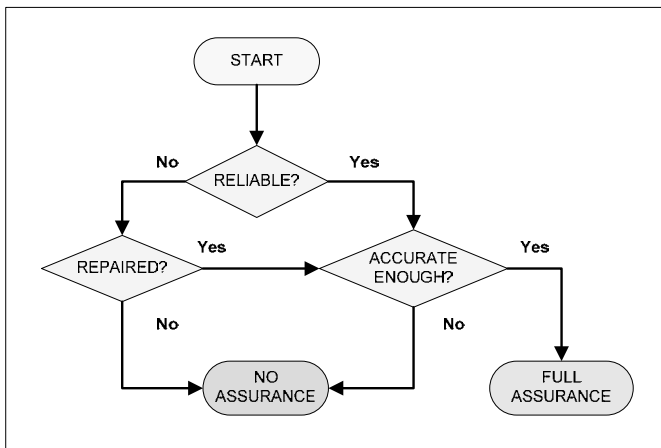


FIGURE 13 System Assurance (Confidence) Diagram

Determining whether the GVLS is operating with a specified level of assurance is a multi-step process, as illustrated in Figure 13. Hence, assurance is a function of GVLS availability and GVLS accuracy. GVLS availability, in turn is a function not only of the GVLS reliability but also the 'repairability' of GVLS if it fails.

The required assurance is the product of the performance level multiplied by the availability level. Hence if the performance and availability levels are set equal to each other and the assurance level is 0.99999990 (0.9₇), then these two levels are each 0.99999995 as summarized in Table 7.

The GVLS lateral accuracy requirement under a 0.99999995 confidence level is 0.34 m. This is much more stringent than the 0.40 m accuracy requirement for a 0.999995 confidence level. However, these accuracy requirements apply to the time at which a decision has to be reached whether the maintenance vehicle is on one

of several parallel tracks or next to one of the tracks. Hence, multiple measurements can be combined to achieve the equivalent accuracy of 0.34 m.

For instance, multiple lateral position measurements can be averaged over a period of time. The lateral positions might be

taken directly (straight track) or referenced to one of the tracks (curved track), as determined from the rail database. The time period for which this average is computed would start at the point when the vehicle passed over a turnout, as determined from along-track position and the rail database. The time period would conclude when a decision must be made whether the vehicle is on the correct track. With a single time point

TABLE 7 GVLS System Requirements

Parameter	Requirement
Accuracy Confidence Level	0.99999995
Lateral position accuracy requirement (1 σ):	(0.34 m)
Lateral average position accuracy requirement if 1minute averaging is used (1 σ):	(0.40 m)
Availability Confidence Level	0.99999995
Assurance Level (Performance x Availability)	0.99999990

lateral position accuracy of 0.40 m, an equivalent lateral position accuracy of 0.34 m is achieved after 1 minute of averaging. This calculation is based on the statistics of this problem and using an autocorrelation time constant of 3 min [10].

Based on Table 7, the required availability confidence level is 0.99999995. This value includes not only the availability of the GVLS hardware and software but also the availability of the GPS and HA-NDGPS service that is used.

Currently the GPS service availability is 0.99 while the NDGPS availability will be 0.999 when dual-station coverage is achieved throughout the Continental US in 2008 [27]. While NDGPS uses the GPS service, it is able to achieve the higher availability because it provides a more thorough and timely monitoring of the GPS satellite integrity. Hence, it is anticipated that with use of the HA-NDGPS service dual-station coverage, an availability of 0.999 will be achieved.

If the GVLS mean time between failures (MTBF) is 10,000 hours and its mean time to repair (MTTR) is 22 hours or less, then the availability probability is 0.998, when the GPS/HA-NDGPS service is available. This MTTR could be achieved if a failed GVLS unit is replaced with a new one, rather than attempt to repair the failed unit in the field. If a dual-redundant GVLS system architecture is employed and the two systems are fully independent, then dual-redundant GVLS availability of 0.999995.

Hence if the HA-NDGPS service availability is combined with the basic GVLS availability, a combined system availability of 0.998995 is achieved but not 0.99999995. To achieve a higher availability will require another approach. One strategy is to incorporate a highly reliable, reasonably precise dead reckoning system. This dead reckoning system would then be periodically updated with the HA-NDGPS system measurements. This approach will have to be explored further in the future.

3.7.2 Error Budget

The error budget in Table 8 focuses on the key system error sources. The goal is to constrain the errors such that the total GVLS cross-track position accuracy is less than 0.40 m (1σ) if 1 minute of averaging is employed to get the higher accuracy. This accuracy must be achieved whether GPS is available or not.

TABLE 8 Summary of Cross Track Position Accuracy Requirements (With Map Match)

Data Sources	Required Lateral Position Accuracy (1σ , m)	
	With GPS	Without GPS*
Vehicle Cross-track Position Accuracy:		
Filter Position (0.38 m)	0.38	
Map Match Position		0.38
Rail Database Access Accuracy (No. 33 Turnout):		
Distance (1.5 m for Database Access)	0.04	0.04
Speed (0.0015 m/s)		0.01
Accelerometer ($0.10\text{mg}/\sqrt{\text{Hz}}$)		0.03
Rail Database Survey Accuracy:		
Rail Database:	0.08	0.08
Total GVLS:	0.39	0.39
PTR Requirement (1 min averaging will be used):	0.40	0.40

*Using Map Matching, 150 sec duration

4 INVESTIGATION

4.1 CDGPS ALGORITHM SELECTION

4.1.1 General Algorithm

A review of the extensive carrier phase differential literature has yielded only one reference that addresses both the long-baseline problem of GVLS and also incorporates the future HA-NDGPS reference station network [1]. This reference, which is a doctoral thesis by Dr. Yujie Zhang, reviewed the CDGPS literature and selected a multi-step algorithm as illustrated in Figure 14.

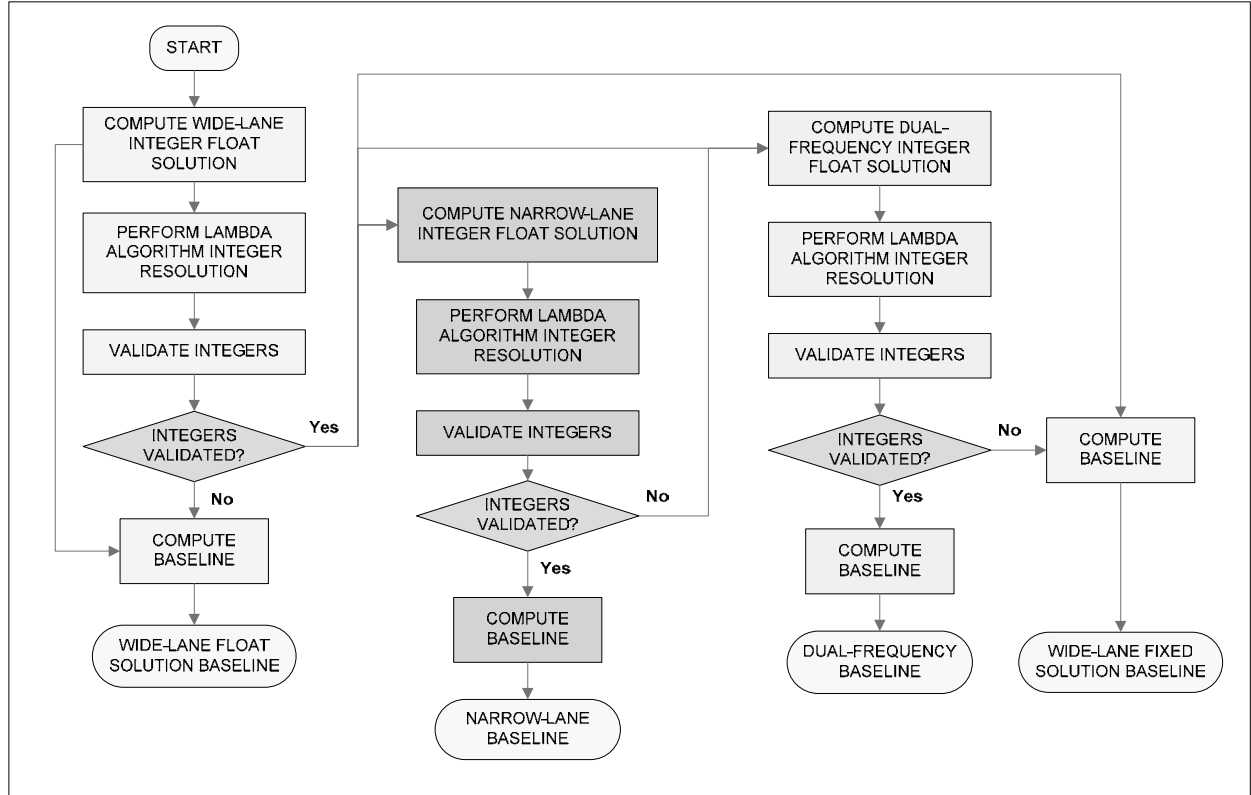


FIGURE 14 Multi-Step Dual-Frequency CDGPS Algorithm Functional Flow

This algorithm uses a three-step approach as indicated by the three columns in Figure 14. The methodology focuses on determining the carrier phase integer ambiguities. Once the integer ambiguities have been determined, determination of the receiver location with carrier phase data becomes as simple as computing the location with GPS code data.

The first column derives the wide-lane integers. These are obtained by adding the L1 and L2 carrier phase measurements to obtain a composite measurement with a wavelength of 86.2 cm. The L1 and L2 wavelengths are respectively 19.0 and 24.4 cm. The significance of the measurement wavelength is that the floating integer solution has to be accurate to a fraction (<0.5) of the wavelength in order for the LAMBDA integer resolution to be successful. Hence, with the larger wide-lane measurement wavelength, there is more tolerance for errors up to about 43 cm.

The second column describes a narrow-lane integer identification process, which is referred to as the iono-free integer process in [1]. The narrow-lane measurement is obtained by subtracting the L1 from the L2 carrier phase measurement. The narrow-lane measurement wavelength is 10.7 cm. Hence the narrow-lane errors must be less than about 5 cm in order for the narrow-lane integers to be resolved. This is accomplished in part by using the wide-lane integers as input.

If the narrow-lane integer resolution is unsuccessful, the dual-frequency integer resolution process in the third column of Figure 14 is pursued. This approach uses the L1 and L2 carrier measurements separately with their respective wavelengths of 19.0 and 24.4. Hence, for the L1 integer to be resolved, the errors in this processing step must be less than 9.5 cm. This is accomplished in part by using the wide-lane integer from the wide-lane integer processing step in the first column.

To summarize the processing performed in Figure 14, the wide-lane integers must be resolved before the narrow-lane integer processing is performed. If the narrow-lane integer processing using the wide-lane integers is unsuccessful, the L1-L2 processing is performed. The L1-L2 processing also uses the wide-lane integers. Hence, this last processing step is a fall-back step, in case the very stringent narrow-lane processing is unsuccessful.

4.1.2 GVLS Algorithm

For the dynamic (kinematic) carrier phase application required by GVLS, the architecture of Figure 15 is proposed. This figure utilizes the first and third processing steps shown in Figure 14.

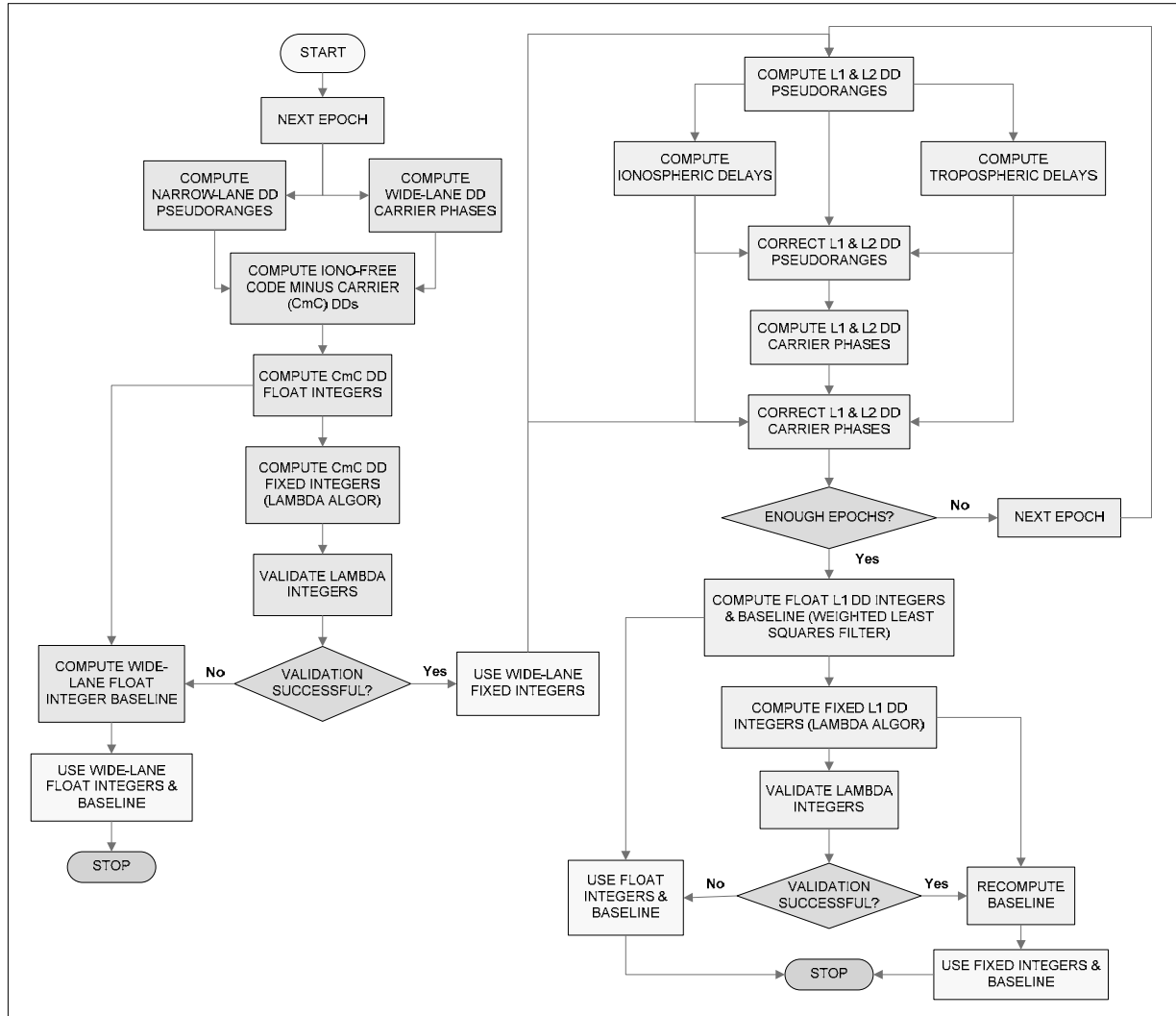


FIGURE 15 GVLS Multi-Step Dual-Frequency CDGPS Algorithm Functional Flow

A summary of the common and alternative approaches to implementing the CDGPS algorithms are summarized in Table 9. Key differences include the use of only two algorithms in the Sensis implementation. Also, the Wide Lane (WL) and Dual Frequency (DF) algorithms incorporate separate Kalman Filters in the Sensis approach. These Kalman Filters are used to not only estimate the float integers but also the undesired multipath errors. In the Zhang approach, multipath is estimated in a separate process called Wavelet Decomposition, an approach similar to discrete Fourier series analysis.

TABLE 9 Comparison of CDGPS Algorithm Implementations

Features	SENSIS Approach	Zhang Approach
CDGPS Algorithms	1. Wide Lane (86 cm) 2. Dual Frequency (19/24 cm)	1. Wide Lane (86 cm) 2. Narrow Lane (11 cm) 3. Dual Frequency (19/24 cm)
Wide Lane Algorithm Filtering	Kalman Filter	None – scalar inversion
Wide Lane Multipath Estimation	Estimated with Kalman Filter	Wavelet Decomposition
Dual Frequency Algorithm Filtering	Kalman Filter	Least Squares Filter
Dual Frequency Multipath Estimation	Estimated with Kalman Filter	Wavelet Decomposition
Fixed Integer Resolution	LAMBDA Algorithm	LAMBDA Algorithm

4.2 CDGPS ALGORITHM IMPLEMENTATION

4.2.1 Input Data Processing

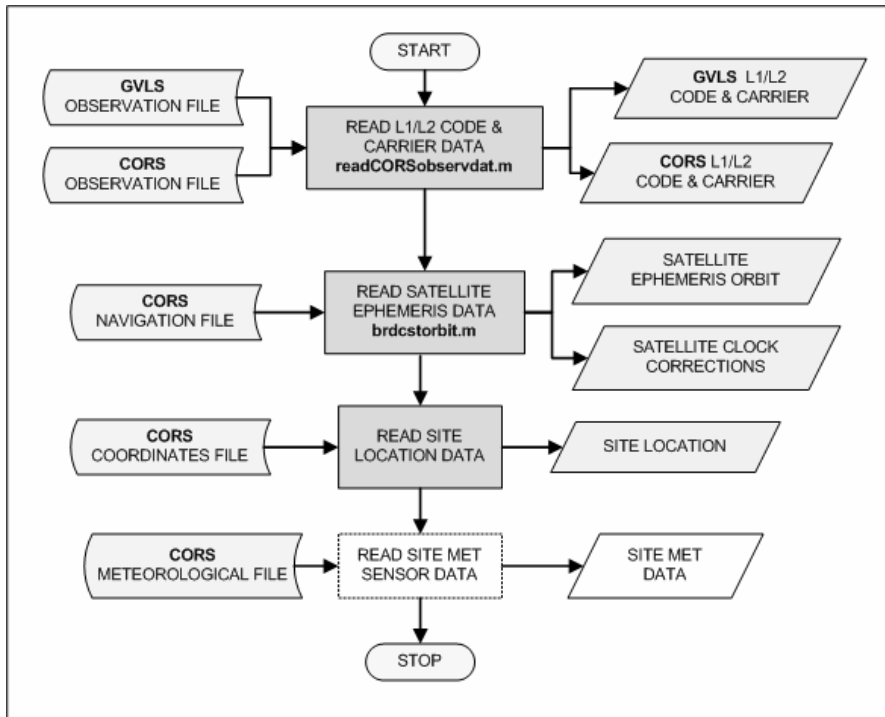


FIGURE 16 Input Data Processing Flow Diagram

The input data comes from different sources as shown in Figure 16. Matlab m-file functions are used to read these data files. The Matlab functions are shown in bold with lower-case names that end with 'm'(.m), in the middle column of Figure 16.

The principal data consists of the dual frequency (L1/L2) code and carrier measurement data that is archived in site-specific observation files.

Another key data source is the ephemeris data required to determine the GPS satellite orbit. The CORS navigation file contains the ephemeris that was broadcast by the satellites at the time the GPS measurements were recorded.

The predicted orbit data that is available is summarized in Table 10. As shown in Table 10, the only viable orbit data sources for GVLS are the real-time sources.

The standard real-time orbit data sources are the GPS broadcast ephemeris or the FAA Wide Area Augmentation System (WAAS) orbit. The former just requires a standard GPS receiver while the latter requires a WAAS-enabled GPS receiver. The International GPS Service (IGS) also offers four levels of orbit data. Only the predicted ultra-rapid orbit, however, is available in real time. Since the accuracy of this orbit is considerably higher than the other real-time sources, this option is very desirable. However, a real-time or near real-time IGS data link must also be used for the GVLS vehicle to take advantage of this accurate orbit data.

As shown in Figure 16, for now both the GPS broadcast ephemeris and the IGS predicted ultra-rapid orbit will be considered and evaluated. The accuracy of the Broadcast Orbit was derived by comparing it to the IGS Final Orbit, as shown in Figure 17 and Table 11. The orthogonal ECEF orbit position errors have been translated to the line-of-sight slant range from a particular receiver site (ZOA1, Oakland, CA).

As can be seen from this figure, the Broadcast Orbit errors have a large mean error but a small (cm-level) random component. With the data processing that will be discussed in the following section, the mean error is minimized by

TABLE 10 Comparison of GPS Ephemeris and Clock Correction Data [1]

Data Source		Real-Time?	Orbit Accuracy (cm)	SV Clock Accuracy (ns, [cm])	User Requirements
GPS Broadcast Ephemeris		Yes	~200	~7 [200]	GPS receiver
WAAS* Corrected Orbit & SV Clock Correction		Yes	~130	~7 [200]	WAAS GPS receiver
IGS**	Predicted Ultra-Rapid Orbit	Yes	~10	~5 [150]	IGS data link
	Observed Ultra-Rapid Orbit	3 hr latency	<5	~0.2 [6]	
	Rapid Orbit	17 hr latency	<5	~0.1 [3]	
	Final Orbit	13 day latency	<5	~0.1 [3]	

* WAAS – Wide Area Augmentation System. ** IGS – International GPS Service

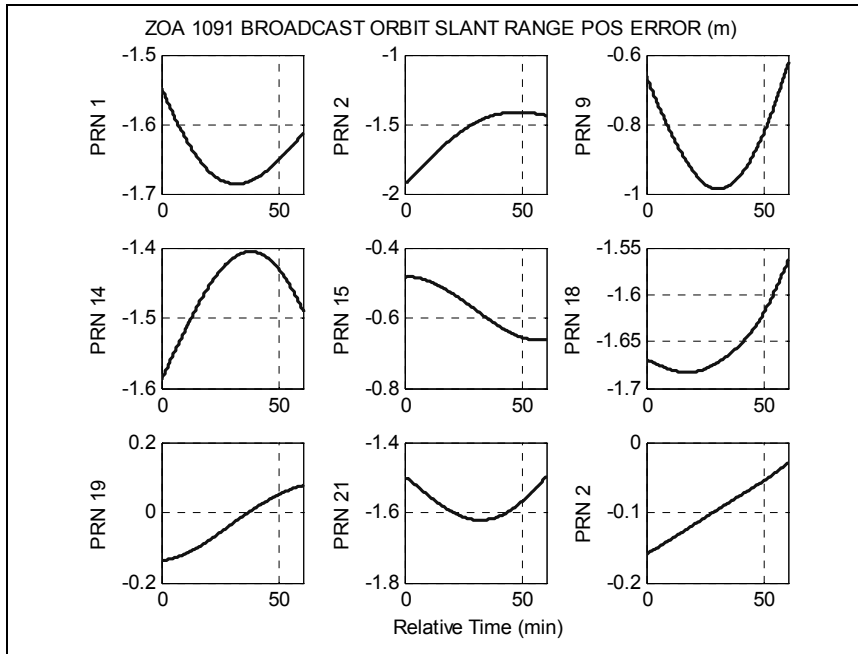


FIGURE 17 Broadcast Orbit Slant Range Error Histories

TABLE 11 Broadcast Orbit Slant Range Error Statistics

PRN	Mean (m)	Sigma (m)
1	-1.648	0.036
3	-1.561	0.161
9	-0.863	0.105
14	-1.459	0.051
15	-0.575	0.065
18	-1.655	0.033
19	-0.033	0.071
21	-1.575	0.039
22	-0.097	0.037

differencing the slant ranges from two receiver sites to the same satellite. Hence, the net Broadcast orbit error is closer to the random position error shown in Figure 17.

4.2.2 Wide-Lane Baseline Algorithm

Once the measurement and auxiliary data has been read and reformatted and the data preprocessing has been completed, the first part of the CDGPS algorithm is evaluated. This algorithm is the wide-lane baseline algorithm that is illustrated in Figure 18.

The CmC measurements are used to determine the float integers for each satellite using a Kalman Filter. This filter not only estimates the integer float solution but also the unwanted multipath errors. This float integer, together with an estimate of the covariance matrix for this integer, is passed to the LAMBDA algorithm to determine candidate fixed integer estimates. If this fixed integer validation is successful, then the wide-lane fixed integers are used. Alternatively, the float integers are used.

In the former case, the fixed wide-lane integers are passed on to the second part of the CDGPS algorithm that is described in the next section. In the latter case, the processing stops and does not progress to the second part of the CDGPS algorithm.

4.2.3 Dual-Frequency Baseline Algorithm

If the wide-lane float integers were successfully fixed, the dual-frequency baseline algorithm will be used as illustrated in Figure 19. This is the second part of the CDGPS algorithm that was illustrated in Figure 15.

As shown in Figure 19, this algorithm starts out again with the time-corrected L1/L2 pseudorange and carrier phase measurements. In addition, the wide line integer, which is a function of the L1 and L2 carrier phase integer, is incorporated from the first processing step.

First, the code and carrier are corrected for estimates of the iono and tropo delays. The iono delays are obtained using the dual-frequency code pseudoranges. The tropo delays are obtained using the Hopfield tropospheric delay model. In addition, the wide-lane integer is applied to the L2 carrier phase measurements. With this step, both the L1 and L2 carrier phases are left with an unknown L1 integer.

Next, the L1 and L2 pseudoranges and carrier phases are individually double-differenced to produce four equations for each satellite. Using a Kalman Filter and the line-of-sight angles to the satellite, the baseline distance between the GVLS receiver and the reference receiver is estimated. This Kalman Filter also estimates the L1 and L2 code multipath errors as well as the float L1 integer.

4.3 CDGPS ALGORITHM PERFORMANCE EVALUATION

4.3.1 Evaluation Approach

Archived GPS data from multiple CORS sites was used to evaluate the performance of the GVLS CDGPS algorithm. Three sites from a set of Northern California CORS sites were selected. The three sites included ZOA1, a FAA WAAS site located in Oakland. The second site is the PPT1 site, a NDGPS site, located on the Pacific Coast, southwest of San Jose at Pigeon Point. Finally, the third site is LNC1, a NDGPS site, located in South Sacramento.

The relative distance between ZOA1 and PPT1 is 52 km – a short baseline case. The relative distance between ZOA1 and LNC1 is 156 km – an intermediate baseline case. Finally, the relative distance between PPT1 and LNC1 is 206 km – a long baseline case.

For these three sites an initial 2 hour period of data was obtained from the CORS archives corresponding to the peak ionospheric delay period of 1-3 PM local time. To simplify the data processing, a set of 9 satellites were selected out of a total of 13 visible satellites. These selected satellites could be seen simultaneously by all three sites. In addition, the code and carrier measurements appeared to be relatively well-behaved – no apparent carrier phase cycle slips nor lost data. With these constraints, a final 1-hour period of data was selected.

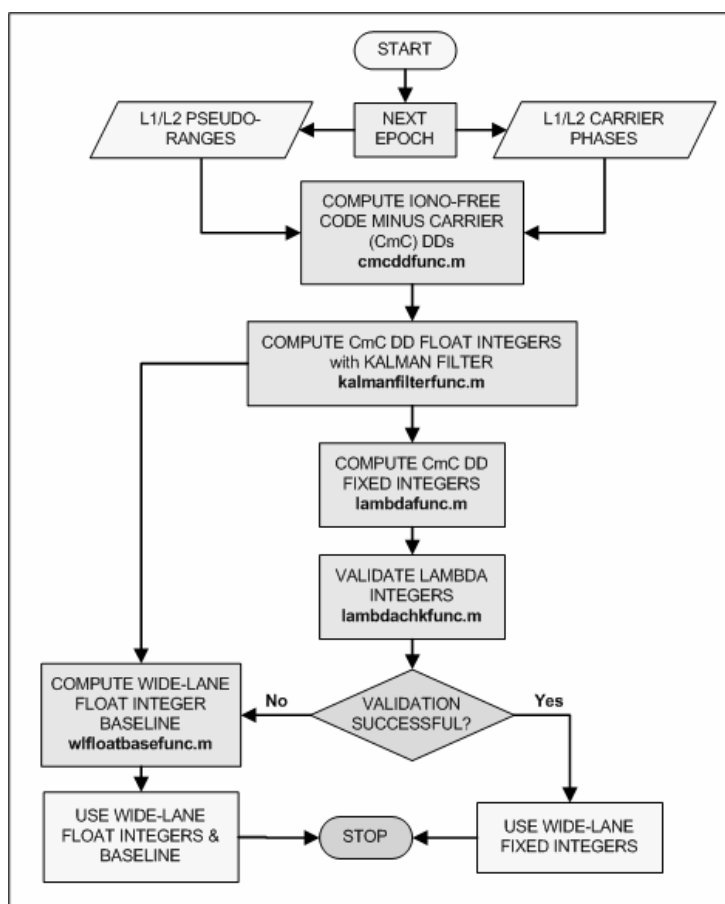


FIGURE 18 Wide-lane Baseline Algorithm Flow Diagram

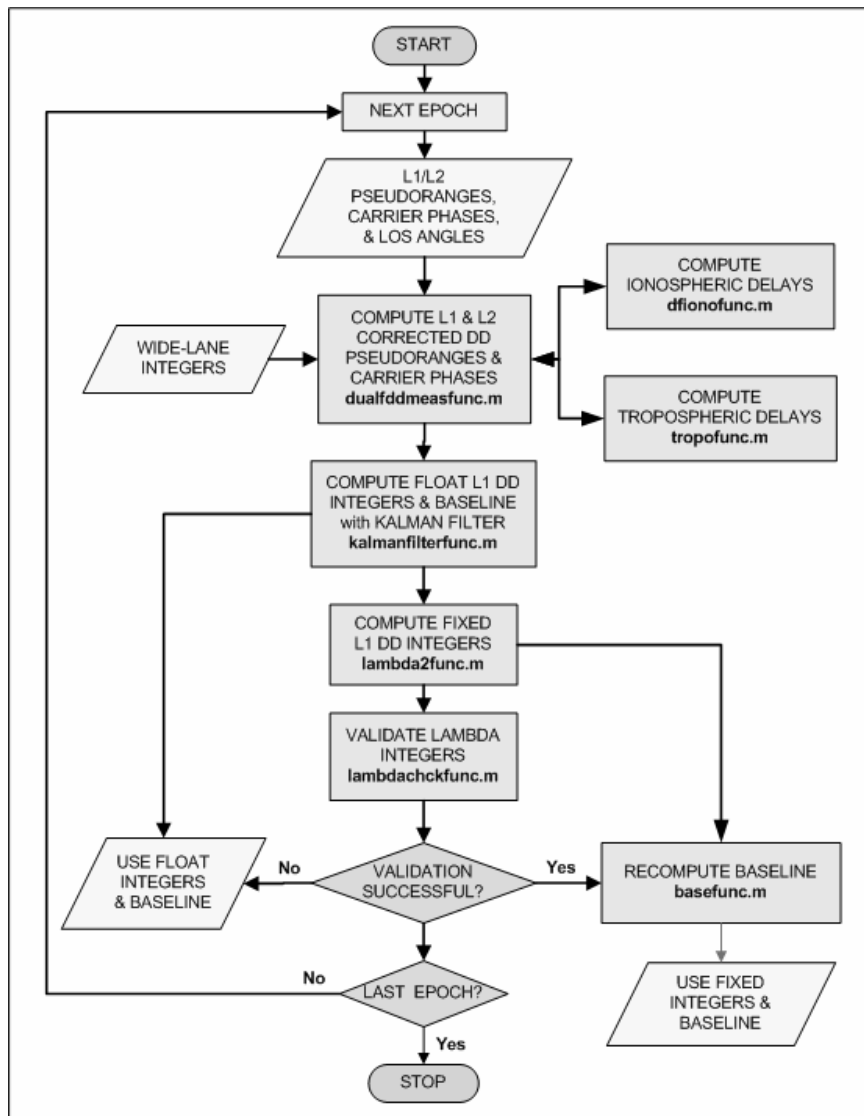


FIGURE 19 Dual Frequency Baseline Algorithm Flow Diagram

The position error statistics are summarized in Table 12 for these three cases. The statistics were computed from 10–60 minutes, assuming a 10-minute initialization period. From the horizontal rms position error results, it can be seen that only the short baseline results meet the required rms position accuracy of 0.38 m.

4.4 WIDE LANE RESULTS

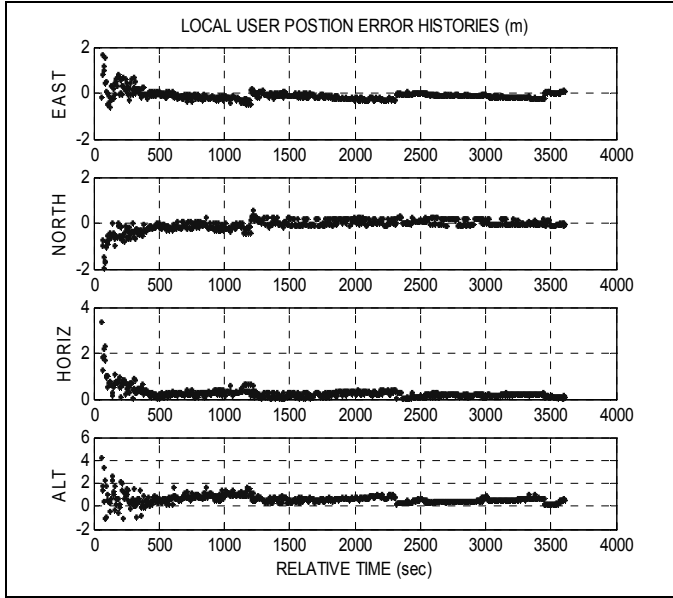
4.4.1 Position Accuracy

The Wide Lane algorithms position error histories are presented in Figure 20 – 22. Figure 20 presents the ZOA1-PPT1 short baseline position error results in North, East, Horizontal, and Vertical position for ZOA1. The histories cover a 1-hour period.

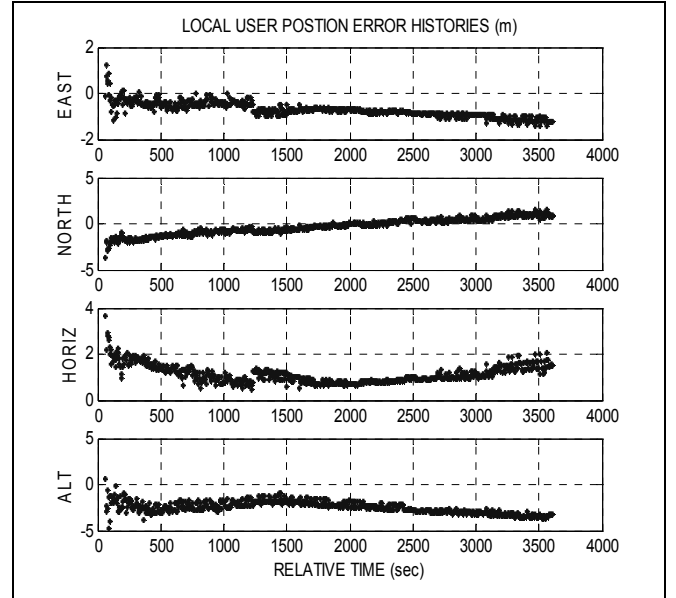
Similarly, Figure 21 presents the intermediate baseline position error histories for ZOA1-LNC1. Finally, Figure 22 presents the long baseline position error histories for the long baseline case corresponding to PPT1-LNC1.

The key error history that is of interest is the horizontal position error history. The goal is to achieve a 1-sigma or root-mean-square (rms) horizontal position error of less than 0.38 m. For a 95% confidence limit, this requires that the horizontal position error histories be approximately within 0.76 m.

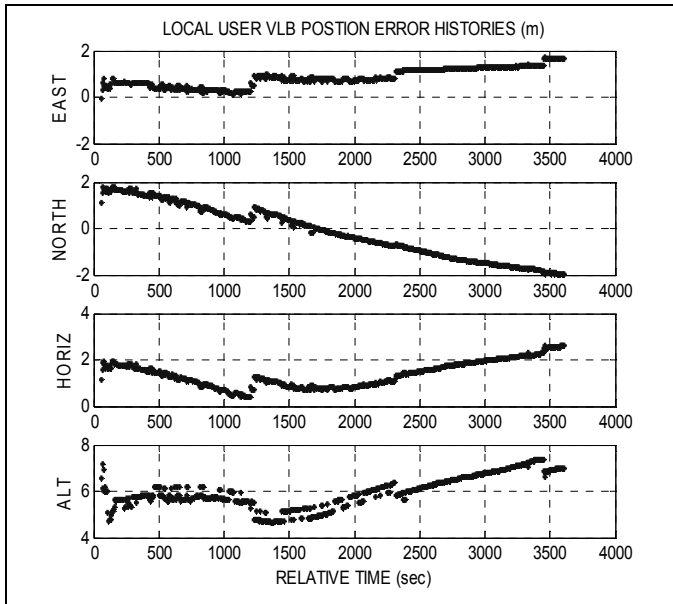
As can be seen, only the short baseline case is able to meet this criterion throughout the 1-hour data period. It should be noted, however, the primary function of the WL algorithm is to provide good double-difference integer ambiguity estimates to the DF algorithm. Hence, the position error histories for the WL case only provide a quality check on the WL results.



**FIGURE 20 WL User Position Estimate Errors
(52 km Baseline)**



**FIGURE 21 WL User Position Estimate Errors
(156 km Baseline)**



**FIGURE 22 WL User Position Estimate Errors
(206 km Baseline)**

TABLE 12 Wide Lane User Position Error Statistics (m, After 10 min Initialization)

	Short Baseline (52 km)			Intermediate (156 km)			Long Baseline (206 km)		
Error	Mean	Sigma	RMS	Mean	Sigma	RMS	Mean	Sigma	RMS
East	-0.142	0.104	0.176	-0.782	0.253	0.822	0.903	0.403	0.989
North	0.010	0.152	0.152	-0.056	0.678	0.680	-0.463	0.968	1.073
Altitude	0.615	0.258	0.667	-2.515	0.596	2.585	5.994	0.728	6.038
Horizontal	0.209	0.102	0.232	1.028	0.283	1.066	1.334	0.590	1.459
Required Horizontal			0.38			0.38			0.38

4.5 DUAL FREQUENCY RESULTS

Using the integer estimates from the WL algorithm, the DF algorithm determined the necessary DF float integers. This was accomplished with a second Kalman Filter that estimates the baseline, L1 float integers, and the undesired L1 and L2 multipath errors.

The float integers were fixed when the candidate fixed integers satisfied a specified criterion. The criterion is that the second best candidate must be at least 50% different from the best candidate for the best candidate to be accepted. If this criterion was not satisfied, both candidates were rejected and the float integer solution was used.

Using the integer solution to correct the L1 carrier phase measurements, the receiver position estimates were computed for the three sites as shown in Figures 23-25.

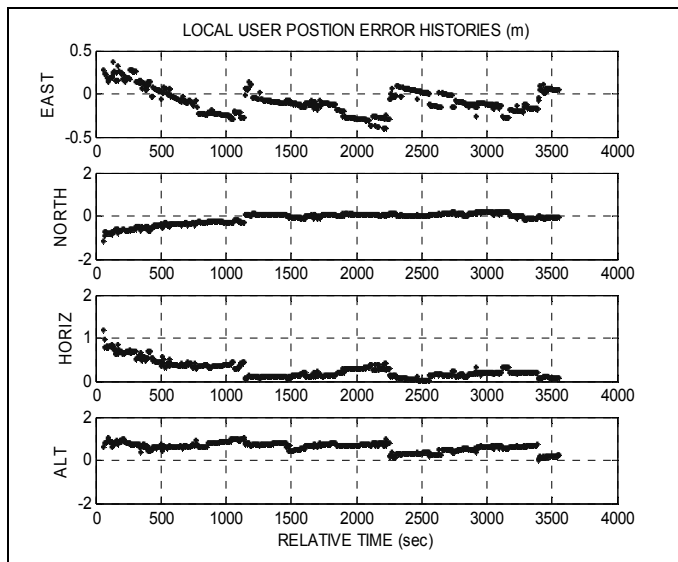


FIGURE 23 DF Receiver Position Estimate Error Histories (52 km Baseline)

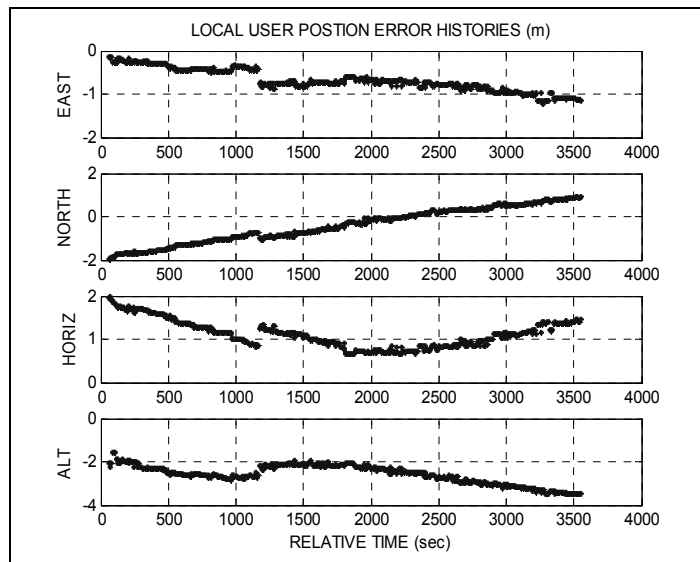


FIGURE 24 DF Receiver Position Estimate Error Histories (156 km Baseline)

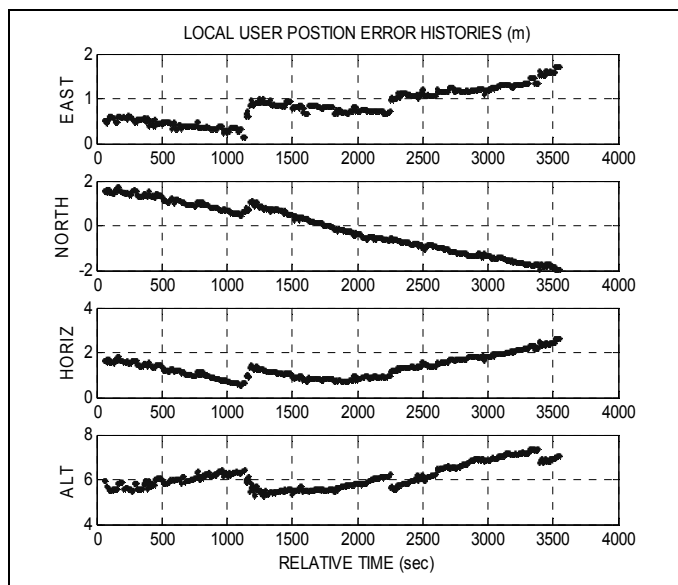


FIGURE 25 DF Receiver Position Estimate Error Histories (206 km Baseline)

The corresponding position error statistics are summarized in Table 13. The statistics in this table correspond to the time interval 10-60 minutes, assuming a 10-minute initialization period.

As can be seen from this Table, only the short baseline results are able to meet the required rms position accuracy of 0.38 m. From the short and intermediate baseline results, the required position accuracy of 0.38 m can be met to a maximum baseline distance of over 70 km.

TABLE 13 Dual-Frequency User Position Error Statistics (After 10 min Initialization)

	Short Baseline (52 km)			Intermediate (156 km)			Long Baseline (206 km)		
Error	Mean	Sigma	RMS	Mean	Sigma	RMS	Mean	Sigma	RMS
East	-0.128	0.109	0.168	-0.751	0.209	0.779	0.904	0.362	0.974
North	-0.021	0.153	0.154	-0.182	0.665	0.689	-0.365	0.948	1.016
Altitude	0.600	0.206	0.634	-2.655	0.431	2.690	6.181	0.570	6.207
Horizontal	0.200	0.109	0.228	1.015	0.225	1.040	1.308	0.519	1.407
Required Horizontal			0.38			0.38			0.38

4.6 ANALYSIS SUMMARY

In comparing the WL and DF position error statistics, as presented in Tables 12 and 13, not much improvement is noted with the DF algorithm processing. One obvious explanation is that the DF requires a good integer estimate from the WL algorithm for it to achieve better accuracy than the WL algorithm.

At this point, it is worthwhile to examine the fundamental WL and DF position algorithms and determine the different error sources that contribute to the horizontal position error for each algorithm. This examination was performed and summarized in Table 14. This table was obtained by determining the linear error sources that contributed to the horizontal position errors.

The input into these linear error models are the one sigma/rms error estimates shown in the second column in Table 14.

In many cases, such as the dual frequency ionospheric delay, integer ambiguity, and receiver noise, the fundamental error source is the receiver code or carrier measurement noise (0.10 and 0.001 m, respectively).

In other cases, such as for the orbit errors, consideration had

TABLE 14 GVLS CDGPS Error Budget with Broadcast Orbit

Error Source	Error (Code/ carrier, m)	DD WL Algorithm Error	DD DF Algorithm Error
Broadcast Orbit	2.00	0.10*	0.10*
Satellite Clock	0	0	0
Dual Frequency Ionosphere	0.28	0.56 (0.42**)	0.42**
Troposphere	0.04	0.08	0.08
Multipath (P/ Φ)	1.4/0.014	0.04	0.03
Integer Ambiguity		0.16	0.03
Receiver Noise (P/ Φ)	0.10/0.001	0.003	0.002
Total Range		0.60 (0.47)	0.45
Horizontal Position	DD HDOP: 0.8	0.48 (0.38)	0.36
Required Horizontal Position		0.38	0.38

* [1], DD for 300 km baseline

** Filtering assumed to reduce raw dual-frequency DD ionospheric delay error by 25%

to be given to the effect that double differencing has on the input errors. Hence, while the orbit error is approximately 2.00 m, as shown in Table 10, most of the error is a bias error as shown in Figure 17 and Table 11. This bias error is eliminated for closely-spaced (short baseline) receiver pairs and has only a small contribution for longer baseline cases.

One of the key error sources that remain is the ionospheric delay error. When using L1 and L2 code measurements, the ionospheric delay estimate is corrupted by a number of errors. In addition to the code measurement noise errors, the estimate is corrupted by multipath errors and receiver and transmitter line bias errors. By double-differencing the ionospheric delays, the line biases are eliminated, the multipath errors may be somewhat reduced, but the measurement error is compounded as shown in the third and fourth column. Hence, one approach is to filter the slowly-changing ionospheric delay estimates to estimate the unwanted multipath errors and to smooth the code measurement noise errors. In Table 14, this filtering approach is estimated to improve the ionospheric delay accuracy by about 25%.

When the total range error is scaled by the double difference Horizontal Dilution of Precision (HDOP), both the WL and the DF algorithms are able to achieve the required horizontal position accuracy, assuming the ionospheric delay estimates are filtered. Note that while the HDOP is typically around 1.5 (1.2-2), the double-difference HDOP is smaller: 0.8 (0.6-1.0). This latter estimate was computed directly for the 1 hour of data processing that was done. Based on the results presented in Table 14, it appears that the GVLS performance requirements could be satisfied with a single WF algorithm.

5 PLANS FOR IMPLEMENTATION

5.1 FUTURE DEVELOPMENT EFFORTS

The current contract explored the feasibility of the GVLS concept. This was accomplished by focusing on the key technology that will determine the concept feasibility – the CDGPS position algorithm. As shown in Figure 26, there are at least two additional development steps required before a finished product is available for the marketplace.

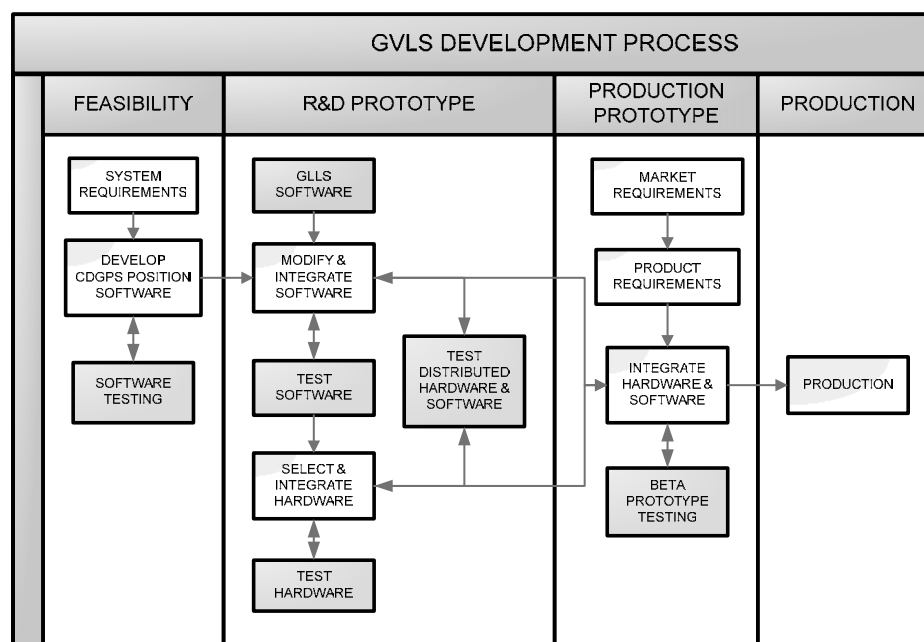


FIGURE 26 GVLS Development Process

The second step is the development and testing of an R&D prototype. The key features of this prototype are that all the prototype hardware is assembled and tested. Also, all the prototype software is assembled and tested on a laptop computer. Finally, the R&D prototype hardware and software are tested together.

Having validated the feasibility of the GVLS system using the R&D prototype hardware and software, the final step is to develop and test a production prototype GVLS system. The design of this production prototype will be driven by the results obtained with the R&D prototype. In addition, it will be driven by the current and future market requirements at the start of the production prototype system development. After the production prototype requirements have been established, the hardware and software will be integrated. Next, the production prototype will be tested internally, as an alpha prototype. This will be followed by a beta prototype testing where pre-production units will be made available to prospective customers for testing. The results of all the tests will then be incorporated in the final prototype design.

5.2 FUTURE TECHNOLOGY DRIVERS

The GVLS concept involves a number of supporting technology developments. First, the HA-NDGPS network must be deployed. If the necessary funding is obtained annually for its continued deployment, dual-redundant receiver coverage will be available by 2010. This suggests that by 2008, single station coverage will be available. This would be sufficient to field the GVLS technology.

Second, the GVLS concept assumes that reasonably priced dual-frequency receivers will be available at the time the GVLS product is available on the market. This requires that the next generation of GPS satellites, which broadcast a civilian L2 signal in addition to the currently available L1 signal, will be in orbit in sufficient numbers.

While the satellites were originally expected to be launched starting in 2003 [28], the first one was not launched until September 2005. If the original deployment rate of these satellites continues starting September 2005, at least 12 of the 24 GPS satellites would be in orbit by 2009. This would mean that in 2009, of the typically 12 visible satellites, half would broadcast the civilian dual frequency signal. This number is sufficient for the CDGPS algorithms discussed in this report to perform the necessary position estimation. However, the accuracy will not be as high as when the full 24 modernized satellite constellation is available, around approximately 2013.

6 SUMMARY

6.1 CONCLUSIONS

Under this contract, a candidate CDGPS algorithm has been identified to meet the stringent PTC position performance requirements needed for maintenance vehicles operating in PTC territory. The algorithm uses a 2-step sequential approach to determine the precise GVLS cross-track position with GPS code and carrier phase measurements. This algorithm is a modified version of a 3-step algorithm, described in an Ohio University PhD thesis, which meets the stringent GVLS accuracy requirements for the longer baseline distances to the nearest reference station.

The CDGPS algorithm takes measurements from a dual-frequency GVLS receiver and combines them with real-time dual-frequency measurements from a network of reference receivers. These reference receivers are provided by the high accuracy upgrade to the current National DGPS network – the HA-NDGPS network.

The decision to switch from a single frequency GVLS receiver to a dual-frequency receiver was motivated by the need to provide precise position accuracy under normal GPS conditions as well as during the periods of higher solar activity. Under these higher solar conditions (peaking every 11 years), the ionospheric delay corrections must be determined directly rather than with models, since this is one of the largest GPS error sources. With dual-frequency receivers, this error source can be determined directly and removed, within the accuracy of the measurements that are used to compute it. As a result, the CDGPS position accuracy will be nearly independent of the baseline separation distance between the GVLS receiver and the reference receiver.

Currently the key drawback to using dual-frequency GPS receivers is that they are considerably more expensive than single frequency receivers. This price difference will be significantly reduced when a full constellation of the new generation of GPS satellites is available in the 2010-2014 time frame. The reason for this expected dual-frequency receiver price reduction is based on the fact that with the new GPS satellites, non-proprietary algorithms can be used to determine the ionospheric delay.

The performance capability of the proposed GVLS CDGPS algorithm was evaluated with archived dual-frequency GPS data. This data was obtained free over the Internet from the Continuously Operating Reference Station (CORS) network. Based on the CORS data analysis for short (52 km), intermediate (156 km), and long baselines (206 km), it was not possible to demonstrate the required position accuracy for the intermediate and long baseline cases with the proposed GVLS CDGPS algorithm.

Based on linear error analysis of the GVLS CDGPS algorithms, it is believed that the GVLS CDGPS algorithms have the potential for meeting the stringent position accuracy requirements for the longer baselines. Furthermore, the GVLS CDGPS approach, consisting of a Wide Lane (WL) algorithm followed by a Dual-Frequency (DF) algorithm might be able to achieve the required accuracy with just the WL algorithm. This requires that the estimated ionospheric delay computed from dual-frequency code measurements be filtered to remove unwanted multipath errors and reduce the code noise errors.

6.2 RECOMMENDATIONS

It is recommended that a field test be performed to evaluate the CDGPS algorithm with vehicle (dynamic) GPS measurement data. This could be accomplished by taking a dual-frequency receiver and placing it on a hi-rail vehicle. The tests would be performed with the help of one of the railroads and involve driving the hi-railer on different closely-spaced parallel tracks as well as right next to one of the tracks.

The vehicle GPS receiver data would be recorded for post-processing. The recorded data would then be processed using archived CORS network reference receiver data that was collected by the CORS network during the same time as the vehicle test data collection. To assure that the maximum benefit is obtained from this test, the field test might be repeated to incorporate any changes to the CDGPS algorithm or the test procedures after the first field test. With this approach, the CDGPS algorithm would be tested in an operational environment.

In preparation for these field tests, it is also recommended that additional research is performed to enhance the performance capability of the current GVLS CDGPS 2-step algorithm. In particular, this algorithm would incorporate a third Kalman filter to reduce the errors of the dual-frequency code-derived ionospheric delay estimates and remove the unwanted code multipath errors. The viability of a GVLS CDGPS 1-step algorithm, focusing on the Wide-Lane algorithm, might also be investigated.

When the feasibility of the revised GVLS CDGPS algorithm has been established with field data, the remaining software and hardware developments required for a GVLS R&D prototype development and testing can be pursued.

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