# ANALYSES OF HEADWAYS FOR ASSESSMENT OF RISK REAR- END COLLISION 

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#### Abstract

The paper presents the results of analyses of headways between vehicles on road sections, which can be used as an indirect measure of traffic safety. Such an assumption is derived from a classic model of risk, where the number of accidents and collisions is a product of the values of risk exposure and probability of risk events and their severity. Time To Collisions (TTC) and Time available for Driver's Reaction (TDR) were taken by the authors as indirect measures of risk. To evaluate the TTC the authors used a typical physical model. Deceleration values and lengths of vehicles were adopted in reference to their type (passenger cars and heavy vehicles).The aim of the analysis was to indicate the impact of several factors, not only traffic flow intensity, on the share of unsafe headways between vehicles, which can affect rear-end collision risk. Particular attention was paid to the impact of the following factors on unsafe headways in traffic flow: - traffic volume and traffic flow composition and its speed, - different drivers' behaviour at night-time, - characteristics of road and road development, - random character of drivers' behaviour, expressed by variability of reaction time.

The data coming from measures on 219 road sections in Poland were involved in the evaluation. There were sections of different cross-section types, with different speed limits, and of different localisation (e.g. roads through built-up area, suburban area and rural area). Additionally, entries of intersections and sections of approaches to pedestrian crossings were also taken into account. The analyses did not include the sections on entries to intersections with light signalling, which due to their specific functioning need a separate study, as is presented in foreign literature (Archer, Young, 2009; Cunto, Saccomanno, 2009). The conducted analyses resulted in building up a database, comprising records of traffic parameters for over 2.6 million vehicles. 24-hour measurements (automatic traffic recorders) covered: time of vehicle's appearance, speed and length of vehicles. The paper introduces a possible method of assessing the probability of rearend collision occurrence.


Keywords: road safety, headways, time to collision,

## INTRODUCTION

A correlation between unsafe headways share and rear-end collision occurrence is assumed. According to Fiorani et al. (2005), in Europe rear-end collisions make up to ca. $13 \%$ of all accidents, which is ca. $3.5 \%$ of fatal accidents, ca. $5.5 \%$ accidents with the seriously injured and ca. $14 \%$ with the slightly injured. The rate of such conflicts is even higher in the USA, amounting to ca. $30 \%$ of all collisions.
Drawing on the study of pertinent literature (Elvik et al., 2009; Son et al., 2009), it was assumed that the probability of a traffic event called "rear-end collision" is connected with traffic intensity and the share of unsafe headways between vehicles. Unsafe headways are those that are too short for drivers to respond timely to unexpected situations occurring on the road, including rapid deceleration by the leading vehicle.

The values of unsafe TTC can be equal to or lower than those in Time To Collision critical TTCc calculated from the well-known dependence (Brill, 1972) presented below (Formula 1). Additionally, given the specificity of the collected data, the formula incorporates the length of the leading vehicle, which allows determination of the value of the time gap between two analysed vehicles. Incorporating the impact of the leading vehicle plays an important role, especially in the case of heavy vehicles (long) moving at low speeds:

$$
\begin{equation*}
T T C_{c}=\frac{V_{i+1}^{2}}{7,2 \cdot V_{i}\left(a_{i+1}+\mu\right)}-\frac{V_{i}}{7,2 \cdot\left(a_{i}+\mu\right)}+\frac{V_{i+1}}{V_{i}} \cdot t_{r}+\frac{3,6 \cdot l_{i}}{V_{i}} \tag{1}
\end{equation*}
$$

where:
$T T C_{c}$ : critical time headway between analysed vehicles [s],
$V_{i}$ : speed of leading vehicle $[\mathrm{km} / \mathrm{h}]$,
$V_{i+1}$ : speed of following vehicle $[\mathrm{km} / \mathrm{h}]$,
$a_{i}$ : deceleration of leading vehicle $\left[\mathrm{m} / \mathrm{s}^{2}\right]$,
$a_{i+1}$ : deceleration of following vehicle $\left[\mathrm{m} / \mathrm{s}^{2}\right]$,
$\mu$ : coefficient of friction [-],
$t_{r}$ : reaction time of the driver of following vehicle [s],
$l_{i}: \quad$ length of leading vehicle $[\mathrm{m}]$.
The problem of unsafe distances and TTC is extensively treated in literature, mainly using the (Brill, 1972; Davis et al., 2006; Vogel, 2003; Oh et al., 2006) rear-end collision model. In this model, exogenous variables were vehicle speeds, vehicle accelerations/delays and reaction time.

Incorporating another explanatory variable, namely vehicle length, into the analyses actually allows assessing TTC rather than headways between vehicles, which has a serious impact on the occurrence of dangerous situations. The analyses of the sections around intersections were discussed by Vogel (2003).

Accident risk is particularly increased at night-time due to limited visibility. Their level of fatality occurs to be also considerably higher. Analysis of risk exposure conducted by Fors et al. (2009),
proves that the risk of rear-end collisions occurring at night is more than twice bigger than during daytime. This may result from:

- higher rate of unsafe TTCs between vehicles at night-time, i.e. a tendency to drive in platoons even at lower traffic volume rates,
- decreased psycho-physical capabilities of drivers at night-time, e.g. extended reaction times.

Driver's reaction time is a significant parameter in assessing the risk of rear-end collisions, with values tending to differ between day and night time. The analyses of Fors et al. (2009) show that the minimum reaction time at daytime is 0.2 sec ., while at night time 0.6 sec . On the other hand, Hartmann's studies (1979) indicate that reaction time is likely to range between $0.35-1.4 \mathrm{sec}$. at daytime, and $0.4-1.8 \mathrm{sec}$. at night (the lower limits include ca. $0.2 \%$, and the top ones ca. $99 \%$ of the recorded drivers). It was also found that the reaction time values of ca. $80 \%$ drivers between day and night differed (ca. 0.2 sec.). Extended reaction time at night time is likely to occur among drivers of heavy vehicles, who frequently are excessively tired.

Works from the Czech Republic (1985) indicate the mean reaction time of 0.84 sec . in situations when the driver has a front view of the object. Shrestha (2009) observed that the perception and reaction time according to many researchers was found in the range from 0.5 sec . to 1.52 sec .
The longer the driver's reaction time the longer the $\mathrm{TTC}_{\mathrm{c}}$ s between vehicles ought to be. Furthermore, the additional factor determining whether TTCs between vehicles are safe or not appears to be diversity of their braking distances (resulting from different both speed and braking deceleration values). Consequently, defining the value of $\mathrm{TTC}_{\mathrm{c}}$ is a complex problem and both the random character of drivers' behaviour together with diversity of speed values of vehicles and their dynamic features should be taken into consideration.

A thorough analysis of $\mathrm{TTC}_{\mathrm{c}} \mathrm{s}$ formation will definitely improve traffic management and allow limiting frequency of unsafe TTCs, e.g. by introducing automatic monitoring and fining too short TTCs between vehicles. Yet, such measures demand an accurate way of indicating the unsafe value of TTC. Too short TTCs appear to be particularly unsafe on two-lane roads with two driving directions, on sections where frequent exit manoeuvres cause traffic smoothness disruption (e.g. urban and suburban areas, intersections with the right of way, single exits on rural roads). The analyses presented by the authors also include widely known dependencies, translated into the Polish reality.

Apart from accidents, rear-end collisions appear to be frequent as well, although a detailed analysis of this occurrence is hampered due to the way collisions are recorded in accident database. An incomplete accident record in the police database is common in both Poland and Central and Eastern Europe. It is estimated that about $50 \%$ of all accidents are not included in the database. Numerous collisions result in serious material losses (of vehicle owners) and socioeconomic losses, connected with traffic limitations on sections where collisions have just taken place.

The lack of complete information about collisions results in an increased importance of indirect measures in traffic safety evaluation, including unsafe TTCs.

The literature outlined here does not fully incorporate other factors which can have a material impact on the share of unsafe TTC, which can constitute a measure of risk. The identification of additional factors determining the occurrence of unsafe TTCs in streams of vehicles has become the main aim of analyses. Special attention was paid to the impact of the following factors on unsafe TTCs risk in traffic flow:

- traffic volume and traffic flow composition and its speed,
- different drivers' behaviour at night-time,
- characteristics of road and road development,
- random character of drivers' behaviour, expressed by variability of reaction time.

Some of the above factors have been hitherto analysed on a very narrow front but have not documented in pertinent literature.

The analyses included the database of the project conducted by Gaca et al. (2002-2008), covering records of traffic parameters for over 2.6 million vehicles on 219 road sections.

## UNSAFE HEADWAYS AND DATABASE FOR THEIR ANALYSES

To evaluate unsafe TTCs between vehicles the authors used a physical model of vehicles driving at a given (recorded during the analysis) speed, which brake with appropriate deceleration, depending both on driver's reaction time and length of the leading vehicle. Deceleration values and lengths of vehicles were adopted in reference to their type (passenger cars and heavy vehicles).

The analyses conducted involved only two types of vehicles, i.e. passenger cars and heavy vehicles. The lengths of vehicles, recorded by magnetic and pneumatic detectors, were considered as a classifying criterion. The critical length, separating the two types of vehicles, was 3.1 m distance between the axles of vehicles, recorded by pneumatic detectors, and 6.5 m socalled 'electric distance', recorded by magnetic detectors. Analysing merely two types of vehicles is a kind of simplification, as dynamic characteristics of vehicles (speed, deceleration) appear to have a more diverse character. However, evaluation of the sensitivity of Formula 1 to changes of values of these parameters led to the assumption that the accepted simplification at the phase of initial analyses is undeniably permissible. It refers particularly to cases of little dispersion of speed values and relatively rare occurrence of a pair of vehicles (a passenger car and a heavy vehicle) driving close to each other. In the case of occurrence of such passenger car and heavy vehicle pairs in a traffic flow, the difference of the assumed values of deceleration of these vehicles is of great importance in finding the critical value of unsafe TTC. For example, when changes of the deceleration values of a passenger car vary from $5 \mathrm{~m} / \mathrm{s}^{2}$ to $8 \mathrm{~m} / \mathrm{s}^{2}$ and for a heavy vehicle from $3 \mathrm{~m} / \mathrm{s}^{2}$ to $5 \mathrm{~m} / \mathrm{s}^{2}$, the determined critical values $\left(\mathrm{TTC}_{\mathrm{c}}\right.$ ) resulted in the changes of unsafe TTC rate in a traffic flow by no more than $3 \%$ for empirical data. However, Formula 1 is rather sensitive to the values of reaction time $t_{r}$. The analyses distinguishes two separate cases of applying reaction time $t_{r}$ values:

- different values of time as mean values representing drivers response to various traffic conditions,
- reaction time $t_{r}$ as a random variable of empirical distribution.

Due to the fact that all traffic measurements were conducted on level terrain and dry road surfaces, Formula 1 included a uniform coefficient of friction $\mu=0.8$.

Considering the sensitivity of Formula 1 to the value of reaction time $t_{r}$ and the fact that reaction time is a random variable, the rate of unsafe TTCs was estimated taking empirical distribution of this variable into account. For this purpose, the initial part of the analysis aimed at calculating for each pair of vehicles the so-called Time available for Driver's Reaction (TDR), taking into consideration the difference of braking distances of both the leading and following vehicles. The time was calculated by the following formula:

$$
\begin{equation*}
T D R=\left[T T C-\left(\frac{V_{i+1}^{2}}{7,2 \cdot V_{i}\left(a_{i+1}+\mu\right)}-\frac{V_{i}}{7,2 \cdot\left(a_{i}+\mu\right)}\right)-\frac{3,6 \cdot l_{i}}{V_{i}}\right] \cdot \frac{V_{i}}{V_{i+1}} \tag{2}
\end{equation*}
$$

The meaning of the symbols is the same as in Formula 1.
The applied technique of analysis was to compare each of the recorded TTC between vehicles with the calculated (using Formula 1) critical value $\left(\mathrm{TTC}_{\mathrm{c}}\right)$ for each of these vehicles, and then to calculate the rate ( $U_{T T C_{C}}$ ) of unsafe TTCs using the following simple formula:

$$
\begin{equation*}
U_{T T C_{C}}=\frac{N_{u}}{N} \tag{3}
\end{equation*}
$$

where:
$N_{u}$ : number of TTCs classified as unsafe in the applied analysis interval,
$N$ : number of all TTCs between vehicles in the applied analysis interval.

The calculated value of time TDR for each pair of vehicles ought to be compared with randomly selected values of reaction time and this would result in classifying it as safe or unsafe TTC. Next, the value $U_{T T C_{C}}$ ought to be calculated, considering the random character of reaction time, using the following formula:

$$
\begin{equation*}
U_{T T C_{C}}=\frac{N \cdot \int_{t 1}^{t 2} f(T D R) \cdot q(T D R) d t}{N}=\int_{t 1}^{t 2} f(T D R) \cdot q(T D R) d t \tag{4}
\end{equation*}
$$

where:
$f(T D R)$ : function of density of TDR distribution, $q(T D R)$ : function representing probability that $t_{r}<T D R$,
$t 1, t 2$ : the lower and top limits of estimated time available for reaction of the driver TDR

The analyses involved estimating $U_{T T C_{C}}$ using the general Formula 4, with the application of empirical distribution for $f(T D R)$ and $q(T D R)$. For function $q(T D R)$ the following assumption was made:

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for \(T D R \leq t_{r}^{0,02} q(T D R)=1\),
for \(t_{r}^{0,02}<T D R<t_{r}^{0,99} q(T D R)=1-q\left(t_{r}\right)\), where \(q\left(t_{r}\right)\) is cumulative frequency of
        reaction time distribution \(t_{r}\),
for \(T D R>t_{r}^{0,98} q(T D R)=0\)
```

Evaluation of unsafe TTCs and factors determining the rates of these time gaps in traffic flows involved using data from 219 measurement sites located on Polish roads. The sites were sections of different cross-section type, with different speed limits, and of different localisation (built-up areas, crossing through small town, suburban area, rural area). The conducted analyses resulted in building up a database, comprising records of traffic parameters for over 2.6 million vehicles. 24hour measurements (automatic traffic recorders) covered: time of vehicles' appearance, their speed and length. The measurements were possible owing to the use of both magnetic and pneumatic recorders and were performed during 27 sessions every few months in 2002-2008 on dry road surfaces.

## ANALYSES OF DIFFERENT VARIABLES IMPACT ON UNSAFE HEADWAYS RATE WITH REACTION TIME CONSTANT VALUES

## Impact of Traffic Volume

The impact of traffic volume on TTCs between vehicles has already been analysed before, however, analyses mostly tend to focus only on TTC distribution. Considering the way of defining unsafe TTCs that would distinguish speed values and type of vehicle, it is most important to undertake further studies concerning the impact of traffic volume on unsafe TTCs. Basing on model (1), the rate of unsafe TTCs between vehicles in the function of traffic volume has been identified under the assumption that $t_{r}=1.5 \mathrm{sec}$. (Figure 1). As expected, the presence of unsafe TTCs is closely related to traffic volume and an increase in values of traffic volume causes the rate of unsafe TTCs to rise. The empirical data set (Figure 1) presenting the measurements of 2.6 million TTCs is sufficient to find a mathematical relationship between the analysed variables, represented by a logarithmic curve. The results of evaluation of unsafe TTCs rate measured for empirical data were compared with the results of the simplified version of $U_{\Delta t}$ evaluation, supposing that the variable of TTC between vehicles has an exponential and lognormal distribution.

For the exponential distribution the diversity of vehicles' deceleration and their speed values were omitted, which means removing elements concerning the braking manoeuvre from Formula 1. Assuming that the mean length of a vehicle is 8.5 m (weighted average taking into account different types of vehicles), the value of $U_{T T C_{C}}$ was calculated, depending on traffic volume (exponential curve in Figure 1). For the simplified calculations the same values of reaction time were taken as for empirical data analysis. The analyses including the mentioned assumptions revealed the fact that the values of $U_{T T C_{C}}$, calculated with the given simplifications, considerably differ from the values resulting from the empirical data which were fully based of Formula 1. It means that, especially for traffic volume values higher than $1000 \mathrm{P} / \mathrm{h} / \mathrm{lane}$, applying the
simplified version of the theoretical model leads to a considerable underestimation of unsafe TTCs rate in comparison with the empirical data.

Such an underestimation was avoided when a log-normal distribution of TTC between vehicles was assumed. This distribution is an advanced version of the theoretical model and gives the best fit of the TTC empirical distribution, as no simplifications were made. The value of $U_{T T C_{C}}$ was calculated for several different volume values. The log-normal curve in Figure 1 gives similar rates of unsafe TTCs between vehicles as the empirical model.

All the measurements were taken at traffic volumes lower than traffic capacity, which ensured that proper functioning of the physical model was not affected (no stops within the flow caused by exceeded traffic capacity). Considerable dispersion of the results of $U_{T T C_{C}}$ estimation indicates the existence of other factors affecting unsafe TTCs rate. Therefore further analyses focused on the effects of traffic characteristics, road features and time of the day on changes of the unsafe TTCs rate.


Figure 1 Impact of traffic volume on unsafe TTC rate
As it has been presented before, reaction time $t_{r}$ has a considerable impact, resulting from both drivers' behaviour and physical model (Formula 1), on unsafe TTCs rate. The impact of various values of reaction time, assumed by the authors, on the estimated values of $U_{T T C_{C}}$ have been illustrated in Figure 2. Different reaction time values affect the achieved coefficient of determination where $U_{T T C_{C}}$ depends on volume $Q$. It rises together with the increased values of the assumed reaction time.

Due to the fact that assuming a certain value of reaction time may be questionable, further analyses consider reaction time as a random variable as well.


Figure 2 Impact of traffic volume on unsafe TTC rate for different reaction time values
In further analyses of the impact of other independent variables on $U_{T T C_{C}}$ the value of reaction time is 1.5 sec .

## Impact of Road Development

The analyses of unsafe TTCs involved both qualitative and quantitative independent variables. The only analysed quantitative variable was road development intensity. Development intensity was classified according to the assessment of residential development along the road. Due to its specific character, this variable ought to have a considerable impact on unsafe TTCs rate, as an increase of road development intensity is closely related to an increase of accessibility to the adjoining area. Consequently, frequent exit manoeuvres occur, interfering with the traffic flow.

The analyses were limited to the measurements taken from ca. 0.2 million vehicles. The qualitative division of road development included scattered, medium-intensive and intensive character of road surroundings development. The presented relations (Figure 3) are set in the range of volume changeability from 100 to $900 \mathrm{veh} / \mathrm{h} /$ lane.

The analyses confirmed the predicted impact of qualitative characteristics of road development on $U_{\text {TTC }}$. For the value of traffic volume of $400 \mathrm{veh} / \mathrm{h} /$ lane the unsafe TTCs rate is similar and amounts to about $26 \%$, regardless of road development density. If the values of volume are higher, the unsafe TTCs rate is, as expected, the highest for intensive road development. The obtained results prove that the values of $U_{T T C_{C}}$ for scattered and medium-intensive development are comparable. If the values of volume are lower than $400 \mathrm{veh} / \mathrm{h} / \mathrm{lane}$ the unsafe TTCs rate is the highest for scattered development, which may be related to the impact of speed on $U_{T T C_{C}}$. On
sections with scattered development speed values are generally higher than in intensive development.


Figure 3 Relation between traffic volume and unsafe TTCs rate for changeable density of road development

## Impact of Average Speed and Rate of Heavy Vehicles

The above studies and the presented physical model (taking dynamics and speed of vehicles in pairs of vehicles into account) indicate a need to include scalar variables, i.e. speed of vehicles and the rate of heavy vehicles, into analyses of unsafe TTCs. Evaluation of the impact of average speed $\left(V_{a}\right)$, the rate of heavy vehicles $\left(H_{V}\right)$ and traffic volume $(Q)$ on the rate $U_{T T C_{C}}$ of unsafe TTCs involved nonlinear estimation. Evaluation of parameters of models (5 and 6) was obtained with the use of least squares estimates (Table 1). The regression analyses resulted in obtaining, for the following relationships of $U_{\text {TTC }}^{C}$ with different explanatory variables (presented in Figure 4 and 5), a set of 2.6 million vehicles ( 8685 time intervals - 30 minutes each):
a) for explanatory variables $Q$ and $V_{a}$

$$
\begin{array}{r}
U_{\text {TTC }}=-0,499+0,341 \cdot \widetilde{Q^{0,162}}-0,001 \cdot V_{a} \quad[-]  \tag{5}\\
R^{2}=0,60
\end{array}
$$

b) for explanatory variables $Q$ and $H_{V}$

$$
\begin{array}{r}
U_{T T C_{C}}=-0,552+0,261 \cdot \widetilde{Q^{0,191}}-0,05 \cdot H_{V} \quad[-]  \tag{6}\\
R^{2}=0,57
\end{array}
$$

Table 1 Parameters of models (5) and (6)

| Model (5) |  |  |  |
| :---: | :---: | :---: | :---: |
| Coefficient of <br> formula | Standard error | Value of t | p-value |
| -0.499 | 0.13914 | -3.585 | 0.00034 |
| 0.341 | 0.10695 | 3.191 | 0.00142 |
| 0.162 | 0.02727 | 5.946 | 0.00000 |
| -0.001 | 0.00005 | -24.831 | 0.00000 |
| Model (6) |  |  |  |
| Coefficient of <br> formula | Standard error | Value of t | p-value |
| -0.552 | 0.10594 | -5.208 | 0.00000 |
| 0.261 | 0.07494 | 3.488 | 0.00049 |
| 0.191 | 0.02632 | 7.259 | 0.00000 |
| 0.050 | 0.00477 | 10.534 | 0.00000 |

Variables $V_{a}$ and $H_{V}$ in models (5 and 6) appear to be statistically significant and they cause the value of determination coefficient to rise by 0.05 and 0.02 , respectively.


Figure 4 Impact of traffic volume and traffic flow average speed on unsafe TTCs rate (Formula 5)


Figure 5 Impact of traffic volume and heavy vehicles rate in a flow on unsafe TTCs rate (Formula 6)

The results of the presented studies form the basis for the assumption that the rate $U_{T T C_{c}}$ of unsafe TTCs decreases with the growth in average speed of vehicles. If the speed value increases by $10 \mathrm{~km} / \mathrm{h}$, reduction amounts to $1 \%$. The increase in gap times between vehicles (fewer unsafe TTCs) related to growth of speed values, may be closely connected with drivers' tendency to maintain constant level of risk in traffic.

The other scalar variable characterising dynamic properties of vehicles and occurring indirectly in model (6) is the rate of heavy vehicles. The increase in the rate of heavy vehicles causes an increase in unsafe TTCs rate. The growth in the heavy vehicles rate by $10 \%$ causes the unsafe TTCs rate to increase by $0.5 \%$. The results meet predictions, and they are connected with the difference of dynamic properties of vehicles, especially for pairs of passenger car-heavy vehicles. It is worth mentioning that drivers particularly prone to driving at unsafe TTCs in passengerheavy pairs are those who are strongly affected by permissible speed values. On rural roads, with the $90 \mathrm{~km} / \mathrm{h}$ speed limit, the rate of unsafe TTCs for such pairs is relatively low and does not depend on the rate of heavy vehicles (Figure 6a). However, on roads passing through towns with the $50 \mathrm{~km} / \mathrm{h}$ speed limit, the rate of unsafe TTCs of such pairs is considerably higher and rises with the increase of heavy vehicles rate (Figure 6b). This is confirmed by the low determination coefficient value which is 0.01 and 0.08 , respectively:
a) rural roads


Figure 6a Impact of heavy vehicles rate on unsafe time TTCs rate between vehicles in types of vehicle pairs 'heavy-heavy' and 'passenger-heavy'
b) roads passing through small towns


Figure 6b Impact of heavy vehicles rate on unsafe time TTCs rate between vehicles in types of vehicle pairs 'heavy-heavy' and 'passenger-heavy'

## Impact of Time of Day

To evaluate the impact of time of the day on unsafe TTCs rate between vehicles in the recorded flows on rural roads, this rate was compared for the following cases:
a) assumption of the same values of reaction time 1.0 sec . for both day and night-time, with restricting the comparisons to similar traffic volume values $100-500 \mathrm{P} / \mathrm{h}$. In this case various values of $U_{T T C_{C}}$ for day and night-time result from a higher rate of heavy vehicles (Figure 7a),
b) assumption of different reaction time values for day and night-time, resulting from psycho-physical conditions and tiredness of drivers at night-time. As in case a), the comparisons were restricted to similar volumes at night-time ( $100-500 \mathrm{P} / \mathrm{h}$ ). Assuming the reaction time of 1.0 for daytime and 1.2 for night-time, considerably different values of $U_{T T C_{C}}$ (Figure 7b) were obtained at the same traffic volume values. In this case, the increase in unsafe TTCs rate at night-time results from both higher tendency to drive in platoons (presented in item a) and the effect of assuming a lower value of reaction time in Formula 1.

Driving may be more risky at night-time as a result of wrong estimation of the distance to the leading vehicle at night-time and drivers' unawareness of the risk related to insufficiently long TTCs between vehicles.


Figure 7a Comparison of relationships of unsafe TTCs rates at daytime and night-time The same reaction time of 1.0 sec for daytime and night-time


Figure 7 Comparison of relationships of unsafe TTCs rates at daytime and night-time Reaction time of 1.0 sec for daytime and 1.2 sec for night-time

## Rate of Unsafe TTCs with Random Values of Reaction Time

The assumption of a constant value of reaction time $t_{r}$, close to the average value from the empirical distribution of this variable when estimating $U_{T T C_{c}}$ is one of the simplifications that can make engineering practice more feasible. The model, more complex, though closer to real drivers' reactions, is a model of $U_{T T C_{C}}$ estimation including random values of reaction time.

Acknowledging that simplifications in models are necessary in practice, the authors decided to perform an evaluation of their impact on the results of calculations of $U_{T T C_{C}}$. This evaluation involved comparison of the results of two cases: when the reaction time value $t_{r}$ is assumed to be constant, and when it is a random variable of a distribution determined by empirical studies. Calculations of $U_{T T C_{c}}$ to applying reaction time as a random variable were performed in accordance with the procedures presented in the initial part of the paper. The comparisons were made for traffic at both daytime and night-time, i.e. for different distribution of time available for drivers to react TDR (Figure 8) and for different reaction time distribution.


Figure 8 Comparison of histograms of time available for drivers to react TDR in daytime and at night-time

The analyses resulted in the statement that applying the method of estimating the unsafe TTCs rate (constant value of reaction time vs. treating this time as a random variable) has a significant impact on the obtained results. Figure 9 presents the comparison of estimations of $U_{T T C_{C}}$ for the same empirical data but for a different way of assuming reaction time values. The constant value of time applied to calculations corresponded approximately to the average from the empirical distributions $t_{r}$.

Applying a constant value of reaction time $t_{r}$ results in obtaining higher values of unsafe TTCs rate $U_{T T C_{C}}$ than in the case of considering reaction time as a random variable.


Figure 9 Comparison of estimations of $U_{T T C_{C}}$ with a constant value of reaction time vs. treating it as a random variable

## DISCUSSION

The analyses presented as simple models point at many factors affecting the rate of unsafe TTCs between vehicles that can be a surrogate measure of road traffic safety. To assess unsafe TTCs rate it is important to know both traffic factors as well as those connected with traffic participants' behaviour and the development of road surroundings. One way of such assessment is to include in calculations the approach in which distribution of random variables used in Formula 1 are taken into account (a physical model of headways between vehicles). On the basis of available data it is possible to determine various shapes of distribution of random variables. These variables include: headways between vehicles in traffic flow, vehicles speeds, reaction time, structure typology for variable road conditions. The different distribution shapes can next be applied in determination of the probability of unsafe TTCs. Such analyses can be performed using WinBugs software (Davis, 2006; Davis, 2007). Basing on the rate of unsafe TTCs between vehicles the collision probability can be assessed after the methodology presented below.

Due to incomplete information in databases on collisions (in Poland) it is not possible to use standard quantitative descriptions applying, for example, regression calculus tools. Moreover, these data describe events from the past, while some traffic safety determinants have changed considerably in the meantime. Thus regressive dependencies describing in general terms the relationship accident indicators $=\mathrm{f}$ (set of reasons) has a limited prognostic value. As shown by Durth et al. (1988), more credible projections of road traffic risks can be generated by means of risk analysis methods. In risk analyses, the interdependency accident indicators $=\mathrm{f}$ (set of reasons) may be replaced by a model easy to decompose, a model which assumes the following shape:

$$
\begin{equation*}
R=\int_{S} S \cdot F(S) d S \tag{7}
\end{equation*}
$$

where:
$R$ : assumed measure of risk factoring in the probability and scale of the consequences of a road event,
$F(S)$ : probability density function of loss valued at $S$,
$S: \quad$ size of loss - the consequences of a road event.
To solve Equation 7 it is imperative to know the probability density function of an event having certain consequences. $F(S)$ depends on numerous factors related to road and traffic conditions. Mostly the functions have multiple parameters. It is much easier to solve Equation 7 if and when the $F(S)$ functions are determined separately for different types of road events. Additionally, applying a categorisation of independent variables of the $F(S)$ function in specific cases of loss, the integral in Equation 7 is replaced by a simple summation after each case of loss. When this approach is used, risk $R$ is assessed as the sum of risk of different critical situations which can occur in traffic.

The probability of critical situations can be assessment by means of a simplified traffic model developed following the fault tree analysis (FTA) described by Ericson (2000). It is a model composed of elements corresponding to different traffic events and elements describing transitions between these situations (Figure 10). The probability of each situation manifesting itself tends to depend on preceding situations and external factors underlying traffic.


Figure 10 Traffic model for assessment of probability of critical traffic event
For the diagram of traffic event shown in Figure 10, when we assume we know the probability of initial situations $P\left(E^{l}\right)$ and $P\left(E^{2}\right)$, the probability of subsequent situations $P\left(E^{i}\right)$ is computed by means of using the probability of transition $P_{i, i+1}$ from $i$ situation to $i+1$. With this assumption, in the example defined in Figure 10 we can compute:
$P\left(E^{3}\right)=P\left(E^{1}\right) \cdot P_{1,1-3}+P\left(E^{2}\right) \cdot P_{2,2-3}$
$P\left(E^{4}\right)=P\left(E^{2}\right) \cdot P_{2,2-4}$
$P\left(E^{5}\right)=P\left(E^{3}\right) \cdot P_{3,3-5}+P\left(E^{4}\right) \cdot P_{4,4-5}$
By generalising this example, one must state that the probability of concurrent occurrence of multiple mutually independent traffic events is computed as the product of probabilities of occurrence of subsequent situations. It must be noted, however, that the probability of occurrence of subsequent situation factors in the condition of occurrence of antecedent situations. This can be expressed by the following formula:

$$
\begin{equation*}
P\left(E^{1} \cap E^{2} \cap \ldots \ldots \cap E^{n}\right)=P\left(E^{1} \cdot \prod_{i=2}^{n} P\left(E^{i} \mathrm{I} E^{1}, \ldots \ldots ., E^{i-1}\right)\right. \tag{8}
\end{equation*}
$$

where:
$E^{i}$ : an i situation of a traffic event.

The probability of occurrence of certain situations and the probabilities of transitions between situations are not expressed by single values, but are, in fact, matrices of probabilities dependent on various parameters.

Using the general assumptions presented above, the authors analysed the risks of accidents involving running into the back of a car on a stretch of road passing through a locality. A simplified traffic diagram of traffic situations and transitions between these situations for this model has been illustrated in Figure 11. This is a highly simplified approach illustrating only the general rule of conduct to solve the problem. Additionally, it is essential to factor in e.g. interactions between vehicles. In the analyses conducted by the authors it was assumed that rearend collisions can occur when the following situations happen concurrently: 1) there appears an obstacle on the road (traffic event $E_{3}$ ), 2) the vehicle approaches the obstacle so closely that it needs to decelerate rapidly or collides with the obstacle and comes to a halt with heavy deceleration (traffic event $\mathrm{E}_{4}$ ), 3) the distances between the subsequent vehicles behind the rapidly decelerating vehicle are too short to permit stopping the vehicles (traffic event $\mathrm{E}_{7}$ ).

On the basis of the fault tree above a possible way of assessing the probability of transition from normal traffic to a rear-end collision under a situation involving the appearance of an obstacle on the road and a need to decelerate rapidly has been presented. Obstacles might include e.g. a vehicle pulling out from the side, a pedestrian entering the road, a vehicle stopping to make a left turn from the main road.

The probability that the vehicle will approach the obstacle so close that it will need to decelerate abruptly is a random variable described by a multi-dimensional distribution dependent on various parameters. These parameters include independent parameters as well as correlated parameters. To assess this probability use can be made of the model traffic event and transitions between subsequent situations as illustrated in Figure 11. The logical sequence of traffic events and transitions between them is developed following the rules underlying the development of the fault tree (Durth et al., 1988).

To ultimately determine the probability of rear-end collision it is essential to know the probability of the occurrence of subsequent events shown in the diagram in Figure 11. With these values (eg. assessed on the basis of traffic conflicts) and knowing the probability of unsafe TTCs occurrence (based on WinBugs results) calculations can be made of the probability of rear-end collision after Equation 8.


Figure 11 Situational diagram illustrating occurrence of rear-end collision

## SUMMARY

The paper presents analyses of TTCs between vehicles which may indirectly influence traffic safety. The research conducted resulted in indicating road-traffic factors, time of day and the effects of drivers' psycho-physical behaviour on rate of unsafe TTCs in traffic flow. The analyses gave foundation to the following conclusions:

- Rate of unsafe TTCs is related to traffic volume and it is possible to represent this relationship by a logarithmic curve. Applying a theoretical model of TTCs between vehicles based on exponential distribution results in a considerable underestimated unsafe TTCs rate, when compared with empirical data. Log-normal distribution gives results very similar to those obtained from empirical data.
- The analyses proved that road development, characterised by intensity of development, is a factor affecting unsafe TTCs rate. For traffic volume values higher than 400veh/h/lane the rate of unsafe TTCs is the highest for intensive development and comparable with both scattered and medium-intensive development, which may be connected with the impact of speed on $U_{T T C_{C}}$. On sections with scattered road development speed values tend to be higher than on roads with intensive development.
- The impact of variables characterising traffic (i.e. mean values of speed, and rate of heavy vehicles) on unsafe TTCs rate was found to be statistically significant. Unsafe TTCs rate decreases with the increase in average values of speed. The reduction is $1 \%$ for the growth of speed by $10 \mathrm{~km} / \mathrm{h}$, which may be connected with drivers' tendency to preserve the constant level of traffic risk. An opposite effect is observed when the heavy vehicles rate is considered: its increase results in growth of unsafe TTCs rate. An increase in heavy vehicles share in traffic by $10 \%$ causes the unsafe TTCs rate to rise by $0.5 \%$, which is related closely to dynamic properties of these vehicles.
- On rural roads with the $90 \mathrm{~km} / \mathrm{h}$ speed limit the unsafe TTCs rate in 'passenger-heavy' pairs of vehicles is relatively low and does not depend on heavy vehicles rate. However, on roads passing through small towns with the $50 \mathrm{~km} / \mathrm{h}$ speed limit the rate of unsafe TTCs between 'passenger-heavy' pairs is considerably higher and it rises with the growth in the heavy vehicles rate.
- At night-time the rate of unsafe TTCs is higher by approx. 4\% than at daytime, regardless of traffic volume. The difference increases if psycho-physical conditions of drivers and their tiredness are taken into consideration, with the assumption that at night-time reaction time value is higher by 0.2 sec . than at daytime.
- Drivers' reaction time plays a decisive part in the model of estimation of unsafe TTCs. The results of the analyses prove that assuming reaction time value close to the mean value obtained from the distribution of this variable underestimates the rate of unsafe TTCs when compared with the calculated values assuming the random character of reaction time.

A thorough analysis of factors determining the occurrence of unsafe TTCs is an extreme necessity, particularly in the field of traffic management, as it would allow formulation of additional rules which would be aimed at reduction of unsafe TTCs rate and, consequently, traffic safety improvement. Moreover, application of relevant methods of unsafe TTCs rate estimation would allow a direct and more comprehensive traffic safety prediction.

An assessment of the probability of rear-end collision occurrence is possible on the basis of distribution curves of random variables: headways between vehicles in traffic flow, vehicles speeds, reaction time, typology structure for variable road conditions and the probability of occurrence of an event disturbing the traffic, which needs further study.

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