ANALYZE BICYCLE-CAR MIXED FLOW BY SOCIAL FORCE MODEL FOR COLLISION RISK EVALUATION

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ABSTRACT

Based on the interaction mechanization of vehicles and the basic hypothesis of social force model, a dynamics model of bicycle-car mixed flow has been established. The bicycles and cars are affected by the inner and outer forces, by which almost all of the traffic actions of the bicycles and cars have been affected synchronously, and the inner forces derive from the drivers' or cyclists' desire of moving with the best velocity while the outer forces derive from the affection of the other traffic units and the borders of the road. The inner and outer forces which the bicycles and cars have been affected are analyzed quantitatively. By the probability of the sudden braking, while the bicycles and cars flow in a urban road section has been simulated, the traffic collision risk of the bicycle-car traffic flow has been analyzed, from which we come to the conclusion that the risk of the bicycles and cars potential collision increase while the traffic density becomes high. For the bicycle-car mixed traffic flow, with the motor vehicle traffic density decreasing, the velocity and the probability of the bicycles driving into the motor vehicle lane will increase, and the probability of the traffic accident between the bicycles, which rides into the motor vehicle lane, and the motor vehicle with a high speed will sharply increase. When bicycle's density remains constant, the probability that bicycles may run into vehicle lane decreases with the increase of cars' density due to the mutual interference between motor vehicles get stronger, therefore the risk of vehicle's crash are becoming higher, which mainly due to the aggravated interference between motor vehicles.

Key words: bicycle-car flow, safety, social force, collision risk, simulation.

1 INTRODUCTION

The bicycle-car flow is a universal phenomenon of transport system in many countries, and a lot of relative researches have been done (Liu and Guan, 2000; Jia et al., 2008). Many scholars have analyzed the principle of the inter-action between vehicles in different by different ways, and have proposed many models of traffic flow simulation, such as car-following model, continuous model, aerodynamics model, cellular automaton model (Jiang and Wu, 2006) and social force model (Helbing, 1995), all of which have gained some outcomes at different scenes in the practical application. However, the continuous model and the aerodynamics model are macroscopic model, which are applied to simulate macroscopic traffic and are not suitable to analyze the interference of the bicycle-car traffic flow. The car-following model is well used in explaining the one dimensional vehicle action of 'acceleration-deceleration-following' in road segement. For the bicycle-car traffic flow, it is impossible to make a reasonable explain to the two-dimensional vehicle actions (Jiang and Wu, 2002). Each cellular of the cellular automaton model is homogenous and obeys to the same rules while the bicycle-car traffic flow is non-homogeneous, so the cellular automaton model is not applicable to analyze the traffic unit's action of the bicycle-car traffic flow. The social force model can give a reasonable explain to the traffic action of each unit in the traffic flow, in which each unit is impelled by inner and outer forces and all of the traffic action of the unit can derive from the resultant force of the inner and outer forces. Now the social force model is mainly applied to simulate the pedestrian traffic in the public place and can give a reasonable explain to all kinds of phenomenon of the pedestrian traffic, while it is seldom applied to solve the problem in the field of vehicle and bicycle-car traffic flow. Based on the basic method of the social force model, a social force model of bicycle-car traffic flow is simulated in the article, and the mutual interference of the car and the bicycle in the traffic flow are also analyzed. Phenomenon, such as the stop-and-go action of vehicle flow and the basic diagram of road capacity and traffic density, are all recurred in the research, although here has to omitted them due to consideration of concentrating on expressing collision risk evaluation between the bicycle and the motor vehicle clearly. So simulation of bicycle-car traffic flow by social force model is also adopted to reveal the mechanism of collision between the bicycle and the car due to the bicycle rides into the motor vehicle lane. Assuming a bicycle riding into the motor vehicle lanes suddenly, if the car behind does not brake in time, a collision will happen between the bicycle and the car. Even if the car behind braked in time, it may have a collision with the other cars followed due to the sudden braking. For my best knowledge, there are a few researches, especial quantitative analysis, on evaluation of collision risk between bicycle and car. The probability of the sudden braking of the motor vehicle is proposed to measure the collision risk of the bicycle-car traffic flow. Results of the simulation show: with the motor vehicle traffic density decreasing, the velocity and the probability of the bicycle riding into the motor vehicle lane will increase, and the probability of the traffic accident between the bicycle, which rides into the motor vehicle lane, and the motor vehicle with a high speed will sharply increase. So, the risk of the traffic potential collision of the bicycle-car traffic flow when the motor vehicle density is low, is higher than the case when the motor vehicle density is high. By the social force model for bicycle-car traffic flow, the safety evaluation of bicycle-car traffic flow on the urban road section can be gained deeply from a new view, and the method is also suitable for the safety evaluation of mixed traffic flow, such as car and trunk flow, by extending to consider more factors than the case of the bicycle-car traffic flow.

In the following paper, social force model of car traffic flow, single bicycle traffic flow and bicycle-car mixed traffic flow is presented in section 2-4 respectively. Analyzing force of different types of border is described in section 5 and three annotations are followed. In order to illustrate the proposed model for the purpose of evaluation of collision risk, an example is analyzed in section 7. Some conclusions and future researches are presented in the last of the paper.

2 SOCIAL FORCE MODEL OF CAR TRAFFIC FLOW

The basic theory of social force model is that, people's travel activities first through the perception of the external environment, thereby making behavioral decision for the maximum effectiveness, the whole trip process is formatted by cycles of above process. The traffic behavior of a motor vehicle in motion is actually the external manifestation that the driver makes behavior decision in complex environment when driving, thus the basic theory of pedestrian social force model can also apply for the car traffic flow.

The external environment perception of motor vehicle driver in motion can be shown by the outer force external environment made on vehicle, while maximum benefit of vehicle in motion can be realized by the following two assumptions:

Assumption 1: Drivers expect traveling at the optimal speed.

Assumption 2: Drivers expect reaching the destination.

Assumption 1 ensures that vehicles travel at the optimal speed by continuous acceleration and deceleration behavior, and assumption 2 ensures that vehicles travel along the shortest path by continuously changing direction. Assumption 1 and assumption 2 reflect that vehicles travel along the shortest path at the optimal speed under the influence of its internal forces.

The forces which the car is influenced during traveling can be divided into two kinds: the inner force and the outer one. The inner force is the forward driving force out of driver's expectation to run on his desire speed and the outer one mainly includes the interaction force between cars and force which the borders of road segment act to the car. All of these forces will be analyzed in the following one by one.

2.1 Forward driving force

The car traveling on the road segment wants to arrive at the destination as soon as possible, so car driver prefers to choose the best travel route at his desire speed. It assumes that the anticipated travel direction of the car at time t is the direction of the tangent of lane's middle line of road segment $\vec{e}_\alpha(t)$.

If a car is not disturbed by extraneous factors, it will travel with the anticipated travel direction $\vec{e}_\alpha(t)$ and the optimized velocity v_α^0. However, due to the influence of the inevitable deceleration progress and some unanticipated interference factors, there is a deviation between the actual velocity $\vec{v}_\alpha(t)$ and the anticipated optimal velocity $v_\alpha^0 \cdot \vec{e}_\alpha(t)$. Car driver will reduce this deviation by acceleration as soon as possible, and so a forward driving force will be generated, by which the car starts to accelerate. Forward driving force is a function of the difference between car's actual speed and expected optimum speed at time t, if car's actual speed equals the expected optimum speed at time t, that is $\vec{v}_\alpha^0 \cdot e\ (t) - \vec{v}_\alpha(t) = \mathbf{0}$, the forward driving force is zero vector. The forward driving force of car α at time t can be represented as follows:

$$\vec{F}_\alpha^0(t) = \frac{1}{\tau_\alpha}(v_\alpha^0 \cdot \vec{e}_\alpha(t) - \vec{v}_\alpha(t)) \tag{1}$$

Here, τ_α is the response time of the driver in the paper.

2.2 Interactional force between cars

The car is influenced by the other cars within its driver's visible range in the process of travel. The sight of vehicle α is shown in Figure 1. From Figure 1, we know that vehicle β_1 is in the sight of vehicle α, while the vehicle β_2 is not.

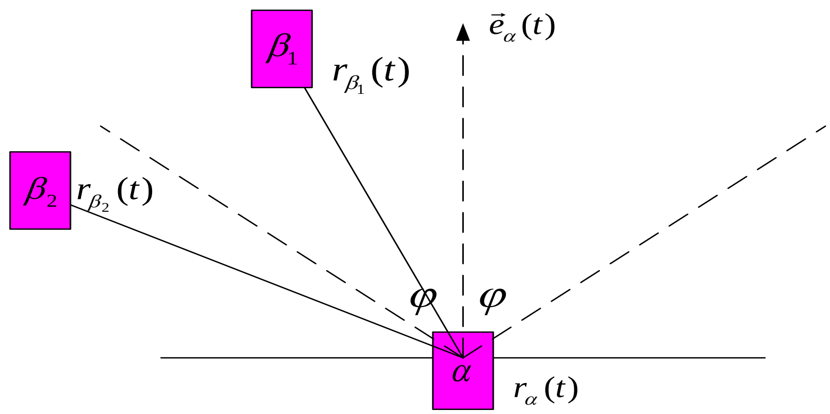


Figure 1 Schematic picture of visual range of a driver

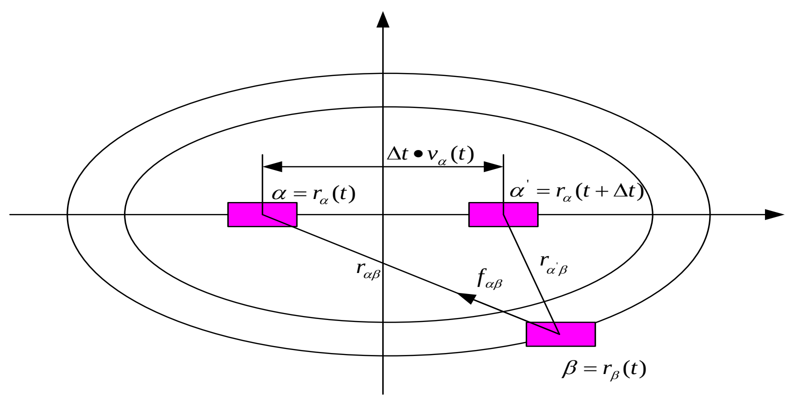


Figure 2 Schematic picture of interaction force

Car requires a certain safety space during moving, which depends on the current traffic density and speed. Assuming that the car β is in the visible range of car α, the car α needs some safe space for traveling in the next time step, but part of which is occupied by the car β which can be measured by the variable $b(t)$. The variable $b(t)$ can be calculated as follows (Helbing, 1995):

$$b_{\alpha\beta}(t) = \frac{1}{2}\sqrt{(\|\vec{r}_{\alpha\beta}(t)\| + \|\vec{r}_{\alpha\beta}(t) - v_\beta(t)\cdot\Delta t\cdot\vec{e}_\beta(t)\|)^2 - [v_\beta(t)\cdot\Delta t]^2} \tag{2}$$

$$\vec{r}_{\alpha\beta}(t) = \vec{r}_\alpha(t) - \vec{r}_\beta(t) \tag{3}$$

Here, $\vec{r}_\alpha(t)$ and $\vec{r}_\beta(t)$ represents the position vector of car α and β at time t respectively, $v_\beta(t)$ represents the velocity of car β at time t, Δt represents the unit time interval, $\vec{e}_\beta(t)$ is the anticipated travel direction of car β at time t, $\|\vec{r}\|$ represents the norm of the vector $\vec{r}$, which represents the distance between traffic units measured from center to center of traffic units. Although few vehicle dimension is included to be considered for our analysis purpose in this article, other vehicle dimensions should be added to analyze the interaction force between vehicles while there are heavy vehicles in traffic flow, such as passenger cars and trucks.

The value of the force, which the car β acts to the car α, is proportional to the space which is occupied by the car β in the forward direction of car α, and so it is proportional to the value of the variable $b(t)$, and it can be calculated as follows:

$$\vec{f}_{\alpha\beta}(t) = -\nabla_{\vec{r}_{\alpha\beta}(t)} V_{\alpha\beta}\left[b_{\alpha\beta}(t)\right] \tag{4}$$

Here, $\nabla_{\vec{r}}$ represents the unit vector of $\vec{r}$, $V_{\alpha\beta}\left[b_{\alpha\beta}(t)\right]$ is a function with the independent variable $b_{\alpha\beta}(t)$.

2.3 Border force

The cars travel in a special range of road segment which is restricted by the road border, so they are influenced by the road border. The forces which the road borders act to a car can be divided into two types, one of which is the nearest points of the border acting to the car and the other are the farthest points acting to the car. As displayed in Fig 3 and Fig 4, the farthest and the nearest points are the point 1,2 and the point 3,4 respectively.

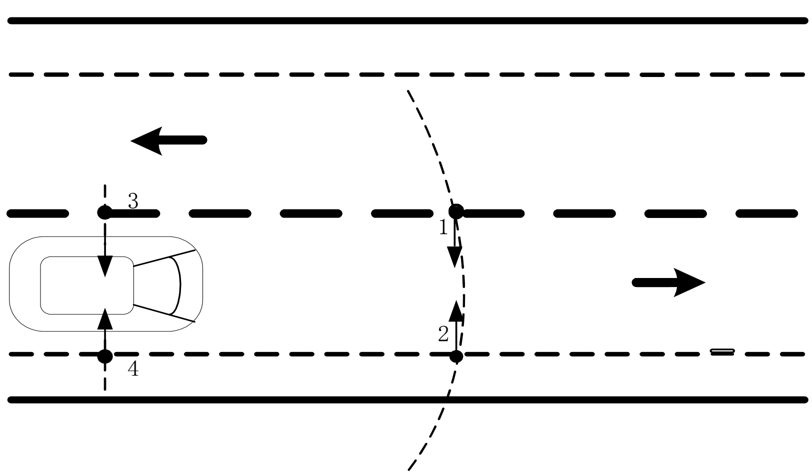
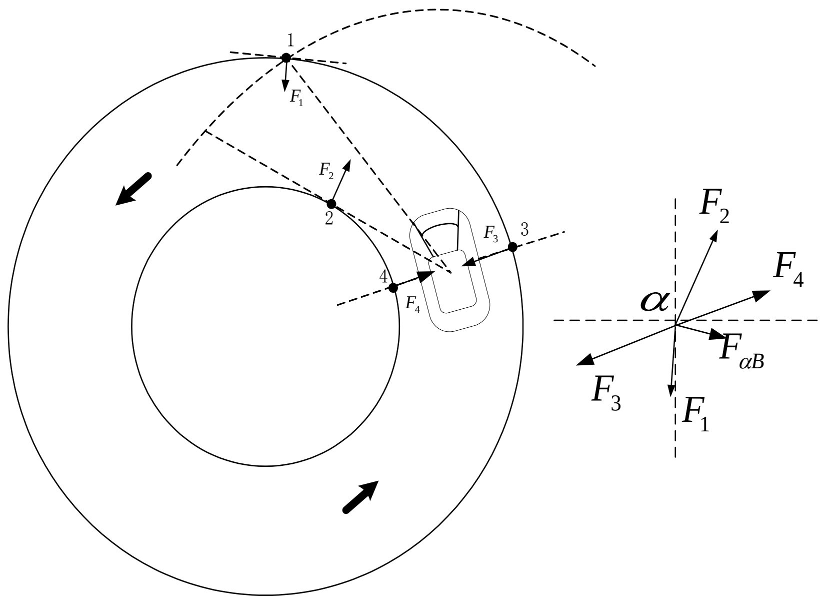
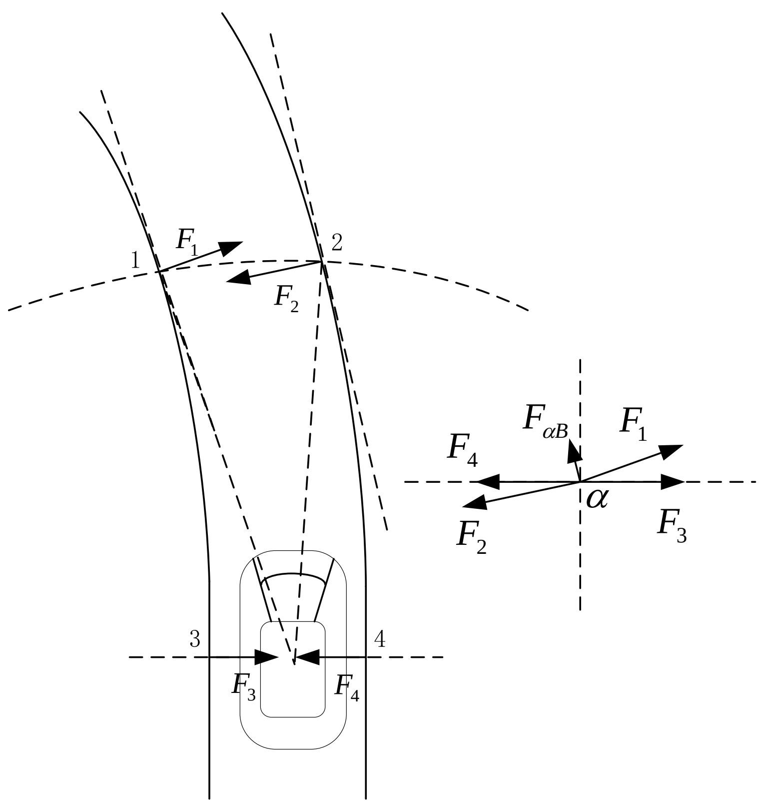


Figure 3 Schematic picture of forces in straight road



a



b

Figure 4 Schematic picture of border forces in bend road

Let $\|\vec{r}_{\alpha i}(t)\|$ represents the distance between the car α and the point i ($i = 1,2,3,4$)of the border at time t. The forces which the borders act to the car α are proportional to the value of $\|\vec{r}_{\alpha i}(t)\|$, and can be denoted as follows:

$$\vec{F}_{\alpha B}(t) = -\sum_i \nabla_{\vec{r}_{\alpha i}} U_{\alpha i}(\|\vec{r}_{\alpha i}(t)\|) \tag{5}$$

Here,

$$\vec{r}_{\alpha i}(t) = \vec{r}_{\alpha}(t) - \vec{r}_i^{\,\alpha}(t) \tag{6}$$

$\vec{r}_{\alpha}(t)$ represents the position vector of car α at time t, $\vec{r}_i^{\,\alpha}(t)$ represents the position vector of border point i ($i = 1,2,3,4$) which acts to car α, $U_{\alpha i}(\|\vec{r}_{\alpha i}(t)\|$ is a function with the independent variable $\|\vec{r}_{\alpha i}(t)\|$.

Cars are influenced by all of the forces above in the process of traveling. So we can get the total force which is the resultant force of all the forces above, and can be denoted as follows from equation 1,4-5:

$$\vec{F}_{\alpha}(t) = \vec{F}_{\alpha}^{0}(t) + \sum_{\beta} \vec{f}_{\alpha\beta}(t) + \vec{F}_{\alpha B}(t) \tag{7}$$

The social force model of single car freely traveling can be denoted as follows:

$$\frac{d\vec{v}_{\alpha}(t)}{dt} = \vec{F}_{\alpha}(t) + r(t) \tag{8}$$

Here, $r(t)$ is the random variable in the process travel of car. The actual speed $\vec{v}_{\alpha}(t)$ of car cannot excess maximal velocity of the car, which is discussed in section 6.

3 SOCIAL FORCE MODEL OF SINGLE BICYCLE TRAFFIC FLOW

Traffic behavior of cycling actually is the result of combined effects of individual characteristics of bicycle riders (including trip purpose, preferences, habits, etc.) and road environment, and it is the external manifestation of behavior decision when cyclist faces a kind of complex road environment, and some of the fundamental social force model theory can also apply for the bicycle flow.

Without considering the mutual influence between different types of vehicles, bicycle flow freely travels in bike lanes, and continuously changes the path and speed at the role of “social force” which generated by driveway borders and other vehicles. The social force model of single bicycle freely traveling is similar to the car. Forward driving force of bicycle l relates to the difference between actual speed $\vec{v}_l(t)$ and expected speed $v_l^0 \cdot \vec{e}_l(t)$ at time t, as the following calculation:

$$\vec{F}_l^{\,0}(t) = \frac{1}{\tau_l}(v_l^0 \cdot \vec{e}_l(t) - \vec{v}_l(t)) \tag{9}$$

Here, τ_l represents the response time of the bicycle rider, other symbols are similar as that in function (1). The force which the bicycle m acts to the bicycle l can be denoted as follows:

$$\vec{f}_{lm}(t) = -\nabla_{\vec{r}_{lm}} V_{lm}\left[b_{lm}(t)\right] \tag{10}$$

Here,

$$b_{lm}(t) = \frac{1}{2}\sqrt{(\|\vec{r}_{lm}(t)\| + \|\vec{r}_{lm}(t) - v_m(t)\cdot\Delta t\cdot\vec{e}_m(t)\|)^2 - [v_m(t)\Delta t]^2} \tag{11}$$

The force which the borders act to the bicycle l can be calculated as follows:

$$\vec{F}_{lB}(t) = -\sum_i \nabla_{\vec{r}_{li}} U_{li}(\|\vec{r}_{li}(t)\|) \tag{12}$$

Bicycles are influenced by all of the forces above in the process of traveling. So we can get the total force which is the resultant force of all the forces above, and can be denoted as follows from equation 9-12:

$$\vec{F}_l(t) = \vec{F}_l^{\,0}(t) + \sum_m \vec{f}_{lm}(t) + \vec{F}_{lB}(t) \tag{13}$$

The social force model of single bicycle free traveling can be denoted as follows:

$$\frac{d\vec{v}_l(t)}{dt} = \vec{F}_l(t) + s(t) \tag{14}$$

Here, $\vec{v}_l(t)$ is the velocity of bicycle l at time t; $s(t)$ is the random variable in the process of travel of bicycle. The actual and maximal velocity of bicycle obeys the same restrictions with cars, which is also discussed in section 6.

4 SOCIAL FORCE MODEL OF BICYCLE-CAR MIXED TRAFFIC FLOW

Characteristics of bicycle-car mixed traffic flow are much more complex than single-bike or single-car. Bikes move normally along “snake path” with large swing, so bikes will impacts cars seriously if urban roads aren’t separated physically. In the bicycle-car mixed traffic flow, interference that comes from bicycles and cars as well as the borders of road segments. Jia (2008) divided interference between bicycles and motor vehicles into two types: friction interference and block interference.

The friction interference of the bicycles to vehicles is due to either bikes or cars, the process of moving will occur more or less waver from side to side, while the bike is even worse. So cars need greater security space than the width of cyclists. Therefore, car driver will still feel the side safety pressure when he/she travel closely to a bike though he/she doesn’t drive beyond road dividing line. If the speed of car is faster, the driver will reduce the speed carefully in the situation.

Block interference of cyclists on cars is due to bike traveling beyond the bike lane's line and into the traffic lane, and produces block interference to motor vehicle. In this case, the interaction between bicycle and motor vehicles can be divided into three stages.

4.1 The first stage

When the cyclist drives into the motor vehicle lane, in order to keep certain space with the bicycle driving into the motor vehicle lane, the car behind had to slow down and follows the bycycle, the case is shown in Figure 5.

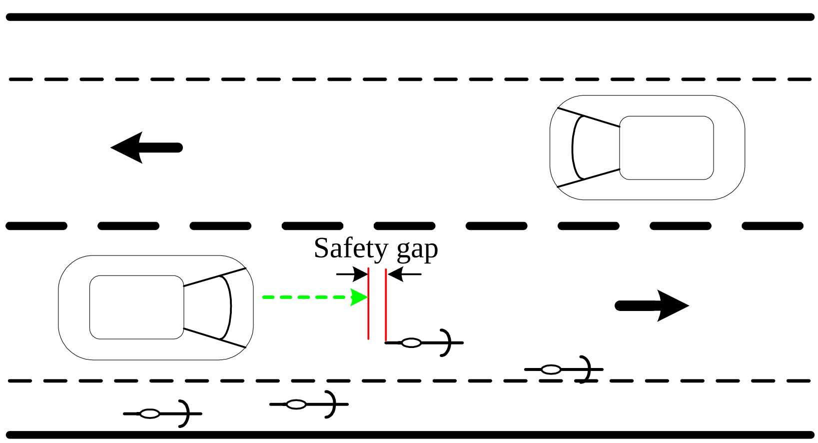


Figure 5 Schematic picture of the interaction force of the motor vehicle(first stage)

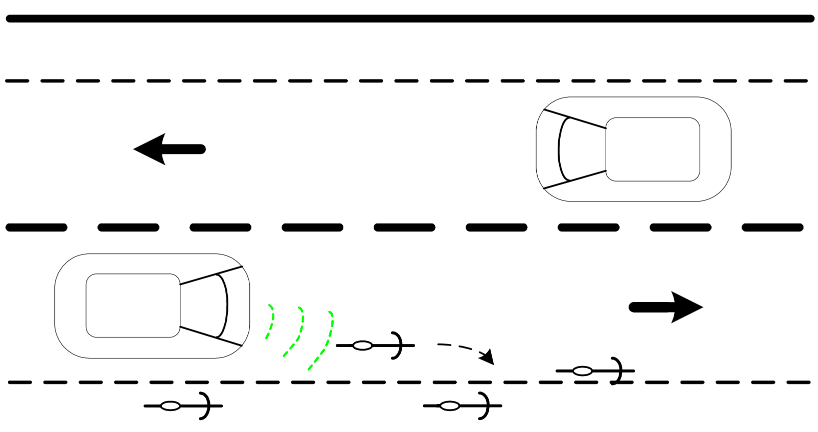


Figure 6 Schematic picture ot he interaction force of the motor vehicle(second stage)

4.2 The second stage

After the bicycle drives into the motor vehicle lane, part of the safe space, which is needed by the car in the driving process, is occupied, so the car will give a warning (whistling) to the cyclist in order to impel the cyclist rides back into the bicycle lane. In order to keep a safe space with the car behind, the cyclist, which rides into the motor vehicle lane, will ride back into the bicycle lane gradually by a series of accelerating or decelerating behavior (it is depended on the social forces of the bicycle), the case is shown in Figure 6.

4.3 The third stage

For avoiding the collision with the bicycle, the safe space of the car will be updated. If the safe space is enough in the advancing direction or the near lane, the car will overtake the cyclist which drives into the motor lane, and which can be shown in Figure 7. All the traffic behaviors of the car are implemented with the influence of all kinds of the social forces.

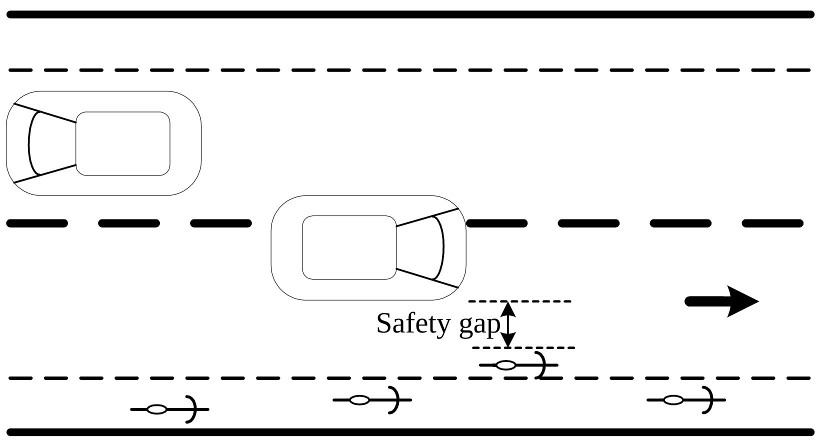


Figure 7 Schematic picture of the interaction force of the motor vehicle(third stage)

Because of different types of the force between bicycles and cars, the social force of the bicycle-car traffic flow is much more complex than the single motor vehicle or bicycle social force model. Almost all of the traffic behavior of the bicycle-car traffic flow can be illustrated by the interactive force (including the force between the bicycle and car, and the force between traffic unit and the borders), so the social force model of bicycle-car traffic flow can be constructed by considering these details. Let the repulsive forces, which the bicycle l acts to the car α and the vehicle α acts to the bicycle l, be $\vec{f}_{\alpha l}(t)$ and $\vec{f}_{l\alpha}(t)$ respectively and they can be given as:

$$\vec{f}_{\alpha l}(t) = -\nabla_{\vec{r}_{\alpha l}} V_{\alpha l}\left[b_{\alpha l}(t)\right] \tag{15}$$

$$\vec{f}_{l\alpha}(t) = -\nabla_{\vec{r}_{l\alpha}} R_{l\alpha}\left[b_{l\alpha}(t)\right] \tag{16}$$

Here,

$$b_{\alpha l}(t) = 1/2\sqrt{(\|\vec{r}_{\alpha l}(t)\| + \|\vec{r}_{\alpha l}(t) - v_l(t)\cdot\Delta t\cdot\vec{e}_l(t)\|)^2 - [v_l(t)\Delta t]^2} \quad (17)$$

$$b_{l\alpha}(t) = 1/2\sqrt{(\|\vec{r}_{l\alpha}(t)\| + \|\vec{r}_{l\alpha}(t) - v_\alpha(t)\cdot\Delta t\cdot\vec{e}_\alpha(t)\|)^2 - [v_\alpha \Delta t(t)]^2} \quad (18)$$

$V_{\alpha l}[b_{\alpha l}(t)]$ and $R_{l\alpha}[b_{l\alpha}(t)]$ are the functions with the independent variable $b_{\alpha l}(t)$ and $b_{l\alpha}(t)$ respectively. Based on the analysis above, we can get the resultant force which the bicycle l is influenced in the process of traveling from equation 12,13 and 16 :

$$\vec{F}_l(t) = \vec{F}_l^{\,0}(t) + \sum \vec{f}_l(t) + \vec{F}_{lB}(t) \quad (19)$$

Here,

$$\sum \vec{f}_l(t) = \sum_m \vec{f}_{lm}(t) + \sum_\alpha \vec{f}_{l\alpha}(t) \quad (20)$$

The resultant force which the car α is influenced in the process of traveling can be denotes as follows from equation 7,13 and 15:

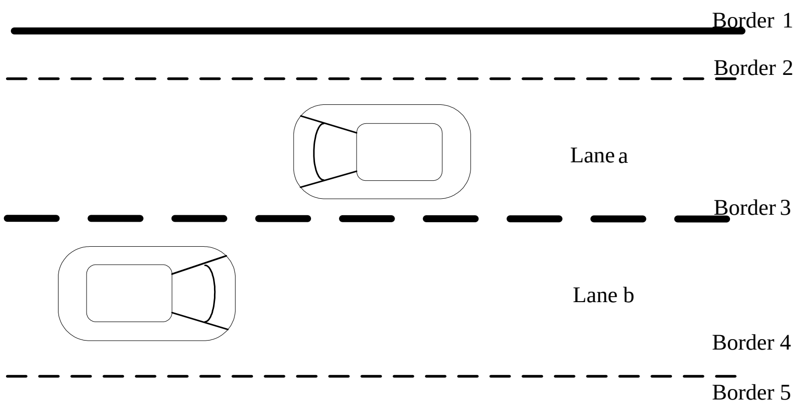
$$\vec{F}_\alpha(t) = \vec{F}_\alpha^{\,0}(t) + \sum \vec{f}_\alpha(t) + \vec{F}_{\alpha B}(t) \quad (21)$$

Here,

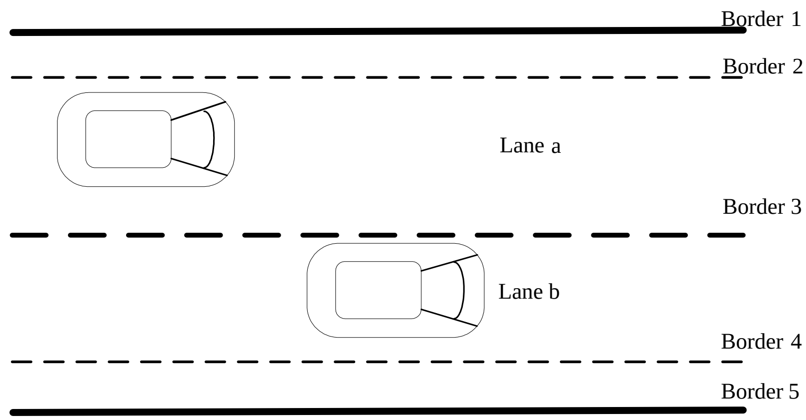
$$\sum \vec{f}_\alpha(t) = \sum_\beta \vec{f}_{\alpha\beta}(t) + \sum_l \vec{f}_{\alpha l}(t) \quad (22)$$

5 ANALYSIS FORCE OF DIFFERENT TYPES OF BORDER

In the social force model, vehicle which includes car is restricted by the road borders, which can be divided into the real border and the virtual border (the partition line on the surface of road). As revealed in Figure 8a, the border 1 and 5 are the real borders, which vehicle cannot get through. When the distance between the real border and vehicle is less than the safe distance, the forces which the border acts to the vehicle will increase infinitely. The border 2, 3 and 4 are the virtual borders which vehicle can get through in the process of moving.



a) Reverse driving



b) Co-rotating driving

Figure 8 Schematic of different types of border

According to the running direction of the vehicle, the forces which the virtual border acts to the vehicle can be divided into two types: the co-rotating influence and the reverse influence. The vehicle is influenced by the two borders of the road lanes simultaneously. In Figure 8a, the vehicle α is running in the lane a, and is influenced by the border 2 and 3, and the interactional force between vehicle α and vehicle β belong to the reverse influence. If vehicle α runs into the lane b influenced by all kinds of social forces, the vehicle α is restricted by the border 2 and 3 as well as the reverse force of vehicles in lane b and will come back into the lane a finally.

In Figure 8b, the force between vehicles in the traffic flow within the lane a and lane b is the co-rotating type. When the vehicle α runs into the lane b from the lane a, it will not restricted by the borders of the lane a, and will go on traveling under the influence of the border of the lane b (borer 3 and 4).

6 THREE ANNOTATIONS

(i) The asymmetry of the forces that cars impose on bicycles and bicycles impose on cars
Obviously, it is asymmetry of the forces that cars impose on bikes and bikes impose on cars. For safety reasons, cars tend to exert a much stronger influence on bikes than bikes do on cars, which also depends on the proportion between cyclists and the drivers. We can deal with this asymmetry by setting different parameter values of the forces of cars and bicycles.

(ii) The maximal steering angle
Because of the influence of the kinds of social forces, the car will be steered in the process of driving, and the steering angle may exceed the value of the maximal steering angle which is determined by the characteristics of the car. So it is necessary to set a limitation of the actual steering angle of the car in the unit time interval: φ and φ_{max} denotes the actual and the maximal steer-

ing angle respectively, if $\varphi > \varphi_{\max}$, let $\varphi = \varphi_{\max}$.

(iii) The optimal and maximal velocity of the car and bicycle

The car will travel with the optimized velocity v_α^0 if it is not disturbed by extraneous factors. At the same time, there is a maximal velocity $v_{\max}$, which is determined by the characteristics of the car, and normally $v_\alpha^0(t) \le v_{\max}$ (the difference of them is determined by the characteristics of the car driver). The actual velocity of the car influenced by the social forces will never exceed the value of the maximal velocity $v_{\max}$. If $v_\alpha(t) > v_{\max}$, let $v_\alpha(t) = v_{\max}$. The optimal and maximal velocity of bicycle obeys the same restrictions with cars.

7 EXAMPLE ANALYSIS

Bicycle-car mixed traffic flow is a typical feature of the urban traffic in many countries, such as China, British, Netherlands and et al. In most of sceneries, the lanes of the motor vehicle and the bicycle are separated by road barriers, though there are many local roads are used by bicycle and car without road barriers. According to an investigation, the road barriers has made a great contribution to regulate the bicycle-car mixed traffic flow to run on their own way, but the non-motorized vehicle, such as bicycle still drives into the motor vehicle lane frequently. The bicycles, which ride into the motor vehicle lanes, will have a strong effect to the driving action of the motor vehicle and the probability of sudden braking which will cause potential collision risk among cars, bicycles and both between bicycles and cars will increase. Here, the potential collision risk is divided into two parts. The one is some cars may have a collision with the bicycles, which ride in the motor vehicle lanes, due to the braking is not in time. And the other is some cars may have a collision with other cars followed due to the sudden braking. We will evaluate the collision risk of the bicycle-car mixed traffic flow under different traffic density and different ratio of the bicycles and the cars through a simulation of the bicycle-car mixed traffic flow by the social force model which is constructed in this paper. The collision between bicycles usually is not serious due to the low speed of bicycle, so in the paper the kind of collision risk is not the main research concern.

(i) Evaluation of the traffic collision risk

In the bicycle-car mixed traffic flow, the bicycle will ride into the motor vehicle lane frequently if there is no segregate installation. Assuming that a bicycle rides into the motor vehicle lane suddenly and if the car behind does not brake in time, a collision will happen between the bicycle and the car. However, even if the car behind braked in time, it may have a collision with the other cars due to the sudden braking. Both of the two types of collision which are introduced above are the most amount potential collision risk and both of them have a strong relation with the braking behavior of the car. So we can use the “probability of the sudden braking” of the car to measure the collision risk of the bicycle-car mixed traffic flow.

In this paper, we have adopted two methods to judge whether a car brakes or not: a) whether the value of the velocity difference of a car $\Delta v = v(t_1) - v(t_2)$ in the two end of a unit time exceeds a critical value $\hat{v}$ or not. b) Whether the braking deceleration at a certain time $a(t)$ exceeds a critical value $\hat{a}$ or not, such as equation 23.

$$\hat{a} = \frac{\hat{v}}{t_2 - t_1} \tag{23}$$

Through a statistical analysis of the sudden braking under different traffic density and different ratio of car and bicycle, we can get the “probability of the sudden braking” in the different traffic scenes. The bigger of the value of the “probability of the sudden braking” is, the higher of the traffic potential collision risk will be.

(ii) Parameter calibration of model

In order to evaluate the collision risk of the bicycle-car mixed traffic, we have set up a social force model for the bicycle-car mixed traffic flow of urban road section with length of 1000 meter. The start speed of cars and bicycles are set as zero at the beginning end of road section, which is based on consideration that the speed of cars is very low after they traversed an intersection the road section followed. By setting frequency of the vehicle driving into the road section, we can get different vehicle density and different proportion of bicycles and cars. The width both of the cycle lane and motor vehicle lane is 3.5m. In the scene simulation, the behavior of changing lane of the car is limited. The bicycle is permitted to driving into the motor vehicle lanes while the car is not driving into the cycle lane. When the traffic density is great, the cars will steadily car-following. During the scene simulation by the social force model, the value of the maximum velocity and the anticipated optimal velocity of the cars are 17m/s and 15m/s while the one of the bicycles are 8m/s and 4m/s~7m/s respectively.

The riding process of the cyclist can be divided into two states according to cyclist’s position: a) driving in the cycle lane, b) driving into the motor vehicle lane. The interaction force between bicycle and car under the first state is much smaller than the one under the second state. So the calculate method of the interaction force under different state should be different.

The social force of the car α and the bicycle l, $\sum \vec{f}_{\alpha}(t)$ and $\sum \vec{f}_{l}(t)$ which is due to the other traffic units can be calculated as follows:

$$\sum \vec{f}_\alpha(t) = \sum_\beta \vec{f}_{\alpha\beta}(t) + \sum_l \vec{f}_{\alpha l}(t) = \sum_\beta V_{\alpha\beta} e^{-b_{\alpha\beta}/\sigma_c} + \sum_l V_{\alpha l} e^{-b_{\alpha l}/\sigma_{cb}} \quad (24)$$

$$\sum \vec{f}_l(t) = \sum_m \vec{f}_{lm}(t) + \sum_\alpha \vec{f}_{l\alpha}(t) = \sum_m V_{lm} e^{-b_{lm}/\sigma_b} + \sum_\alpha V_{l\alpha} e^{-b_{l\alpha}/\sigma_{bc}} \quad (25)$$

At the first state, we set $V_{\alpha\beta} = 20m^2/s^2$, $V_{\alpha l} = 8m^2/s^2$, $V_{lm} = 8m^2/s^2$, $V_{l\alpha} = 8m^2/s^2$, $\sigma_c = 3.75m$, $\sigma_b = 1.2m$, $\sigma_{cb} = 1.2m$, $\sigma_{bc} = 1.2m$.

At the second state, we set $V_{\alpha\beta} = 20m^2/s^2$, $V_{\alpha l} = 8m^2/s^2$, $V_{lm} = 8m^2/s^2$, $V_{l\alpha} = 8m^2/s^2$, $\sigma_c = 3.75m$, $\sigma_b = 1.2m$, $\sigma_{cb} = 3.75m$, $\sigma_{bc} = 1.4m$.

When the bicycle is driving in the cycle lane, it is restricted by the border 1 and border 2(referring to Figure 9), and the value of the border force is proportional to the distance between the bicycle and the border. The border force of the bicycle l can be calculated as follows:

$$F_{lB} = \sum_i^B U_{lB_i}^0 \cdot e^{-r/R_b} \quad (26)$$

In the social force model, the road borders can be divided into the real border and the virtual border (the partition line on the surface of road). For the real border, there have $U_{lB_1} = 10m^2/s^2$, $R_b = 0.3m$, and for the virtual border, there have $U_{lB_2} = 3m^2/s^2$, $R_b = 0.3m$.

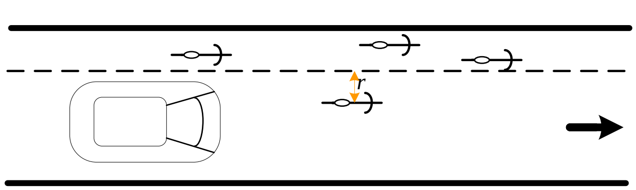


Figure 9 Schematic of the mixed traffic flow of bicycle/motor vehicle

If the cyclist rides into the motor vehicle lane, as is shown in Figure 9, the farther the distance between border and the bicycle which drives into the motor vehicle lane is, the bigger the border force which is due to the border 2 will be. So border force which due to the border 2 when the cyclist rides into the motor vehicle lane can be calculated as follows:

$$F_{lB} = \sum_i^B U_{li}^0 \cdot e^{\lambda \cdot r/R_b} \quad (27)$$

As it is not permitted to drive into the cycle lane, so the border force $\vec{F}_{\alpha B}(t)$ of the motor vehicle α can be calculated as follows:

$$F_{\alpha B} = \sum_{i}^{B} U_{\alpha i}^{0} \cdot e^{-r/R_c} \qquad (28)$$

Here, $U_{\alpha i} = 15m^2/s^2$, $R_c = 0.7m$.

In the process of the analysis of the traffic collision risk, the critical acceleration is $\hat{a} = -3m/s^2$. If $a(t) \le \hat{a}$, a sudden brake happens. Here, $a(t)$ represents the acceleration of the motor vehicle at time t.

(iii) Results

The traffic density is a set as a scale parameter, which is set as the ratio of the actual density and the jam density in the paper, and the jam density of bicycle and car is 0.4 bic/m^2 and 133 veh/km respectively. In the sudden braking process of car, its actual velocity will decrease sharply in a short time. When the deceleration is less than - 3m/s^2, it is shown that the car had an emergency braking behavior in the simulation. So the variety of the velocity is the external expression of the emergency braking of the motor vehicle, which is different corresponding to different density of the bicycle and car, and the results can be seen from Figure10~13, which are corresponding to the change of the car's velocity within time period [0, 60] second when the car's density ρ_c is 0.75\0.5\0.35\0.1875 respectively, and each of the figures concludes four sub-graphs (a)/(b)/(c)/(d), from which we can see the change of the car's velocity within time period [0,60] when the bicycle's density is 0.41/0.36/0.21/0.17 respectively. In Figure10~Figure13, ρ_c represents the density of car.

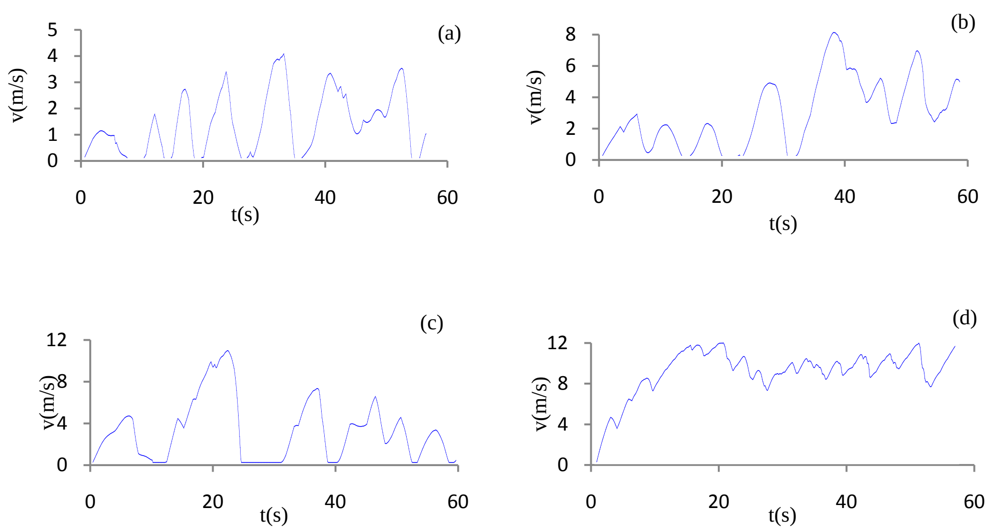


Figure 10 Schematic of the velocity-time curve（ρ_c =0.75）

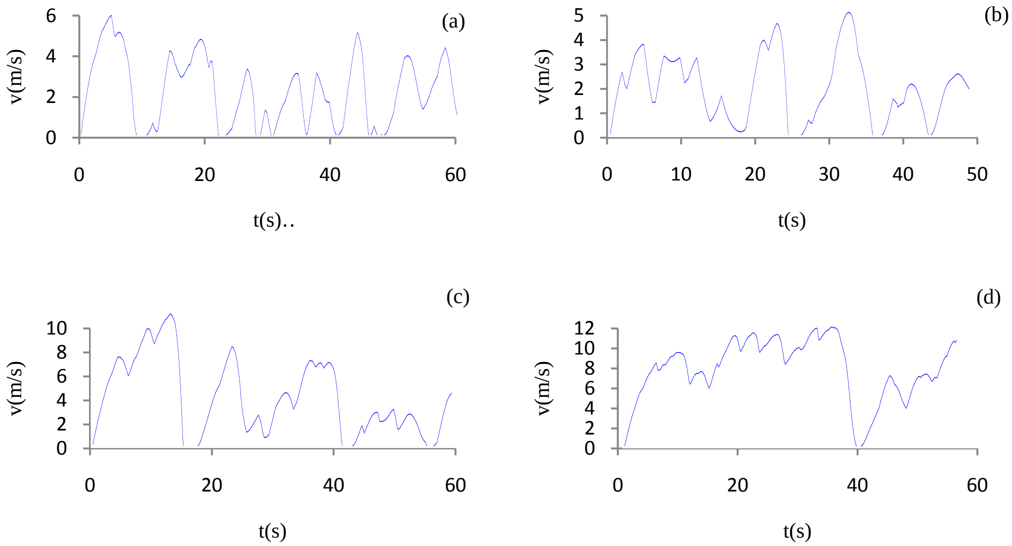


Figure 11 Schematic of the velocity-time curve（ρ_c =0.5）

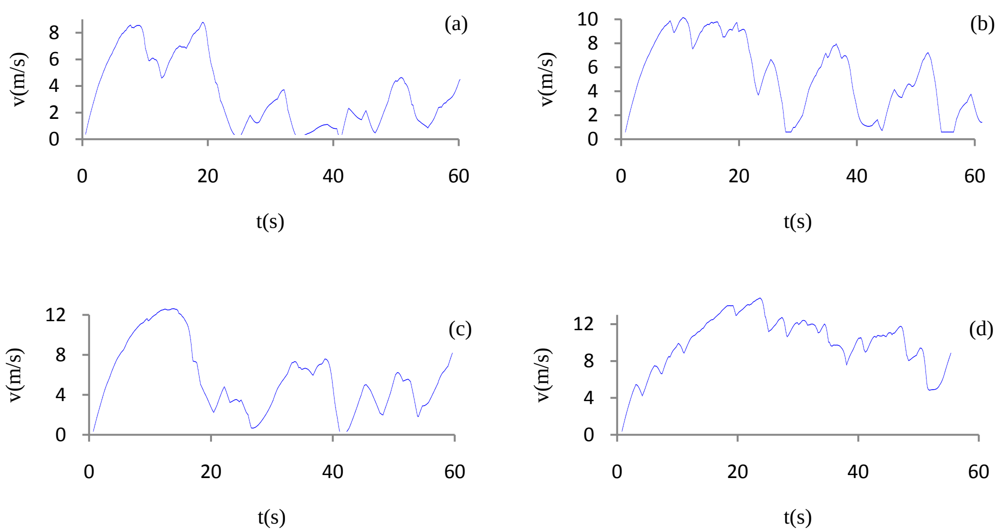


Figure 12 Schematic of the velocity-time curve（ρ_c =0.35）

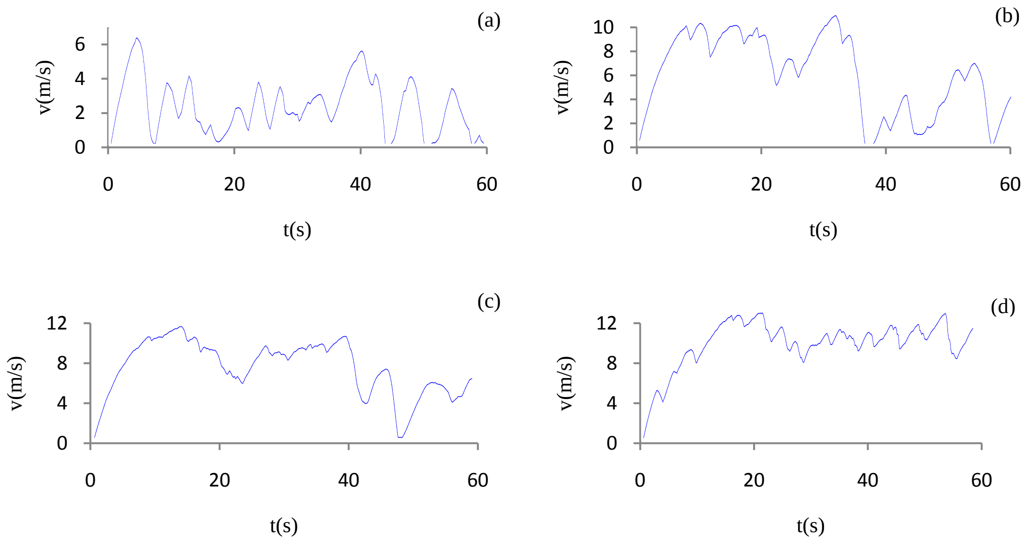
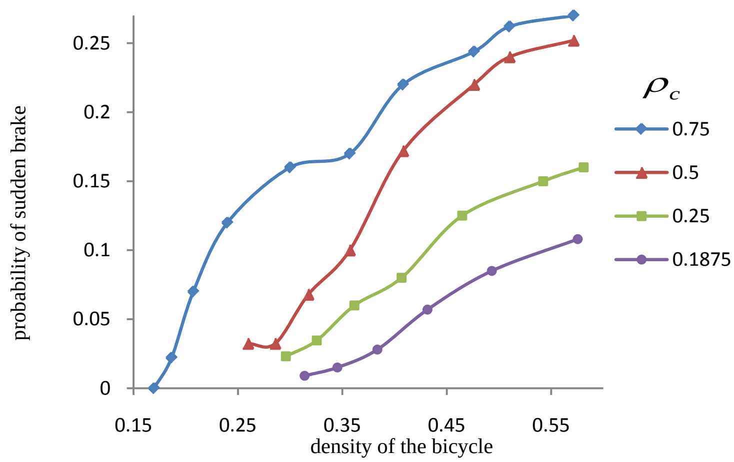


Figure 13 Schematic of the velocity-time curve（ρ_c =0.1875）

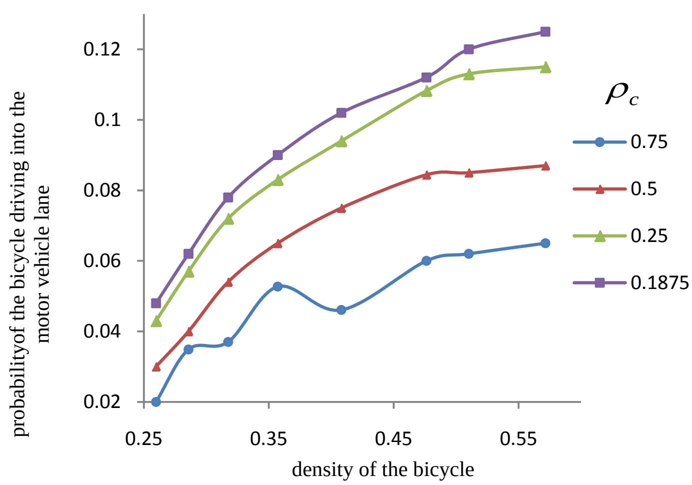
The acceleration/deceleration of car at a time is the slope of car speed curve in Figure10~Figure13. If the car's speed dropped sharply within short time, it expresses that in the certain period the braking deceleration of car is very large, which shows that the vehicles had an emergency braking behavior. Contrasting the four son picture of Figure10, we can see, in case of constant density of cars and with the lower density of bicycles, the times of car's sudden brake reduced. For example, in Figure10 (d), the density of bicycles is 0.17, while the car shows fewer of the emergency braking behavior than that in Figure10 (a). From Figure11 ~ Figure13, the relation between the times of car's emergency braking and bicycle's density is shown the same trend with Figure10.

In the simulation process, the deceleration times during different time period of all cars has been counted under different car's density, as the change of bicycle density and the probability of the bike being into the motor lane. We count and analyze such cases, and the statistical results are shown in Figure14 below. According to Figure14 sub-figure a and b, when the car's density remains constant, the probability of car's sudden brake and bicycle riding into motor vehicle lane would become higher while the bicycle's density increased, which is due to, with the bicycle's density becoming higher, the interference between bicycles will increase. Therefore, the probability of bicycle moving into vehicle lane impelled by the interference will become higher. After bicycle moved into the motor vehicle lane, it will seriously interfere with car's moving. With the number of bicycles that moved into motor vehicle lane increase, the number of sudden brake is inevitable to increase, therefore, the probability of car's sudden brake will become higher.

When the bicycle's density remains constant, and with car's density rising, the probability of bicycle moving into motor vehicle lane will cut down, however, the probability of car's sudden brake will become higher. The reason is that, bicycle density remaining constant, when the vehicle density increase, the number of cars within the unit distance will rise. The interference between vehicle and bicycle will become stronger. Even if bicycles have strong tendency to move into vehicle lane, but only to drive in the original cycle lane due to car's social force to them. So the probability of bicycles moving into motor vehicle lane is relatively small, the result is showed in Figure 14(b). On the other hand, the interference between cars will rapidly increase along the rising of car's density. With the interference among cars, the probability of car's sudden brake will rise. In the case, the potential collision risk among cars is larger than that between bicycles and cars. The same result that crashes have high relationship with vehicles' density is also revealed by other researchers (Lord, Maner and Vizioli 2005; Golob, Recker and Palis, 2008) by data analyzing method.



(a)



(b)

Figure 14 Schematic picture of the simultion results analysis

In Figure 14, ρ_c also represents the density of cars, based on the above analysis, we can get the changes of the probability of car's brake and bicycle moving into vehicle lane, in the case of different vehicle density and rate of bicycle. In the reality scene, the rate of traffic accidents is closely related to probability of car's sudden brake and bicycle moving into vehicle lane, they show a positive correlation. That is, when the car's density remains constant, the rate of traffic accidents increases as the bicycle's density becomes high. The higher the density is, the higher the rate of the traffic accidents becomes. And when bicycle's density remains constant, although the density of cars increases and the probability that bicycles may run into motor vehicle lanes decreases. In the scene, the mutual interference between cars gets stronger, therefore the collision risks of cars are becoming higher, which mainly due to the aggravated interference between cars.

Based on the analysis above, we can come to the conclusions:
(a) When car's density remains constant, the risk of traffic collision increases with the bicycle's density. The higher the density of the bicycle is, the higher the rate of the traffic accidents becomes. The increment of the risk of traffic collision is largely due to the intensified interference between cars and bicycles.
(b) When bicycle's density remains constant, although the density of cars increases and the probability that bicycles may run into vehicle lane decreases, the mutual interference between motor vehicles get stronger, therefore the risk of vehicle's crash are becoming higher, which mainly due to the aggravated interference between motor vehicles.
(c) For the bicycle-car mixed traffic flow, with the density of cars decreases, the motor vehicle speed will increase accordingly, but the probability that bicycles may run into motor vehicle lane will increases greatly, which may increase the collision probability between bicycle and car rapidly.

8 CONCLUSIONS

Based on the basic theory of the social force model, the article has constructed a social force model of bicycle-car mixed traffic flow, and gives a quantitative description for the three main forces which car and bicycle are influenced in the process of traveling. A systemic method has been established to analyze the traffic motion of bicycle-car mixed traffic flow. By the probability of the sudden brake, the bicycle-car mixed traffic flow in a urban road section has been simulated, and the collision risk of the bicycle-car mixed traffic flow has been analyzed, from which we come to the conclusion that the risk of the bicycle-car mixed traffic's potential collision increases with increment of the traffic density. For the bicycle-car mixed traffic flow, with the car's density decreasing, the probability of bicycles driving into the motor vehicle lane will increase, and the probability of the traffic collision between the bicycle, which drives into the motor vehicle lane, and the car with a high speed will sharply increase. When bicycle's density remains constant, the probability that bicycles may run into vehicle lane decreases with the increase of cars' density due to the mutual interference between motor vehicles get stronger, therefore the risk of vehicle's crash are becoming higher, which mainly due to the aggravated interference between motor vehicles. The conclusion of the simulation suggests that the model constructed in this article is reasonable and effective. However, the practical application and modification of this model needs further study in future. Phenomenon of the other mixed traffic flow, such as traffic flow including passenger car and truck, are left to exploit in future.

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