# Bayesian Trajectory-Based Reconstruction of Rear-Ending Events Using Naturalistic Driving Data 

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#### Abstract

Road safety engineering studies in the past typically involved regression-based crash prediction models relating crash frequency to road design or operational configurations. In addition to this conventional approach there has been an ongoing interest in using microscopic simulation to predict the safety consequences of engineering decisions, similar to how microscopic models that are used to predict operational performances. Historically, a major obstacle to the development of such crash-inclusive simulation models has been a scarcity of data on driver behavior in crash and near-crash conditions. In the recent years however, with more advanced and sophisticated data collection techniques emerging, it has been possible to collect individual vehicle data under naturalistic driving conditions. In this study we illustrate using vehicle based data to show how trajectory based modeling technique can be implemented to reconstruct crash related events, and in turn estimate the posterior distribution of important event parameters such as braking accelerations, reaction time, and critical headways for a given set of trajectory data. Our results suggest that for a rear ending event it is possible to obtain precise estimates of the interaction between the leading and the following vehicle and identify the critical driving conditions that cause the event. Given sufficiently large samples of crash and near-crash events, this method could be used to compile distributions for these inputs, which could in turn be used to include realistic crash features in a microscopic simulation model.


## INTRODUCTION

Rational planning involves selecting actions based on their expected consequences, and in road safety engineering this requires an ability to predict the frequency or severity of road crashes expected to result from engineering actions. The Highway Safety Manual (HSM) is expected to provide road safety professionals with tools for making at least some of the needed predictions. The dominant methodology in the first edition of the HSM is based on regression analysis of generalized linear models to relate the crash frequency to traffic flow at some given baseline conditions and then empirically determined accident modification factors (AMFs) are multiplied to the expected crash frequency to reflect the changes from the baseline conditions. Bonneson and Lord (2006) have pointed out an interesting similarity with development of the Highway Capacity Manual, where first- generation regression models of intersection delay which used naïve specifications of independent variables were subsequently replaced by regression models where the forms of the independent variables were based on theoretical arguments. These regression-based models were then later replaced by models, such as Webster's delay model, whose functional forms were developed from theoretical considerations.

In addition to regression-based approaches there has been an ongoing interest in using microscopic simulation to predict how roadway changes affect safety. These include efforts to identify safety-related surrogates from the output of simulation models (Gatteman and Head, 2003), the Roadside Safety Analysis Program (RSAP) (Mak and Sickling, 2003), and recent efforts to allow for crash occurrence in microscopic car-following models (Xin et al., 2008; Hamdar and Mahmassani, 2008). Historically, a major obstacle to the development of crashinclusive simulation models has been a scarcity of data on driver behavior in crash and nearcrash conditions, especially since events involving more than one vehicle can arise out of the interaction between the behaviors of the individual drivers. In particular, although there has been considerable recent interest in aggressive driving and driver inattention as crash risk factors, it is unclear how prevalent these factors are or how to include such considerations in a simulation model. During the past several years however an improved capability for collecting data in naturalistic conditions promises to at least partially fill this need. The most prominent of these efforts is the naturalistic driving component of the SHRP 2 safety program, but notable past efforts include Virginia Tech Transportation Institute's 100-Car Study (Dingus, et al., 2006) and the Automotive Collision Avoidance System field test conducted by the University of Michigan Transportation Institute. In these studies volunteers drive instrumented vehicles which continuously collect and record measurements such as vehicle position, speed, direction, acceleration, as well as radar-based range and range-rate measurements for other vehicles. An important and as yet unresolved question concerns the degree to which naturalistic driving data can support the development of realistic crash simulation models.

In this paper we will describe how Bayesian analysis can be used in combination with data from naturalistic driving studies to identify and validate tractable yet plausible models of driver behavior in car-following situations.

## Review of Structural Modeling for Crash Related Events

To illustrate our approach let us take a simple example of a car-following interaction, where a leading vehicle and following vehicle successively brake to stops. Suppose the initial speed and braking deceleration of the leading vehicle are denoted by $v_{1}$ and $a_{1}$ respectively, $v_{2}$ and $a_{2}$ denote the initial speed and braking deceleration of the following vehicle, and $h_{2}$ and $r_{2}$ denote the following driver's headway and reaction time. As pointed out by Brill (1972), a collision occurs when the stopping distance available to the following driver is less than that needed to stop without colliding with the lead vehicle. Brill's collision condition can be expressed formally as

$$
\begin{equation*}
v_{2} r_{2}-\frac{v_{2}^{2}}{2 a_{2}}>h_{2} v_{2}-\frac{v_{1}^{2}}{2 a_{1}} \tag{1}
\end{equation*}
$$

Alternatively, the position of a vehicle initially traveling at a constant speed and then braking to a stop with a constant deceleration can be expressed

$$
\begin{gather*}
v_{k} t, \quad t \leq t 0_{k} \\
y_{k}(t)=v_{k} t 0_{k}+a_{k}\left(t-t 0_{k}\right)^{2} / 2, \quad t 0_{k}<t \leq t 0_{k}+v_{k} /\left(-a_{k}\right)  \tag{2}\\
v_{k} t 0_{k}-v_{k}^{2} / 2 a_{k}, \quad t>t 0_{k}+v_{k} /\left(-a_{k}\right)
\end{gather*}
$$

where $y_{k}(t)$ denotes the (one-dimensional) position of vehicle k at time $\mathrm{t}, v_{k}$ denotes the initial speed of driver $k, a_{k}$ denotes his or her braking acceleration and $t 0_{k}$ denotes the time at which braking began. This model can be connected to the rear-end collision model described above by noting that the reaction time of driver k is simply

$$
\begin{equation*}
\mathrm{r}_{\mathrm{k}}=\mathrm{t} 0_{\mathrm{k}}-\mathrm{t} 0_{\mathrm{k}-1} \tag{3}
\end{equation*}
$$

while, the initial following headway between vehicles $k$ and $k-1$ when driver $k-1$ began braking is

$$
\begin{equation*}
\mathrm{h}_{\mathrm{k}}=\left(\mathrm{y}_{\mathrm{k}}\left(\mathrm{t} 0_{\mathrm{k}-1}\right)-\mathrm{y}_{\mathrm{k}-1}\left(\mathrm{t} 0_{\mathrm{k}-1}\right)\right) / \mathrm{v}_{\mathrm{k}} \tag{4}
\end{equation*}
$$

A collision would then occur when

$$
\begin{equation*}
y_{k}(t)-y_{k-1}(t)<0 \tag{5}
\end{equation*}
$$

The main point we wish to emphasize is that a structural model consists of a set of input variables and one or more equations which give predicted outputs as functions of the input variables. Certain values for those inputs can be indicators of more qualitative driving states. For example, particularly for vehicles closely following each other, atypically long reaction times can indicate driver inattention while atypically short following headways or atypically high speeds can indicate aggressive driving. The question then is how estimates of a model's inputs can be obtained from observations of its output.

## METHODOLOGY

The simple brake to stop model discussed in the previous section allowed only for a constant speed followed by a single deceleration phase. One of the objectives of this research is to develop a more robust model that can capture more complicated scenarios such as mult-staged acceleration. One basic assumption we will make is that a driver's behavior can be modeled as a piecewise-constant series of accelerations, which are then treated as inputs into a dynamic trajectory model. The vehicle state at a given time is its location and speed, and the trajectory model then takes the acceleration input sequence and numerically integrates the associated differential or difference equations to produce time histories of vehicle locations and speeds. For discrete-time data, the trajectory model can be conveniently represented using the generic linear state-space form

$$
\begin{align*}
& \mathrm{x}(\mathrm{t}+1)=\mathrm{Ax}(\mathrm{t})+\mathrm{Ba}(\mathrm{t})  \tag{6}\\
& \mathrm{y}(\mathrm{t})=\mathrm{Cx}(\mathrm{t})
\end{align*}
$$

where, $x(t)$ is vector of state variables (position and speed), $a(t)$ is a vector input variables (accelerations), and $y(t)$ is the vector of observed variables (following vehicle's speed, range and range rate). $\mathrm{A}, \mathrm{B}$, and C in equation (1.6) stand for matrices of coefficients.

The nature of $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{x}(\mathrm{t})$ and $\mathrm{y}(\mathrm{t})$ will vary depending on the class of events being modeled and the sort of data that is available. For two vehicles following on a straight road, the simplest trajectory model would consist of two state variables for each vehicle, its location and speed, with linear acceleration values as inputs. That is, if $\Delta$ denotes the basic time interval of the data, then the deterministic progression for a leading and following vehicle can be captured by the linear equation

$$
\left[\begin{array}{c}
x 1(t+1)  \tag{7}\\
x 2(t+1) \\
v 1(t+1) \\
v 2(t+1)
\end{array}\right]=\left[\begin{array}{llll}
1 & 0 & \Delta & 0 \\
0 & 1 & 0 & \Delta \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right] \cdot\left[\begin{array}{c}
x 1(t) \\
x 2(t) \\
v 1(t) \\
v 2(t)
\end{array}\right]+\left[\begin{array}{cc}
0 & 0 \\
0 & 0 \\
\Delta & 0 \\
0 & \Delta
\end{array}\right]\left[\begin{array}{c}
a 1(t) \\
a 2(t)
\end{array}\right]
$$

For naturalistic driving scenarios where forward radar produces range and a range rate measurement for a leading vehicle, the observation equation takes the form

$$
\left[\begin{array}{l}
y 1(t)  \tag{8}\\
y 2(t) \\
y 3(t)
\end{array}\right]=\left[\begin{array}{cccc}
0 & 0 & 1 & 0 \\
1 & -1 & 0 & 0 \\
0 & 0 & 1 & -1
\end{array}\right] \cdot\left[\begin{array}{c}
x 1(t) \\
x 2(t) \\
v 1(t) \\
v 2(t)
\end{array}\right]
$$

Here $\mathrm{x} 1(\mathrm{t})$ and $\mathrm{x} 2(\mathrm{t})$ give the locations of the leading and following vehicles at time t , $\mathrm{v} 1(\mathrm{t})$, $\mathrm{v} 2(\mathrm{t})$ are the corresponding speeds, and a1(t), a2(t) are accelerations. For motion in two directions a similar structure can be used but with state and input variables for each direction.

Given initial values for the state variables and the time history for the inputs, the trajectories of both vehicles can be replicated. The above relationship between position, speed and acceleration which are all function of time can be expressed in the form a set of ordinary differential equations as shown below. This is typically referred as initial value problem in the ordinary differential equation solution which describes the evolution of a state over time for a given initial condition.

$$
\begin{align*}
& d z / d t=v(t) \\
& d v / d t=a c c l \tag{9}
\end{align*}
$$

where $z(t)$ defines the position of the vehicle at any time $t, v(t)$ represents speed of the vehicle at time t and accl states the acceleration of the vehicle which was treated as a piecewise function over different time intervals depending on the diver's behavior.

Given estimates of a driver's initial speed, the times at which he or she changed acceleration and the corresponding accelerations, the differential equations can be solved to give predicted time histories of that vehicle's position and speed and predicted values for the observations. Fitting a trajectory model then involves searching plausible combinations of values for these input quantities to find those that best account for the data.

Major Parameters of Interest:

- Piecewise constant acceleration
- Change points (where the driver changed from one acceleration mode to another)
- Reaction time ( time elapsed between the leading vehicle decelerating and the following vehicle taking the evasive action as its reaction to the preceding vehicle's behavior)
- Critical headway (distance between the two vehicles as the following vehicle begin to take the evasive action)

Bayesian analysis using Markov Chain Monte Carlo (MCMC) simulation was used to estimate the parameters. (Robert and Casella, 1999) The differential equations were numerically solved using WinBUGS differential equation interface (Lunn et al. 2000). This provided compiled procedures that can be included in a WinBUGS model specification which numerically solve ordinary differential equations using Runge-Kutta methods. Solutions from the differential equations were then fitted into our model as predicted values for instantaneous position and speed. As is our standard practice in working the MCMC estimation, we first conducted exploratory analyses using frequentist methods, in this case nonlinear least-squares, implemented using MATLAB (2002). This was done in order to understand the complexity of the acceleration model suggested by a given data set and to get reasonable starting values for the MCMC simulation. Bayes estimates for model parameters were then computed using WinBUGS.

## Brief Model Description from Bayesian Perspective

The basic data model used in our reconstruction as follows:

$$
\begin{equation*}
V_{\text {obs }} \sim \operatorname{Normal}\left(\hat{V}, \tau_{1}\right) \tag{10}
\end{equation*}
$$

Range $\sim \operatorname{Normal}\left(\hat{R}, \tau_{2}\right)$
RangeRate $\sim \operatorname{Normal}\left(R \hat{R}, \tau_{3}\right)$
where, $V_{\text {obs }}$, Range and RangeRate are the observations obtained from instruments every 0.1 secs interval and taul,tau 2 and tau 3 are the precisions (inverse of variance)
$\hat{V}, \hat{R}$, and $R \hat{R}$ are the predicted speed, range and range rate values obtained by solving the differential equations governing the relationship between location, speed and acceleration of the vehicle (refer to Equation 9). As mentioned one of our assumptions in the model is the driver behavior as piecewise constant accelerations, which means that driver's acceleration changes over discrete time interval. These acceleration and change points are also treated as random variables which are used as initial conditions for each stage of the driver's behavior into the WinBUGS differential equation solver. Vague normal priors were assumed for acceleration inputs; however more informative priors were used for change points based on the frequentist's approach of nonlinear least square estimates from MATLAB.

## Data Acquisition and Reduction

In order to develop and test our estimation methods we requested a set of example data originally collected by Virginia Tech Transportation Institute (VTTI) during the 100-Car Study (Dingus et al., 2006). The original data mainly consist of time series of measurements from the in-vehicle sensors, along with the videos from the forward camera, for about 30 seconds (secs) preceding and including the crash or near-crash event. For the instrumented vehicle which is the following vehicle, the data available consist of speedometer output, lateral and longitudinal accelerations, yaw, heading, and indications of the status of the turn signal, the brake, and the accelerator, recorded at 10 Hz . For the lead vehicle, the available data consist of range, range-rate, and azimuth obtained from the forward-viewing radar, also recorded at 10 Hz . The most critical information that was used for analysis was speedometer readings from the following vehicle, range and range rate data.

## Case Studies

## Case 1

In this case the instrumented vehicle (i.e., the following vehicle) at the start of the video took a right turn and continued to follow the leading vehicle. But then the leading vehicle decelerated and came to a complete stop. This forced the following vehicle also to decelerate, resulting in a conflict. However, the following driver's deceleration was sufficient to enable the vehicle to come to a complete stop without a collision. Although the total length of the video was 19 seconds, the event actually happened within first 8 seconds. For the remaining period of time, both the vehicles were in stopped condition.

The speed of the following vehicle was obtained directly from the speedometer of the instrumented car. For exploratory purposes an approximate speed of the leading vehicle was calculated by adding the speed of the following vehicle and the range-rate data obtained from radar. Final estimation however was done using the original data, i.e., follower speed and leader range and rangerate. Figure 1 shows the range and range-rate data obtained for this event.


Figure 1 Range and range-rate data for case 1
For the following (instrumented) vehicle, a three-stage model was (see Figure 2), where a period of initial acceleration lasting about 2.5 seconds was followed by a period of strong deceleration, which was followed by a short period of another deceleration over a period of 2 seconds until the vehicle stopped completely.


Figure 2 Proposed 3-block model for the following vehicle
Similarly, for the leading vehicle a 3-stage model was proposed (see Figure 2), where first two mild deceleration periods that lasted for 2.5 secs was followed by stronger deceleration until the vehicle came to a complete stop.

After initial estimates of the change points and accelerations were obtained from MATLAB, the trajectory model was in WinBUGS for final estimates. Table 1 gives the final MCMC simulation estimates of the parameters.

At the time the radar acquired the leading vehicle the initial speeds of the following and leading vehicle were 25.66 feet/sec and 26.07 feet/sec respectively. The leading vehicle decelerated in three different stages. The first two deceleration stages were characterized by mild deceleration followed by a very steep negative acceleration ( $-24.29 \mathrm{feet} / \mathrm{sec}^{2}$ ) bringing the leading vehicle to a complete stop. Subsequently, the following vehicle initially was accelerating for 2.626 seconds, and then it accelerated at -21.76 feet $/ \mathrm{sec} / \mathrm{sec}$ followed by a third negative acceleration of $-2.87 \mathrm{feet} / \mathrm{sec}^{2}$. The predicted piece-wise acceleration model was compared by fitting the observed data. The MCMC estimates of the parameters from Table 1 were then provided as inputs to the differential equation solver to obtain predicted positions, range and speed of the vehicles. The range and speed of the following vehicle was fitted as shown Figures 3 and 4 .

Table 1 WinBUGS Estimates for the Model Parameters

|  | Mean | Stand. $\mathrm{Dev}^{1}$ | 2.50\% | Median | 97.50\% |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Following Vehicle |  |  |  |  |  |
| Initial Speed (feet/sec) | 25.66 | 0.3869 | 24.92 | 25.66 | 26.45 |
| First Acceleration (feet/sec ${ }^{2}$ ) | 1.567 | 0.2429 | 1.072 | 1.566 | 2.027 |
| Second Acceleration(feet/ $\mathrm{sec}^{2}$ ) | -21.76 | 0.7101 | -23.06 | -21.79 | -20.3 |
| Third Acceleration (feet/sec ${ }^{2}$ ) | -2.876 | 0.3197 | -3.461 | -2.891 | -2.212 |
| First Change (sec) | 2.626 | 0.02057 | 2.572 | 2.629 | 2.657 |
| Second Change (sec) | 3.811 | 0.03753 | 3.755 | 3.806 | 3.891 |
| Reaction time (sec) | 1.059 | 0.04373 | 0.9932 | 1.052 | 1.161 |
| Leading Vehicle |  |  |  |  |  |
| Critical headway (feet) | 14.7 | 0.6635 | 14.04 | 14.23 | 15.57 |
| Critical Speed (feet/sec) | 26.87 | 1.052 | 25.51 | 26.39 | 28.75 |
| Initial Speed (feet/sec) | 26.07 | 0.4079 | 25.27 | 26.07 | 26.88 |
| First Acceleration (feet $/ \mathrm{sec}^{2}$ ) | -2.973 | 0.2788 | -3.54 | -2.971 | -2.434 |
| Second Acceleration(feet/sec ${ }^{2}$ ) | -6.172 | 0.4611 | -7.155 | -6.153 | -5.319 |
| Third Acceleration (feet/sec ${ }^{2}$ ) | -24.29 | 0.7389 | -25.61 | -24.34 | -22.71 |
| First Change (sec) | 2.752 | 0.02641 | 2.674 | 2.755 | 2.793 |
| Second Change (sec) | 3.362 | 0.01218 | 3.339 | 3.361 | 3.387 |

Note: ${ }^{1}$ Standard deviation


Figure 3 Comparing observed and predicted range data


Figure 4 Predicted and observed instrumented speed
For this event the leader and follower were initially traveling at similar speeds, and the estimated reaction time of the follower (about 1.06 seconds) is typical of what has been observed in surprise braking experiments (Fambro et al., 1997). The follower's headway at time of his reaction was only 14.7 feet, suggesting the aggressive car following behavior.

## Case 2

In this event the two vehicles were closely following each other. The leading vehicle accelerated and then traveled at uniform speed before it decelerated to almost zero speed, which resulted in almost a rear-end crash. The following vehicle follows the same pattern as the leading vehicle, shown in Figure 5.


Figure 5 Speed trajectory of the leading and following vehicle for case 2

Range and range data obtained from the radar is shown in Figure 6.


Figure 6 range and range-rate data for case 2
Exploratory analysis in MATLAB suggested a four stage model for the following vehicle's speed trajectory. For the first two stages the following vehicle had two different acceleration values, the second being the weakest, which was then followed by two deceleration stages. Similarly for the leading vehicle another four stage acceleration model was proposed. First stage was characterized by a strong acceleration over a period of six seconds and then the vehicle traveled with almost constant speed for little more than 2 seconds, followed by two deceleration stages where the last one being the strongest. Table 2 shows the WinBUGS estimates.

Initially the following vehicle was traveling at $11.21 \mathrm{feet} / \mathrm{sec}$ and then accelerated at $3.15 \mathrm{feet} / \mathrm{sec} / \mathrm{sec}$ for 5.36 seconds. Then it traveled at almost constant speed for another 5 seconds before decelerating at -2.419 feet $/ \mathrm{sec}^{2}$ for 4.3 seconds followed by a strong negative acceleration of -10.74 feet $/ \mathrm{sec}^{2}$ and finally came to a stop at 16.47 seconds. A similar pattern was observed for the leading vehicle, which had an initial acceleration stage of 8.16 feet $/ \mathrm{sec}^{2}$ for 1.764 seconds, followed by a period of 8.277 seconds of almost constant speed and then two deceleration stage with the final negative acceleration rate as high as $-9.502 \mathrm{feet} / \mathrm{sec}^{2}$. The similar speed profile of the two vehicles seems reasonable as they were following closely each other in this case.

Predicted versus observed speed plotted (see Figure 7) indicates reasonable fit for the model.

Table 1.2 WinBUGS Estimates for Case 2

|  | Mean | Stand. Dev ${ }^{1}$ | 2.50\% | Median | 97.50\% |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Following Vehicle |  |  |  |  |  |
| Initial Speed (feet/sec) | 11.21 | 0.2399 | 10.74 | 11.21 | 11.67 |
| First Acceleration (feet $/ \mathrm{sec} / \mathrm{sec}$ ) | 3.149 | 0.06246 | 3.034 | 3.146 | 3.275 |
| Second <br> Acceleration(feet/sec/sec) | 0.7217 | 0.03928 | 0.6431 | 0.7219 | 0.7981 |
| Third Acceleration (feet $/ \mathrm{sec} / \mathrm{sec}$ ) | -2.419 | 0.1075 | -2.635 | -2.415 | -2.22 |
| Fourth Acceleration (feet/sec/sec) | -10.74 | 0.3316 | -11.38 | -10.74 | -10.07 |
| First Change (second) | 5.364 | 0.06755 | 5.236 | 5.362 | 5.495 |
| Second Change (second) | 10.71 | 0.05586 | 10.61 | 10.71 | 10.83 |
| Third Change (second) | 14.3 | 0.03184 | 14.23 | 14.3 | 14.36 |
| Time when stopped (second) | 16.47 | 0.05112 | 16.37 | 16.47 | 16.57 |
| Reaction time (second) | 0.4847 | 0.04604 | 0.3996 | 0.4829 | 0.5788 |
| Leading Vehicle |  |  |  |  |  |
| Critical Range (feet) | 12.79 | 0.3077 | 12.11 | 12.86 | 13.47 |
| Critical Speed (feet/sec) | 24.43 | 0.226 | 23.96 | 24.43 | 24.84 |
| Initial Speed (feet/sec) | 14.07 | 0.3193 | 13.46 | 14.07 | 14.71 |
| First Acceleration (feet/sec/sec) | 8.16 | 0.2434 | 7.679 | 8.164 | 8.631 |
| Second <br> Acceleration(feet $/ \mathrm{sec} / \mathrm{sec}$ ) | -0.00214 | 0.04673 | 0.09826 | -0.00121 | 0.08024 |
| Third Acceleration (feet $/ \mathrm{sec} / \mathrm{sec}$ ) | -1.45 | 0.07088 | -1.591 | -1.448 | -1.312 |
| Fourth Acceleration (feet/sec/sec) | -9.502 | 0.3231 | -10.12 | -9.506 | -8.857 |
| First Change (second) | 1.764 | 0.03767 | 1.69 | 1.763 | 1.841 |
| Second Change (second) | 8.277 | 0.1001 | 8.073 | 8.28 | 8.461 |
| Third Change (second) | 13.81 | 0.03837 | 13.74 | 13.81 | 13.88 |

Note: ${ }^{1}$ Standard deviation

Again, leader and follower were traveling at similar initial speeds, and the follower's estimated reaction time (about 0.5 seconds) definitely does not suggest driver inattention. However, the follower's estimated critical headway, as less as 12.79 feet, at the beginning of his evasive action suggests strong aggressive following behavior and which almost resulted in a rear-end collision in this case. Such an event can be qualified as a useful crash surrogate.


Figure 7 Predicted and observed instrumented speed for Case 2

## Case 3

In this case the instrumented vehicle was traveling in the right-most lane of an arterial and continued to travel until it was forced to a complete stop to avoid a rear-end collision with the leading vehicle, which was waiting for a gap to change lanes at a merging section of the arterial. The total duration of the video was 35 seconds and the event occurred at about 23 seconds. Respective speed trajectories for the leading and following vehicles were plotted in Figure 8. The radar could only mange to capture the leading vehicle's information for about 5 seconds.


Figure 8 Speed trajectories for the leading and the following vehicles for Case 3

A two-stage model was proposed for both the leading and the following vehicles. Table 3 shows the WinBUGS estimates. Initial speed of the following vehicle was 60 feet $/ \mathrm{sec}$ compared to the initial speed of 30.93 feet $/ \mathrm{sec}$ for the leading vehicle. This speed is the estimated speed of the leading vehicle when for the first time the radar captured information about the leading vehicle. A two-stage model was proposed for the following vehicle where in the first stage the vehicle accelerated at -7.57 feet $/ \mathrm{sec}^{2}$ until 3.92 seconds then it shifted to a stronger negative acceleration of -16.16 feet $/ \mathrm{sec}^{2}$. The leading vehicle's trajectory was also fitted with a two-stage model with 10 feet $/ \mathrm{sec}^{2}$ of acceleration in the first stage, for 1.714 seconds, and -4.332 feet $/ \mathrm{sec}^{2}$ acceleration in the second stage.

Table 3 WinBUGS Estimates for Case 3

|  | Mean | Stand. Dev ${ }^{1}$ | 2.50\% | Median | 97.50\% |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Following Vehicle |  |  |  |  |  |
| Initial Speed (feet/sec) | 60.12 | 0.3074 | 59.54 | 60.12 | 60.74 |
| First Acceleration (feet $/ \mathrm{sec} / \mathrm{sec}$ ) | -7.574 | 0.1257 | -7.821 | -7.573 | -7.333 |
| Second <br> Acceleration(feet/sec/sec) | -16.16 | 0.372 | -16.86 | -16.17 | -15.41 |
| First Change (seconds) | 3.922 | 0.04541 | 3.823 | 3.929 | 3.991 |
| Time when stop (seconds) | 5.805 | 0.027 | 5.754 | 5.804 | 5.861 |
| Reaction time (seconds) | 2.207 | 0.0607 | 2.083 | 2.209 | 2.322 |
| Leading Vehicle |  |  |  |  |  |
| Critical Headway (feet) | 94.81 | 2.269 | 91.36 | 94.95 | 98.57 |
| Critical Speed (feet) | 47.23 | 0.5369 | 46.28 | 47.26 | 48.21 |
| Initial Speed (feet/sec) | 30.93 | 0.437 | 30.12 | 30.91 | 31.81 |
| First Acceleration (feet $/ \mathrm{sec} / \mathrm{sec}$ ) | -10.86 | 0.3323 | -11.57 | -10.84 | -10.26 |
| Second <br> Acceleration(feet/sec/sec) | -4.332 | 0.1572 | -4.648 | -4.331 | -4.03 |
| First Change (seconds) | 1.714 | 0.05528 | 1.602 | 1.716 | 1.816 |
| Time when stop (seconds) | 4.559 | 0.07132 | 4.424 | 4.557 | 4.704 |

The above estimates from Table 3 suggests that longer reaction time of 2.207 secs may not be attributed to the driver's inattention as the critical headway ( 94.81 feet) in this case is much higher than the other cases probably giving the following driver more time to react to the situation and eventually avoid the collision.

The piece-wise model seems to be plausible as shown Figure 9, comparing the observed with the predicted speed for the instrumented vehicle.


Figure 9 Predicted and observed instrumented speed for Case 3

## CONCLUSION

In this study we have used a microscopic modeling approach to investigate crash event where driver actions together with initial speeds and vehicle locations are treated as inputs to a physical model describing vehicle motion. One of the main objectives of this study is to demonstrate how in-in vehicle base trajectory data can be used to understand the interaction between individual vehicles and if possible to identify the underlying mechanism for any crash or near crash event. We have illustrated how at least for situations where direction of travel is roughly constant, trajectory-based reconstruction of crash-related events, where trajectory data are used to fit parsimonious models of driver behavior, is feasible using vehicle-based data. The product of such a reconstruction is a set of estimates of when and to what extent drivers changed their acceleration, and the background conditions associated with these changes. This approach is especially helpful in studying crash-related events involving two or more vehicles. And our methods can be used to produce estimates of driver reaction times and following behavior. These estimates can in turn be used to characterize events as to the degree to which driver inattention or aggressive driving may have been present, where information on the behavior of drivers in noninstrumented vehicles is required

Given sufficiently large samples of crash and near-crash events, this method could be used to compile distributions for these inputs, which could in turn be used in traffic simulation models. Such a strategy could advance the use of realistic crash features in a microscopic simulation model. One potential real time application can be possible identification of traffic conditions prior to any event and thus enabling traffic engineers to take necessary action to alert drivers.

In some cases there are strong possibilities that the residuals obtained after fitting a trajectory model showed serial correlation. When serial correlation is present but unaccounted for, the
standard errors and confidence intervals associated with parameter estimates can be biased, that is, although the trajectory model fitting may appear reasonable there will be greater uncertainty associated with the parameter estimates than being acknowledged. For further investigation, time-series models, such as first order autoregressive models, should be used.

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