DETECTING SECONDARY ACCIDENTS IN FREEWAYS

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Abstract

Current research on secondary crash identification links the likelihood of secondary crashes occurrence to primary incidents using pre-defined spatial and temporal criteria. This paper extends past research on secondary crash detection by defining dynamic thresholds of the influence area of a primary incident using detailed real-time collected traffic data from upstream loop detectors on a freeway. Results suggest influence curves of different characteristics with respect to the prevailing traffic conditions before the occurrence of the primary incident, as well as the crash characteristics such as the number of vehicles involved or the number and type of blocked lanes. The results of the proposed approach are compared to those of two prevailing methods for detecting secondary accidents and similarities and differences are identified.

Keywords: secondary accidents, dynamic thresholds, ASDA/FOTO model, freeway operations

INTRODUCTION

Incidents impose social and personal costs on drivers, passengers and generally on the users of the urban networks. Severe incidents that cause delays are also associated with secondary accidents. Secondary crashes are generally defined as crashes that occur within a predefined spatiotemporal region of a primary incident and reduce roadway capacity (Moore et al. 2004, Zhang and Khattak 2010). Secondary crashes have increasingly been recognized as a major source of freeway incidents and their occurrence causes extra traffic delay. Compared with the primary ones, secondary accidents are much more severe, and are more significant with respect to the traffic management resource allocation implications; as a consequence, their identification may help maintain and increase the safety and operability levels of freeways.

Most research links secondary crashes to primary incidents according to some pre-defined spatial and temporal criteria without considering prevailing traffic flow conditions and incident characteristics. Raub (1997) proposed a methodology for the temporal and spatial analysis of incidents on urban arterials in order to identify the secondary crashes. For his analysis, Raub (1997) assumed accident effect duration of 15 minutes plus clearance time. He also set a distance of effect of less than 1,600 meters (1 mile). In other words, if an accident occurred within these fixed spatiotemporal thresholds, the accident was considered to be secondary. Moore et al. (2004) developed and applied a method for identifying secondary accidents on Los Angeles freeways that relied on special data resources, and finally estimated secondary accident rates. Specifically, secondary accidents were defined as those occurring upstream of the initial incident, in either direction, within or at the boundary of the queue formed by the initial incident. A static threshold of 3,218 meters (2 miles) and 2 hours was used for forming this boundary.

Zhan et al. (2008) described a method for identifying secondary accidents and their contributing factors using incident and traffic data. Their method is based on a cumulative arrival and departure traffic model for estimating the maximum queue length and traffic delay for incidents with lane blockages. Using these data they define the spatial and temporal boundaries in contrast with previous studies which used pre-defined thresholds. Still they don't take into consideration the evolution traffic conditions during the primary incident. Zhang and Khattak (2010) proposed a dynamic queue based methodology to identify the influence area of a primary accident; if a spillback is observed, the queue length is estimated by a deterministic D/D/1 model with estimated arrival and departure flow rates based on Highway Capacity Manual methodologies. Sun and Chilukuri (2010) presented a methodology in order to improve upon the existing method of static thresholds by formulating dynamic boundaries based on video based reported queue data. An incident progression curve is used to indicate the evolution of the queue throughout the entire incident period, related to the severity of the accident, and for different values of the volume over capacity ratio.

As for the modeling of secondary accidents, various attempts can be traced in the literature. Karlaftis et al. (1999) examined the primary crash characteristics that influence the likelihood of secondary crash occurrence. The authors suggested that clearance time, season, type of vehicle involved, and lateral location of the primary crash were the most significant factors. Later, Vlahogianni et al. (2010) proposed an advanced Bayesian framework for combining crash and queue information on freeways in order to reveal the traffic and crash related determinants for secondary crashes occurrence with respect to different distance ranges of secondary crashes from primary freeway incidents. Finally, Zhang and Khattak (2010) developed an ordered logit model to address the interrelations between the secondary crash probability occurrence and the primary accidents characteristics. The duration of a primary accident, the number of vehicles involved, the lane blockage, the segment length but not curvature, were found influential.

In this paper we extend past research on secondary crash detection by defining dynamic thresholds of the influence area of a primary incident using detailed real-time collected traffic data from upstream loop detectors in a freeway. The proposed detection methodology relates the propagation of the influence of the primary incident with the secondary crash occurrence and is based on continuously tracking the upstream and downstream fronts of an induced bottleneck. If within the defined influence area another accident occurs, then it is considered as a secondary. The proposed approach will result in determining a set of secondary accidents that will be contrasted to prevailing methods for detecting secondary accidents. Moreover, a preliminary analysis of the relationship between queue evolution and incident duration is provided with respect to the different initial – just before the primary accident occurrence - traffic conditions, as well as the characteristics of the primary accident.

METHODOLOGY

In order to evaluate whether an incident that occurred both temporally and spatially adjacent to a primary one is secondary, a methodology for identify the spatio-temporal evolution of traffic flow upstream of the primary incident is required. Literature indicates three prevailing methods for tracking and defining the spatio-temporal influence of traffic disturbance in freeways: cumulative plots, speed threshold algorithm and the ASDA model. A comparative analysis of the three approaches can be found in Li and Bertini (2010).

Cumulative plots may be used for visually comparing transformed curves of cumulative vehicle arrival number versus time measured at neighboring loop detectors (Cassidy and Windover 1995, Cassidy and Bertini 1999, Munoz and Daganzo 2002). Through the curves' observation it is possible to study the traffic flow propagation over time and space and to identify the detectors affected by the incident occurrence. By definition, the horizontal distance between two cumulative curves indicates the vehicular trip time between the corresponding detectors while the vertical distance indicates the number of vehicles present on the segment between the two detectors at some time *t*.

The second method, speed threshold algorithm developed by Chen et al. (2003), uses observed speed drop for defining the influence area. Speed differentials between two detectors and maximum speed for the upstream detector are used to identify bottlenecks. The algorithm sets a speed differential between the two detectors of 20mph and a maximum speed of 40mph for the upstream detector. The identification of the influence area is based on identifying the last pair of detectors at which the two above criteria are fulfilled. Despite the fact that the above two methods reveal the traffic flow evolution and changes due to the primary incident occurrence, they don't estimate accurately the start and the end point of the formulated bottleneck. It can be said that are more visual methods. Therefore the spatiotemporal boundaries of the influence area cannot be accurately defined.

The ASDA model (Kerner et al., 1998; Kerner and Rehborn,2000) is a robust approach to perform automatic tracking of the propagation of moving traffic jams. More specifically, ASDA may be used to track a moving jam at all times (Kerner et al., 2004). Let (Q_o, Q_n) be two consecutive loop detectors on a freeway road section and let L be their respective distance. After the moving jam has been observed at the detector Q_n at time t_o, the ASDA model starts to calculate continuously the positions of the upstream front, $x_{up}^{(jam)}$ (t). After the downstream front of the moving jam is registered at the detector Q_n at the later time t_1, the ASDA model starts to calculate continuously the positions of the downstream front, $x_{down}^{(jam)}$ (t) (Kerner et al., 2004).

In freeways, where several detectors following one another, the positions of both the fronts of the formulated wide moving jam caused by the primary incident are determined by using the following two equations:

$$x_{up}^{(jam)}(t) = L_{i+1} - \int_{t_o^{(i+1)}}^{t} \frac{q_0^{(i)}(t) - q_{min}}{\rho_{max} - \left(q_0^{(i)}(t)/w_0^{(i)}\,(t)\right)} dt \qquad (1)$$

$$x_{down}^{(jam)}(t) = L_j - \int_{t_1^{(j)}}^{t} \frac{q_{out}^{(j)(jam)}(t) - q_{min}}{\rho_{max} - \left(q_{out}^{(j)(jam)}(t)/w_{max}^{(j)}\,(t)\right)} dt \qquad (2)$$

where index "i" and "j" represent detectors, whose time values at time t have to be used, L_{i+1}, L_j are the coordinates of the corresponding detectors; $t_0^{(i+1)}$ indicates the time when the upsteram front of the moving jam has been observed at the detector "i+1"; $t_1^{(j)}$indicates the time when the downstream front of the jam has been observed at the detector "j"; $q_0{}^{(i)}$(t) and $w_0^{(i)}$(t) are the measured flow rate and the average vehicle speed at the detectors "i" upstream of a wide moving jam; $q_{out}^{(j)(jam)}$ and $w_{max}^{(j)}$ are the measured flow rate and the average vehicle speed at the detector "j" downstream of the wide moving jam (Kerner et al., 2004).

In the present analysis, where the ASDA model is applied, the time when the upstream and the downsteram front are observed at each detector is estimated based on the speed threshold algorithm.Due to the fact that in Attica Toll way high speeds are observed, a maximum speed threshold equal to 60kph was set. Additionally, the speed differential between two successive detectors was selected equal to 20kph.

More specifically, the upstream front of the moving jam is considered to have reached one detector at some time if the three following criteria are fullfilled:

1. The speed at the detector is below the maximum speed threshold,
2. The speed drop at the detector is greater than 35%,
3. The difference between the speeds at the detector and the next downstream detector is greater than the speed differential.

If the traffic conditions within the primary incident occurred are congested, only the speed drop criterion is applied since in such cases the speeds are already below 60kph and the speeds between successive detectorsrange at the same levels.
Using the two equations, presented above, it is possible to calculate accurrately the position of the upstream and the downstream front of the wide moving jam at any time. Consequently, it is also possible to estimate the jam width $L_s = x_{down}^{(jam)} - x_{up}^{(jam)}$ at any time, the queue duration and the maximum queue length caused by the primary incident occurrence. Therefore the spatiotemporal boundaries of the influence area are fully defined. Any accident falling within this area is considered as secondary.

In the next step a preliminary analysis will be conducted in order to try to model the relationship between the spatial distance from the primary incident and the duration of an incident. For this purpose different statistical distributions are fitted to the various primary accident influence areas. Fitting was conducted based on the Levenberg-Marquardt algorithm, a robust nonlinear least-squares curve-fitting procedure, usually met in transportation problems as a neural network learning optimization method (Vlahogianni et al. 2007). Evaluation is based on: (a) the square of the correlation between the response values and the predicted response values R-square(a value closer to 1 indicates that a greater proportion of variance is accounted for by the model), (b) the adjusted R-square statistic, and (c) the root mean squared error.

DATA ANALYSIS

The Study Area

The available data come from Attica Toll way (http://www.aodos.gr), an urban motorway which connects 2 major interurban motorways, the Athens international airport and the city center. Attica Toll way features hundreds of CCTV cameras connected to the Traffic Management Centre (TMC). Apart from the cameras, traffic monitoring and management is conducted via inductive loop detectors placed every 500m in open road and every 50 m in tunnels, overheight detection system at entrance points and electronic variable message signs. This equipment helps in automatically detecting any incidents occurring on the motorway, informing the intervention and maintenance patrol units and providing assistance and estimating queues and travel time.

For the present analysis, the data consists of 856 accidents records for the year 2007 along with volume and speed data from all the available loop detectors upstream of the accident. The accidents are described by the following information:

- Location of the accident including the post mile and the lane where the accident occurred,
- Start/end time of the accident and thus its duration,
- Severity of the accident (damages, injuries, fatality),
- Type of vehicle(s) involved,
- Number of vehicles involved,
- Season (month),
- Type of day(weekday, weekend),
- Time period (peak, off- peak).

Moreover, for each accident record, draft information on the maximum queue length induced by the accident is also provided by the operators of the Attica Tollway TMC using the installed video cameras. This information may have several shortcomings, for example missing maximum queue estimations in some accident records, inaccuracies stemming from the cameras positioning with respect to the accident's location, as well as lack of "dense" temporal and spatial information on the evolutions of the queue. However, this information will assist the comparison of the proposed approach for identifying the dynamic evolution of the accidents traffic impacts.

Defining the spatiotemporal influence area of a primary incident

A primary analysis of the available dataset, in order to define the spatial and temporal thresholds of the influence, was conducted by using cumulative plots. Figure 1 depicts (a) the cumulative *N(x,t)* curve and (b) the temporal evolution of travel speed for an incident with duration more than 30 minutes. As can be observed, at some point in time, cumulative curves diverge indicating the spatiotemporal evolution of upstream traffic caused by the primary incident.

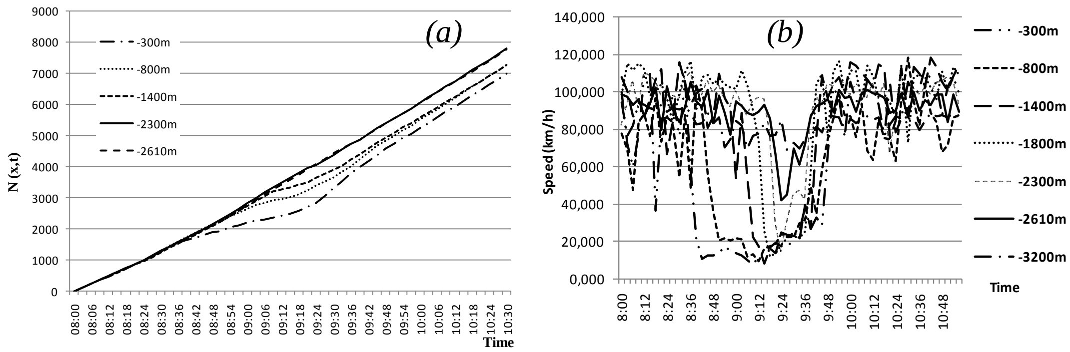

Figure 1: (a) Cumulative count curve and (b) temporal evolution of the travel speed for an incident with duration equal to 58 min.

Based on the above approach, the influence area of a primary accident can be quantitatively determined as seen in Figure 2. The boundaries of the influence area of a crash on the spatiotemporal evolution of traffic over time, defines two areas. Accidents falling within the curve's boundaries are considered as secondary.

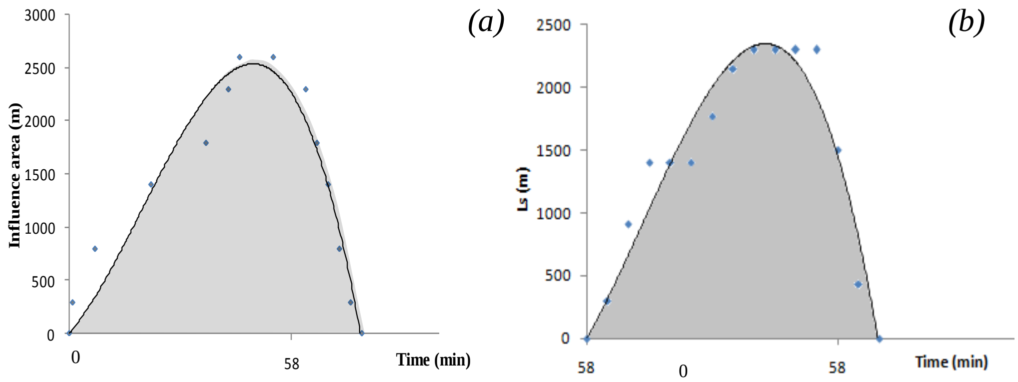

Figure 2: Boundaries of the influence area of an incident with duration equal to 58 min using as method of analysis a) the cumulative plots and b) the ASDA algorithm.

Using the cumulative plots and the speed - time diagram, the influence area of the crash is approximately defined, as the specific method doesn't provide any information about the start and the end point of the formulated bottleneck as well as the accurate location of the upstream and downstream fronts at some time during the incident. On the other hand, the equations formulated by the ASDA model, provide enough information about the spatiotemporal evolution of traffic flow and the propagation of the traffic disturbance upstream of the incident. They calculate accurately the position of both fronts of the moving jam and consequently it is possible to estimate the jam's width, $L_s = x_{down}^{(jam)} - x_{up}^{(jam)}$ at any time. Plotting the jam's width versus time the boundaries of the influence are determined as seen in Figure 2. These spatiotemporal boundaries define two areas; accidents falling within the curve's boundaries are considered as secondary.

Observing the two curves (Figure 2), it is found that both spatial and temporal boundaries are quite different using the above two methods. Specifically, according to the cumulative plots' method the last detector influenced by the primary incident is located 2.6 km upstream from the point of the incident and the influence duration was approximately 80 minutes. By using the ASDA model the spatiotemporal boundaries of the influence area are 2.3 km and 70 minutes respectively.

Calculating the jam's width and formulating the curve for the 856 accidents of the available database, using the equations provided by ASDA model, a set of 30 secondary accidents was identified. This equals to the 3.5% of the total accidents observed during the year.

We also attempt to compare the secondary crashes detected via the proposed approach with those that may be detected using other approaches previously issued in literature; the static spatiotemporal thresholds of Raub (1997) who proposed a temporal fixed threshold equal to accident effect duration of 15 minutes, plus clearance time and spatial fixed threshold equal to 1,600 meters (1 mile) and Moore'sfixed boundaries of 2 hours and 3,218 meters (2 miles) from the primary crash (Moore 2004).According to Raub (1997) only 14 accidents, out of the 856 analyzed, can be considered as secondary that equals to the 1.6% of the total incidents observed. Moore's fixed boundaries result in detecting 28 secondary accidents that equals to the 3.27% of the available total set of incidents.

Comparing the above results with the results of the proposed dynamic spatiotemporal thresholds methodology, it is found out that Raub's methodology underestimates the number of secondary crashes; however, all of the secondary accidents detected using Raub's approach are also identified as secondary by the proposed dynamic approach. This significant underestimation is probably due to the fact that the spatiotemporal boundaries Raub proposes, are limited enough to conclude accidents occurring within a broader spatiotemporal region.

This does not apply to the case of Moore's secondary crash detection method (Moore 2004); Moore's approach slightly diverges from the detected number of secondary crashes using the proposed approach since the spatiotemporal thresholds define a bigger influence area. However, only 22 out of 28 secondary crashes could be identified using the proposed method. This can be possibly explained by the fact that the spatiotemporal influence of a primary incident may be less than 3.2 km and 2 hours. As a consequence Moore's approach identifies as secondary accidents, crashes occurring out of the primary's spatiotemporal influence area but within the fixed thresholds he proposes. The existing slight diversion, corresponding to the number of secondary accidents detected, is due to the fact that this approach does not include accidents occurring within a much broader region.

Analysis of the Observed Influence Areas

Analysis as described in the previous section results in a set of influence areas seen in Figure 3; these influence areas refer to a vast range of primary incident durations, accidents with different characteristics, as well as to different initial traffic conditions. As can be observed, there is a significant dispersion with respect to the maximum queue length observed, as well as the total time for queue dissipation. The intense dispersion observed is evident that there are certain factors influence the manner a traffic disturbance, occurring due to a primary incident, is propagating upstream of an incident.

The incidents were then classified in three categories with respect to their duration: incidents with duration (a) less than 30 min,(b) from 30 to 60 min and (c) larger than 60 min. The curves fitted to data of each category are shown in Figure 4.

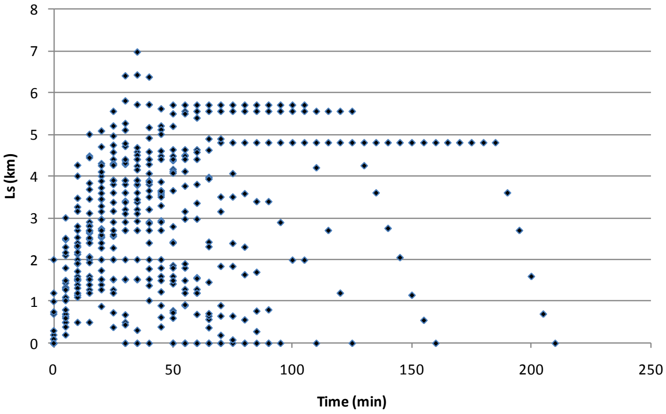

Figure 3: Influence evolution for all accidents.

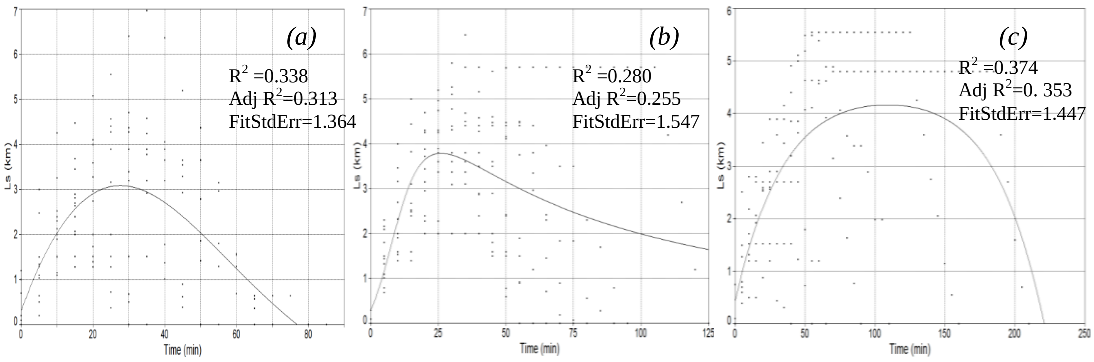

Figure 4: Influence evolution for accidents with duration (a) less than 30 min (b) ranging from 30 min to 60 min and (c) more than 60 min.

The curve (a) is best described by Weibull distribution (Appendix, Equation 1), for curve (b) Pearson IVdistribution (Appendix, Equation 2) provides the best fit and, curve (c) is described by SDS distribution (Appendix, Equation 3).

Because of the low values of the statistical diagnostics, indicating an insufficient fit, the incidents of each category were further classified with respect to the initial prevailing traffic conditions before the incident occurrence. Similar to previous research, this classification was done according to three criteria (Li and Bertini, 2010):

1. If the speed is high, the traffic flow is free,
2. If the speed is middle, the traffic flow is synchronized,
3. If the speed is low, the traffic flow is congested.

<u>Incidents with duration less than 30 minutes</u>

Analysis showed that classifying the incidents related to the initial traffic conditions results in an improved fit as shown in Figure 5 where the fitted influence curves for accidents with duration less than 30 min under (a) free flow, (b) synchronized and (c) congested initial traffic conditions are depicted.

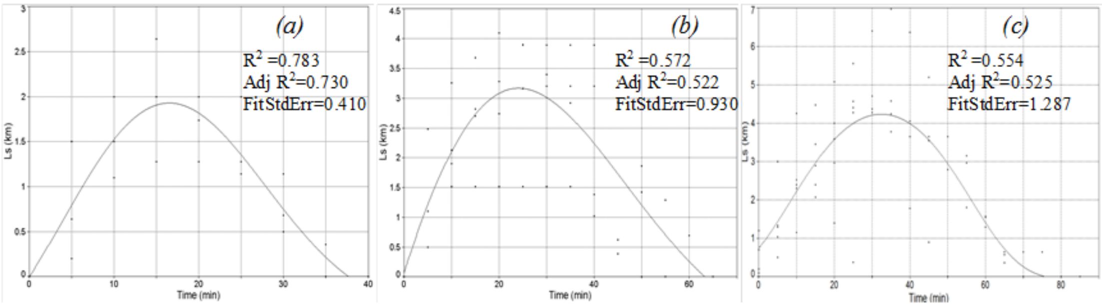

Figure 5: Influence evolution for accidents with duration less than 30 min under (a) free flow, (b) synchronized and (c) congested initial traffic conditions.

In free flow and synchronized traffic conditions the spatio-temporal evolution of traffic flow upstream of an incident is described by Weibull distribution with different parameters (Appendix, Equations 4 and 5 respectively). Congested traffic conditions are described by Complementary Error Function Peak distribution (Appendix, Equation 6).

<u>Incidents with duration ranging from 30min to 60 min</u>

Figure 6 depicts the fitted curves for incidents with duration ranging from 30 min to 60 min under free flow, synchronized and congested initial traffic flow conditions. Similarly to the previous category, the fit is improved as indicated by the statistical diagnostics when compared to the fit depicted in Figure 5.

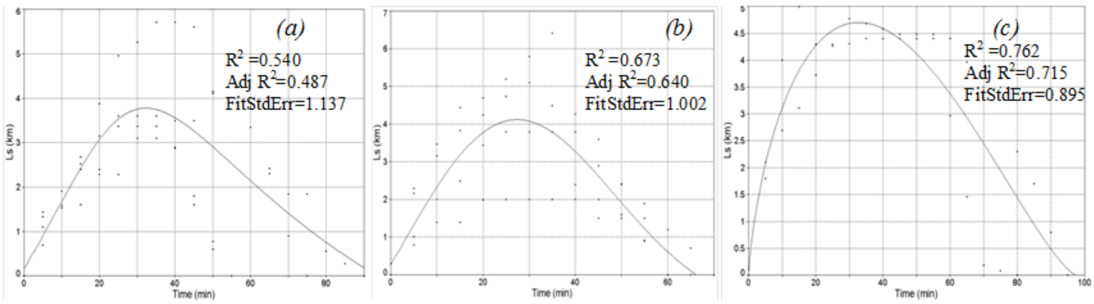

Figure 6: Influence evolution for accidents with duration ranging from 30 min to 60 min under (a) free flow, (b) synchronized and (c) congested initial traffic conditions.

For accidents with duration ranging from 30min to 60 min under free flow conditions, Pearson IV distribution best fits the data (Appendix, Equation 7). The curve (b), depicting synchronized conditions, is described by Weibull distribution (Appendix, Equation 8), whereas for curve (c) Beta distribution provides the best fit (Appendix, Equation 9).

<u>Incidents with duration more than 60 minutes</u>

For accidents with duration more than 60 minutes, free flow conditions were not observed. For incidents under synchronized and congested initial traffic flow conditions, the fitted curves are shown in Figure 7. For incidents with duration more than 60 min, Beta and Asymmetric Double Gaussian Cumulative distributions describe best the data for synchronized and congested traffic conditions respectively (Appendix, Equations 10 and 11 respectively).

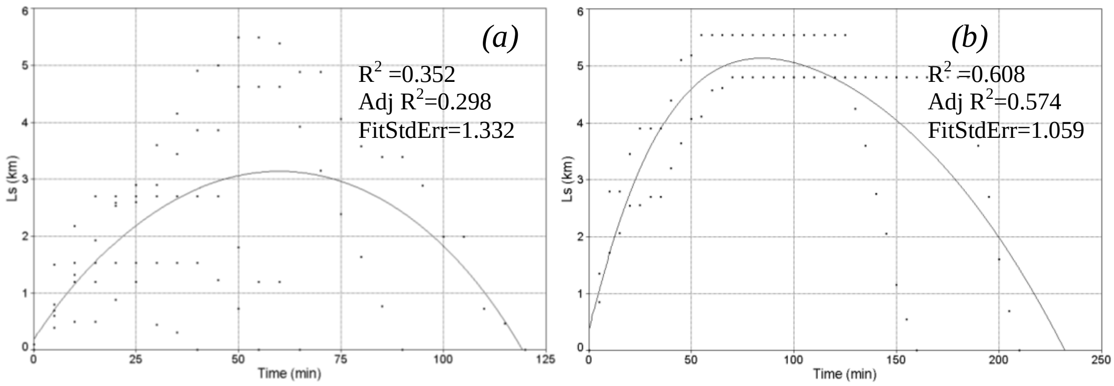

Figure 7: Influence evolution for accidents with duration more than 60 min under (a)synchronized and (b)congested initial traffic conditions.

A critical look to the approximation curves in Figures 5 to 7 shows that there is still some unaccounted dispersion, due to other influential factors that have not been taken into consideration. An attempt to improve fit is undertaken by considering factors such as the number of blocked lanes during the primary accident, as well as the number of vehicles involved. As shown in Figure 8, curves (a) show that for the same number of vehicles involved, the number of blocked lanes is important for the influence evolution of a primary incident causing larger influence areas. Specifically, increased number of blocking lanes causes higher values of jam's width at any time as well as larger influence duration, i.e. total time of queue dissipation.

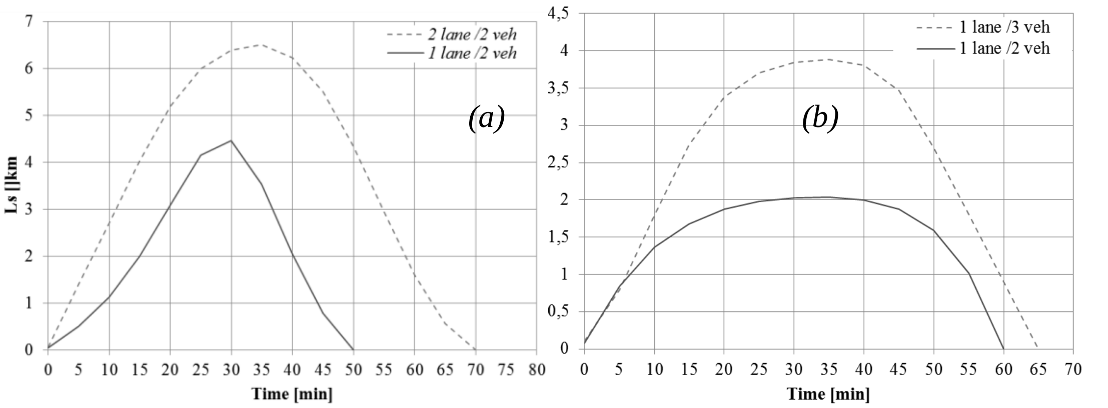

Figure 8: Influence evolution for incidents (a) with 2 vehicles involved for different numbers of blocked lanes and (b) occurring in one lane with different numbers of vehicles involved

Similarly, curves (b) indicate that for the same number of blocked lanes, increased number of vehicles involved has larger effects. The value of max queue length is much higher (in this case a double value is observed)whereas the influence duration is slightly larger. Both cases also reveal that for increased number of blocked lanes and number of vehicles involved respectively, the wave speed for the upstream evolution of traffic is higher causing faster formation of the queue.

CONCLUSIONS

The present paper extended past research on secondary crash detection and presented a methodology for defining dynamic thresholds of the influence area of a primary incident using detailed real- time collected traffic data from upstream loop detectors in a freeway. For defining the boundaries of the influence area, the ASDA model was used providing adequateanalytical information on the spatiotemporal evolution of traffic flow and the propagation of the traffic disturbance upstream of the incident. Using this approach, it was possible to accurately calculate the position of the upstream and downstream front of the moving jam and, consequently, to estimate a crash's influence propagation with time. The findings indicated that the set of secondary accidents that were identified using the proposed approach differs from those that were detected by previous approaches using fixed spatiotemporal boundaries;the use of fixed boundaries led to either an underestimation of the number of secondary accidents,or to false consideration of some crashes as secondary, whereas some truly secondary accidents were not identified.

Moreover, the modeling of the relationship between queue evolution and duration of the influence of a primary accident resulted in revealing the effect of various determinants of the primary accident, such as the initial traffic conditions, the number of blocked lanes and vehicles involved in the crash; these determinants significantly alter the form and magnitude of the spatio-temporal effect of a primary accident, and, consequently, the likelihood of a secondary crash occurrence.

From a conceptual view, the proposed approach enables the analytic estimation of the dynamic boundaries for detecting secondary accidents using real - time traffic data from loop detectors located upstream of the incident. The analytical consideration on the manner the traffic propagates through a disturbance enables to accurately estimate queue formation and dissipation. Further research is needed towards two distinct directions. First, the research should be extended to the identification of the prevailing traffic, geometric and accident related determinants that may increase the likelihood of secondary incident occurrence. Second, the extension to a real-time application for dynamic secondary incident detection should be further investigated. The proposed approach that may operate with historical data, in cases of data collection system malfunctions and encompass no computationally demanding calculations, may be considered as an efficient candidate for real-time applications.

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REFERENCES

Bertini, R. L., Fernandez, R. Wieczorek, J. and Li, H. Using Archived ITS Data to Automatically Identify Freeway Bottlenecks in Portland, Oregon. In *Proc., 15th World Congress on ITS*, New York, 2008.

Cassidy, M.J. and Windover, J.R. (1995), "Methodology for assessing dynamics of freeway traffic flow," *Transp. Res. Rec*. 1484, pp. 73-79.

Cassidy, M.J. and Bertini, R.L. (1999), "Some traffic features at freeway bottlenecks," *Transp. Res.,* vol. 33B, no. 1, pp. 25-42.

Chen, C., Skabardonis, A. and Varaiya, P. Systematic Identification of Freeway Bottlenecks. In Transportation Research Record: Journal of the Transportation Research Board, No. 1867, Transportation Research Board of the National Academies, Washington, D.C., 2004, pp. 46–52.

Garib, A., Radwan, A.E. and Al-Deek, H. (1997). Estimating Magnitude and Duration of Incident Delays.*Journal of Transportation Engineering* 123.6: 459–466.

Giuliano, G. (1989). Incident Characteristics, Frequency, and Duration on a High Volume Urban Freeway.*TransportationResearch*23A.5: 387–396.

Karlaftis, M.G., Latoski, S.P. Richards, N.J. and Sinha, K.C. (1999). ITS Impacts on Safety and Traffic Management: An Investigation of Secondary Crash Causes. *ITS Journal* 5: 39–52.

Kerner, B.S., Rehborn, H., Kirschfink, H. "Method for the automatic monitoring of traffic including the analysis of back- up dynamics", German Patent DE 196 47 127 C2, Dutch Patent: NL1007521C, USA Patent US 5861820, 1998.

Kerner, B.S., Rehborn, H., (2000). German patent publication DE 198 35 979 A1, day of notification: 08.08.1998, day of publication: 10.02.2000.

Kerner, B.S., Rehborn, H., Aleksic, M., Haug, A. "Recognition and Tracing of Spatial-Temporal Congested Traffic Patterns on Freeways", Trans. Rec. C, Vol. 12, 2004, pp. 369-400.

Lee, C., Hellinga, B. and Saccomanno, F.(2003). Proactive Freeway Crash Prevention Using Real Time Traffic Control.*Canadian Journal of Civil Engineering* 30: 1034–1041.

Li, H., and Bertini, R.L. (2010).Comparison of Algorithms for Systematic Tracking of Patterns of Traffic Congestion on Freeways in Portland, Oregon."*Transportation Research Record, 2178*, 101–110.

Lindley, J. (1987). Urban Freeway Congestion: Quantification of the Problem and Effectiveness of Potential Solutions. *ITE Journal*, 57(1), 27–32.

Moore, J.E., Giuliano, G. and Cho, S.(2004). Secondary Accident Rates on Los Angeles Freeways.*Journal of Transportation Engineering* 130.3: 280–285.

Morales, J. (1987) .Analytical Procedures for Estimating Freeway Traffic Congestion.*ITE Journal*, 57(1), 45–49.

Munoz, J.C. and Daganzo, C.F. (2002).“ Fingerprinting Traffic from Static Freeway Sensors,” *Cooperative Transportation Dynamics*, 1, 1-11.

Raub, R.A. (1997). Secondary Crashes: An important component of Roadway Incident Management. *TransportationQuarterly*51.3: 93–104.

Skabardonis, A., Petty, K., Noeimi, H., Rydzewski, D. andVaraiya, P.(1996). I-880 Field Experiment: Database Development and Incident Delay Estimation Procedures. Transportation Research Record 1554: 204–212.

Smith, B., Qin, L. and Venkatanarayana, R.(2003). Characterization of Freeway Capacity Reduction Resulting from Traffic Accidents.*Journal of Transportation Engineering* 129.4: 362–368.

Sullivan, E. (1997).New Model for Predicting Freeway Incidents and Incident Delays.*Journal of TransportationEngineering*123.4: 267–275.

Sun C. and Chilukuri V. Secondary Accident Data Fusion for Accessing Long – Term Performance of Transportation Systems, Proceedings of the 2005 Mid-Continent Transportation Research Symposium, August 2005, Ames , Iowa.

Sun, C. and Chilukuri V. (2010).Dynamic Incident Progression Curve for ClassifyingSecondary Traffic Crashes, Journal of Transportation Engineering,Vol. 136, No. 12, 1153-1158.

Vlahogianni, E., Karlaftis, M., Golias, J. Halkias, B. (2010). Freeway Operations, Spatiotemporal Incident Characteristics and Secondary Crash Occurrence, Transportation Research Record: Journal of the Transportation Research Board, 2178, 1-9.

Vlahogianni, E.I., Karlaftis, M.G., and Golias, J.C. (2007), Spatio-Temporal Short-Term Urban Traffic Volume Forecasting Using Genetically Optimized Modular Networks, Computer-aided Civil and Infrastructure Engineering, 22(5), 317-325.

Zhan, C.J., Shen, L. O., Hadi, M., Gan, A. Understanding the Characteristics of Secondary Crashes on Freeways, in 87th Annual Meeting of the Transportation Research Board, TRB CD-ROM, January, 2008, Washington DC.

Zhang H. and Khattak A. (2010). What Is the Role of Multiple Secondary Incidentsin Traffic Operations?, Journal of Transportation, Engineering, Vol. 136, No. 11, November 1, 986-997.

APPENDIX: DISTRIBUTIONS

$$y = a\left(\frac{d-1}{d}\right)^{\frac{1-d}{d}} \left[\frac{x-b}{c} + \left(\frac{d-1}{d}\right)^{1/d}\right]^{d-1} \exp\left[-\left(\frac{x-b}{c} + \left(\frac{d-1}{d}\right)^{1/d}\right)^{d} + \frac{d-1}{d}\right] \quad (1)$$

a = - 1.677,b =4.765, c =27.649, d =53.960

$$y = \frac{a}{\left(1+\frac{4(x-b)^2\left(2^{1/d}-1\right)}{c^2}\right)^d} \quad (2)$$

a= - 1.068, b= 4.857, c= 25.947, d= 11.877

$$y = \frac{a\left[1+\exp\left(-\frac{c}{2d}\right)\right]\left[1+\exp\left(\frac{c}{2d}\right)\right]\exp\left(-\frac{x-b}{d}\right)}{\left[1+\exp\left(-\frac{x-b+c/2}{d}\right)\right]\left[1+\exp\left(-\frac{x-b-c/2}{d}\right)\right]} (3)$$

a=-21.682, b=25.850, c=108.790, d=319.200

$$y = a\left(\frac{d-1}{d}\right)^{\frac{1-d}{d}} \left[\frac{x-b}{c} + \left(\frac{d-1}{d}\right)^{1/d}\right]^{d-1} \exp\left[-\left(\frac{x-b}{c} + \left(\frac{d-1}{d}\right)^{1/d}\right)^{d} + \frac{d-1}{d}\right] \quad (4)$$

a = - 0.451,b =2.382, c =16.520, d =25.958

$$y = a\left(\frac{d-1}{d}\right)^{\frac{1-d}{d}} \left[\frac{x-b}{c} + \left(\frac{d-1}{d}\right)^{1/d}\right]^{d-1} \exp\left[-\left(\frac{x-b}{c} + \left(\frac{d-1}{d}\right)^{1/d}\right)^{d} + \frac{d-1}{d}\right] \quad (5)$$

a = - 1.251, b =4.419, c =24.132, d =42.098

$$y = a\, erfc\left[\left(\frac{x-b}{c}\right)^2\right] \quad (6)$$

a=-0.080, b=4.314, c=32.459

$$y = \frac{a}{\left(1+\frac{4(x-b)^2\left(2^{1/d}-1\right)}{c^2}\right)^d} \quad (7)$$

a= - 11.147, b= 14.926, c= 32.226, d= 25.966

$$y = a\left(\frac{d-1}{d}\right)^{\frac{1-d}{d}} \left[\frac{x-b}{c} + \left(\frac{d-1}{d}\right)^{1/d}\right]^{d-1} \exp\left[-\left(\frac{x-b}{c} + \left(\frac{d-1}{d}\right)^{1/d}\right)^{d} + \frac{d-1}{d}\right] \quad (8)$$

a= - 1.068, b= 5.193, c= 27.329, d= 45.709

$$y = \frac{a\left[\frac{x-b+\frac{c(d-1)}{d+e-2}}{c}\right]^{d-1} \left(1-\frac{x-b+\frac{c(d-1)}{d+e-2}}{c}\right)^{e-1}}{\left(\frac{d-1}{d+e-2}\right)^{d-1} \left(\frac{e-1}{d+e-2}\right)^{e-1}} \quad (9)$$

a= - 0.005, b= 4.708, c= 32.856, d= 97.734, e= 2.391

$$y = \frac{a\left[\frac{x-b+\frac{c(d-1)}{d+e-2}}{c}\right]^{d-1} \left(1-\frac{x-b+\frac{c(d-1)}{d+e-2}}{c}\right)^{e-1}}{\left(\frac{d-1}{d+e-2}\right)^{d-1} \left(\frac{e-1}{d+e-2}\right)^{e-1}} \quad (10)$$

a= - 4.428, b= 7.569, c= 60.025, d= 164.061, e= 1.702

$$y = \frac{a}{2}\left[1 + \mathrm{erf}\left(\frac{x-b+{}^{c}/_{2}}{\sqrt{2}d}\right)\right]\left[\frac{1}{2} - \frac{1}{2}\mathrm{erf}\left(\frac{x-b-{}^{c}/_{2}}{\sqrt{2}d}\right)\right] \quad (11)$$

a= - 42.154, b=48.267, c= 159.653, d= 470.438