EVALUATING ROADWAY SAFETY IMPROVEMENT IN A TRAFFIC ASSIGNMENT FRAMEWORK

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ABSTRACT

In this study, a traffic equilibrium analysis conceptual framework is proposed along with related modeling components pertaining to stochastic capacity due to probabilistic incidents, with a specific focus on crashes. This paper deviates from empirical, regression-based methods to predict crashes. Crash probabilties are first estimated with a conceptual logit model, and the commuters in the network are catagorized into two classes: (1) travelers with some degree of knowledge of average travel time and incident conditions across different days, and (2) travelers with access to perfect pre-trip traffic information every day. Within a gap function framework for describing the user equilibrium under different information availability, a static traffic assignment framework is first presented to describe the route choice behavior of the perfect

information (PI) and expected travel cost (ETC) user classes under stochastic day-dependent travel time under probabilistic incident traffic conditions. The model is applied to a simple corridor to illustrate the effectiveness of varying safety improvement strategies. A more realistic dynamic traffic assignment framework is further proposed for incorporating safety prediction for large-sclae networks with multipler user classes.
Keywords: Crash prediction, route choice behavior, stochastic link capacity, traffic assignment

INTRODUCTION

Traffic accidents result in high human and economic losses and lead to traffic congestion. Traffic congestion leads to a wide variety of adverse consequences such as traffic delays, travel time unreliability, increased noise pollution, as well as deterioration of air quality. To combat these inherent consequences of roadway vehicular traffic, safety has emerged as a priority in the transportation planning process. Safety conscious planning, which is based on the idea that safety should be considered as an explicit part of the transportation planning process, is gaining momentum in transportation practice, particularly as a result of requirements in recent transportation funding legislation. Considerable research has already been devoted to crash prediction modeling, which could be adaptable to planning activities (Hauer 1986, 1997, and 2001; Abdel-Aty and Radwan 2000; Ulfarsson and Shankear 2003; Kweon and Kockelman 2000; Lord and Persaud 2000; Lord et al. 2005; Ma and Kockelman 2006; Karlaftis and Rarko 1998; Shankar et al. 1998; Khattak et al. 2006; HSM 2009; NCHRP Report 546 2006).

Policy, planning, and design have historically taken a more nominal approach to safety. The real implications of policy and project decisions made in an effort to improve safety have often been subjectively, qualitatively, or relatively defined. Likewise, the response to an incident aimed at mitigating the impact of that incident on traffic has also been difficult to quantify ahead of time. If outcomes of implementing safety strategies, including the outcomes on incident-induced traffic congestion, could be accurately predicted prior to construction or implementation of a mitigation strategy, then their level of cost effectiveness could be more thoroughly considered by decision makers.

Review of crash estimation models

The evolution of crash estimation methods that are adaptable to a traffic equilibrium analysis conceptual framework has included crash estimation using observed crash frequencies and crash rates, indirect safety measures such as surrogate measures, and statistical analysis techniques to predicted the expected number of crashes (a long-term property of a road segment or intersection) (HSM Part A pg 3-17). For many years, the staple in the crash prediction analysis has been statistical models estimated using cross sectional data. Such models are developed by obtaining a database of historical accident and roadway characteristics, selecting an appropriate model, and using regression analysis to estimate the values of the coefficients in the model. These models have used various forms of regression techniques including; multiple linear regression, Poisson, negative binomial, and others (see Lord and Mannering 2010 and Savolainen, Mannering, Lord and Quddus 2011 for a comprehensive review of modeling crash frequency data).

Count regression models such as the Poisson and the negative binomial have been used to observe the link between crash frequency and other relevant factors such as traffic volume, ramp spacing, speed limit, and number of lanes (Miaou et al. 1993; Miaou and lum 1993; Miaou 1994, 1996 and 2001; Fridstrom et al. 1995; Johansson 1996; Vogt and Bared 1998; Vogt 1999; Balkin and Ord 2001; Zegeer et al. 2002; Pernia 2004;Lord et al. 2005; Shankar et al. 1997; Garber and Wu 2001; Lee and Mannering 2002; Kumara and Chin 2003; Miaou and Lord 2003; Rodriguez et al. 2003; Shankar et al. 2003; Noland and Quddus 2004; Qin et al. 2004; Xie and Zhang 2008; Liu et al. 2008; Li and Zhang 2008; Naderan and Shahi 2009; Turner et al. 2011; Aguero-Valverde and Jovanis 2010; Quddus et al. 2010; Anastasopoulos and Mannering 2011; Naveen et al. 2010; Schultz et al. 2010; Lord 2010; Savolainen 2011). The models have been taken even further to account for unobserved factors, injury counts and types, as well as links to safety improvements (Kweon and Kockelman 2000; Karlaftis and Rarko 1998; Shankar et al. 1998; Chin and Quddus 2003; Ladron de Guevara and Washington 2004; Bijleveld 2005; Ma and Kockelman 2006; Li et al. 1999; Christiansen et al. 1992; MacNab 2003; Miaou and Song 2005; Liu et al. 2005; Pawlovich et al. 2006; and Washington and Oh 2006; Milton et al. 2007; Tarko et al. 2008; Elvik 2008; Golob et al. 2008; Ma et al. 2008; Jonsson et al. 2009; Elvik 2009; Davis and Morris 2009; Ossenbruggen et al. 2010; Lord 2010; Jo et al. 2011; Savolainen 2011).

Regardless of the regression model used, parameter estimates represent statistical correlations between specific roadway or traffic characteristics and crashes. An inherent problem is that this correlation may not necessarily have a cause-and-effect relationship. Additionally, if an independent variable has a strong correlation with another variable, then it will be difficult to separate out their independent effects. Likewise, if a variable in the model is correlated with another unseen or unknown variable, then the coefficient in the model may represent something entirely other than what it is intended (i.e., ommitted variable bias). Thus, the value of a coefficient in the model may be a good representation of a specific roadway characteristic, or it may be modeling some other unintended variable.

Before-and-after studies have been used for many years to evaluate the effectiveness of highway improvements in reducing crashes. What has become known as the naive before-and-after design studies have flaws such as regression to the mean (HSM pg 3-15) leaving decision makers unsure of whether the results of the treatment were a result of a random fluctuation in traffic crashes or a direct result of the treatment itself. Hauer (1997) has proposed the Empirical Bayes technique as a way to address the problem of regression to the mean. If this bias can be overcome then the before-after evaluation is a good method for analyzing the safety effect of a specific treatment on a roadway, and may get closer to a cause-effect relationship than multivariate regression models of cross sectional data.

Needs for incorporating crash prediction in an equilibrium analysis framework

A recent road safety modeling synthesis and strategic planning document characterized count regression models, estimated using aggregated accident data combined with roadway inventory data, as limited in terms of the insights gained from modeling results (Transportation Research Circular in press; Lord and Mannering 2010). This paper deviates from the typical statistically based regression methods for crash prediction models and combines the statistical measures obtained for roadway safety with user equilibrium equations. General stochastic user equilibrium traffic assignment models have been developed where travel times are predicted based on varying traveler information and time-variant road capacity (Yang 1998; Yang, et. al. 2001; Yin and Yang 2003). De Palma and Picard (2005) further developed a graphical user equilibrium model that was able to consider two types of user classes. Li et al. (2011) further proposed a deterministic static traffic assignment model under stochastic capacity conditions, in which traffic capacity variations are assumed to be given externally, in order to provide a multi-day analysis framework for evaluating ATIS strategies.

Our research builds on various models that have evolved, such as stochastic capacity analysis and dynamic traveler behavior modeling, within the classical user equilibrium analysis. This paper expands the analysis technique proposed by Li et al. (2011) to examine how roadway safety can be examined through the gap function-based formulation for user equilibrium. The proposed model specifically considers the stochastic nature of network capacity for two different scenarios while considering the impact of crash probabilities and facility specific attributes. After establishing a conceptual logit model for predicting incident probabilities, we examine the effect of three safety improvements; (1) Reduce incident probability through improved geometric design, (2) Reduce capacity reduction due to incidents thought the use of a shoulder lane or quick response, and (3) better information provided to drivers by way of real-time traveler information, on a theoretical network and the impact that each has on the equilibrium of the system. The proposed model is able to provide a link between the statistical crash prediction methodology and crash mitigation techniques in a user equilibrium model.

CONCEPTUAL EQUILIBIRUM ANALYSIS FRAMEWORK

The conceptual modeling framework is illustrated in Fig. 1 using a simple corridor with a single origin-to-destination pair and two paths p=1 for the primary path, $p=2$ for the alternative path, where p is the path index. As each path only has one link, path 1 is denoted as link $a=1$ with a free-flow travel time of 20 minutes, and path 2 is denoted as link a=2 with a free-flow travel time of 30 minutes, where a is the link index. Following a similar analysis setting in the study by De Palma and Picard (2005) and Li et al. (2011), this example considers two different conditions: "bad" days with one incident (d=1), and "good" days with zero incidents (d=0). and the peak hour demand is Q=8000 vehicles per hour on each day.

"Bad" days have a reduced capacity due to the incident for the primary route, and "good" days have their full capacity available. As detailed in Table 1, the primary path has the following capacity values:

- On bad days it is 3,000 vehicles per hour (vph) per link.
- On good days it is 4,500 vph per link.

The alternative path is assumed to have a fixed capacity of 3,000 for all days. The probability of incident is included as a function of the V/C ratio with a specific risk, to be discussed later.

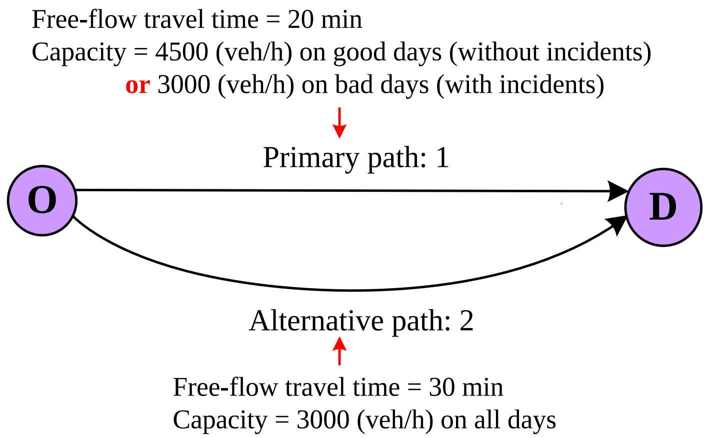

Figure 1. Simple network used as an illustrative example

To setup a mathematical programming model for steady-state traffic equilibrium, the non-negative flow variables $f_{p,d}$ is considered as the traffic flow using path p on day d. Obviously, the path flow distribution should ensure the total demand constraint on each day:

$$f_{1,d} + f_{2,d} = Q, \forall d = 0,1 \quad (1)$$

Let $T_{p,d}$ be defined as the travel time on path p on day d, which can be calculated from the BPR function such as

$$T_{a,d} = FFTT_a(1 + \alpha \left[\frac{f_{a,d}}{c_{a,d}}\right]^{\beta}) \quad (2)$$

Where $FFTT_a$ is the free-flow travel time of link a. Coefficients α and β are set to commonly used default values 0.15 and 4, respectively.

Table 1 Day-dependent capacity values

<table>
<tr><th></th><th></th><th></th><th>Bad Day (with incidents)</th><th>Good Day (without incidents)</th></tr>
<tr><td rowspan="2">Day Dependant Capacity</td><td>Daily Capacity on Path/Link 1 (veh/h) $c_{a=1,d}$</td><td>Path 1</td><td>3000*</td><td>4500</td></tr>
<tr><td>Daily Capacity on Path/Link 2 (veh/h) $c_{a=2,d}$</td><td>Path2</td><td>3000</td><td>3000</td></tr>
<tr><td rowspan="3">Base Case:</td><td rowspan="2">Flow (veh/hour/link)</td><td>Path 1</td><td>5322</td><td>5322</td></tr>
<tr><td>Path 2</td><td>2678</td><td>2678</td></tr>
<tr><td colspan="4">Travel Time: 32.9 min</td></tr>
<tr><td rowspan="3">Scenario I (Reduce Probability of Incident through Design)</td><td rowspan="2">Flow (veh/hour/link)</td><td>Path 1</td><td>5669</td><td>5669</td></tr>
<tr><td>Path 2</td><td>2331</td><td>2331</td></tr>
<tr><td colspan="4">Travel Time: 31.6 min</td></tr>
<tr><td rowspan="3">Scenario II (Increased Post-Incident Capacity)</td><td rowspan="2">Flow (veh/hour/link)</td><td>Path 1</td><td>5920</td><td>5920</td></tr>
<tr><td>Path 2</td><td>2080</td><td>2080</td></tr>
<tr><td colspan="4">Travel Time: 31.0 min</td></tr>
<tr><td rowspan="3">Scenario III (Consider ITS and 15% PI Conscious Users)</td><td rowspan="2">Flow (veh/hour/link)</td><td>Path 1</td><td>4926</td><td>5962</td></tr>
<tr><td>Path 2</td><td>3074</td><td>2038</td></tr>
<tr><td colspan="4">Travel Time for ETC Users: 34.8 min
Travel Time for PI Users: 32.2 min
Travel Time for all Users: 34.4 min</td></tr>
</table>

*reduced capacity

Now the two different degrees of traveler knowledge can be examined.

Expected travel cost (ETC) knowledge-based user equilibrium

As there are different realized capacity values $c_{a,d}$ due to incidents on different days, the travel times on different links can be viewed as a set of stochastic variables. In reality, most travelers rely on their expected travel times and their knowledge and experience on incident occurrence probability over different days to make route choices. The expected travel time can be considered as the long-run average, or more precisely, the probability-weighted sum of the possible travel time values from different days. Under a user equilibrium condition with ETC users, the expected travel times on used routes in the network are assumed to be the same, and accordingly, an ETC user selects the same route every day, regardless the actual traffic conditions.

The expected travel time for link a with random capacity c_a over different days can be represented as travel time on each day d for link a. $T_{a,d}(f_{a,d}, c_{a,d})$ is a function of the prevailing flow and capacity on that particular day. For link a=1 in the illustrative example,

$$\overline{T_a} = \rho_a T_{a,d}(f_{a,d}, c_a^R) + (1 - \rho_a) T_{a,d}(f_{a,d}, c_a^F) \tag{3}$$

Where ρ_a is the probability of having an incident on link a, c_a^R and c_a^Fcorresponds to the reduced and full capacity on link a.
In this study, we consider incident rate as a function of flow volume and capacity

$$\rho_a = \frac{exp\left(\frac{f_{a,d=0}}{c_a^F} \times \gamma + \delta\right)}{1 + exp\left(\frac{f_{a,d=0}}{c_a^F} \times \gamma + \delta\right)} \tag{4}$$

Where f_a is the average volume on link a across different days. γ is the coefficient associated with the volume-over-capacity (V/C) ratio in the logit model , and link volume $f_{a,d=0}$ under no incident conditions and full capacity are used to calculate the V/C ratio. δ is link-specific risk constant (default 0), associated with different link type, # of lanes and other related geometric design features.

$$\overline{T_a} = FFTT_a \times \left\{ \frac{1}{1 + exp\left(\frac{f_{a,d=0}}{c_a^F} \times \gamma + \delta\right)} \times \left(1 + \alpha \left[\frac{f_{a,d}}{c_a^R}\right]^\beta\right) + \left(\frac{exp\left(\frac{f_{a,d=0}}{c_a^F} \times \gamma + \delta\right)}{1 + exp\left(\frac{f_{a,d=0}}{c_a^F} \times \gamma + \delta\right)}\right)\left(1 + \alpha \left[\frac{f_{a,d}}{c_a^F}\right]^\beta\right) \right\} \tag{5}$$

This research aims to generalize Wardrop's first principle to describe the equilibrium conditions for travelers relying on their expected travel time to make route decisions: travelers with the same origin-destination pair experience the same and minimum expected travel time along any used paths on different days, with no unused path offering a shorter expected travel time. Obviously, when there is a single capacity value without incidents, then the above conditions are consistent with the standard user equilibrium with deterministic capacity, as the expected travel time devolves to the travel time on the single day.

The corresponding Karush-Kuhn-Tucker optimality conditions can be re-written as

$$gap^{ETC} = f_1^{ETC} \times (\overline{T}_1 - \overline{\pi}) + f_2^{ETC} \times (\overline{T}_2 - \overline{\pi}) = 0 \tag{6}$$

where $\overline{\pi}$ is the least expected travel time between the given OD pair over a multi-day horizon (with and without incidents) satisfies

$$\overline{\pi} = \min(\overline{T}_1, \overline{T}_2) \tag{7}$$

When $gap^{ETC} = 0$, it can be shown that if $f_a^{ETC} > 0$, then $\bar{T}_a = \bar{\pi}$. That is, the selected routes by expected travel time information users between an OD pair have equal and minimum costs. On the other hand, if $f_a^{ETC} = 0$, then $\bar{T}_a \geq \bar{\pi}$, which indicates that all unused routes by ETC users have greater or equal costs (compared to the used path costs). These two conditions further imply that no individual trip maker with expected travel time information can reduce his/her expected path costs by switching routes on any given day, under a user equilibrium condition.

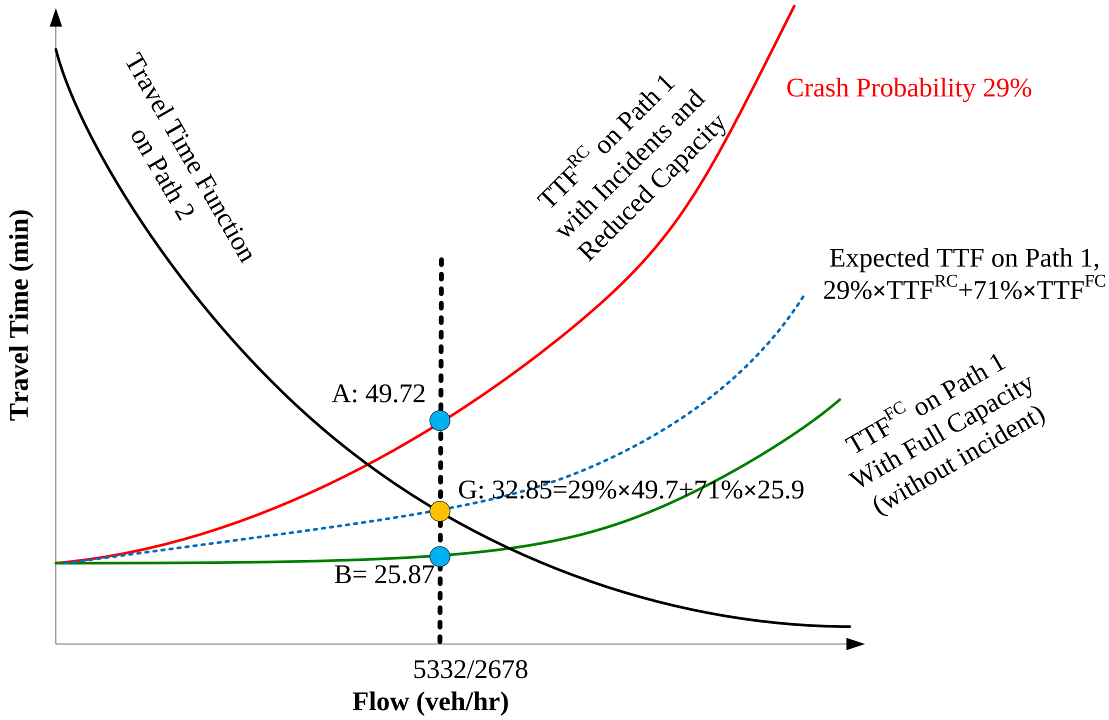

Figure 2. Solutions with 100% ETC information users and $\gamma = -0.1$ and $\delta = -1$, incident probability = 29.3%

Figure 2 graphically demonstrates the travel time of each path at full capacity (green line for path 1 and black line for path 2) and path 1 at reduced capacity due to an incident (red line). The expected travel time function (TTF) is generated by assigning a 29.3% weight to TTF with reduced-capacity (RC) days and an 70.7% weight to TTF with full-capacity (FC). The ETC-based user equilibrium corresponds to the intersection (in orange) of expected TTF on path 2 and path 1. 5322 vehicles are using link 1 and 2678 vehicles are using link 2 each day.
Point A: travel time = 49.72 min on link 1, reduced-capacity days,
Point B: travel time = 25.87 min on link 1, full-capacity days.
Point G: travel time = 32.85 min on link 2 every day, and the expected travel time on link 1 is the same 32.85 min.

If it is assumed that all users rely on ETC information in the simple corridor, then the ETC-based user equilibrium assigns about **5322** vehicles (66.5%) on path 1, and about **2678** vehicles on path 2, leading to different travel time on good and bad days. As both paths carry positive flows, their average travel times over multiple day horizon are the same at **32.9** min.

Perfect Information (PI) based user equilibrium

Every day, perfect travel time estimates with zero prediction error for all links are available for the traveler to make route decisions, and travelers can switch routes every day in response to an incident and its resulting non-recurring delay. According to Wardrop's first principle of user equilibrium, for a specific origin-destination pair, travelers with perfect information experience the same and minimum travel time along any used paths on each day *d*, with no unused path offering a shorter travel time.

To construct the objective function in the optimization model, the following gap function (for each day d) can be used to characterize the KKT optimality conditions (Wiki, 2010) required for reaching the user equilibrium for perfect information users.

$$gap_d^{PI} = f_{1,d}^{PI} \times (T_{1,d} - \pi_d) + f_{2,d}^{PI} \times (T_{2,d} - \pi_d) = 0, \forall d \quad (8)$$

Where $f_{1,d}^{PI}$ and $f_{2,d}^{PI}$ are path flow rate of PI users on paths 1 and 2, respectively, on day *d*, where π_d is minimum path travel time on day *d*

$$\pi_d = \min(T_{1,d}, T_{2,d}), \forall d \quad (9)$$

EVALUATING SAFETY IMPROVEMENT STRATEGIES USING STATIC TRAFFIC ASSIGNMENT METHOD

Now that the base case has been established, the travel times obtained from the modified BPR function could be easily calibrated. Following the steps lined out by Huntsinger and Rouphail (2011), one can calibrate and validate a BPR function using available sensor data. Because the proposed network is only hypothetical, there was no way to calibrate it, but in a real world application, the model would need to be validated. After the crash prediction model is calibrated, the inputs to the modified function can be altered to reflect proposed changes resulting from safety enhancing methodologies. The results from the enhanced methodologies can be compared to the base case and give meaningful results about how the safety enhancing features will improve the actual travel time within the link or network.

Alternative 1: Reducing incident probability through improved geometric design

The first safety performance enhancement we considered was reducing incident probability through improved geometric design. Theoretically the methodology used here can represent any improvement made to the roadway beyond simply improved geometric design. We changed the mode constant from -1 to -2, meaning that, incident probability is reduced for the same V/C ratio (1.183). The probability of incident dropped from the base case of 29.3% to **13.2%.**

After traffic equilibrium in this case, more flow switches to route 1, from 66.5% to **70.9%.** Because of the increased travel time, the V/C ratio is increased to 1.26 which in turn results in a slightly higher incident rate from the base case (**1.26**). Despite the increase in incident rate, the average travel time is reduced to **31.6 min**.

Alternative 2: Reducing capacity reduction due to incidents thought the use of a shoulder lane or quick response

The second safety performance enhancement we considered was reducing capacity reduction due to incidents thought the use of a shoulder lane or quick response. In this alternative, the lane capacity reduction on day 1 is increased to 1300 veh/lane/hr instead of 1000 veh/lane/hr in the other scenarios. The same mode constant was used as in the other scenarios and the resulting probability of incident was **29.6%.**

After traffic equilibrium in this case, more flow switches to route 1, up 7.5% over the base case (66.5% to **74%**). Because of the increased incident flow, the V/C ratio on route 1 is increased to 1.32 which is consistent with the slightly increased probability of crashes. Again, despite the increase in incident rate, the average travel time is again reduced to **31.0 min**.

Alternative 3: Real-time traveler information provision

The third safety performance enhancement we considered was better information provided to drivers by way of real-time traveler information. The market penetration rate we considered for these PI drivers was 15%. The benefit of having drivers with perfect information is that they can help the equilibrium of the system by being aware of congestion conditions on both routes and adjusting their route choice in response to the current conditions.

In this alternative, and on good days, more flow switches to route 1 due to the lower travel time. This increased the flow on route 1, up 8% over the base case (66.5% to **74.5%**). Similarl to the other scenarios, the increase in V/C ratio on route 1 led to an increased probability of incident, **29.6%.** When incidents occurred, the PI drivers were lured to route 2 by the lower travel time which helped lower the average travel time. The helpfulness of the PI drivers under incident conditions was not enough to overcome the damage they had done by using route 1 when it was incident free which resulted in a high incident rate. Interestingly, the overall average travel time was increased over the base case to **34.4 min**, up from 32.9 min. This additional system-wide average travel time is contributed by the increased link volume $f_{a,d=0}$ in Eq. (4) on "good" days due to "smart" PI drivers, as higher V/C ratios lead to higher probabilities of traffic breakdown and incidents as well as additional delays. On the other hand, thanks to real-time traffic information, PI drivers still enjoy lower travel time (32.2 min) than ETC drivers (34.8 min).

GENERAL MATHEMATICAL PROBLEM FORMULATION AND SOLUTION ALGORITHM

This section extends the above conceptual framework and the mathematic programming model proposed by Li et al. (2011) to a general network with with endogenous crash prediction functions and with variable road capacity, under different safety improvement strategies.

Formulation

The sets and subscripts, parameters and decision variables in the proposed flow assignment model are introduced as follows:
Indices:

i = index of origins, $i = 1, \ldots, I$, where I is the number of origins

j = index of destinations, $j = 1, \ldots, J$, where J is the number of destinations

p = index of paths, p=1, …, P, where P is the number of paths between an OD pair i and j

a = index of links, a=1, …, A, where A is the number of links in networks

d = index of days, d=1, …, D, where D is the number of days over analysis horizon

Input Parameters:

c_a^F = Full capacity of link a

c_a^R = Reduced capacity of link a

$q^{i,j}$ = OD demand volume between an OD pair i and j

$\delta_{p,a}$ = path-link incidence coefficient, $\delta_{p,a}$=1, if path p passes through link a, and 0 otherwise

γ = market penetration rate of the perfect information (PI) users as a function of the total OD demand

Decision variables:

$f_{p,d}^{PI,i,j}$ = flow of PI users on path p for OD pair (i, j) on day d

$f_p^{ETT,i,j}$ = flow of ETC users on path p for OD pair (i, j) (flow rates are the same across different days)

$v_{a,d}$ = total flow on link a on day d

ρ_a = Probability of having incidents on link a

$c_{a,d}$ = capacity of link a on day d

$T_{a,d}$ = Travel time on link a on day d

$U_{a,d}$ = generalized disutility on link a on day d, which is a function of capacity $c_{a,d}$ and link flow $v_{a,d}$

$U_{p,d}^{i,j}$ = generalized disutility of path p between OD pair (i, j) on day d

$\overline{U_p^{i,j}}$ = expected disutility of path p between OD pair (i, j) over the multi-day horizon

$\pi_d^{i,j}$ = day-dependent least path disutility between OD pair (i, j) on day d

$\overline{\pi^{i,j}}$ = Least expected disutility between OD pair (i, j) over the multi-day horizon

The proposed model incorporates the two user classes into a traffic assignment framework under stochastic capacity due to incidents that varies on a daily basis during the peak hour. The objective function aims to minimize the total gap for users with perfect traffic information and users with imperfect information based on expected travel time.

Objective function:

$$\min Gap = \sum_{d}\sum_{i}\sum_{j}\sum_{p}\left[f_{p,d}^{PI,i,j} \times \left(U_{p,d}^{i,j} - \pi_{d}^{i,j}\right) + f_{p}^{ETC,i,j} \times \left(\overline{U}_{p}^{i,j} - \overline{\pi}^{i,j}\right)\right] \quad (10)$$

PI flow constraints:

$$\gamma \times q^{i,j} = \sum_{p} f_{p,d}^{PI,i,j} \qquad \forall i, j, d \quad (11)$$

ETC flow constraints

$$(1-\gamma) \times q^{i,j} = \sum_{p} f_{p}^{ETC,i,j} \qquad \forall i, j \quad (12)$$

Path - link flow balance constraints

$$v_{a,d} = \sum_{i}\sum_{j}\sum_{p}\left(f_{p,d}^{PI,i,j} \cdot \delta_{p,a}\right) + \sum_{i}\sum_{j}\sum_{p}\left(f_{p}^{ETC,i,j} \cdot \delta_{p,a}\right) \qquad \forall a, d \quad (13)$$

Path- link cost connection

$$U_{a,d} = T_{a,d}(v_{a,d}, c_{a,d}) \qquad \forall a, d \quad (14)$$

$$U_{p,d}^{i,j} = \sum_{a}\left(U_{a,d} \cdot \delta_{p,a}\right) \qquad \forall i, j, d, p \quad (15)$$

Crash/capacity reduction probability constraint:

$$c_{a,d} = \rho(v_{a,d}, c_{a}^{F}, c_{a}^{R}) \qquad \forall a, d \quad (16)$$

Average disutility definitional constraint:

$$\overline{U}_{p}^{i,j} = \frac{1}{D}\sum_{d} U_{p,d}^{i,j} \qquad \forall i, j, p \quad (17)$$

Least disutility definitional constraints:

$$\pi_{d}^{i,j} \le U_{p,d}^{i,j}, \qquad \forall i, j, p, d \quad (18)$$

$$\overline{\pi}^{i,j} \le \overline{U}_{p}^{i,j} \qquad \forall i, j, p \quad (19)$$

Constraints (11) and (12) show the relationship between OD demand and path flows for each information class. Eq. (13) aggregates path flows from two different user classes to link flows. Eqs. (14-15) calculate the path disutility for each path on day d. Function $\rho(v_{a,d}, c_{a}^{F}, c_{a}^{R})$ in Eq. (16) represents a generic crash prediction probability based on prevailing link volume on day *d*, and the prediction results could be either full capacity c_{a}^{F} and reduced capacity c_{a}^{R} due to a crash.

Eq. (17) defines the average disutility for each path across different days, which will be used in the gap function for ETC users in objective function (10). The solution algorithm executing the above steps is depicted in Figure 3.

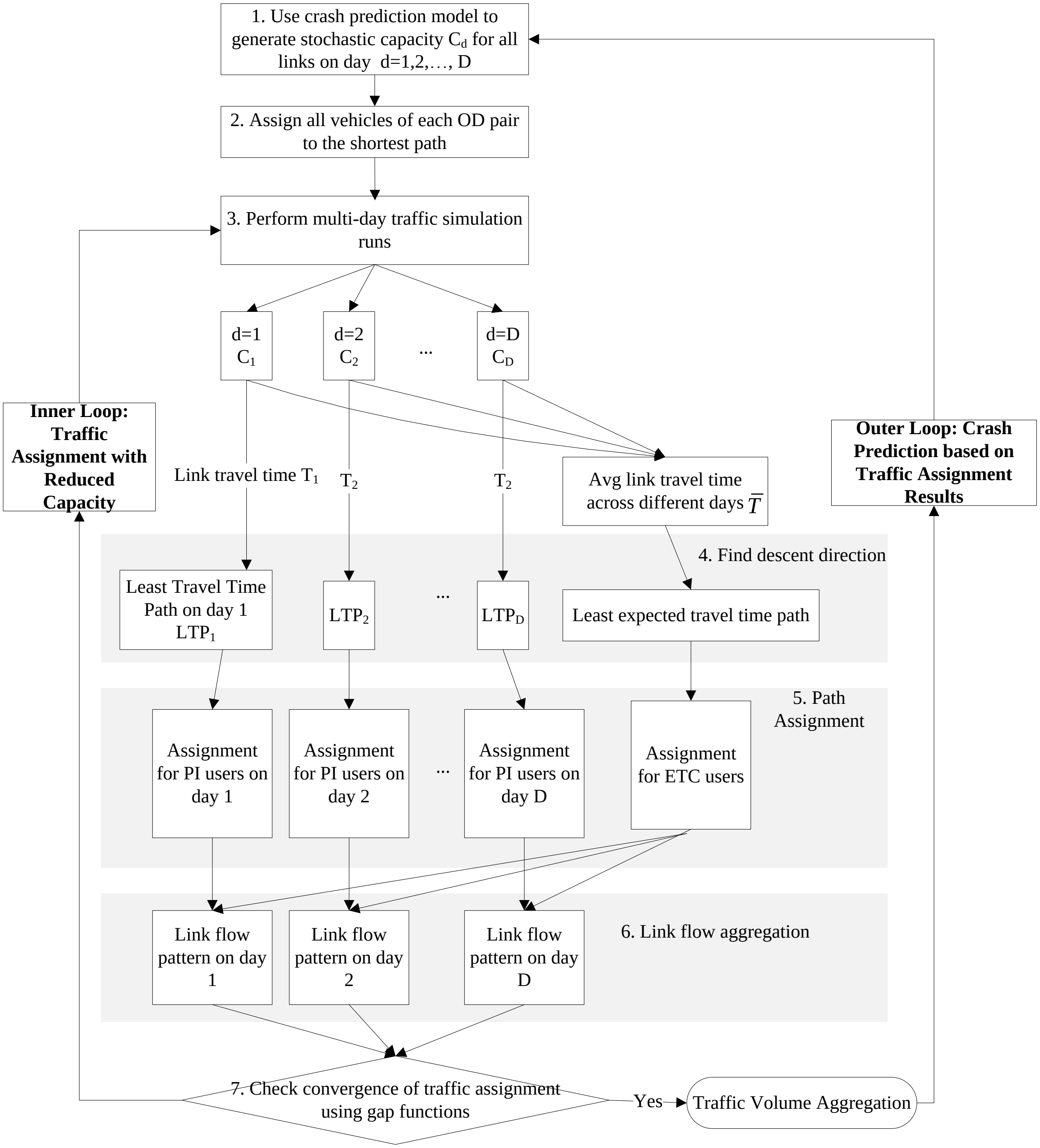

Figure 3. Solution algorithm for integrated crash prediction and traffic assignment with both PI and ETC users

In order to iteratively reduce the overall gap in the proposed optimization problem for a general network with multiple origins and destinations and with variable capacity due to incidents, we extend a descent search solution framework developed by Lu et al. (2009), which used a path-based gap function to describe the dynamic traffic equilibrium pattern. Figure 4 presents the iterative procedure for solving the multi-class static traveler assignment problem under stochastic and endogenous capacity conditions. The proposed procedure adds day-dependent simulation, path finding and assignment dimensions to the existing static traffic assignment algorithm that typically assumes deterministic road capacity conditions. In this study, we implement the proposed algorithm within a mesoscopic traffic assignment framework, which represents flow as vehicles with origin, destination and path attributes. Recall that, in conventional assignment programs, a vehicle is associated with a single path. In the proposed multi-day traffic assignment algorithm, an ETC vehicle still follows a single path across different days, but a PI vehicle can use and store different (day-dependent) paths on different days.

The main steps of the solution procedure are described as follows:

Step 1: Outer Loop: Use crash prediction model to generate day-dependent capacity.
Generate road capacity vector $C_d = \left[c_{a,d} \right]$ for all link a=1, 2, …, A, on day d=1, 2, …, D, according to a given crash prediction models and stochastic capacity distributions. The outer loop stops after a pre-specified number of iterations.

Step 2: Inner Loop: Traffic assignment with given stocahtic capacity
For each OD pair, compute the shortest path (in travel times) and assign both PI and ETC vehicles to the corresponding shortest path.

Step 3: Multi-day traffic simulation with stochastic capacity.
On each day d =1, 2, …, D, for given link flow patterns, generate day-dependent link travel times according to stochastic capacity vector C_d. The simulation results generate link travel time $T_{a,d}$ for link a=1, 2, …, A, on day d=1, 2, …, D.

Step 4: Find descent directions
Find the Least Travel time Path (LTP) using day-dependent link travel time $T_{a,d}$ on each day d, for link a=1, 2, …, A.

Find the Least Expected Travel time Path (LETP) using average link travel time $\overline{T_a} = \frac{\sum_d T_{a,d}}{D}$, for link a=1, 2, …, A.

Step 5: Path assignment for PI and ETC vehicles
For each day d, a certain percentage of PI vehicles are assigned to the least travel time path according to the Method of Successive Average (MSA).

Step 6: Link flow aggregation
For each day d, calculate the aggregated link volume $v_{a,d}$ using PI flow volume on day d and ETC flow (across every day).

Step 7: Convergence checking for traffic assignment
Calculate the gap function as shown in Eq. (10), if $Gap < \epsilon$ convergence is achieved, where ϵ is a pre-specified parameter. If convergence is attained, go back to Step 1 for crash prediction stage. Otherwise, go to Step 3.

EXTENSION TO DYNAMIC TRAFFIC ASSIGNMENT FRAMEWORK

With the goal of integrating improved transportation safety impacts into the region-wide planning process, this section aims to develop and articulate the specific method for representing both the propensity for crashes and the effects of these crashes on congestion. We will first explore several approaches to modeling crash probabilities $c_{a,d} = \rho(v_{a,d}, c_a^F, c_a^R)$ in a dynamic traffic network simulation environment including: probability distributions, conditional probabilities, time sequential models, and liner regression models.

Approach 1: Introduce link-specific incident probability functions, which can be calibrated from historical crash record GIS databases, typically maintained by state DOTs.

Approach 2: Estimate incident rates based on traffic conditions and underlying geometric design characteristics through quantitative crash probability and regression models. For example, one potential tool that can be integrated in the proposed framework is the macroscopic PlanSafe model that forecasts the safety impacts of socio-demographic changes and safety countermeasures.

Approach 3: Provide an external crash/incident probability input interface for a subarea study that focuses on high crash locations. State DOT engineers can first identify potentially hazardous roadway locations and designs, and key in crash probability changes (based on expert knowledge) for strategies to reduce the contributing factors that lead to crash occurrences under given roadway and/or environmental characteristics.

Approach 4: Integrate the mesoscopic simulator in dynamic traffic assignment models with fine grained microscopic simulation tools for evaluating new design proposals. For example, SSAM can derive surrogate measures of safety from existing microscopic traffic simulation models for intersections. This is a more computational challenging method and the feasibility of this hybrid micro-mesoscopic simulation approach will be examined in this project, but the corresponding implementation subtask remains as optional due to limited time and resources in this project.

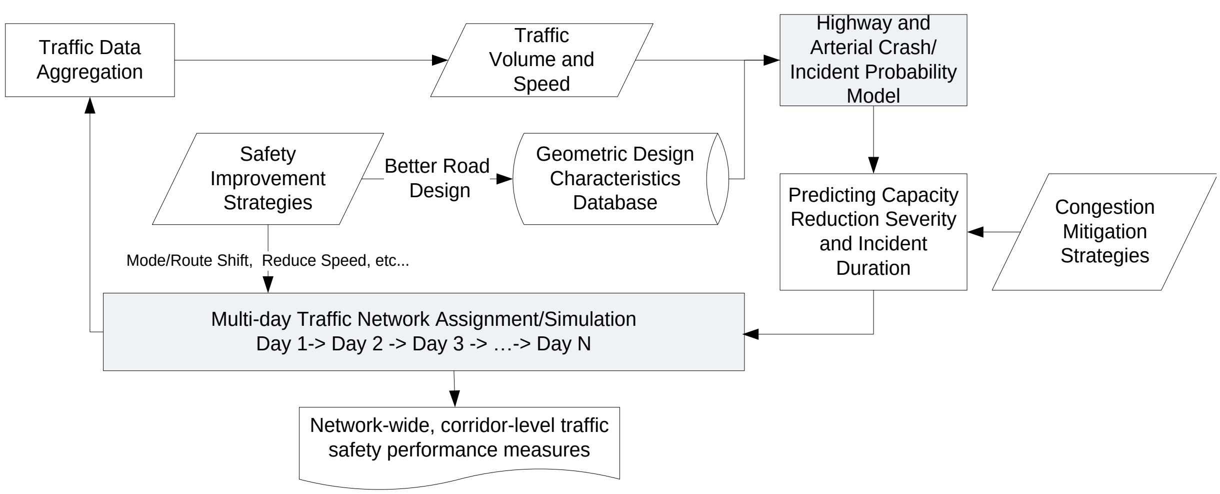

Figure 4. Incorporating Safety Prediction in DTA Framework

In addition to capacity reduction severity, we also need to predict incident duration that corresponds to the time period of capacity reduction. To enable an effective project selection processes that respond to various safety improvement strategies, a proposed modeling framework is shown in Figure 4.

The approach relies on an integrated feedback loop that will allow planners to evaluate different safety enhancement strategies for reducing the frequency and effects of events that cause travel times to fluctuate in an unpredictable manner. Because crashes and resulting incidents are small-probability events, it requires the dynamic traffic assignment program to perform simulation for multiple weeks or multiple months, in order to obtain the statistically sound measures for evaluating safety improvement strategies, include but not limited to: enhancing work zone safety, improve intersection and interchange safety. To generate reliable traffic condition measures for crash probability prediction, the traffic data (e.g. speed and volume at different times of day) need to be aggregated over a period of time covering several weeks or several months (depending on the data resolution requirement from the crash prediction model). The traffic condition statistics are further fed into the crash prediction model to predict the locations, time stamps and severity of crashes, as well as the resulting capacity reduction magnitude and duration in the traffic network system.

EXPERIMENTS ON MEDIUM-SCALE NETWORKS

The following numerical experiments are performed on two medium-scale network data sets. The proposed algorithm is implemented in C++ on the Windows Vista 64-bit platform and evaluated on a computer with an Intel Xeon CPU with 4 2.33 GHz processors and 9 GB memory. The proposed algorithm has been incorporated into an open-source traffic assignment package available at https://sites.google.com/site/dtalite.

Table 2. Test network characteristics and computational performance with 30 day samples, 20 iterations and 10% PI vehicles.

	Anaheim, California	**Chicago Sketch Network**
# of nodes	416	933
# of links	914	2950
# of OD zones	38	387
Total OD Volume	104K	1,261K

As shown in Table 2 and Figure 5, the Anaheim, California network contains about 38 zones, and 0.1 million vehicles, and the Chicago sketch network, an aggregated representation of the Chicago region, has 387 zones with 1.2 million vehicles. Under a setting of 10% PI users, 20 assignment iterations and 30 days of random road capacity, the Anaheim network uses about 30 minutes, and the Chicago sketch network takes about 8 hours of CPU time and 2.6G memory.

The original data sets use the BPR function to describe travel time performance, and a single valued mean capacity is specified for each link. To evaluate different incident response/management strategies with different degrees of random road capacity reduction due to incidents, we consider the impact of two alternative schemes with low and high capacity reduction variation values. In particular, improved geometric design, the use of a shoulder lane or quick response can reduce the variations of capacity reduction due to incidents.

Table 3. Evaluating safety improvement strategies: unit (min)

Peak-hour Capacity Approximation Scheme	Anaheim network			Chicago Sketch Network		
	ETC Travel Time	PI Travel Time	Average Relative Travel Time Saving (%)	ETC Travel Time	PI Travel Time	Average Relative Travel time Saving (%)
Low Capacity Reduction Variations with CV= 6.4%	12.903	12.864	0.302%	17.348	17.242	0.611%
High Capacity Reduction Variations with CV= 12.8%	13.233	13.157	0.574%	17.794	17.469	1.827%

CONCLUSIONS

In this document, an equilibrium-oriented conceptual framework is proposed along with related modeling components pertaining to stochastic capacity due to incidents, travel time performance functions and different degrees of traveler knowledge. Within a gap function framework (for describing the user equilibrium under different information availability) a mathematical programming model is formulated to include crash probability as a function of the V/C ratio as well as a facility specific safety constant. The model was applied to a simple 2-link corridor to illustrate the ability of the model to provide a basis for comparing different safety enhancing strategies. This comparison allows the user to systematically predict and evaluate the cost / benefit of a proposed strategy and compare the results to other scenarios. The user is thus able to have a foundation on which to decide the best locations and types of safety investments to make.

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