JACKKNIFING WARNING FOR ARTICULATED VEHICLES BASED ON A DETECTION AND PREDICTION SYSTEM

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ABSTRACT

According to statistics, articulated vehicles like tractor-semitrailers represent over 55% of heavy goods traffic in France. In some cases, this type of truck may have unstable behavior, resulting in high risks of accidents. Jackknifing is defined by a loss of yaw stability of the articulated system. It most commonly occurs when the semi-trailer is empty. This paper presents the development of an intelligent jackknifing safety system. The objective is to develop a strategy to detect a critical situation (jackknifing) and trigger a warning message under dangerous conditions such unstable yaw. A new system for articulated vehicles jackknifing detection and prediction is designed. This system uses on one hand, a nominal model of articulated vehicle making it possible to determine the vehicle dynamic state such as the relative yaw angle. On the other hand, detection algorithm is based on a jackknifing criterion and on the prediction function of jackknifing, in view of estimating the time to jackknifing. The validation of the results is done by using PROSPER, which is a very detailed commercial simulator.

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INTRODUCTION

Road safety is a priority for transport policy. According to recently published studies on accidents, the number of killed people on road is decreasing, despite the considerable increase in traffic in the last ten years. This may be due to safety policies, road design and in large part to the development of on-board safety systems (Anti-lock braking system “ABS”, Electronic Stability Program “ESP”, Adaptive cruise control “ACC”, ...). Nevertheless, the number of accidents remains high for heavy vehicle (type articulated). Moreover, accidents involving heavy vehicles can have very serious consequences for victims. The statistics show that articulated vehicles like tractor-semi trailers represent over 55% of heavy goods traffic in France (Desfontaines, 2003) (Bouteldja, 2005). In some cases, this type of truck may have unstable behaviour, resulting in high risks of accidents.

Jackknifing is defined by a loss of yaw stability of the articulated system (Ervin et al., 1998) (Weidner, 2007) (Boumediene, 2001). It most commonly occurs when the semi-trailer is empty. However, equipment failure (especially brake failure), improper braking and poor road conditions can entail jackknifing.

Several studies have been realized to analyze the risk of instability in yaw dynamics of articulated vehicles Some works proposed a yaw control active systems of tractor and semi-trailer by acting independently or by combining the braking or steering actions (Ackermann et al., 1999) (Peng et al., 1999) (Chen et al., 1997). Moreover, indicators (detectors) of the jackknifing were little discussed in the literature. Currently, this type of accident is part of the research areas under development. Nevertheless, (Timothy et al., 1981) proposed a mathematical model describing the jackknifing situation. Subsequently, this model was adapted in (Bouteldja, 2010) (Bouteldja et al.,2006) to develop a dynamic indicator of the jackknifing risk. Other methods are presented in (Boumediene, 2001) (Grainer et al., 2004) (Mikulick, 1971) to identify jackknifing situations from the evaluation of friction coefficient and the yaw relative angle between tractor and semi-trailer. However, these research works suppose that the parameters such as for example the lateral dynamics and the friction coefficient and variable dynamics, are measured even if obtaining such information remains a difficult task.

This paper presents an analytical study of the dynamics of tractor-semi-trailer vehicles, with particular regard to the jackknifing phenomenon. This work concerns detection and prediction of articulated vehicle jackknifing. The objective is to develop a strategy to detect a critical situation (jackknifing) and trigger a warning message under dangerous conditions such unstable yaw.

A new system for articulated vehicles jackknifing detection and prediction is designed. This system uses on one hand a nominal model of articulated vehicle and on another hand an algorithm of detection of jackknifing. The numerical model makes it possible to determine the dynamic state of the vehicle in real time such as the angle of relative yaw. The detection algorithm is based on a jackknifing criterion and on a prediction function, which estimates the time remaining before jackknifing. This system could greatly improve the safety of the articulated vehicle. Some maneuvers where the system successfully detect and predict jackknifing are presented in this paper.

The validation of the results is done by using a software called Prosper (http://www.sogeclair.com/filiales/sera/index.php). It was used throughout the work to verify theory, to validate the model, to find critical maneuvers and to test the detection and prediction system developed.

The paper is divided into four parts. The first part deals with articulated vehicle modeling, where suspension and tire models as well as chassis models are considered. The dynamics equations derived by Lagrangian approach are presented. The second part is devoted to the design of detection algorithms for the articulated vehicle in view of avoiding jackknifing risk. The third part presents the prediction system, which allows estimating the time remaining before the jackknifing. The last part evaluates the detection and prediction system developed in this paper. Simulation results are presented and discussed.

PROBLEM STATEMENT

The jackknifing is defined by a loss of stability in the yaw motion of the articulated system (Figs.1 and 2).

Figure 1. Jackknifing accident

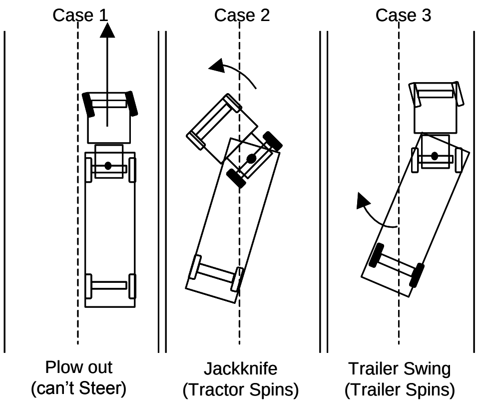


Figure 2. Cases of jackknifing

It occurs when the rear wheels of the tractor are blocked, when the vehicle applies in turn the brakes abruptly or when the road is slipping (low skid resistance). This phenomenon is more frequent for an empty vehicle or with a bad useful charge distribution. The side forces resulting from control in turn and the state of the roadway are important factors for jackknifing. Theoretically, jackknifing occurs when the angle between the tractor and the semi-trailer becomes superior to 90°. The driving wheels of the tractor lose their skid resistance and are involved towards the right-hand side or the left because of the force exerted by the trailer. To avoid a situation of jackknifing, one needs to slack off the accelerator or the brake pedal, according to the cause of loss traction (Figure 2).

ARTICULATED VEHICLE MODELLING

This section provides details of a mathematical model in the context of its development.
The type of heavy vehicle considered in this work is a tractor semi-trailer with 5-axels (2-axels for tractor and 3- axels for semi-trailer). In order to detect and predict the jackknifing situations, we adopt a configuration of vehicle representing tractor with two axles and one chassis body, and semi-trailer with one wheel axle and one chassis. In this configuration, we consider only the dynamics of the two bodies (ie tractor and semi-trailer's chassis). The roll angle is considered around the tractor roll axis at the fifth wheel. The pitch and bounce will be neglected in the model (Bouteldja, 2005) (Chen and al., 1997).

To develop the dynamics equations of the model, the motion of the two sprung masses in the coordinate system is considered (Figure 3).

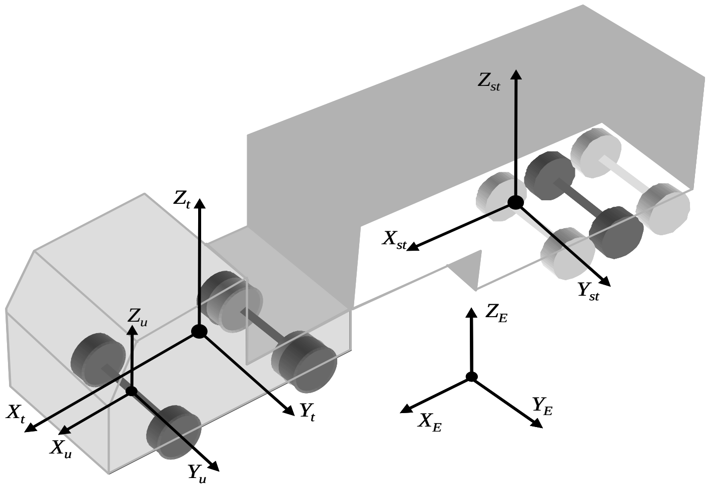


Figure 3. Vehicle coordinate systems

The previous description of the vehicle motion allows the calculation of the translation and rotational velocities of each body-mass at center of gravity by studying the kinematics with respect to different references frames. Then, the total kinetic and potential energy is calculated in the inertial reference frame (X_E,Y_E,Z_E): The expressions of kinetic energy and potential energy are used to derive the vehicle model based on the Lagrangian mechanics approach. Thus, the model of the vehicle takes the following form:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = F_g \tag{1}$$

where q_i is the i^{th} generalized coordinate and q is the generalized coordinate vector defined as $q = [x, y, z, \psi, \phi, \psi_f]$. F_g represents the vector of generalized forces. M is the inertial matrix that is symmetric positive definite, $C(q,\dot{q})\dot{q}$ is the Coriolis and Centrifugal forces and G is the gravity vector. The generalized forces summarized vector F_g represents the effect of the external forces acting on the vehicle body. These forces are from the tire-road interface and suspensions defined in terms of the longitudinal and lateral tire forces and vertical forces. The locations of these external forces could be considered at each wheel of the three axles (Chen et al., 1997).

The suspension is modeled as the combination of a nonlinear spring and damper element (Bouteldja, 2005) (Chen et al., 1997). The tire model describes the relation ship between lateral and longitudinal force. Pacejka tire model (magic formula) is adopted. In this model the lateral and longitudinal forces are determined as function of side slop angle, longitudinal slip, and vertical load (Pacejka, 1996).

All states of the vehicle, for example velocities, are considered known unless stated otherwise (from Prosper simulator). In a real environment these states are estimated using observers and sensors. The forces and loading conditions of the articulated vehicle are considered unknown, since they are not available in a commercial articulated vehicle. One of the most important variables in vehicle modeling is the coefficient of friction. This is also considered unknown in this paper.

JACKKNIFING WARNING SYSTEM SOLUTION

Description of the warning system

The detection and prediction system of risk is assessing the jackknifing risk level based on instantaneous inputs of driver (steering, braking, etc.), on vehicle dynamic behaviour and infrastructure characteristics. These information are used by the risk module to alert the driver upstream in order to avoid a critical situation. The system is composed of three blocks (figs. 4 and 5):

- The first block contains information from sensors embedded in the vehicle about its dynamic state, and the information from a digital map database coupled with a GPS (geometrical and physical data of the road).
- The second block is constituted of the software sensors, which allows observing the unknown states from the data provided by sensors embedded and estimating the uncertain parameters of articulated vehicle.
- The last block is the risk evaluation module. It incorporates information from other blocks in order to assess the risk level of jackknifing on the predicted trajectory and warn the driver.

It is necessary to have the characteristics of the road for risk prediction and it is assumed that the embedded database on the vehicle contains the road profil information and the road curvature.

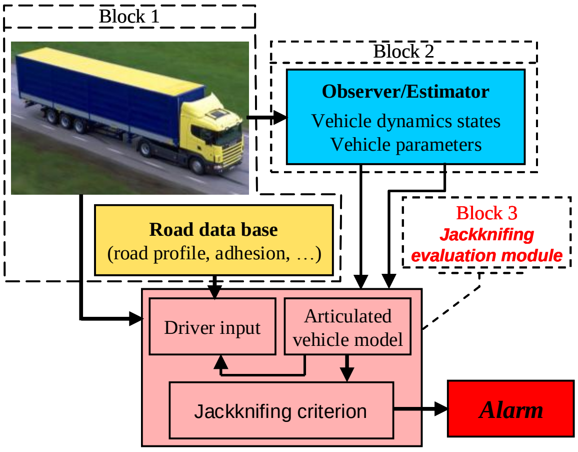


Figure 4. Warning system architecture

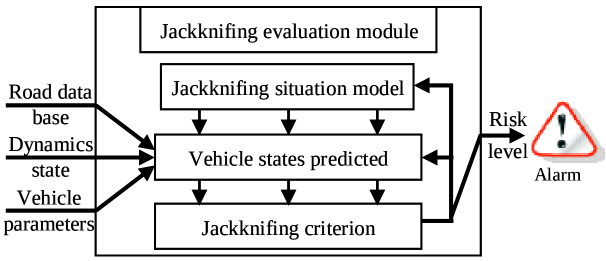


Figure 5. Risk evaluation module

The risk detection is based on a jackknifing criterion. At each moment, the vehicle is located using the database infrastructure The dynamics state prediction allows the risk evaluation module "stability analyze of the vehicle behavio". If an early jackknifing risk is predicted in an horizon time, the time prediction corresponds to time before the accident, assuming that the driver remains the same actions on the controls (driver commands).

Jackknifing detection algorithm – definition of the jackknifing criterion

Mathematically, the jackknifing can be defined by a cross product as follows. The figure 6 represents the articulated vehicle in the reference frame (O, X_E, Y_E) with the coordinates of the points P_1 and P_2.

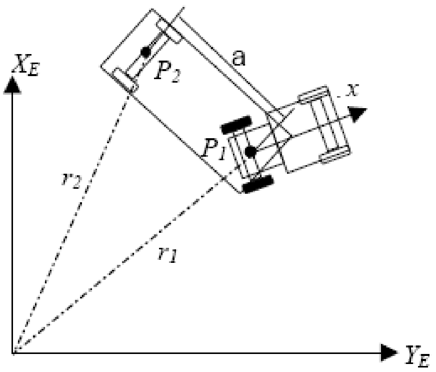


Figure 6. Articulated vehicle position

Suppose that a tractor is pulling a semi-trailer, which is a unit long. r_1 will be represented according to time t. We would like to be able to predict r_2. That is, for a given path $r_1 = r_1(t)$, we want to know if the truck-trailer will jack-knife. Alternatively, we want to determine what conditions must be imposed on $r_1(t)$ to prevent jackknifing (Erwin et al., 1998). We model the motion of r_2 as follows. First, the semi-trailer length $|r_1 - r_2| = 1$ is constant, which shows that:

$$(r_1 - r_2) \cdot (r_1 - r_2) = 1 \tag{2}$$

Where "." is the vector dot product. Also, the wheels of the semi-trailer constrain the vector r_2 so that its velocity vector is directed along the semi-trailer lateral axis $(r_1 - r_2)$. That is

$$\dot{r}_2 = \lambda (r_1 - r_2) \tag{3}$$

for some λ, where $\dot{r}_2 = \frac{dr2}{dt}$. After differentiating (2), we obtain $2(\dot{r}_1 - \dot{r}_2).(r_1 - r_2) = 0$ and so

$$\dot{r}_1 \cdot (r_1 - r_2) = \dot{r}_2 \cdot (r_1 - r_2) \tag{4}$$

Taking the dot product of (3) on both sides by $(r_1 - r_2)$ gives $\dot{r}_2 \cdot (r_1 - r_2) = \lambda (r_1 - r_2) \cdot (r_1 - r_2) = \lambda$ and so by (4), $\lambda = \dot{r}_1 \cdot (r_1 - r_2)$. Therefore, (3) becomes

$$\dot{r}_2 = [\dot{r}_1 \cdot (r_1 - r_2)](r_1 - r_2) \tag{5}$$

For convenience, let $r = (r_1 - r_2)$, so that (5) can be rewritten as $\dot{r}_2 = (\dot{r}_1 \cdot r) \cdot r$.

It the cab is moving forward, we say the cab and semi-trailer are jackknifed if $C_m = \dot{r}_1 \cdot r < 0$; otherwise, we say they are unjackknifed. If the cab is braking up, the conditions remain valid but the above situation is reversed (when $C_m = \dot{r}_1 \cdot r > 0$, the tractor and trailer are jackknifed). For more detail the authors may consult the following (Bouteldja, 2005) (Bouteldja et al.,2006) (Timothy et al., 1981).

We consider that we have the coordinates of unspecified point of the tractor or semi-trailer. In the reference frame related to the fifth wheel, we consider $(\vec{e}_x, \vec{e}_y, \vec{e}_z)$ the base of the principal axis.

Let O the origin of the reference frame, the coordinates of the points P_1 and P_2 are written in this reference frame by

$$\begin{cases} P_1 = x\vec{e}_x + y\vec{e}_y \\ P_2 = (x - a\cos(\psi_f))\vec{e}_x + (y + a\sin(\psi_f))\vec{e}_y \end{cases}$$

According to the relation of the field speed in a solid, the speed vector of the point P_1 is defined by these components.

$$\vec{V}_{P_1} = (\dot{x} - y\dot{\psi})\vec{e}_x + (\dot{y} + x\dot{\psi})\vec{e}_y \quad (6)$$

While introducing the vector displacement and equation (6) in C_m, then

$$C_m = \dot{r}_1 \cdot r = \vec{V}_p \cdot \left(\overleftrightarrow{Or}_{P_1} - \overleftrightarrow{Or}_{P_2}\right)$$

Which takes into account that $\vec{V}_P \cdot \left(\overleftrightarrow{Or}_{p1} - \overleftrightarrow{Or}_{p2}\right) \leq 0$, after development, we obtain:

$$C_m = \frac{\dot{x} - y\dot{\psi}}{\dot{y} + x\dot{\psi}} = \tan(\psi_f) \quad (7)$$

This equation represents the criterion of truck jackknifing. To be able to detect and predict a jackknifing situation, it is necessary to define zones of stability for yaw angle. This zone is defined around the ψ_f articulation angle; it varies between [-85°, +85°]. We showed previously that $C_m = \tan(\psi_f)$, therefore by framing we obtain the following inequality. It represents the zone of stability for jackknifing situation.

$$\tan(-85) < C_m = \frac{\dot{x} - y\dot{\psi}}{\dot{y} + x\dot{\psi}} < \tan(85) \quad (8)$$

Jackknifing prediction algorithm

To avoid Jackknifing, it is necessary to early detect any evolving conditions that may lead to this situation. The question is how predicting accident risks sufficiently early? A prediction method was introduced. This method consists in evaluating the remaining time before the accident.
The prediction of risk situations is done in two steps: the first step is state prediction and the second step is a prediction based on risk criterion as shown in figure 7.

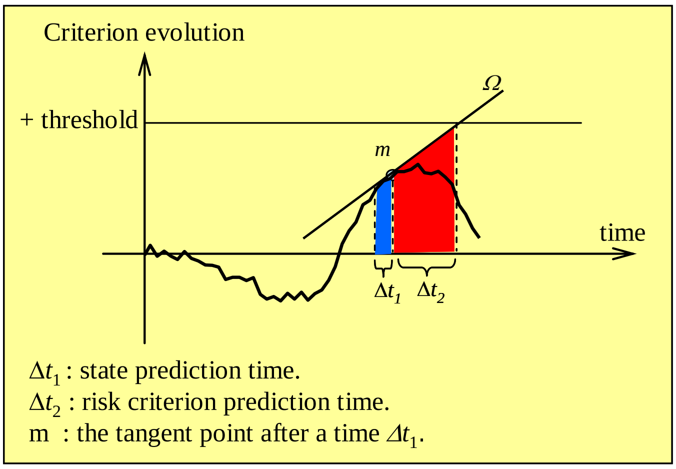


Figure 7. Risk situation prediction principle

<u>State prediction</u>

An extended Kalman filter (Julier et al., 1995) is used to estimate the articulated vehicle state. The model (1) is represented by a non-linear state-space description incorporating state and measurement difference equations:

$$\begin{cases} X_{k+1} = g(X_k, u_k, w_k) \\ Y_k = h(X_k, n_k) \end{cases} \quad (9)$$

In equations (9) the non-linear function g relates the state vector X and the input vector u at time step k to the state at step k+1. The measurement vector h relates the state to the measurements Y_k.($Y_k = [x, \dot{x}, y, \dot{y}, z, \psi, \dot{\psi}, \phi]$) Vectors w_k and n_k denote the superimposed process and measurement noise respectively. The state vector X_k is defined as $X_k = [x, \dot{x}, y, \dot{y}, z, \dot{z}, \psi, \dot{\psi}, \phi, \dot{\phi}, \psi_f, \dot{\psi}_f]$.

The extended Kalman filter (EKF) uses the measurement Y_k at time kT_e to predict the next process state $X_{k|k+1}$ at time $(k+1)T_e$. T_e represents the sampling period and k corresponds to the sampling time. The discrete-time EKF algorithm is implemented as adopted from reference (Maybeck, P. S. (1989). The EKF measurement update equations are shown below:

$$K_k = D_k^- H_k^T \left(H_k D_k^- H_k^T + R_k\right)^{-1} \quad (10)$$

$$\hat{X}_k = \hat{X}_k^- + K\left(Y_k - h\left(\hat{X}_k^-, 0\right)\right) \quad (11)$$

$$D_k = D_k^- \left(I - K_k H_k\right) \quad (12)$$

while the EKF time update equations are as follows:

$$D_{k+1}^- = A_k D_k A_k^T + Q_k \quad (13)$$

$$\hat{X}_{k+1}^- = g\left(\hat{X}_k, u_k, 0\right) \quad (14)$$

Equations (10) to (14) form the complete set of equations for the EKF algorithm. The EKF propagates the state and error covariance estimates (14) and (13) by computing the filter gain matrix (10) and by updating the state and covariance estimates based on the measurement residuals (11) and (12). Matrices A_k and H_k are computed by linearizing equations (9) around X_k at each time step. Matrix A and H are respectively the Jacobian matrix of partial derivatives of g( ) and h( ) with respect to X.

The filter is initialized with a state estimate corresponding to the true state and a large covariance matrix. Kalman predictor allows determining the system state at instant $k+1$ based on system state at instant k. With this prediction method, a time $\Delta t_1 \approx 2s$ before accident is determined.

<u>Risk criterion based prediction</u>

Accidents risks prediction method (jackknifing) is based on the evolution of the criterion, calculated with respect to the predicted state. The tangent line of the evolution curve is determined for each time path.

Thus, on each point of the curve, which defines the risk level (figure 8), the duration between the current time and the instant corresponding to the intersection axis between the tangent line

and the threshold value is calculated. An example of evaluation of $\Delta t_2 \approx 0 \, to \, \infty$ is presented in figure 8 with Ω is the tangent line.
The total accident risk prediction time (Δt : Time Left Before the Accident: TLBA) is the sum of the state prediction time (Δt_1) and the risk criterion prediction time (Δt_2).

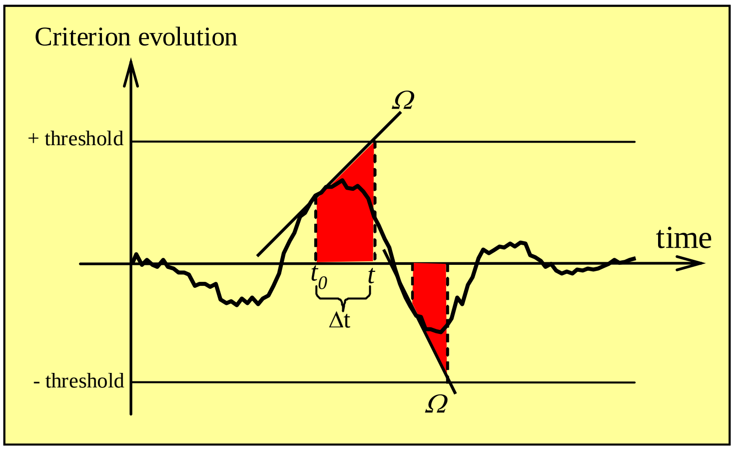


Figure 8. Risk criterion prediction (determination of time left before the accident TLBA)

The risk criterion based prediction time duration is determined according to:

$$\begin{cases} \Omega = C_m^{'}(t_0)(t - t_0) + C_m(t_0) = C_m^{'}(t_0)\Delta t + C_m(t_0) \\ \Omega = threshold \end{cases} \quad (15)$$

This gives $threshold = f'(x_0)\Delta t + f(x_0)$, and:

$$\Delta t = \frac{threshold - C_m(t_0)}{C_m^{'}(t_0)} \quad (16)$$

SIMULATION RESULTS

The effectiveness of the criterion developed and the prediction system proposed above is evaluated through numerical simulations. The tool used for building the warning system is Matlab/Simulink software. For simulation, Prosper software is used. This is a very detailed simulator of heavy vehicle with correct suspension and force models and full degrees of freedom chassis models. The simulation results are very similar to those that would be produced with a real articulated vehicle. It was used throughout the work to verify theory, to validate the model, to find critical maneuvers and test the system developed.

Model validation

The results of simulation obtained with Prosper are compared to the results of the model proposed in the warning system. The parameters of articulated model are obtained from Prosper. The simulations are performed on a trajectory characterized by a strong lateral excitation (succession of curves with a small radius of curvature) at a constant speed U of 90km/h.

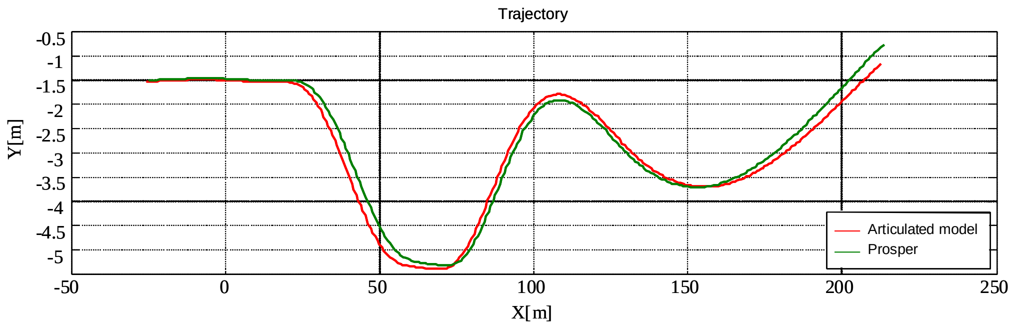


Figure 9. Comparison of the trajectories

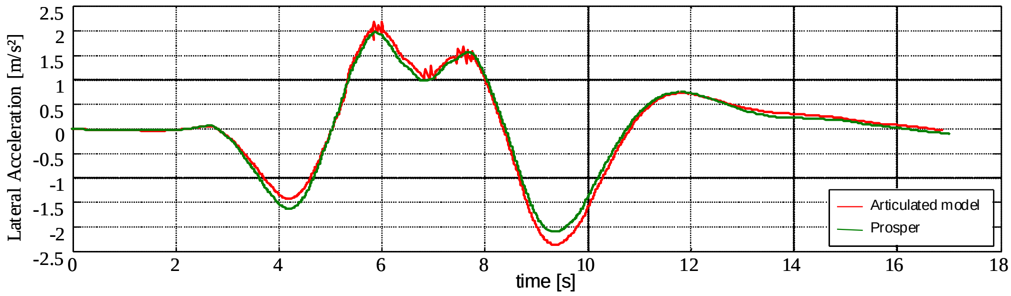
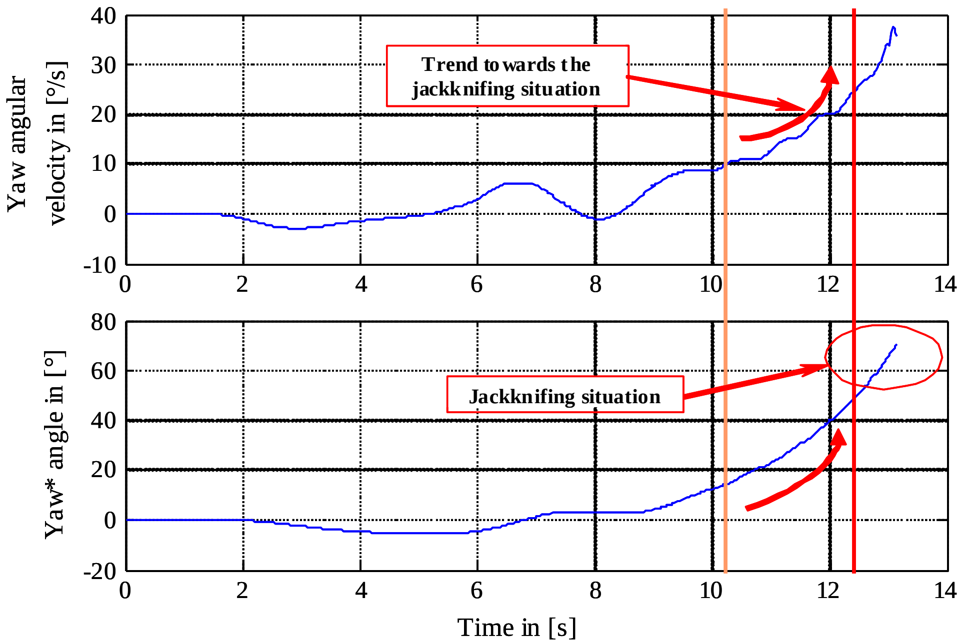


Figure 10. Comparison of the lateral accelerations

Figures 10 and 11 exhibit typical results obtained for the most unfavorable case. Good agreement between the model and Prosper is obtained.

Warning system testing

In the second part of the test, the detection and prediction system is validated. Figure 12 shows the evolution of angular velocity and yaw angle between tractor and semi-trailer on the fifth wheel according to time.



*: Yaw angle between tractor and trailer at fifth-wheel

Figure 11. Yaw angular velocity and yaw angle at fifth wheel

Figure 12 shows that yaw angle and the yaw angular velocity tend to increase strongly after 10 seconds. These values become higher than the thresholds values of stability. The articulated vehicle is in jackknifing situation. Figure 13 presents the evolution of the indicator Cm, which tends to exceed the thresholds value after 10 seconds. So, C_m is a good indicator for Jackknifing detection.

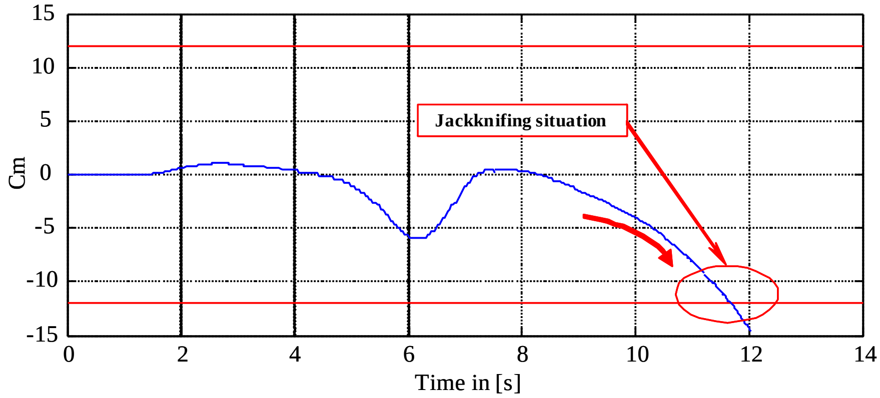


Figure 12. Yaw angular velocity and yaw angle at fifth wheel

The evolution of the slope of the yaw observed angle leads to make possible the prediction of a jackknifing situation. After 10 seconds, the curves representing yaw angle and yaw angular velocity tend to diverge. This behaviour is representative of a high risk of jackknifing around 11 seconds (figs. 12 and 13).

Application

The last part of the study is devoted to the development of an application using digital software Prosper and Matlab / Simulink. In this framework, the software Prosper is used as an emulator of the heavy vehicle and Matlab plays the role of on-board computer system (figure 14).

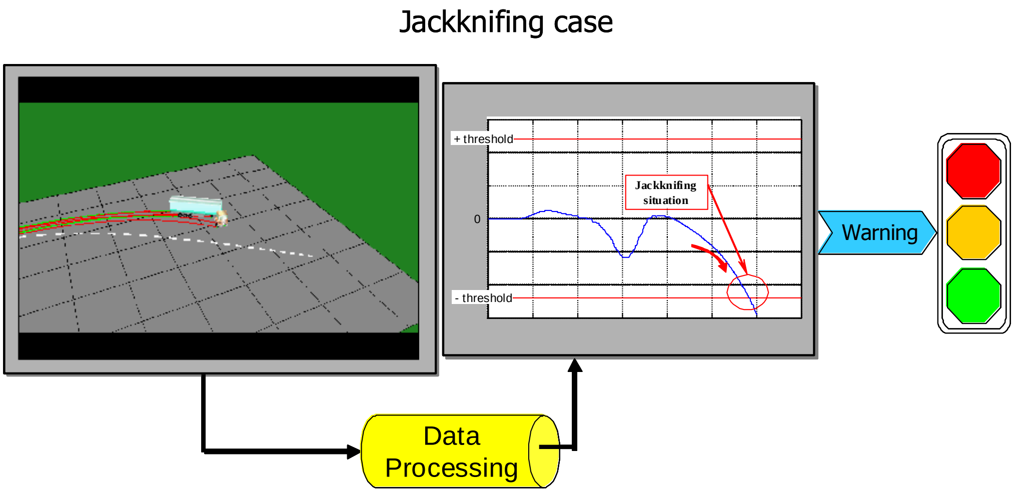


Figure 13. Jackknifing warning system

The presentation of results from these "quasi-experimental system" is not easy considering the real-time prediction. In fact, it only makes sense regarding the dynamic evolution of the vehicle until reaching a critical situation.

Figure 14 simulates a jackknifing case. The system detects very well the different levels of risk. During movement of the vehicle on the road, the system evolves by adjusting various thresholds. The alarm level ranges from green (no danger) to red (danger). In this validation, Δt is estimated at 2.24s.

CONCLUSION

In this paper, a new system for articulated vehicles jackknifing detection and prediction is developed. This system uses a nominal model of articulated vehicle to determine the dynamic state of the vehicle and an algorithm for detection based both on a criterion of jackknifing and on a function of prediction of jackknifing in view of estimating the time to jackknifing. The system is successfully detecting and predicting jackknife.

Design and implementation of the detection and prediction system for an articulated vehicle present challenges. Indeed, articulated vehicle dynamics is complex to model and several factors may cause jackknifing (environment, driver, driving condition). Moreover, jackknifing detection system needs access to some data (road geometry, heavy vehicle parameters, etc;). Despite these difficulties, the system developed may be an efficient safety system of the next generation of heavy trucks. Lastly, additional research work is on process in order to improve this detector/predictor and make it more reliable and robust.

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