ESTIMATION OF CRITICAL HEADWAYS AT UNSIGNALIZED INTERSECTIONS - A MICROSCOPIC APPROACH

Luís Vasconcelos
Adjunct Professor, Polytechnic Institute of Viseu, Civil Engineering Department
Viseu, Portugal, e-mail: vasconcelos@estv.ipv.pt

Ana Bastos Silva
Professor of Civil Engineering, University of Coimbra, Civil Engineering Department
Coimbra, Portugal, e-mail: abastos@dec.uc.pt

Álvaro Seco
Professor of Civil Engineering, University of Coimbra, Civil Engineering Department
Coimbra, Portugal, e-mail: aseco@dec.uc.pt

Gilberto Rouxinol
Adjunct Professor, Institute for Systems Engineering and Computers at Coimbra
Coimbra, Portugal, e-mail: rouxinol@estv.ipv.pt

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ABSTRACT

Understanding gap-acceptance behavior is fundamental to safety and capacity analyses of two-way controlled intersections and roundabouts. Capacity models based on gap-acceptance theory rely on the *critical headway* parameter, whose estimation raises some difficulties: it depends on several external variables and cannot be observed directly. Existing methods, based on field observations, produce estimates that are mostly site-specific, which limits their applicability under different geometric and operational conditions. In this paper an alternative estimation method is proposed. The model is based on the interactions between the entry and opposing vehicles, described at microscopic level. It captures the complex interactions between the driver/vehicle dynamics and the intersection geometry, but is simple enough to be easily implemented in a spreadsheet. The model was calibrated based on a video recording of a roundabout in Coimbra, Portugal, and validated at other roundabouts, with different geometric and traffic characteristics. The resulting estimates are close to the results of conventional estimation methods, suggesting that the proposed model is a promising alternative to field observations, particularly for the study of non-conventional intersections.

Keywords: critical headway, gap-acceptance, roundabout, maximum likelihood, car-following, homography, direct linear transform.

INTRODUCTION

A gap-acceptance maneuver is a complex and risky driver behavior that has a strong relation with traffic safety and operations. Understanding this behavior is fundamental to analyze and predict accidents (Alexander et al., 2002; Spek et al., 2006; Yan et al., 2007) and to model capacity and delays. In fact, capacity models based on the gap-acceptance theory have been developed since the early 1960s, especially in Germany and England (Harders, 1968; Siegloch, 1973; Tanner, 1962) and are used now in many countries for analyses of two-way stop-controlled intersections and roundabouts.

The critical headway, or critical gap in older studies, is one of the most important parameters in these capacity models (Vasconcelos et al., 2011). It is defined for a specific vehicle/driver and a specific movement at an intersection and represents the minimum time interval in the priority stream that a minor street driver is ready to accept for crossing or entering the major stream conflict zone (Brilon et al., 1999). The critical headway is usually regarded as resulting from a driver's decision process, where the inputs are subjective estimates of explanatory variables such as the waiting time, time of the day, vehicle types, circulating speed, driver's age and gender, and maneuver type (Kaysi and Abbany, 2007). The critical headways of a driving population cannot be observed directly and have to be estimated from a vast set of observations using methods such as binary logit regression and maximum likelihood estimation (Brilon et al., 1999). These observations are usually collected in the field but vehicle simulators have also been used (Rossi et al., 2011; Yan et al., 2007).

The estimation of critical headways is therefore a laborious and expensive task. Studies of this nature tend to be commissioned by transport authorities to establish national or regional reference values (Rodegerdts et al., 2007; Weinert, 2000), which are then published in design guidelines such as the Highway Capacity Manual (TRB, 2010) and used by traffic engineers in practical applications, without further adjustments. This approach naturally requires the subject intersection to have standard traffic and geometry conditions, so it is not recommended, for example, to use the gap-acceptance parameters of a stop controlled intersection for a roundabout, or even to use parameters from a conventional roundabout for a turbo-roundabout. Thus, the analysis of special layouts implies either collecting site-specific gap-acceptance data, or dispensing the classic gap-acceptance theory and relying on microscopic simulation using commercial models.

The current paper describes an alternative approach – a gap-acceptance model that can capture the complex interactions between the driver/vehicle dynamics and the intersection geometry, but simple enough to be easily implemented in a spreadsheet by non-expert users. The model describes the process of gap-acceptance at the individual vehicle level, based on vehicle-driver characteristics and on the geometric characteristics of the maneuver.

THE GAP-ACCEPTANCE MODEL

The proposed gap-acceptance model is based on the evaluation of the interactions between the entry (minor) and opposing (major) vehicles, described at microscopic level. The model takes into account the geometric characteristics of the maneuvers (distance to the conflict point,

curvature of the trajectory) and vehicle-driver characteristics (acceleration profile, reaction time, vehicle width and length). The positions on the trajectories are calculated using the Gipps car-following equations (Gipps, 1981), but other formulations could be used.

Every entry maneuver may be classified as a crossing, a merge, or a combination of crossing/merge movements. For example, a vehicle entering a double-lane roundabout using the left approach lane has to cross the outer circulatory lane and merge into the inner lane. For each of these elementary maneuvers the model returns an individual critical headway that can be directly used in maneuver-specific capacity formulas such as Tanner's generic formula (Hagring et al., 2003; Vasconcelos et al., 2011).

Crossing Maneuvers

Consider the crossing maneuver represented in Figure 1. It is assumed that a minor driver *m* decides to move into the intersection if the time i between a pair of successive opposing vehicles is sufficient to enter the conflict area after the passage of the leader vehicle M_1 (type I conflict) and clear that area before the arrival of the follower vehicle M_2 (type II conflict). This condition returns a minimum interval to which safety margins should be added.

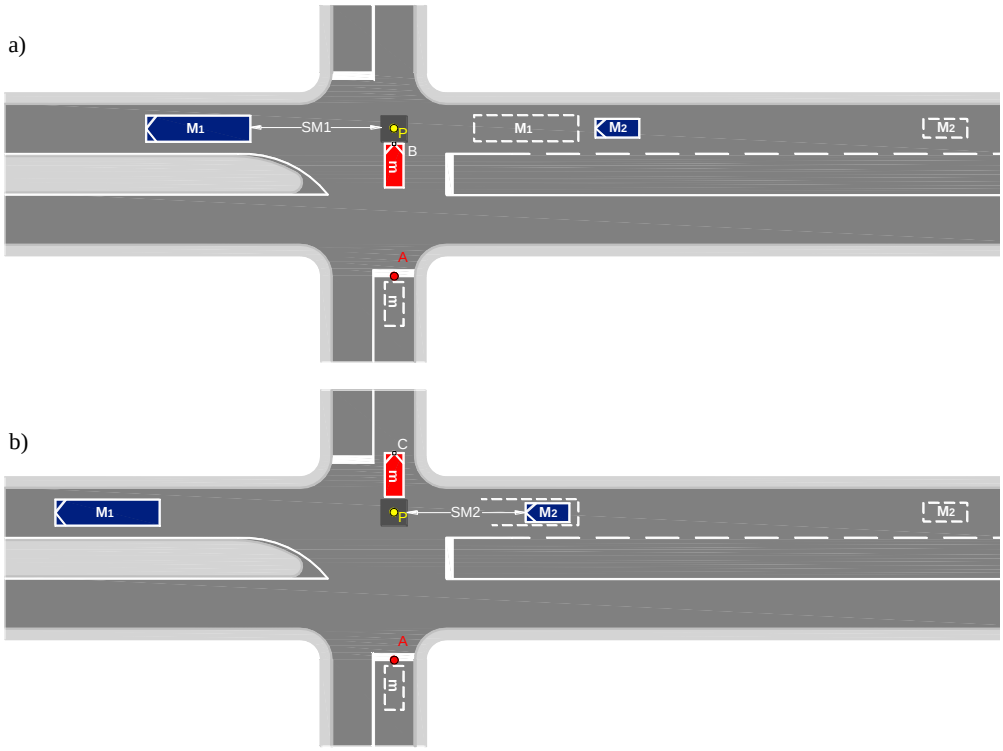

Figure 1: Crossing maneuver: a) Type I conflict (with leader); b) Type II conflict (with follower)

The following procedure sequence returns the critical headway for a crossing maneuver:

1. Find the point P corresponding to the intersection of the trajectories;

2. Find the distance that the entry vehicle *m*, of length l_m, must travel to reach and clear the conflict area. The length of this area in the minor (*m*) and major (*M*) directions is, respectively, $w_M / \sin\alpha$ and $w_m / \sin\alpha$, where *w* is the width of each vehicle and α is the angle between trajectories). In most cases the vehicles can be considered to have the same width *w*. This gives the following distances:

$$d_{A-B} = d_{A-P} - w/\sin\alpha \quad (1)$$

$$d_{A-C} = d_{A-P} + w/\sin\alpha + l_m \quad (2)$$

3. Find the time required by the entry vehicle, initially stopped at point A, to reach and clear the conflict zone (points *B* and *C*). Gipps' equation for free-flow conditions is applied here:

$$v_n(t+\Delta t) = v_n(t) + 2.5\, a_n^+ \Delta t \left(1 - \frac{v_n(t)}{v_n^d}\right)\sqrt{0.025 + \frac{v_n(t)}{v_n^d}} \quad (3)$$

where, for a vehicle *n*,

$v_n(t)$: speed at instant *t*

Δt: time increment

v_n^d: target or desired speed

a_n^+: maximum acceleration.

The position x_n at time $t + \Delta t$ is given by the kinematic equation:

$$x_n(t+\Delta t) = x_n(t) + v_n(t+\Delta t)\,\Delta t \quad (4)$$

Using these two equations it is straightforward to define a function that, given the initial conditions ($t = 0$, $v = 0$, $x = 0$), returns the time used ($t_{A\text{-}B}$ and $t_{A\text{-}C}$) and the end speeds (v_A and v_C);

4. Considering the conflict with the leading vehicle (Type I), the entry driver starts its movement in order to reach the conflict area at the moment the leader vehicle clears it. If the maneuver is undertaken under the safety margin SM_1, at the start of the maneuver the maximum acceptable upstream distance between the leader vehicle and the conflict point is given by

$$d_{M1-P} = v_{M1}\left(t_{A-B} - SM_1\right) - l_{M1} - \frac{w}{2\sin\alpha} \quad (5)$$

where v_{M1} and l_{M1} are the speed and the length of the leader vehicle;

5. Considering the conflict with the follower vehicle, the entry vehicle must have the conflict area cleared the moment the follower vehicle reaches it. If this maneuver is undertaken with safety margin SM_2 (also known as post-encroachment time), at the start of the maneuver the minimum acceptable upstream distance between the follower vehicle and the conflict point is given by

$$d_{M2-P} = v_{M2}\left(t_{A-C} + SM_2\right) + \frac{w}{2\sin\alpha} \tag{6}$$

where v_{M2} is the speed of the follower vehicle;

6. Assuming that the speed in the opposing stream is uniform, then $v_{M1} = v_{M2} = v_M$ and the critical headway h is given by Eq. (7). This headway corresponds simply to the total time required for both the entry and leader vehicles move through and clear the conflict area (added by the safety margins).

$$h = \frac{d_{M2-P} - d_{M1-P}}{v_M} \tag{7}$$

Merge Maneuvers

Consider the merge maneuver represented in Figure 2. It is assumed that the minor vehicle m accepts the headways between successive major vehicles M_1 and M_2 if, at the moment it reaches the conflict area (point B), two conditions are met: i) the entry vehicle m is able to safely react to emergency braking by the leader vehicle M_1; ii) the follower vehicle M_2 is able to safely react to emergency braking by the entry vehicle m.

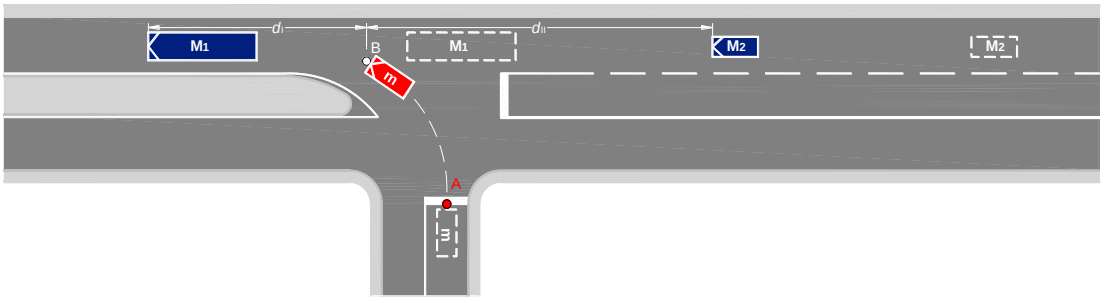

Figure 2: Merge maneuver

The problem is essentially a matter of finding the safety distances for two pairs of vehicles sharing the same lane, according to car-following logic. The following procedure sequence returns the critical headway for a merge maneuver:

1. Find the point B that marks the entry of the minor vehicle into the conflict area and compute its speed at this point (v_{mB}) using Gipps' equation for free-flow conditions;

2. Find the minimum space headway, downstream B, to the leader vehicle M_1. For this calculation it is assumed that:

   i. at the precise moment the entry vehicle *m* reaches point B, the leader vehicle M_1 applies emergency braking, decelerating from the initial speed v_{M1} until a complete stop, using the constant deceleration rate $a^{-}_{\max,M_1}$; the distance travelled by the leader during the deceleration is

$$d_{M1} = \frac{v_{M_1}^2}{2a^{-}_{\max,M_1}} \tag{8}$$

   ii. the deceleration of the entry vehicle in response to this action occurs only after the perception-reaction time. To maintain consistency with Gipps' model it is assumed that this interval is 1.5*T*, where *T* is a model parameter designated as *reaction time*. Therefore, the distance travelled by the entry vehicle, downstream B, is

$$d_m = \frac{v_{mB}^2}{2a^{-}_{\max,m}} + 1.5T\, v_{mB} \tag{9}$$

   Considering, for simplicity's sake, that both vehicles decelerate at the same rate $a^{-}_{\max}$, and assuming that the entry vehicle will require a minimum distance *d* to the leader vehicle (between bumpers), the minimum space headway between entry (*m*) and leader (M_1) vehicles at the moment *m* reaches point B is given by

$$d_I = \max\begin{cases} \dfrac{v_{mB}^2}{a^{-}_{\max}} + 1.5T\, v_{mB} + l_{M1} + d - \dfrac{v_{M1}^2}{a^{-}_{\max}} \\ l_{M1} + d \end{cases} \tag{10}$$

3. The minimum space headway, upstream B, to the follower vehicle M_2 is given by an equivalent formulation:

$$d_{II} = \max\begin{cases} \dfrac{v_{M2}^2}{a^{-}_{\max}} + 1.5T\, v_{M2} + l_m + d - \dfrac{v_{mB}^2}{a^{-}_{\max}} \\ l_m + d \end{cases} \tag{11}$$

   Again, considering that major stream vehicles M_1 and M_2 are travelling at the same speed, the critical headway is:

$$h = \left(d_I + d_{II}\right) / v_M \tag{12}$$

APPLICATION: SINGLE-LANE ROUNDABOUT

This section describes the application of the model to estimate critical headways at roundabout entries. The first part describes the development of a video analysis tool and the estimation of essential parameters; the second part addresses the application of the model, based on calibrated parameters, and its validation against conventional observation methods.

The LUT-VP3 Video Analysis Tool

The model is sensitive to a large number of parameters. Some can be directly measured or easily estimated (distances to the conflict points, vehicles length, average speed of the opposing traffic), but others require specific calibration procedures. For instance, estimating the Gipps parameters requires information about the actual trajectories and conflicts at the roundabout entries. Video analysis software (LUT-VP3) that allows tracing and analyzing vehicle trajectories from video recordings was developed for this purpose. This tool was built on VB.NET. It is essentially a video player application that gives the user full motion control, to which was added the possibility of placing time tagged points at specific locations on the screen via mouse input. For example, the user can pick successive points on the screen representing the head of the vehicle to obtain the trajectory (and, by derivation, the speed profile), or compare the time codes of two markers in similar positions, representing different vehicles, to measure the post-encroachment time. It is also possible to record specific gap-acceptance data using the keyboard and saving, for example, the instants a vehicle arrives at and crosses the give way line. The data is recorded to a text file and can be directly pasted into other applications via the clipboard.

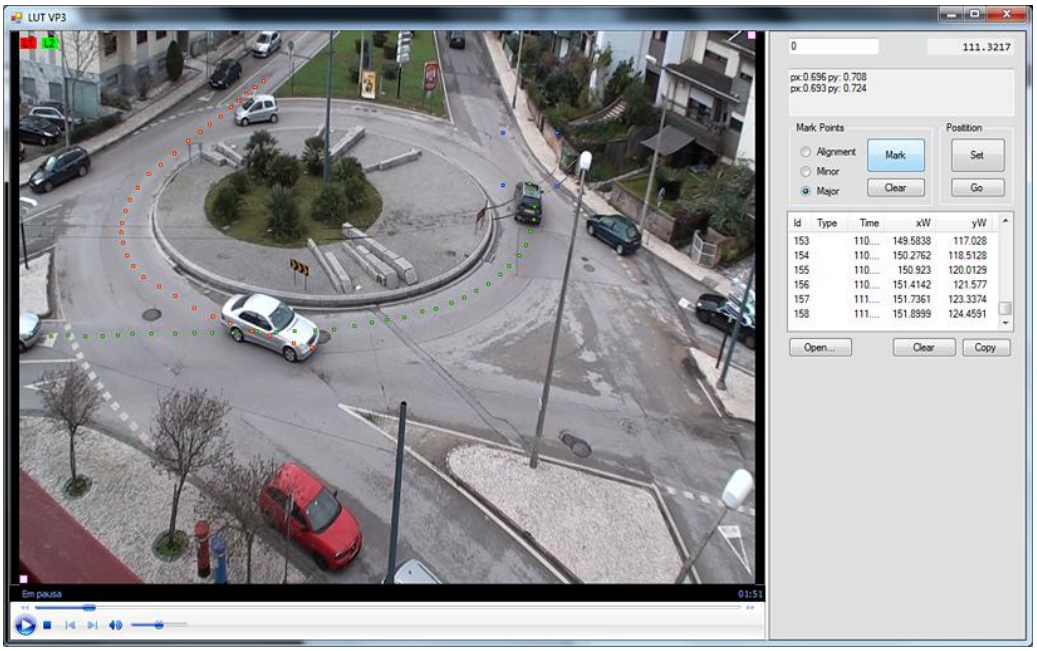

Figure 3: The LUT-VP3 Video analysis tool

The main difficulty of this task was to establish a relationship between the screen and world coordinates so as to correct the perspective distortion. This relationship is given by a transformation matrix, known as homography. It is based on four pairs of homolog points whose

world coordinates are known (usually manholes, light poles, or kerb points). The homography was estimated using the well-known 2D Homography – Projective Transformation method (Hartley and Zisserman, 2003), which is fully described in the appendix.

Model Calibration

A roundabout in Coimbra, Portugal, was chosed for this first calibration. It is located at the intersection of Rua Miguel Torga and Rua Carolina Michaelis (hereafter designated as *Solum Rbt.* (see Figure 4). The main reasons for choosing this location were: a) the video camera could be installed in an adjacent tall building, b) high traffic levels are high at peak periods, and c) traffic operations are relatively simple (one circulating lane). This roundabout has an inscribed circle diameter (ICD) of 35 m and one circulatory lane 6.0 m wide. One of the entries is local traffic only and was excluded from the analysis, as was the right turn from the entry E1 (very low traffic level) and the right lane movements from entry E3 (very low opposing traffic level).

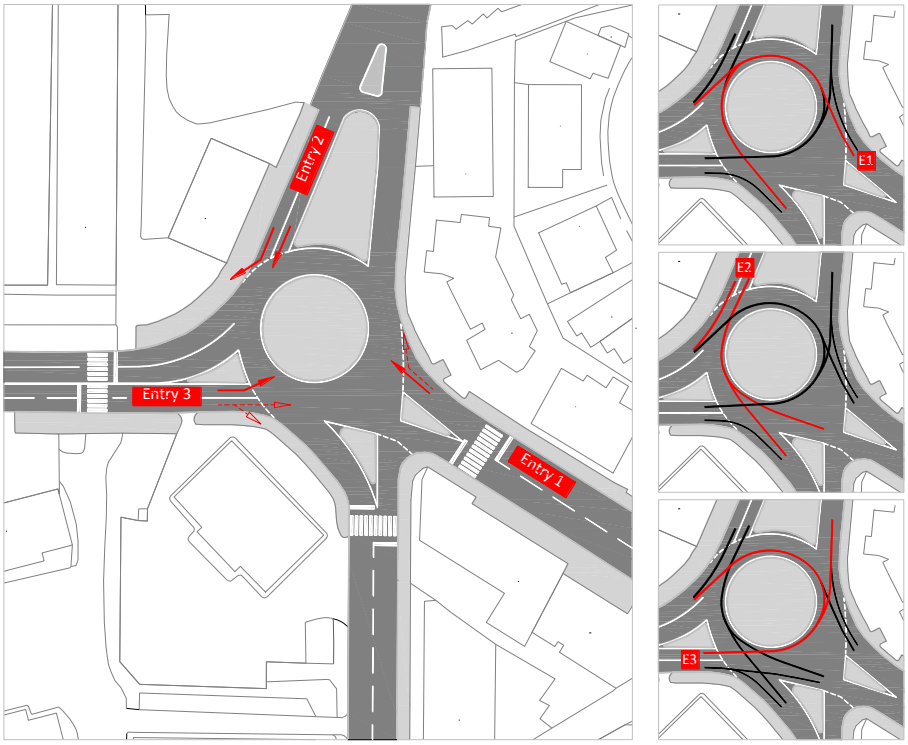

Figure 4: Selected movements for calibration and validation

The procedure allowed the calibration of three main parameters: the maximum acceleration and the desired speed of Gipps' acceleration model, and the safety margin associated with Type I conflicts in crossing maneuvers (SM_1).

Maximum Acceleration and Desired Speed

These parameters describe the acceleration stage of a vehicle and are used in the gap-acceptance model to predict the time a minor vehicle will take to reach and clear a conflict area. The calibration of the acceleration model was based on a set of vehicle trajectories that stopped (or almost stopped) near the give way line (see Figure 5).

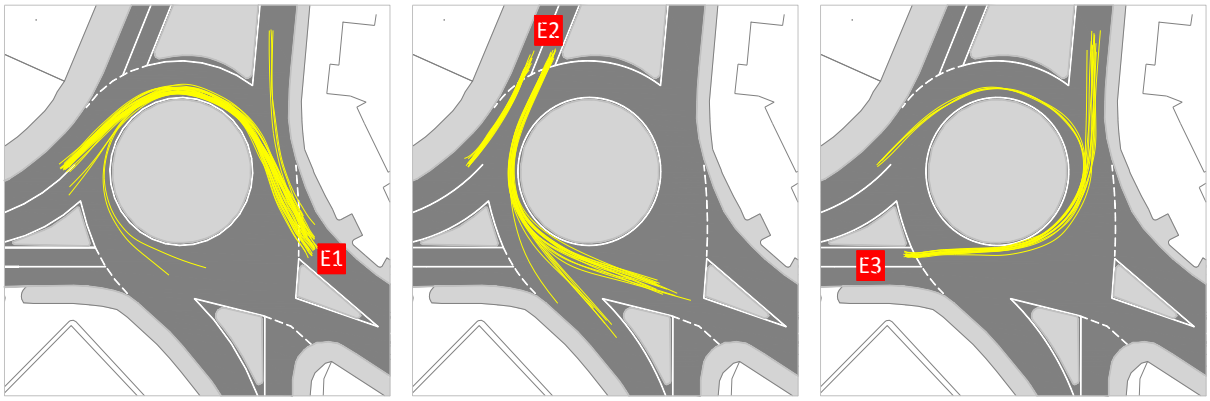

Figure 5: Trajectories used to calibrate the acceleration model

The combination of parameters that minimized the difference between the observed and predicted trajectory plots and followed the speed plot was chosen for each vehicle. This process is illustrated in Figure 6 and Figure 7 for one specific vehicle making a U-turn from entry E3. Between points 1 and 2 the subject vehicle was waiting for a suitable gap in the major stream, and very slowly moving towards the roundabout entry. The main acceleration stage takes place in a relatively short space between points 2 and 3 and is followed by a stable speed stage corresponding to the minimum radius of the trajectory up to point 4. The parameters that best described the main acceleration stage of this specific vehicle were v_d = 6.4 m/s (23.0 km/h) and a^+ = 2.5 m/s^2.

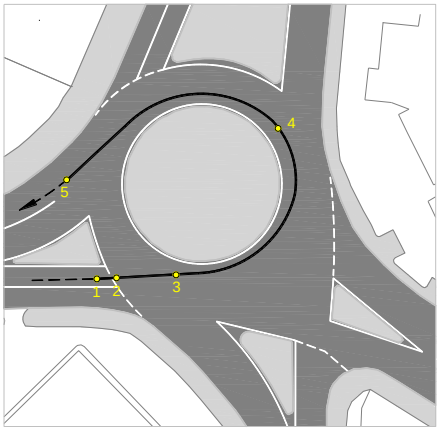

Figure 6: Sample trajectory (U-turn from entry E3)

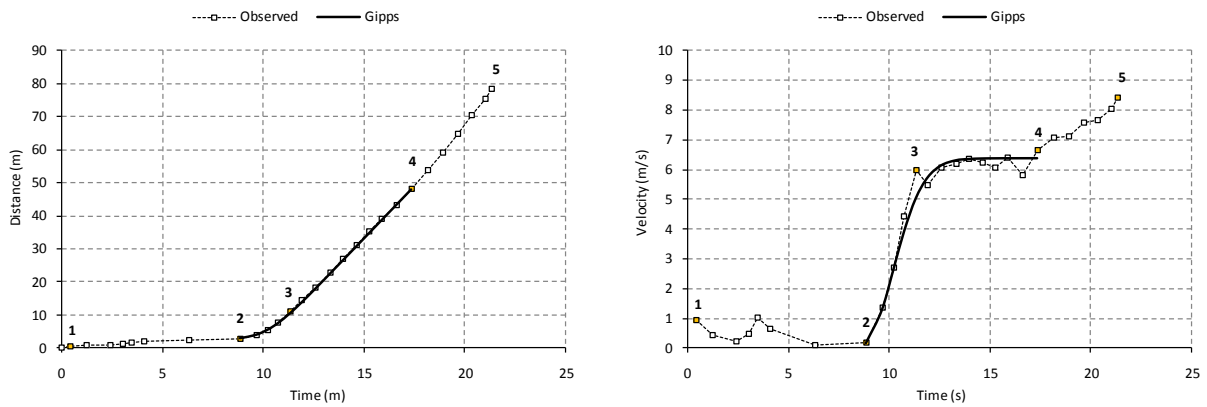

Figure 7: Distance and speed plots for a specific vehicle

The results of this calibration are listed in. The average desired speed is 6.17 m/s (22.2 km/h) in the circulating movements (entries E3-L, E1 and E2-L) and slightly higher - 6.84 m/s (24.6 km/h) in the right-turn movement (entry E2-R). The maximum acceleration rates are fairly similar for the different entries: 2.35 m/s^2 in the circulating movements and 2.13 m/s^2 in the right-turn.

Table 1: Gipps' parameters (vd and a^+) of the tracked vehicles

Mov.	Sample size	v_d (m/s)				a^+ (m/s^2)			
		Average	Standard Dev.	Min	Max	Average	Standard Dev.	Min	Max
E1	21	5.67	0.62	5.0	7.0	1.99	0.45	1.2	3.2
E2-L	16	5.89	0.75	4.5	7.6	2.38	0.68	1.0	3.5
E2-R	18	6.84	1.12	5.0	9.0	2.13	0.40	1.3	2.8
E3-L	27	6.74	1.04	4.8	8.8	2.61	0.75	1.5	4.0

Safety Margin (Type I, SM_1)

This parameter was measured at the crossing locations (E1, E2-L and E3-L), and corresponds to the time difference between the rear-right corner of the major vehicle and the front-left corner of the minor vehicle passing a common spatial zone (see Figure 8). In other locations and under different speed patterns these impact points may differ (Laureshyn et al., 2010). In accordance with the objectives of this study, SM_1 was measured only for minor vehicles that stopped completely at the give way line.

The results are summarized in Table 2. Disregarding some outliers, the SM_1 values are remarkably similar, which suggests that a constant, average value may be used to represent this parameter. However, it should be noted that the observed maneuvers are based solely on accepted gaps, which means that the average SM_1 values are actually top estimates of the real critical values required by the headway model.

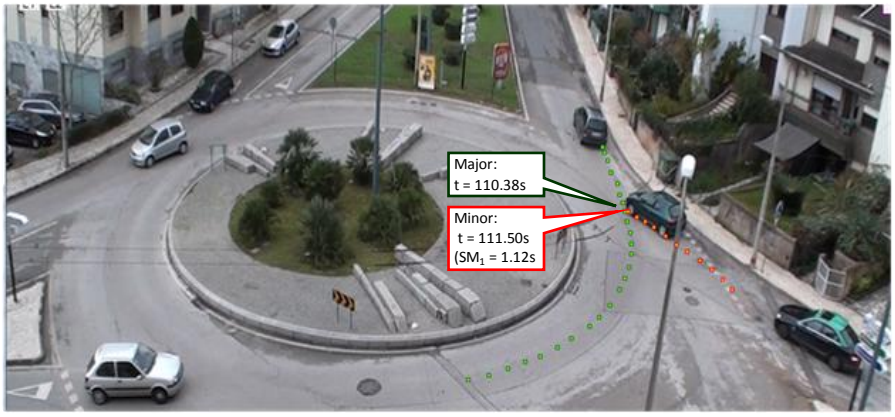

Figure 8: Measurement of the safety margins (Type I, SM_1)

Table 2: Safety margins (Type I)

Entry	Sample size	SM_1 (s)			
		Average	Standard Dev.	Min	Max
E1	40	1.26	0.34	0.5	2.0
E2-L	40	1.15	0.36	0.6	1.9
E3-L	25	1.25	0.32	0.4	2.1

Estimation Results

The model was validated taking the estimates produced by conventional observation models as reference. This was done at each of the *Solum Rbt.* entries (E1, E2-L, E2-R and E3) and, in order to provide independent validation results, at two other roundabouts. The first (*Rainha Santa Rbt.*) has a very large diameter (minimum ICD = 89 m) and a circulatory carriageway of 8.1 m, wide enough for two vehicles side-by-side, but used mostly by single vehicles. Almost all entry maneuvers are crossings. The second location (*Choupal Rbt.*), is a single lane, medium sized roundabout (ICD = 57 m). The first exit is local traffic only, so almost all conflicts are of the merge type.

The parameters were: a) at each roundabout, the opposing traffic speed v_M was observed directly, using the moving observer method (15 laps); b) considering that, at *Solum Rbt.*, the observed opposing traffic speed was very close to the Gipps parameter *desired speed* (v_M = 6.11 m/s, v_d = 6.17 m/s), the equality $v_d = v_M$ was assumed at the independent locations; c) at *Solum Rbt.*, the parameters a^+ and SM_1 were set at the respective values found in the calibration process; at the other locations the average calibration values were adopted; d) the estimation of SM_2 raised several difficulties: as with the critical headway, it can be regarded as a random variable whose critical value cannot be measured directly. Furthermore, the traditional estimation approach, based on the distributions of accepted and rejected margins, cannot be followed because the rejected margins are not observable. A simplified estimation procedure based solely on the distribution of accepted margins (as was done with SM_1) is also impracticable, mainly for two reasons: i) this distribution is highly dependent on the traffic flow levels; ii) while the proposed model takes the initial safety margin as input (at the beginning of the maneuver, assuming constant speed in the opposing stream and average acceleration rates), the observations only

reveal the actual margins when the priority vehicles reach the conflict area; while the initial margins can take very low values (even less than zero), the actual margins always take positive values - under forced entries, major vehicles tend to decelerate to extend the gap ahead and minor vehicles increase their acceleration rates. Therefore, instead of an explicit estimation procedure, an alternative approach was followed: noting that the headway model only depends on the total safety margin (SM_1 + SM_2), not on its individual values, an initial null value was arbitrarily assigned to SM_2 and the feasibly of that choice was checked afterwards by comparing the model estimates with reference values, produced by conventional methods based on observations (such as the maximum likelihood method). After some attempts, it was found that simply assigning a null value to SM_2 provided a very good fit. This is a surprising result which is supported by the abovementioned driver's adaptive behavior. Finally, typical values used in microscopic simulation models (Barceló et al., 2005; Bonsall et al., 2005) were chosen for the parameters of the merge model - reaction time T: 0.9 s, emergency deceleration rate a_{max}^{-}: 6 m/s^2, minimum gap between stopped vehicles d: 1.0 m.

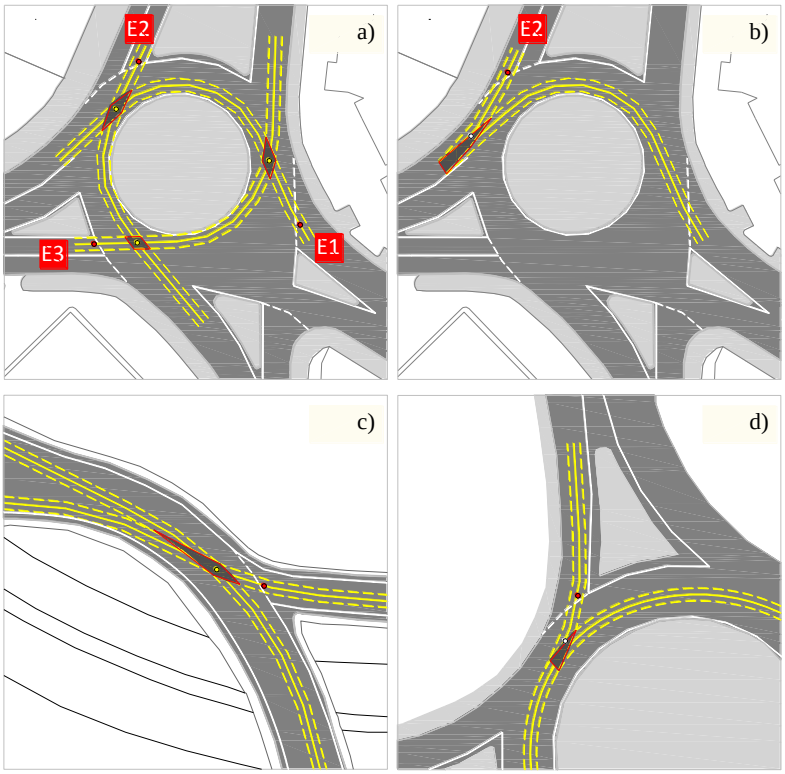

Figure 9: Estimation locations – a) Solum Rbt.: E1, E2-L, E3-L (crossing), b) Solum Rbt.: E2-R, (merge), c) Rainha Santa Rbt.; East (crossing), d) Choupal Rbt.: North (merge)

The main parameters and the resulting critical headways are listed in Table 3 and Table 4. The highest value for the crossing movements is obtained at entry E2-L of the *Solum Rbt.* Two factors contribute to this: a) the short distance from the give way line to the conflict point limits the end speed of the minor vehicle; b) the small angle between trajectories increases the extent of the conflict area and, consequently, the time needed by the minor and major vehicles to clear it. In relation to the merge movements, the higher speed of the opposing traffic at the *Choupal Rbt.*

has two effects: first, it increases the stopping distances; second, it reduces the time required to clear the conflict area. The combined effect is to give a slightly smaller critical headway than at the *Solum Rbt.* These results agree with findings from previous studies (Alexander et al., 2002; Davis and Swenson, 2004; Yan et al., 2007) in which is concluded that drivers accept lower gaps in higher major-road traffic speed scenarios.

Table 3: Estimation results – crossing movements

	Solum Rbt.			**Rainha Santa Rbt.**
Entry	**E1**	**E2-L**	**E3-L**	**East**
a^+ (m/s2)	1.99	2.38	2.61	2.20
v_d (m/s)	5.67	5.89	6.74	11.10
v_M (m/s)	6.11	6.11	6.11	11.10
l_m, l_M (m)	4.2	4.2	4.2	4.2
w_m, w_M (m)	2.0	2.0	2.0	2.0
α (º)	37.0	27.5	50.0	19.0
d_{AP} (m)	12.4	8.1	8.1	10.6
$SM_1 + SM_2$	1.26	1.15	1.25	1.25
d_{AB} (m)	10.7	5.9	6.8	7.6
d_{AC} (m)	18.2	14.4	13.6	17.9
t_{AB} (s)	3.71	2.47	2.52	2.98
t_{AC} (s)	5.12	4.09	3.69	4.47
$d_{M1\text{-}P}$ (m)	9.14	1.75	2.29	11.95
$d_{M2\text{-}P}$ (m)	33.0	27.2	23.9	52.7
h_s (m)	23.8	25.4	21.6	40.7
***h* (s)**	**3.90**	**4.16**	**3.53**	**3.67**

Table 4: Estimation results – merge movements

	Rbt. Solum	***Rbt. Choupal***
Entry	**E2-R**	**North**
a^+ (m/s^2)	2.13	2.20
a^-max (m/s^2)	6.00	6.00
v_d (m/s)	6.84	7.90
v_M (m/s)	7.00	7.90
T (s)	0.9	0.9
l_m, l_M (m)	4.2	4.2
w_m, w_M (m)	4.2	4.2
d (m)	1	1
d_{AB} (m)	7.8	9.0
t_{AB} (s)	3.00	3.17
v_{mB} (m/s)	5.16	5.73
d_I (m)	10.3	10.5
d_{II} (m)	16.5	18.3
h_s (m)	26.8	28.8
***h* (s)**	**3.83**	**3.65**

Validation

Several classic methods, all based on observations, were used to estimate the critical headways at the selected locations (with the exception of *Solum Rbt*, entry E3-L, due to insufficient data), so as to validate the model:

- Raff's method (Raff and Hart, 1950) is probably the best known and easiest to use. According to this model, the critical headway is the value of t at which the functions $F_a(t)$ and 1- $F_r(t)$ intersect, where $F_a(t)$ and $F_r(t)$ are, respectively, the cumulative distribution functions of the accepted and rejected headways;
- Logit methods (e.g. Polus et al (2005)) formulate the headway acceptance problem as a binary choice model, where the deterministic utility of each alternative response is at least dependent on the headway in the opposing stream, but can also be made dependent on other attributes such as the waiting time, destination lane, etc.
- Maximum likelihood method (Tian et al., 1999), which is usually recognized as highly reliable (Brilon et al., 1999) and is used by major transportation agencies to develop design guidelines (Rodegerdts et al., 2007; Weinert, 2000). The method is based on the assumption that all drivers are consistent. It requires recording, for each driver, the maximum rejected headway (r_d) and the accepted headway (a_d) and to specify the general form of the distribution of the critical headways for the population of drivers (usually a log-likelihood distribution). An iterative procedure will then find the parameters of that distribution that maximize the likelihood of each driver's critical headway being between r_d and a_d;
- In addition, the critical headways were estimated by a new method (Wu, 2006), based on the probability equilibrium between the rejected and accepted headways. The author points out several desired properties (solid theoretical background, independent of distribution assumptions, simple calculation, etc.) which make it a promising alternative to the maximum likelihood method.

The results are not fully comparable because the sample varies from method to method. In order to maximize compatibility with the maximum likelihood method, in the other methods the lags (direct entries) were excluded from the sample and only the maximum rejected gap of each driver was considered (i.e. driver consistency was assumed). The results are presented in Table 5.

Table 5: Validation results

	Crossing movements			**Merge movements**	
Roundabout	**Solum**	**Solum**	**Rainha Santa**	**Solum**	**Choupal**
Entry	**E1**	**E2-L**	**East**	**E2-R**	**North**
Sample size (number of decisions)	235	83	542	155	177
Raff	3.59	4.28	3.60	3.51	3.90
Maximum likelihood	3.79	4.50	3.73	3.86	4.28
Logit	3.23	4.05	3.55	3.48	3.74
Wu	3.77	4.35	3.62	3.62	3.75
Proposed model	3.90	4.16	3.67	3.83	3.65

The estimates lie within the variability associated with conventional methods, even at the independent locations.The model performed worse when comparing with the Logit method (maximum error: 0.67 s, at Solum Rbt., entry E1) and best when comparing with Wu's method (maximum error: 0.21 s, at Solum Rbt, entry E2-R). No relevant differences were found between the errors at the calibration and validation locations, which is an indication of good transferability properties.

CONCLUSIONS

The critical-headway model presented in this paper describes the gap-acceptance process at microscopic level. The model is simple to implement in a spreadsheet and explicitly takes the intersection geometry and the vehicle/driver characteristics in the driver's decision process into account. The model was calibrated based on a data sample collected at a one-lane urban roundabout and validated against conventional methods, based on field observations. The results were promising, though it should be noted that the validation process was based on a very small number of locations and observations. The parameter values adopted in this application will very likely be found to be inappropriate under different traffic levels and geometric layouts, especially in relation to the driver's adaptive behavior. With respect to this task, it will be particularly interesting to confirm if the hypotheses $SM_2 \approx 0$ holds for other maneuvers. Future research will therefore focus on the development of simple sub-models for these parameters, based on measurable variables such as the relative speed at the conflict point and the potential impact angle, in order to provide accurate and comparable results for different users. Moreover, future validation tests should include different geometric layouts, such as two-lane roundabouts and two-way stop controlled intersections. But even if the absolute estimates are not sufficiently accurate, the model's explanatory power allows comparative analyses between hypothetical scenarios, which helps, for example, to indentify the effect of the distance between the give way line and the conflict point, the angle between trajectories, opposing vehicle speed and vehicle length. As it stands, therefore, the model provides an evaluation framework that can supplement traditional field or simulator identification techniques.

APPENDIX

The well-known estimation procedure based on the 2D Homography – Projective Transformation (Hartley and Zisserman, 2003) was used to establish a relationship between the screen (video) coordinate P and the world coordinate Q,

$$Q = H\ P \tag{13}$$

where $P = \{X \quad Y \quad Z\}^{\mathrm{T}}$, $Q = \{U \quad V \quad W\}^{\mathrm{T}}$ and H is a nonsingular 3×3 dimension matrix known as a homography matrix, i.e., $H = \{H_1^{\mathrm{T}} \quad H_2^{\mathrm{T}} \quad H_3^{\mathrm{T}}\}$, with $H_i = \{h_{i1} \quad h_{i2} \quad h_{i3}\}$, $i = 1 \text{ to } 3$.

The direct linear transformation (DLT) algorithm was used, with four exactly 2D to 2D correspondences $P_i \rightleftarrows Q_i$ ($i = 1 \text{ to } 4$), resulting in the "minimal" solution. The DLT algorithm was computed by omitting the third equation of the following system for all correspondence points *i*,

$$\begin{bmatrix} \mathbf{0}^{\mathrm{T}} & -W_i P_i^{\mathrm{T}} & V_i P_i^{\mathrm{T}} \\ W_i P_i^{\mathrm{T}} & \mathbf{0}^{\mathrm{T}} & -U_i P_i^{\mathrm{T}} \\ -V_i P_i^{\mathrm{T}} & U_i P_i^{\mathrm{T}} & \mathbf{0}^{\mathrm{T}} \end{bmatrix} h = 0 \tag{14}$$

where $\mathbf{0}^{\mathrm{T}} = \{0 \quad 0 \quad 0\}$ and $h^{\mathrm{T}} = \{H_1 \quad H_2 \quad H_3\}$, resulting in a homogeneous system of dimension (8×9). To solve the system, the coordinates Z_i and W_i take a unitary value (only the coordinates of the images are relevant) and the unknown h_{23} takes the value 1.0 at the beginning of the calculation process, i.e. the set of equations (14) is turned into an inhomogeneous set of linear equations. The solution of h_{ij} can be determined up to a non-zero scale factor. Thus the two lines for each input point correspondences are

$$\begin{bmatrix} 0 & 0 & 0 & -X_i & -Y_i & V_i X_i & V_i Y_i & V_i \\ X_i & Y_i & 1 & 0 & 0 & -U_i X_i & -U_i Y_i & -U_i \end{bmatrix} h' = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \tag{15}$$

where the transpose of the vector h' is $\{h_{11} \quad h_{12} \quad h_{13} \quad h_{21} \quad h_{22} \quad h_{31} \quad h_{32} \quad h_{33}\}$. After finding the solution of the inhomogeneous system, a non-zero scale factor α is determined by imposing the condition in which the norm of the vector h, given by $\{h'_1 \quad h'_2 \quad h'_3 \quad h'_4 \quad h'_5 \quad 1.0 \quad h'_6 \quad h'_7 \quad h'_8\}$ equals the unitary value:

$$\alpha = \left(1.0 + \sum_{i=1}^{8} h_i'^2 \right)^{-\frac{1}{2}} \tag{16}$$

The estimation of the homography matrix yields

$$H = \alpha \begin{bmatrix} h'_1 & h'_2 & h'_3 \\ h'_4 & h'_5 & 1.0 \\ h'_6 & h'_7 & h'_8 \end{bmatrix} \tag{17}$$

In accordance with the definition of the projective transformation, the coordinates of any world point Q, given any screen point P, are given by

$$\begin{Bmatrix} U \\ V \end{Bmatrix} = \beta^{-1} \begin{Bmatrix} H_1 P \\ H_2 P \end{Bmatrix} \tag{18}$$

where H is obtained through Eq. (17) and β is equal to $H_3 P$.

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