

Dynamic Origin-Destination Demand Flow Estimation Utilizing Heterogeneous data sources under Congested Traffic Conditions

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Overview of the proposed approach

Time-dependent origin-destination (OD) demand matrices are fundamental inputs for dynamic traffic assignment (DTA) models to describe network flow evolution as a result of interactions of individual travelers. In the past decades, a rich body of literature, to be presented as follows, has been devoted to the methods of estimating static or time-dependent OD demand tables (e.g., [Yang et al., 1992](#); [Florian and Chen, 1995](#); [Cascetta et al., 1993](#); [Tavana, 2001](#); [Zhou et al., 2003](#); [Zhou and Mahmassani, 2006](#); [Yang, 1995](#); [Balakrishna et al., 2008](#); [Cipriani et al., 2011](#); [Bell et al., 1997](#); [Sherali and Park, 2011](#); [Nie and Zhang, 2008](#); [Nie and Zhang, 2010](#); [Qian and Zhang, 2011](#); [Shen and Wynter, 2011](#)). However, time-dependent OD demand estimation, particularly under congested conditions, remains a critical and challenging problem that is attracting significant attention from transportation researchers to develop theoretically sound and practically deployable approaches.

The contributions of this study to the growing body of literature on dynamic OD demand estimation are as follows.

- Instead of working on the commonly-used OD flow variables, this study presents a new path flow-based optimization model for jointly solving the complex OD demand estimation and UE DTA problems. Specifically, this model simultaneously minimizes (i) the deviation between measured and estimated traffic states, and (ii) the deviation between aggregated path flows and target OD flows, subject to a dynamic user equilibrium (DUE) constraint, which is reformulated using an equivalent gap function. Working in this path-flow dimension, our formulation can

directly aggregate estimated path flows to obtain final OD flow patterns, and obviate explicit dynamic link-path incidences, as opposed to the majority of previous studies.

- By dualizing the difficult DUE constraint into the objective function, this research proposes an effective Lagrangian relaxation-based solution framework. The relaxed problem can be viewed as a simultaneous route and departure time user equilibrium (SRDUE) problem with elastic demand, and the final solution is a set of path flow patterns satisfying “tolled user equilibrium” (Lawphongpanich and Hearn, 2004), where the deviation with respect to traffic measurements can be viewed as an additional penalty for over-estimated or under-estimated path flows. By incorporating heterogeneous real-world measurements in the objective function, such as link densities from video surveillance and road side detectors and link travel times from Bluetooth readings, the proposed estimation model fully utilizes available information to reflect route choices in a congestion network.

- A dynamic network loading (DNL) model that encapsulates Newell’s simplified KW model in a mesoscopic traffic flow simulation framework is proposed to describe congestion phenomena, such as queues formation, spillback, and dissipation. Explicitly using the cumulative arrival and departure curves, Newell’s traffic flow model provides a rigorous mathematical formulation to realistically represent traffic dynamics and capture the impact of shock waves on various macroscopic traffic measures. Compared to standard partial differential equations (PDE)-based DNL models that subdivide a long link into segments with a shorter length, Newell’s model can handle reasonably long links with homogeneous capacity, and its simple form and computational efficiency make it appealing in developing dynamic OD estimation algorithms.

- Based on the proposed DNL model, this research derives analytical, local gradients of different measurement types, such as link flow, density and travel time, with respect to path flows. This valuable gradient information not only considers the dependences of link flow/density/travel time changes on the OD/path flow, but also allows for computing feasible descent directions in an efficient gradient-projection-based method embedded in the Lagrangian relaxation-based solution framework.

Single-level path flow estimation framework

Given sensor data (i.e. observed link flows, densities, and travel times) and target (aggregated historical) OD demand, the proposed single-level path flow estimation model is a nonlinear optimization model with path flows as decision variables. Final OD demand estimates

can be constructed by summing up path flows for each OD pair. To construct a tractable single-level nonlinear optimization model, we first consider the DUE gap function as a side constraint (Lu et al., 2009), and dualize this constraint to the GLS-based objective function with a Lagrange multiplier. The resulting Lagrangian relaxation model is solved by a column-generation-based algorithmic framework consisting of a gradient-projection-based descent direction algorithm for updating path flows, a mesoscopic DNL model for evaluating link and network performances, and a time-dependent least time path (TDLTP) algorithm for generating paths.

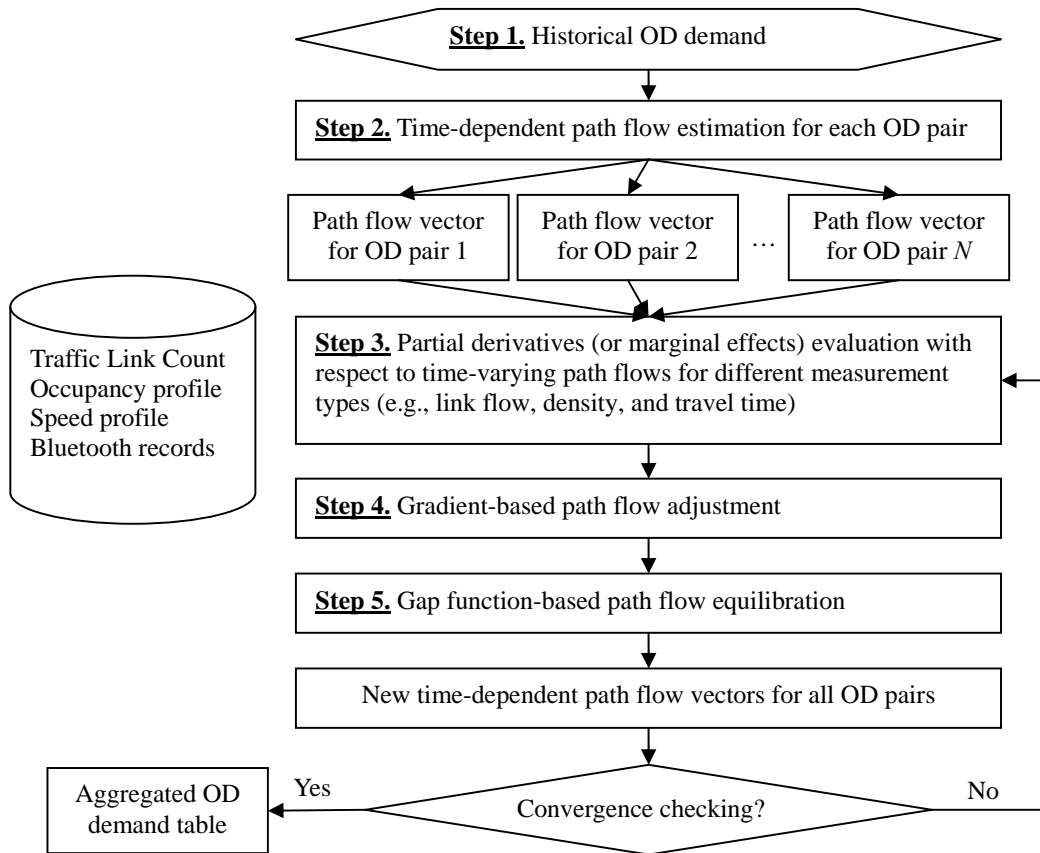


Fig. 1 Flowchart of the time-dependent path flow estimation algorithm

The algorithmic steps of solving the Lagrangian relaxation reformulation are presented in Fig. 1. The basic idea is to iteratively solve a Lagrangian lower bound problem using the gradient-projection-based descent direction method, and update the Lagrange multiplier using a subgradient method, until reaching an optimal path flow vector that can both fit the time-varying observation data and satisfy the DUE conditions (i.e., minimize the gap function). To circumvent the difficulty of path enumeration, the time-dependent least time path (TDLTP) algorithm,

developed by Ziliaskouplas and Mahmassani (1993), is employed to generate new paths in each outer loop iteration n .

Evaluation of partial derivatives with respect to path flow perturbation

Solving the proposed single-level dynamic OD estimation model requires the evaluation of the partial derivatives with respect to time-varying path flows. These partial derivatives represent the marginal effects of an additional unit of path inflow on link (i.e., link flow, density, and travel time) and path performances (i.e., path travel time).

Evaluation of link partial derivatives on a congested link

In this study, link partial derivative is referred to as the change in link flow, density, or travel time, due to an additional unit of link/path inflow. For instance, the link travel time derivative is the travel time contribution of an additional unit of flow on link l at time t'_l to the link travel time $T_{(l,t)}$, where t'_l is in time interval t . Ghali and Smith (1995) presented an analytical approach to evaluate the (local) link marginal travel time (or delay) on a congested link, based on link cumulative flow curves. An illustration of the approach is depicted in Fig. . The key result of their approach is that the link marginal delay equals the grey area.

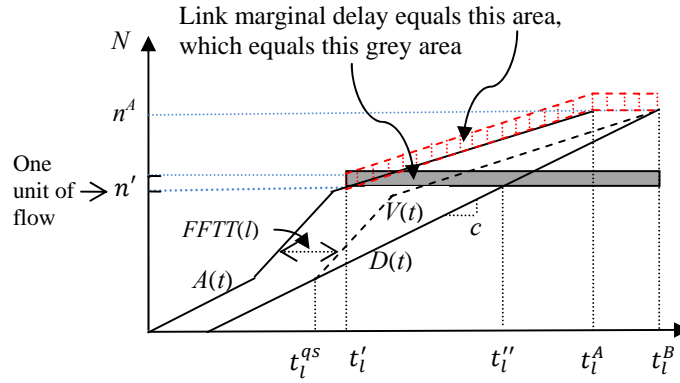


Fig. 2 Illustration of link marginal delay on a congested link

The following propositions can be induced from Fig. for deriving the marginal effects on link flow (inflow and outflow), density, and travel time.

Proposition 1: Under *free-flow* conditions, an extra unit of flow arriving at the upstream end of link l at time t'_l results in the following: (i) the link inflow and outflow increase by 1 at times t'_l and t_l'' , respectively, and the flow rates at the other time intervals do not change; (ii) the link density increases by 1 from t'_l to t_l'' ; (iii) the individual travel times are not changed, and $t_l'' =$

$t'_l + FTT(l)$.

Proposition 2: Under *partially* congested conditions and constant link (outflow) capacity c , an extra unit of flow arriving at the upstream end of link l at time t'_l results in the following: (i) the link inflow and outflow increase by 1 at times t'_l and t_l^B , respectively, and the flow rates at the other time intervals do not change; (ii) the link density increases by 1 from t'_l to t_l^B ; (iii) the flows arriving between t'_l and t_l^A experience the additional delay $1/c$, because it takes $1/c$ to discharge this perturbation flow.

Proposition 3: If the perturbation flow arrives at the upstream end of link when it is *fully* congested, then the link flow and density will remain the same at the maximum flow rate, respectively, and then increase by 1 when the link becomes partially congested.

A common pitfall for deriving the partial derivative of link density, under congested conditions, is to record the increase in density by 1 from t'_l to t_l'' . Proposition 2, induced from Fig. , clarifies that the actual change in link density would last until the queue vanishes at time t_l^B . Proposition 2 also indicates that the change in link outflow, due to an extra unit of flow arriving under congested conditions, occurs at the time t_l^B , rather than t_l'' .

Evaluation of the impact of path flow perturbation on two sequential links

We evaluate the impact of path flow perturbation (i.e., partial derivatives) in the individual link-time level by tracing the changes in link flow, density, and travel time on a sequence of links (or a path) and over different time intervals, due to the addition of unit path flow. [Qian and Zhang \(2011\)](#) conducted a similar analysis for individual path marginal travel times.

Firstly, consider a freeway or an arterial segment with two sequential links, without merges and diverges, say link $l-1$ and link l . Under congested conditions, there are three basic cases of interest, when the additional unit of flow arrives at this segment.

(i) There is a bottleneck on the downstream link l and the queue on link l does not spill back to link $l-1$; that is, link $l-1$ is in free-flow condition while link l is partially congested.

(ii) There is a bottleneck on the downstream link l and the queue on link l spills back to link $l-1$; that is, link $l-1$ is partially congested while link l is fully congested.

(iii) There is a bottleneck on each of the two links, and the two bottlenecks are independent, assuming that both links are sufficiently long so that the queue in the downstream does not spill back to the upstream. This is in fact the case in which both links $l-1$ and l are partially congested.

For cases (i) and (iii), Proposition 2 can be applied to determine the marginal effects of the additional unit of flow on link flow, density, and travel time. For case (ii), there are two possible scenarios. As depicted in Fig. 3(a), one scenario is that the additional unit of flow does not encounter the queue on link $l-1$, so it can enter link l at time $t'_l = t''_{l-1} = t'_{l-1} + FF\!T\!T(l-1)$, when link l is partially congested, i.e., $t'_l < t_l^{q*}$ or $t'_l > t_{l-1}^B$. Another scenario, as depicted in Fig. 3(b), is that the perturbation flow encounters the queue on link $l-1$, i.e., $t_{l-1}^{qS} < (t'_{l-1} + FF\!T\!T(l-1)) < t_{l-1}^B$, so it cannot enter link l until time $t'_l = t''_{l-1} = t_{l-1}^B$. Note that $t_{l-1}^{qS} = t_l^{q*}$, the time at which the queue on link l start to spill back to link $l-1$.

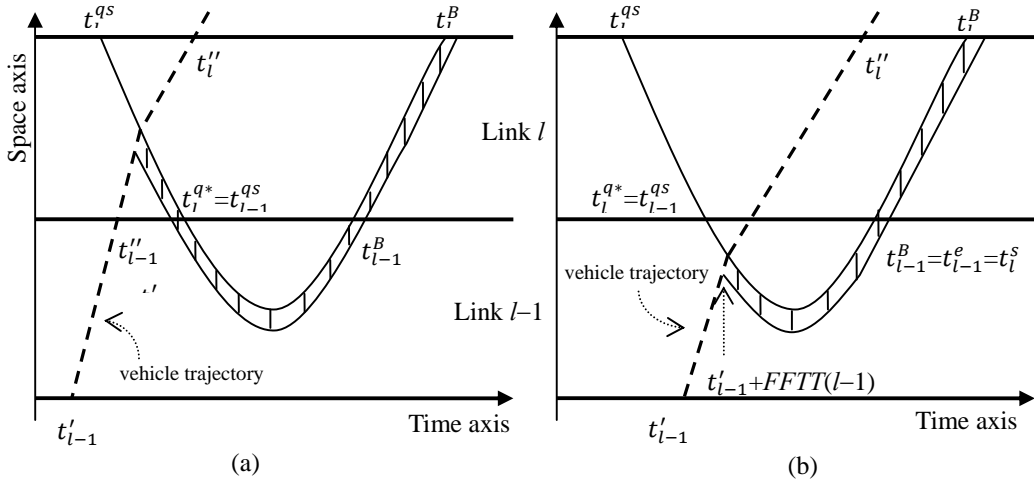


Fig. 3 Link marginal analysis for the case of queue spillback

Experiments on a simple two-link corridor with steady state travel time function

In the first set of experiments, we aim to examine the convergence pattern of the proposed algorithm on a simple corridor with a single O-D pair connected by two parallel links (or paths). A simple linear travel time function is used to perform the traffic assignment which loads a total peak-hour demand, 8000 vehicles/hour (or vhc/hr) to those two paths.

$$T_a = FF\!T\!T_a + r_a / cap_a,$$

where T_a and $FF\!T\!T_a$ are the travel time and free-flow travel time on link/path a , respectively. r_a and cap_a are the flow volume and capacity of link/path a , respectively. Then, the resulting UE assignment results are used as the ground-truth condition to evaluate the path flow estimation performance under various testing conditions.

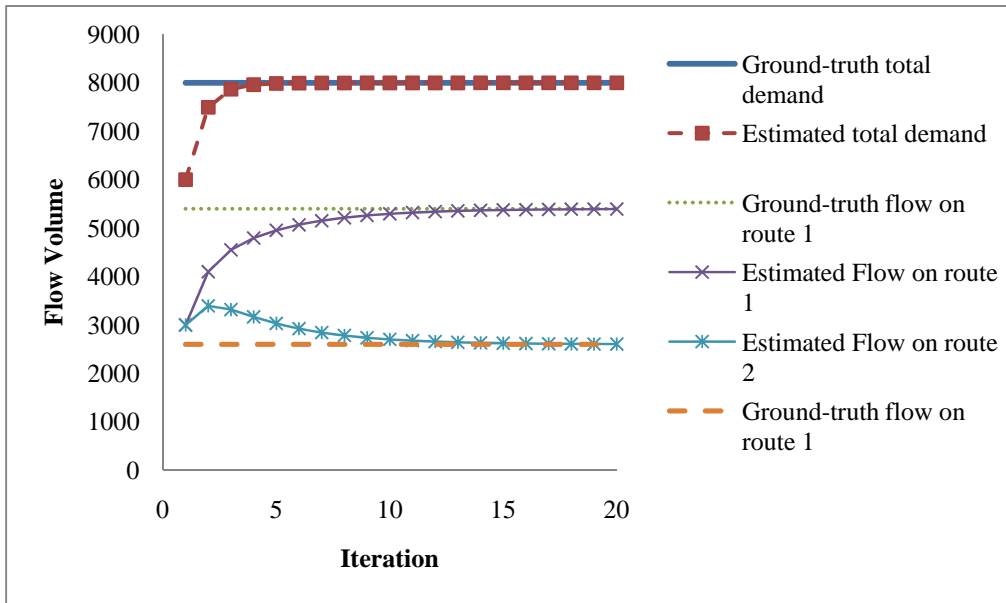


Fig. 4 Path flow volume convergence pattern as a function of inner iteration number

We start with an initial path flow distribution that loads 3000 vhc/hr to each link. The ground-truth demand of 8000 vhc/hr is set as the target demand, and the error-free flow counts ($r_1 = 5400$, $r_2 = 2600$) are used as the observations. Fig. and Fig. demonstrate the convergence patterns of the proposed path flow estimation algorithm in the first 20 inner iterations. We can observe that, after 3 or 4 inner iterations, the total estimated demand is quickly adjusted to a level very close to the ground-truth demand, while the equilibrium processes of path flow distribution and path travel times are relatively slow.

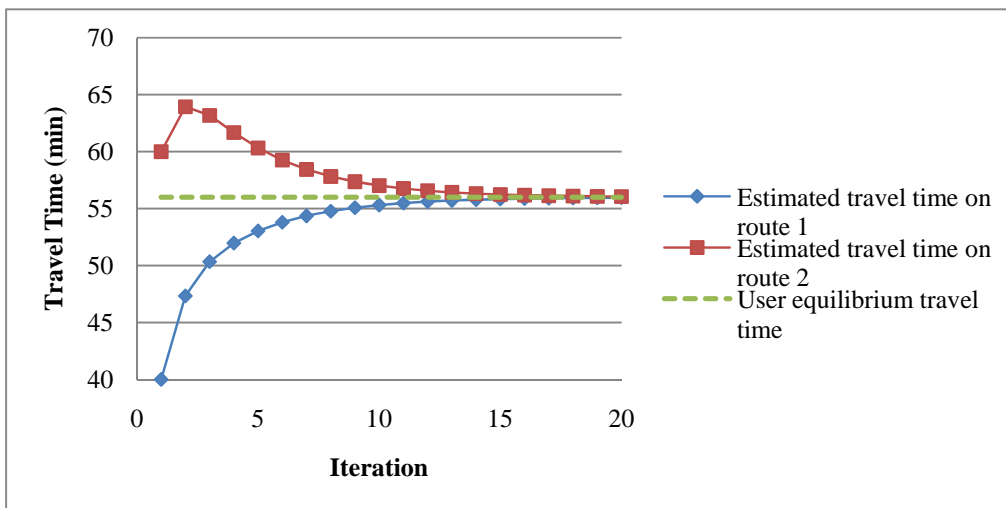


Fig. 5 Path travel time convergence pattern as a function of inner iteration number

Experiments on a freeway corridor with time-dependent sensor data

In the second set of experiments, we test the performance of the proposed algorithm on a freeway corridor with time-dependent real-world sensor data. As shown in Fig. , the freeway corridor of interest is a 2-mile section of I-210 Westbound, located in Los Angeles, CA. This corridor includes three on-ramps and one off-ramp. In the network representation, we ignore the HOV lane and only consider 4 general purpose lanes on the freeway. Traffic speed, flow count and occupancy are measured at 5-mins intervals on freeway and ramp links.

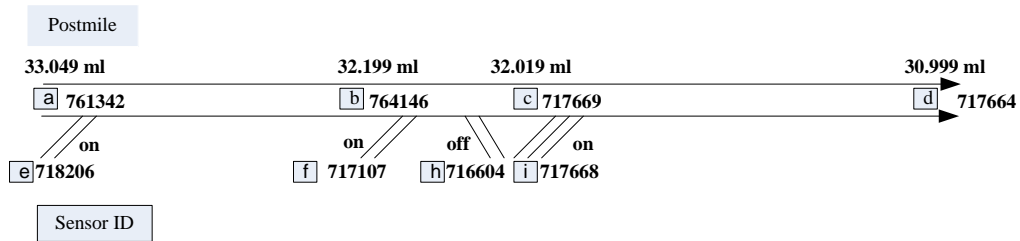


Fig. 6 Network representation of a section of I-210 West bound corridor

In this simple corridor, each OD pair only has a single path, so it does not involve a complex flow equilibration process required for multiple alternative paths. As a result, our focus is on demonstrating how the proposed gradient-based adjustment algorithm adjusts the incoming demand pattern to capture the observed queue formation, propagation and dissipation.

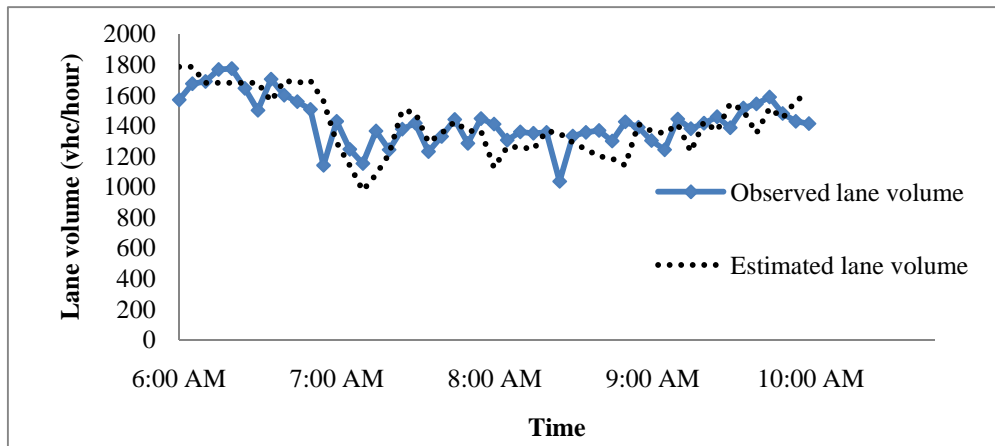


Fig. 7 Observed lane volume on station a vs. estimated lane volume on entrance link

Error! Reference source not found. shows the estimated and observed flow patterns on entry link a , and the corresponding average relative estimation errors are less than 10%. This indicates that the proposed algorithm can adjust a biased, initial demand pattern to match the target demand

volume at the entry point. The estimated space speeds and observed point speeds at station c are plotted in Fig. , which demonstrates that the DNL model is able to accurately reproduce the queue spillback phenomenon along the corridor.

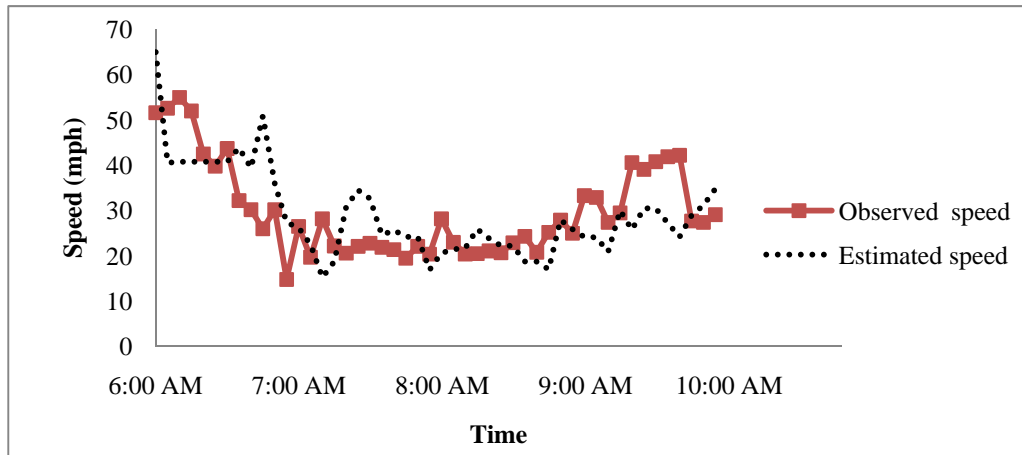


Fig. 8 Observed point mean speed at station c vs. estimated space mean speed on the link from off-ramp h to station c

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