Using “Fair Division” Methods for Allocating Transportation Funds

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Acknowledgments

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Special thanks to Ron Hagquist for his strong support and advice as TxDOT Project Director in Research Project 0-6727: Using “Fair Division” Methods for Allocating Transportation Funds.
1. Brief Overview of TxDOT Project 0-6727
2. Introduction to Fair Division
3. Funding Allocation
4. Fair Division Allocation Model
5. Fair Division Allocation Model Example
6. Conclusions
Two-year research project 0-6727 (FY2012-2013)

Objectives:

- To investigate fair division algorithms and methods for the allocation of transportation funds and/or resources among competing interests at TxDOT.
- To formulate a comprehensive model to enhance the current allocation decision making processes based on fair division concepts.
The problem of dividing a resource in such a way that all recipients believe that they have received a fair share
Fair Division Concepts

- **ENVY-FREE**: Every player thinks that he or she received the largest or most valuable portion of something-based on his or her own valuation- and hence does not envy anyone else.

- **EFFICIENT**: There is no other allocation that is better for one player and as good for all the other players.

- **EQUITABLE**: In which both people win by the same amount over 50 percent by their own valuation scorings. Each person feels they received the same value as the other person.

- **PROPORTIONAL**: Each player receives a piece that he/she perceives to be at least 1/n of the whole.

Source: Pre-Proposal Meeting Presentation by Ron Hagquist
<table>
<thead>
<tr>
<th>Fair Division Method</th>
<th>Allocation Procedure</th>
<th>Characteristics of the Goods</th>
<th>Characteristics of the Procedure</th>
</tr>
</thead>
</table>
| Divide and Choose    | **Trimming Algorithm:** • Selfridge & Conway(1960) | Continuous, divisible good | Not proportional  
Not envy-free  
Non efficient |
|                      | **Moving Knife:** • Dubins and Spanier (1961) • Austin (1982) | Continuous, divisible good | Not proportional  
Not envy-free  
Non efficient  
Two participants |
| Point Allocation Methods | **Knaster Procedure:** • Knaster and Steinhaus (1948) | Indivisible goods | Players submit sealed bids for items  
Capital reserve required as a collateral  
Envy-free for two persons  
Stimulates greediness |
|                      | **Adjusted Winner (AW):** • Brams and Taylor (1994) | Indivisible goods | Players assign points to each good  
Points must total 100  
Envy-free and Efficient for two players |
<table>
<thead>
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</tr>
</thead>
<tbody>
<tr>
<td><strong>Graph Theory</strong></td>
<td>• Aragones(1995)</td>
<td>Indivisible goods</td>
<td>Envy-free</td>
</tr>
<tr>
<td></td>
<td>• Klijn Algorithm (2000)</td>
<td></td>
<td>Assume player’s preferences for money are characterized by linear utility functions</td>
</tr>
<tr>
<td></td>
<td>• Nuchia &amp; Sen Algorithm(2001)</td>
<td></td>
<td>$n$ players</td>
</tr>
<tr>
<td><strong>Linear Programming</strong></td>
<td><strong>Dynamic Programming:</strong></td>
<td>Indivisible goods</td>
<td>Mathematical optimization</td>
</tr>
<tr>
<td></td>
<td>• Dall’Aglio &amp; Mosca (2007)</td>
<td></td>
<td>Solve complex problems by breaking them down into simpler subproblems</td>
</tr>
<tr>
<td></td>
<td><strong>Branch and Bound Algorithm:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Vetschera(2010)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Genetic Algorithms</strong></td>
<td>• Dupuis &amp; Gosselin (2009)</td>
<td>Indivisible goods</td>
<td>Generates useful solutions to optimization and search problems inspired by natural evolution process</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Provides optimal solutions</td>
</tr>
</tbody>
</table>
Fair Division Goal

- Fair division methodology will provide an allocation of transportation funds based on proportionality, envy-freeness, equitability, and efficiency principles
The three main sources of funding for transportation projects are:

- **Federal Funding:**
  - Federal Highway Administration and Federal Transit Administration distributes funds to states using a formula based on population
  - Transportation Equity Act for the 21st Century
  - Economic Stimulus Bill

- **State Funding:**
  - TxDOT receives this funding to provide maintenance and rehabilitation to the roads using formulas

- **Local Funding:**
  - Individual projects funded or awarded to match federal funds
What are TxDOT Critical Tier Allocation Areas?

Federal Funding
- Maintenance and Rehabilitation
- Strategic Priority
- District Discretionary
- Supplemental
- Transportation Enhancements

State Funding
- Urban Corridors
- Non-Traditional
- State Corridors
- Congestion Mitigation/ Air Quality
- Bridges
- Metropolitan Mobility/ Rehabilitation

Local Funding
- Safety
- Supplemental
- District Discretionary
- Strategic Priority
- Maintenance and Rehabilitation

Funding Categories
PROJECT SELECTION

- Funding allocation and project selection process are very complex

- TxDOT selects and funds projects included in the Unified Transportation Program (UTP) based on the goals achieved and benefits generated

- UTP is a listing of projects planned to be constructed over the next 10 years in the state of Texas
Projects selected must fulfill one of the following goals:

- Develop an organizational structure and strategies designed to address the future multimodal transportation needs of all Texans,
- Enhance safety for all Texas Transportation System users,
- Maintain the existing Texas Transportation System,
- Promote congestion relief strategies,
- Enhance system connectivity,
- Facilitate the development and exchange of comprehensive multimodal funding strategies with transportation program and project partners.
Project 0-6727 will be focused on:

- Capacity allocation
  - Population formula
    - Rural Allocation
      - Categories 2, 3, 4, 5, and 7
    - Consensus formulas
  - Metro & Urban Allocations
- Maintenance allocation
  - Formulas
    - District Allocations
      - 70% Lane Mile Basis?
    - Section Allocations

Source: Ron Hagquist Pre-Proposal Presentation- revised by UTEP
<table>
<thead>
<tr>
<th>Rank No.</th>
<th>Funding Category</th>
<th>Amount in Billions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Category 1- Preventive Maintenance and Rehabilitation</td>
<td>10.96</td>
</tr>
<tr>
<td>2</td>
<td>Category 3- Non-Traditionally Funded Transportation Projects</td>
<td>3.68</td>
</tr>
<tr>
<td>3</td>
<td>Category 6- Structures Replacement and Rehabilitation</td>
<td>2.5</td>
</tr>
<tr>
<td>4</td>
<td>Category 12- Strategic Priority</td>
<td>2.47</td>
</tr>
<tr>
<td>5</td>
<td>Category 7- Metropolitan mobility and Rehabilitation</td>
<td>2.03</td>
</tr>
<tr>
<td>6</td>
<td>Category 2- Metropolitan and Urban Area Corridor Projects</td>
<td>1.99</td>
</tr>
<tr>
<td>7</td>
<td>Category 8- Safety</td>
<td>1.24</td>
</tr>
<tr>
<td>8</td>
<td>Category 5- Congestion Mitigation and Air Quality Improvement</td>
<td>1.12</td>
</tr>
<tr>
<td>9</td>
<td>Category 9- Transportation Enhancements</td>
<td>0.65</td>
</tr>
<tr>
<td>10</td>
<td>Category 11- District Discretionary</td>
<td>0.64</td>
</tr>
<tr>
<td>11</td>
<td>Category 10- Supplemental Transportation Projects</td>
<td>0.63</td>
</tr>
<tr>
<td>12</td>
<td>Category 4- Statewide Connectivity Corridor Projects</td>
<td>0.02</td>
</tr>
</tbody>
</table>
Most critical allocation funding categories are related to: maintenance and rehabilitation and increase of capacity of the existing roads

Limited funds are distributed based on demographics and the size of the transportation network

All cities should receive a fair share according to their own necessities and priorities

Fair division methods can provide tools that create distributions among n members considering four characteristics: proportionality, equitability, efficiency, and envy-freeness.
FAIR DIVISION ALLOCATION MODEL
Strives to make all of the recipients believe that they have been allocated a fair share of a resource.

Based on the bid a recipient places on the desired asset.

The resource is to be divided by the participants (i.e., no arbitrator is involved in the allocation).

Börgers (2010) defines fair share of a particular recipient as his/her bidding divided by the total number of participants.
A centralized approach is assumed where envy is an objective to be minimized.

Envy has been observed to be correlated to the deviations of the $\frac{\text{assigned funds}}{\text{expected funds}}$ ratios.

Expected funds are the sum of the cost of all the projects a particular recipient expects to receive.

Assigned funds are what the recipients actually receive.
**BI-OBJECTIVE FORMULATION:**

\[
\min \left[ \sum_{i=1}^{n} \varepsilon_i(\mathbf{X}_i) \right], \max \left[ \hat{r}_p \right]
\]

Subject to:

\[
\sum_{k=1}^{m} x_{ki} \geq 1, \quad \forall i = 1,2,\ldots,n
\]

\[
x_{ki} \in \{0,1\}
\]

Where:

- \( n \) = number of recipients
- \( \mathbf{X}_i = (x_{i,1}, x_{i,2}, \ldots, x_{i,m_i}) \)
- \( m \) = number of funding sources
- \( x_{ki} \) = project that belongs to the \( k^{th} \) funding category requested by the \( i^{th} \) recipient
- \( \varepsilon_i(\mathbf{X}_i) \) = envy sensed by the \( i^{th} \) with respect to the \( j^{th} \) recipient
- \( \hat{r}_p \) = Expected rate of return of the selected portfolio
**Envy Definition:**

$$\min \left[ \sum_{i=1}^{n} \varepsilon_i(X_i) \right]$$

$$\varepsilon_{ij} = \begin{cases} 
|\rho_i - \rho_j|, & \text{if } (\rho_i - \rho_j) < 0 \\
0, & \text{otherwise}
\end{cases}$$

- $\rho_i$: Assigned to Expected Funds ratio of $i^{th}$
- $\rho_j$: Assigned to Expected Funds ratio of $j^{th}$
- $\varepsilon_{ij}$: Envy sensed by the $i^{th}$ with respect to the $j^{th}$
- $n$: total number of participants
Fair division model consists of two main steps, the sequential allocation model (SAM) and the envy finder algorithm (EFA)

- SAM evaluates the prospect recipients’ weighted preferences to determine an allocation of the available divisible goods

- EFA determines the benefit the recipients have received and calculates the total envy produced
**Simple Numerical Example:**

Two heirs John and Mary want to divide 3 items among them:
- House-valued at $100,000
- Boat-valued at $25,000
- Luxury Car-valued at $70,000
- Total value of items = $195,000

<table>
<thead>
<tr>
<th>Person</th>
<th>Points Given to Item</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>House</td>
<td>Boat</td>
</tr>
<tr>
<td>John</td>
<td>30</td>
<td>20</td>
</tr>
<tr>
<td>Mary</td>
<td>60</td>
<td>15</td>
</tr>
</tbody>
</table>
**Sequential Allocation Model:**

- First distribution will allocate items to the person who assign more points to it

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<td>Mary</td>
<td>60</td>
<td>15</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Person</th>
<th>Items assigned (1 if yes, 0 otherwise)</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>House</td>
<td>Boat</td>
</tr>
<tr>
<td>John</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Mary</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
**Envy Finder Algorithm:**

<table>
<thead>
<tr>
<th>Person</th>
<th>Value of Items Allocated, $</th>
<th>Total Allocated</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>House</td>
<td>Boat</td>
</tr>
<tr>
<td>John</td>
<td>-</td>
<td>25,000</td>
</tr>
<tr>
<td>Mary</td>
<td>100,000</td>
<td>-</td>
</tr>
</tbody>
</table>

\[
\text{Total Envy} = (\rho_{\text{John}} - \rho_{\text{Mary}})
\]

\[
\rho_{\text{John}} = \frac{\text{total allocated to John}}{\text{total value of items}} = \frac{95,000}{195,000} = 0.49
\]

\[
\rho_{\text{Mary}} = \frac{\text{total allocated to Mary}}{\text{total value of items}} = \frac{100,000}{195,000} = 0.51
\]

\[
\text{Total Envy} = (\rho_{\text{John}} - \rho_{\text{Mary}}) = (0.49 - 0.51) = 0.02 \text{ minimal envy possible}
\]
CONCLUSIONS

- Envy-free methods have been found to be NP (Nondeterministic Polynomial Time) hard as the number of players increases, model must be able to minimize envy ratios in all participants.

- Individual preferences must be taken into account in order to fulfill the needs of each participant.

- The general fair division model must be able to provide a “fair” allocation of funds among all the participants based on individual needs in order to enhance the current allocation methods.

- Fair Division methods must provide a proportional, envy-free, efficient, and equitable distribution to all the participants.
Questions?