Using "Fair Division" Methods for Allocating Transportation Funds

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Acknowledgments

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Presentation Outline

1. Brief Overview of TxDOT Project 0-6727
2. Introduction to Fair Division
3. Funding Allocation
4. Fair Division Allocation Model
5. Fair Division Allocation Model Example
6. Conclusions

TxDOT Project 0-6727: Using “Fair Division” Methods for Allocating Transportation Funds

Two-year research project 0-6727 (FY2012-2013)

Objectives:

- To investigate fair division algorithms and methods for the allocation of transportation funds and/or resources among competing interests at TxDOT.
- To formulate a comprehensive model to enhance the current allocation decision making processes based on fair division concepts.

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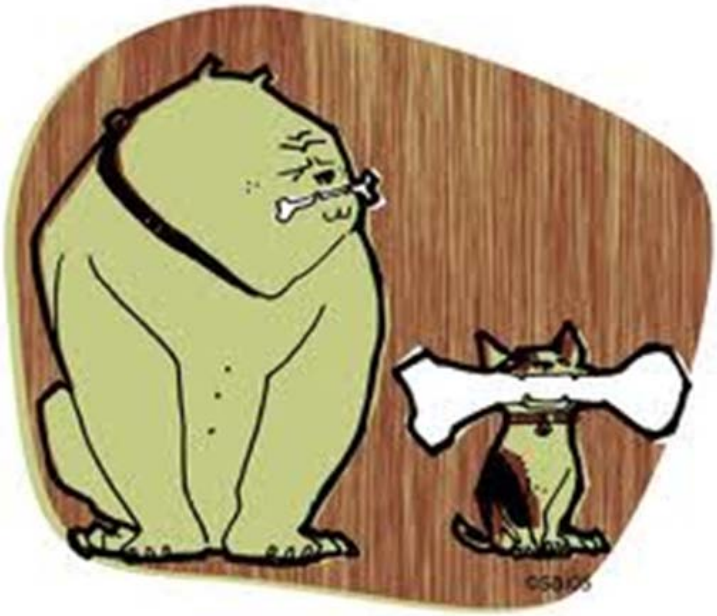
Fair Division: The Problem

The problem of dividing a resource in such a way that all **recipients believe** that they have received a **fair share**



Fair Division Concepts

- **ENVY-FREE:** Every player thinks that he or she received the largest or most valuable portion of something-based on his or her own valuation- and hence does not envy anyone else

- **EFFICIENT:** There is no other allocation that is better for one player and as good for all the other players

- **EQUITABLE:** In which both people win by the same amount over 50 percent by their own valuation scorings. Each person feels they received the same value as the other person

- **PROPORTIONAL:** Each player receives a piece that he/she perceives to be at least 1/n of the whole

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Source: Pre-Proposal Meeting Presentation by Ron Hagquist

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Fair Division Method	Allocation Procedure	Characteristics of the Goods	Characteristics of the Procedure
Divide and Choose	**Trimming Algorithm**: • Selfridge & Conway(1960)	Continuous, divisible good	Not proportional Not envy-free Non efficient
	Moving Knife: • Dubins and Spanier (1961) • Austin (1982)	Continuous, divisible good	Not proportional Not envy-free Non efficient Two participants
Point Allocation Methods	**Knaster Procedure:** • Knaster and Steinhaus (1948)	Indivisible goods	Players submit sealed bids for items Capital reserve required as a collateral Envy-free for two persons Stimulates greediness
	Adjusted Winner (AW): • Brams and Taylor (1994)	Indivisible goods	Players assign points to each good Points must total 100 Envy-free and Efficient for two players

Fair Division Method	Allocation Procedure	Characteristics of the Goods	Characteristics of the Procedure
Graph Theory	• Aragones(1995) • Klijn Algorithm (2000) • Nuchia & Sen Algorithm(2001)	Indivisible goods	Envy-free Assume player's preferences for money are characterized by linear utility functions *n* players
Linear Programming	**Dynamic Programming:** • Dall'Aglio & Mosca (2007) **Branch and Bound Algorithm:** • Vetschera(2010)	Indivisible goods	Mathematical optimization Solve complex problems by breaking them down into simpler subproblems
Genetic Algorithms	• Dupuis & Gosselin (2009)	Indivisible goods	Generates useful solutions to optimization and search problems inspired by natural evolution process Provides optimal solutions

Fair Division Goal

- Fair division methodology will provide an allocation of transportation funds based on proportionality, envy-freeness, equitability, and efficiency principles

Funding Allocation

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Funding Allocation

The three main sources of funding for transportation projects are:

- Federal Funding:
  - Federal Highway Administration and Federal Transit Administration distributes funds to states using a formula based on population
  - Transportation Equity Act for the 21st Century
  - Economic Stimulus Bill
- State Funding:
  - TxDOT receives this funding to provide maintenance and rehabilitation to the roads using formulas
- Local Funding:
  - Individual projects funded or awarded to match federal funds

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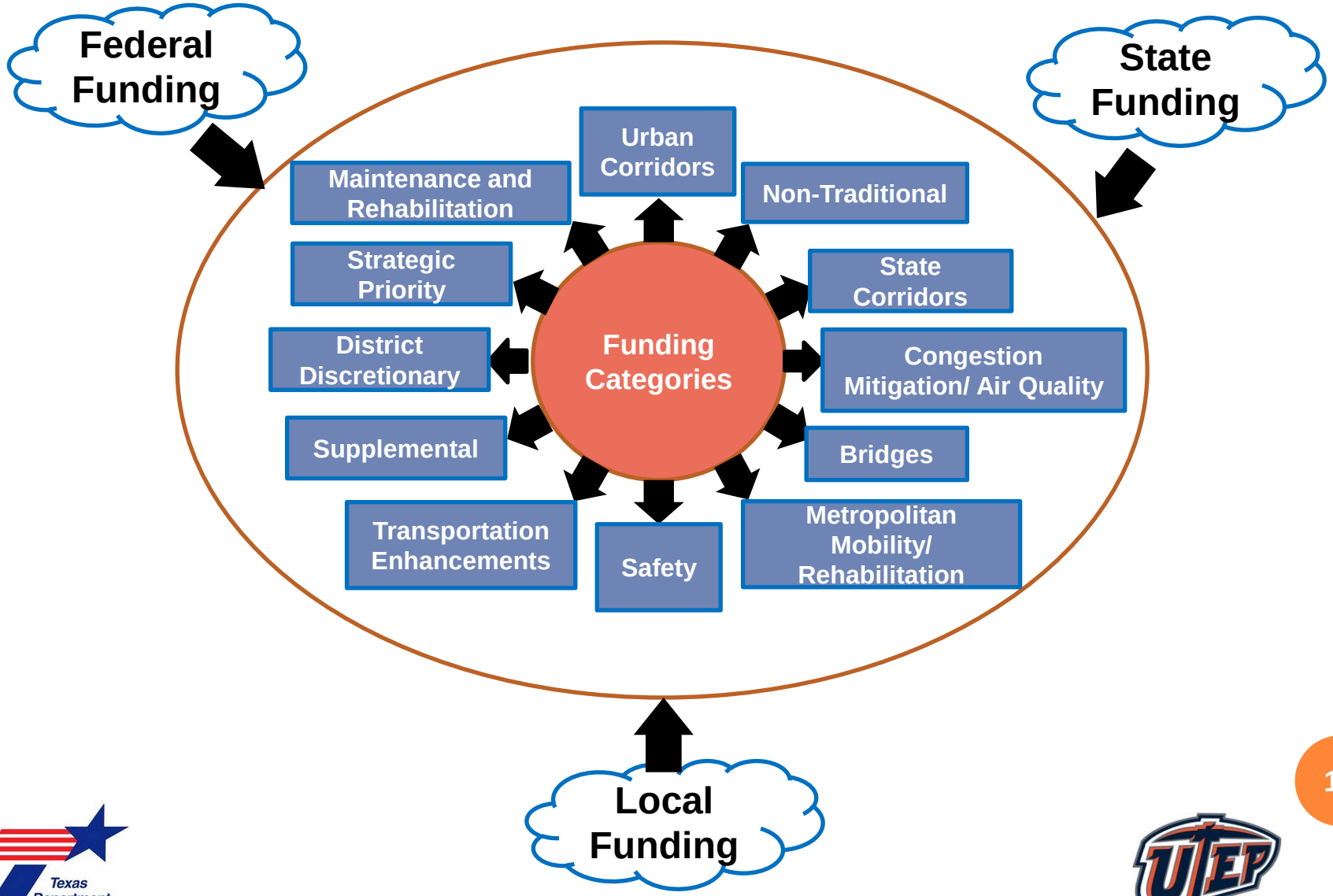
What are TxDOT Critical Tier Allocation Areas?



Project Selection

- Funding allocation and project selection process are very complex
- TxDOT selects and funds projects included in the Unified Transportation Program (UTP) based on the goals achieved and benefits generated
- UTP is a listing of projects planned to be constructed over the next 10 years in the state of Texas

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Project Selection- Overall Criteria

Projects selected must fulfill one of the following goals:

- Develop an organizational structure and strategies designed to address the future multimodal transportation needs of all Texans,
- Enhance safety for all Texas Transportation System users,
- Maintain the existing Texas Transportation System,
- Promote congestion relief strategies,
- Enhance system connectivity,
- Facilitate the development and exchange of comprehensive multimodal funding strategies with transportation program and project partners.

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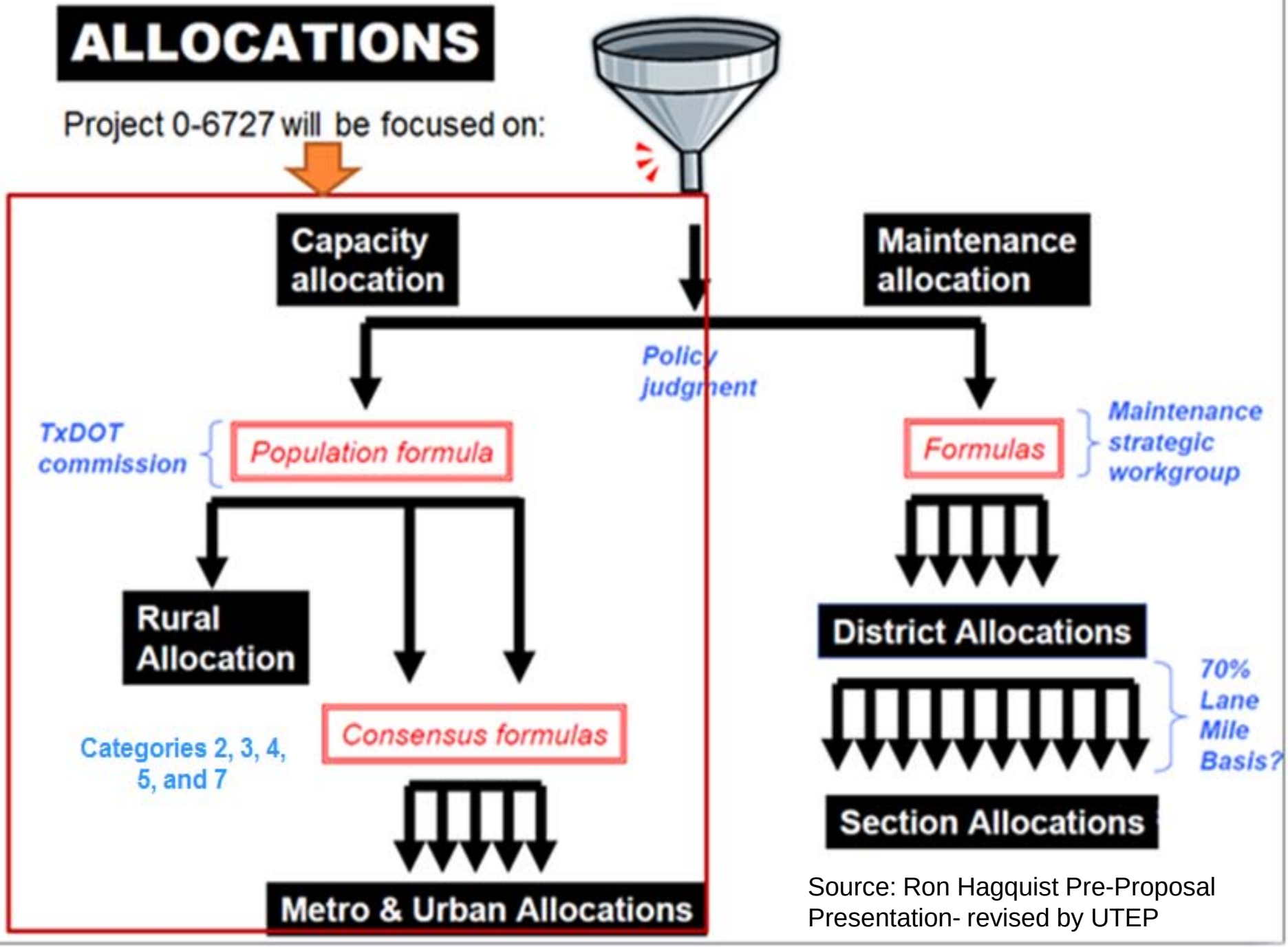

Source: Ron Hagquist Pre-Proposal Presentation- revised by UTEP

Funding Categories Prioritized List

Rank No.	Funding Category	Amount in Billions
1	Category 1- Preventive Maintenance and Rehabilitation	10.96
2	Category 3- Non-Traditionally Funded Transportation Projects	3.68
3	Category 6- Structures Replacement and Rehabilitation	2.5
4	Category 12- Strategic Priority	2.47
5	Category 7- Metropolitan mobility and Rehabilitation	2.03
6	Category 2- Metropolitan and Urban Area Corridor Projects	1.99
7	Category 8- Safety	1.24
8	Category 5- Congestion Mitigation and Air Quality Improvement	1.12
9	Category 9- Transportation Enhancements	0.65
10	Category 11- District Discretionary	0.64
11	Category 10- Supplemental Transportation Projects	0.63
12	Category 4- Statewide Connectivity Corridor Projects	0.02

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COMMENTS:

- Most critical allocation funding categories are related to: maintenance and rehabilitation and increase of capacity of the existing roads

- Limited funds are distributed based on demographics and the size of the transportation network

- All cities should receive a fair share according to their own necessities and priorities

- Fair division methods can provide tools that create distributions among n members considering four characteristics: proportionality, equitability, efficiency, and envy-freeness.

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Fair Division Allocation Model

Fair Share principles

- Strives to make all of the recipients believe that they have been allocated a fair share of a resource
- Based on the bid a recipient places on the desired asset
- The resource is to be divided by the participants (*i.e.* no arbitrator is involved in the allocation)
- Börgers (2010) defines fair share of a particular recipient as his/her bidding divided by the total number of participants.

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Fair Division Algorithm

- A centralized approach is assumed where envy is an objective to be minimized
- Envy has been observed to be correlated to the deviations of the $\frac{assigned\ funds}{expected\ funds}$ ratios
- Expected funds are the sum of the cost of all the projects a particular recipient expects to received
- Assigned funds are what the recipients actually receive

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Bi-objective Formulation:

$$\min\left[\sum_{i=1}^{n}\varepsilon_i(\mathbf{X}_i)\right], \max\left[\hat{r}_p\right]$$

Subject to:

$$\sum_{k=1}^{m} x_{ki} \geq 1, \qquad \forall i = 1,2,\ldots,n$$

$$x_{ki} \in \{0,1\}$$

Where:

n = number of recipients

$\mathbf{X}_i = (x_{i,1}, x_{i,2}, \ldots, x_{i,m_i})$

m = number of funding sources

x_{ki} = project that belongs to the kth funding category requested by the ith recipient

$\varepsilon_i(\mathbf{X}_i)$ = envy sensed by the ith with respect to the jth recipient

$\hat{r}_p$ = Expected rate of return of the selected portfolio

Envy Definition:

$$\min\left[\sum_{i=1}^{n}\varepsilon_i(\mathbf{X}_i)\right]$$

$$\varepsilon_{ij}=\begin{cases}|\rho_i-\rho_j|, & if\ (\rho_i-\rho_j)<0\\ 0, & otherwise\end{cases}$$

ρ_i= Assigned to Expected Funds ratio of ith

ρ_i= Assigned to Expected Funds ratio of jth

ε_{ij}= Envy sensed by the ith with respect to the jth

n= total number of participants

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Fair Division Methodology

- Fair division model consists of two main steps, the sequential allocation model (SAM) and the envy finder algorithm (EFA)
- SAM evaluates the prospect recipients' weighted preferences to determine an allocation of the available divisible goods
- EFA determines the benefit the recipients have received and calculates the total envy produced

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Simple Numerical Example:

- Two heirs John and Mary want to divide 3 items among them:
  - House-valued at $100,000
  - Boat-valued at $25,000
  - Luxury Car-valued at $70,000
  - Total value of items= $195,000

Person	Points Given to Item			Total Points
	House	Boat	Car	
John	30	20	50	100
Mary	60	15	25	100

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Sequential Allocation Model:

- First distribution will allocate items to the person who assign more points to it

Person	Points Given to Item			Total Points
	House	Boat	Car	
John	30	20	50	100
Mary	60	15	25	100

Person	Items assigned (1 if yes, 0 otherwise)			Total Points
	House	Boat	Car	
John	0	1	1	70
Mary	1	0	0	60

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Envy Finder Algorithm:

Person	Value of Items Allocated, $			Total Allocated
	House	Boat	Car	
John	-	25,000	70,000	95,000
Mary	100,000	-	-	100,000

Total Envy= $(\rho_{John} - \rho_{Mary})$

$$\rho_{John} = \frac{total\ allocated\ to\ John}{total\ value\ of\ items} = \frac{95{,}000}{195{,}000} = 0.49$$

$$\rho_{Mary} = \frac{total\ allocated\ to\ Mary}{total\ value\ of\ items} = \frac{100{,}000}{195{,}000} = 0.51$$

Total Envy= $(\rho_{John} - \rho_{Mary})$= (0.49 - 0.51) = 0.02 minimal envy possible

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CONCLUSIONS

- Envy-free methods have been found to be NP (Nondeterministic Polynomial Time) hard as the number of players increases, model must be able to minimize envy ratios in all participants.
- Individual preferences must be taken into account in order to fulfill the needs of each participant.
- The general fair division model must be able to provide a “fair” allocation of funds among all the participants based on individual needs in order to enhance the current allocation methods.
- Fair Division methods must provide a proportional, envy-free, efficient, and equitable distribution to all the participants.

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Questions?