Over the past few years, much attention has been paid to computing flows for multi-class network equilibrium models that exhibit uniqueness of the class flows and proportionality (Bar-Gera et al (2012)). Several new algorithms have been developed such as origin based method (Bar Gera 2002), bush based method (Dial 2006), and LUCE (Gentile 2012), that are able to obtain very fine solutions of network equilibrium models. These solutions can be post processed, in order to ensure proportionality and class uniqueness of the flows. Recently developed, the TAPAS algorithm (Bar Gera 2012) is able to produce solutions that have proportionality embedded, without a post processing. It is generally accepted that these methods for solving UE traffic assignment are the only way to obtain unique route and class link flows. The purpose of this presentation is to show a new discovery that variants of the linear approximation method satisfy these conditions as well.

Statement of Financial Interest

There are no financial interests directly attached to the content of this paper.

Statement of Innovation

The purpose of this presentation is to show a new discovery that variants of the linear approximation method exhibits both class uniqueness and proportionality.
On uniqueness and proportionality in multi-class equilibrium assignment

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Abstract

Over the past few years, much attention has been paid to computing flows for multi-class network equilibrium models that exhibit uniqueness of the class flows and proportionality (Bar-Gera et. al (2012)). Several new algorithms have been developed such as origin based method (Bar Gera 2002), bush based method (Dial 2006), and LUCE (Gentile 2012), that are able to obtain very fine solutions of network equilibrium models. These solutions can be post processed, in order to ensure proportionality and class uniqueness of the flows. Recently developed, the TAPAS algorithm (Bar Gera 2012) is able to produce solutions that have proportionality embedded, without a post processing. It is generally accepted that these methods for solving UE traffic assignment are the only way to obtain unique route and class link flows. The purpose of this presentation is to show a new discovery that variants of the linear approximation method satisfy these conditions as well.

1. Introduction

The convergence of the linear approximation algorithm (Frank-Wolfe (1956)) has been improved recently with a bi-conjugate variant of Mitradjieva and Lindberg (2013). Nevertheless, it has always been considered that in order to obtain class uniqueness and proportionality one has to resort to another method. In this presentation, we show, by comparing the results of a two-class assignment obtained with TAPAS to that of the bi-conjugate variant of the linear approximation method, that the latter exhibits both class uniqueness and proportionality. While there is still not a rigorous proof that this is so, this discovery is very useful since this variant of the linear approximation method can be multi-threaded and executed on multiprocessor computing platforms. Therefore, it provides a far more attractive and computationally efficient method for solving multi-class assignments that bush based methods, for convergence levels used in practice.

2. Computational results

The partial computational experiments reported here in the extended abstract (more computational results will be included in the presentation at increasing levels of congestion) are using the Chicago test database with two classes of traffic, cars and trucks (Boyce and Xie (2013)), that was kindly made available to us by David Boyce. We also had access to class flows obtained by the execution of the TAPAS code (Bar Gera (2010)) to a convergence criterion of relative gap of less than $10^{-12}$. 
This database is widely used as a benchmark for the traffic assignment algorithms (see Figure 1). It has 1,790 zones, 11,192 nodes, and 39,018 links. There are 563 links where trucks are not permitted.

Figure 1. The Chicago test network
We ran the same two-class assignment used with TAPAS by Boyce and Xie, using a multi-threaded variant of the bi-conjugate linear approximation method\(^1\) up to a relative gap of \(10^{-6}\). Figure 2 shows a plot of car link flows versus the flow obtained with TAPAS. Figure 3 shows a plot of truck link flows versus the flow obtained with TAPAS. The computation of this two-class assignment required approximately 5 minutes on a hyper-threaded 16 Xeon, 2.9 Ghz processor computing platform, using 31 threads. The computation results for the same assignment is reported to be about 82 minutes in Boyce and Xin, 2013.

\(\text{Figure 2. Comparison of car flows (bi-conjugate linear vs. TAPAS)}\)

\(^1\) Implemented as SOLA in Emme 4.1 software package.
Figures 2 and 3 clearly show that both tested methods are producing almost the same class flows. Since the assignments are not run to the same precision the fit is not perfect.

We also studied the proportionality property using the two methods of solving for equilibrium. This property assumes that all individual travelers, from all O-D pairs, when facing choices between the same two alternative route segments, they will distribute themselves over two alternative route segments in the same proportions. This assumption is formally known as the condition of proportionality (Bar-Gera and Boyce (1999)) and is a necessary condition that characterize entropy maximizing route flows. To help verify this property, the ratio of travelers
traversing the lower to upper alternative route segment, should form a straight line on the chart that plots O-D demand that uses each segment.

For that purpose we analyzed a pair of alternative segments identified by Bar-Gera, H., Boyce, D. and Nie, Y., in their study report of 2012. The flow on the pair of alternative segments is shown in Figure 4. The O-D pairs that contribute flows to each segment were computed by appropriate path analyses.

![Figure 4. Flows on a pair of alternative segments](image)

The charts from Figure 5 and 6 show plots of the O-D demands that contribute to the flow of each one of the segments for cars and trucks. The relatively straight line of these plots indicates that the condition of proportionality is approximately satisfied. In Figure 5, matrix mf26 was obtained using the bi-conjugate linear approximation whereas matrix mf25 was obtained using TAPAS. In Figure 6, matrix mf16 was obtained using the bi-conjugate linear approximation whereas matrix mf15 was obtained using TAPAS.
Figure 5. O-D pairs using each alternative segment for cars.

Figure 6. O-D pairs using each alternative segment for trucks.
3. Conclusions

From the analyses provided in this paper, it can be safely concluded that the bi-conjugate variant of the linear approximation method yields approximately unique class flows that also satisfy the condition of proportionality. The computing times that can be realized on multi-processor computing platforms for to convergence levels of up to $10^{-6}$, make from this method an attractive alternative for solving large scale multi-class assignments problems on which bush based methods are still relatively untested.

A rigorous proof that this property is always satisfied is under investigation.

4. Acknowledgments

We would like to express our appreciation to David Boyce for providing us with the data for the two-class Chicago assignment instance and with the TAPAS optimal flows for it.

5. Bibliography


