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Real-Time Optimization Model for Dynamic Scheduling of Transit Operations

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A new transit operating strategy is presented in which service vehicles operate in pairs with the lead vehicle providing an all-stop local service and the following vehicle being allowed to skip some stops as an express service. The underlying scheduling problem is formulated as a nonlinear integer programming problem with the objective of minimizing the total costs for both operators and passengers. A sensitivity analysis using a real-life example is performed to identify the conditions under which the proposed operating strategy is most advantageous.

The design and operations of public transit services involve many complicated issues. On the one hand, transit routes are expected to have a fixed schedule or service interval (headway) and to provide broad coverage. On the other hand, the origin-destination (O-D) passenger demands along a route often vary significantly, with some stops, such as those downtown, having a relatively large number of passengers boarding and alighting and others having few passengers. Such an O-D pattern cannot be efficiently serviced by transit with fixed routes and schedules. Balancing the needs of delivering reliable and consistent services and the costs of providing such services has continuously been one of the major challenges facing the transit agencies.

In the past 30 years, many operating strategies have been proposed with the goal of better matching services covered by transit routes with the O-D demand distribution along the routes. One of the central ideas has been flexible routing and scheduling that integrates express services (stop only at a few stops) with local services (stop at all stops). Flexible routing and scheduling strategies can be classified into four general categories (*1*): (*a*) zone scheduling, including restricted zonal service, semirestricted zonal service, and limited-stop zonal service; (*b*) short turning; (*c*) deadheading; and (*d*) dynamic stop skipping. In zone scheduling, the whole route is divided into several zones. The inbound buses make all stops within a single zone and then run without stopping to the terminus, while outbound vehicles operate in the reverse manner. With this operating strategy, passenger travel time and the required numbers of vehicles and drivers may be reduced. This operating strategy, however, has two disadvantages, including reduced service frequency and increased waiting time and the possible requirement of cross-zone transfer. Turnquist (*2, 3*) was among the first to study the zone scheduling method with the objective of simultaneously determining the optimal zone division and vehicle allocation. This model was extended by Jordan and Turnquist (*4*) to consider both the mean and variance of passengers' trip time.

Short turning (*5–8*) strategies consist of a system of short-turn and full-length trips operating along the same route. This express service is particularly suitable for routes in which the O-D demand peaks in a specific zone and decreases substantially outside that zone. The short-turn trips cover only the high-demand zone while the full-length trips run the whole route. Compared with a local service consisting only of full-length trips, short-turn operations require fewer full-length trips and thus fewer service vehicles. The critical design issue in short-turn services is to determine the turn-back point and the route schedule to balance passenger loads among the trips and to minimize the total fleet size and passenger wait time.

Deadheading, another type of express service, involves scheduling some service vehicles to run empty through a number of stations at the beginning or the end of their routes to save time and hence reduce the headways at later stations. Furth and Day (*1*) and Furth (*9*) studied the preplanned deadheading problem, which was formulated to minimize the fleet size required to meet a regular alternating deadheading schedule. More recently, Eberlein (*10*) and Eberlein et al. (*11*) investigated the deadheading problem in the context of real-time transit controls. The objective was to determine which vehicle to deadhead and at which stations. Both problems were formulated as nonlinear quadratic programs.

Dynamic stop skipping, also called expressing, is an operating strategy that has been frequently used in heavy-demand corridors. The basic idea behind this control strategy is to allow those vehicles that are late and behind schedule to skip certain low-demand stops and increase operating speed. In contrast to other strategies, which are mostly off-line strategies and designed at the service planning stage, dynamic stop skipping is an on-line strategy determined in real time. One disadvantage of this control strategy is that passengers with either their origin or destination stop being skipped have to wait for at least another headway to get service.

Eberlein (*10*) formulated the stop-skipping problem as an integer nonlinear programming model (INLP) with both quadratic objective function and constraints. Li and Wu (*12*), Li et al. (*13*), and Li (*14*) focused on the application scenario of a heavily used transit system with short headway (e.g. 2 to 3 min) and proposed two alternative models: a deterministic mixed INLP that assumed known average travel time and demands but variable dwell times, and a stochastic integer program with random travel times and demands but deterministic dwell times. Araya et al. (*15*) proposed a method for the generation of optimal schedules for on-line train traffic control. Lin et al. (*16*) investigated the combined strategy of stop skipping and holding and suggested that tight controls increase passenger travel times and therefore should be avoided. In their approach, a minimum headway could not be guaranteed, especially at some O-D pairs with low travel demand, because no limitation was imposed on dispatching patterns. The solutions of Eberlein (*10*), Li and Wu (*12*), and Li (*14*) were based on either very simplified formulation or heuristic algorithms. Li et al. (*13*) and Li (*14*) compared different models and algorithms and

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concluded that solutions from controlling a small subset of vehicles were very close to those when all vehicles were considered.

In what follows, a new dynamic scheduling strategy is presented that aims to strike an optimal balance between the benefits to operators and those to passengers. In the second section, a mathematical model is developed for the dynamic scheduling problem. Results of a sensitivity analysis are presented in the third section. The last section highlights the conclusions and suggests future research directions.

MODEL FORMULATION

System Definition

Consider a transit route consisting of N stops labeled sequentially from the terminal (Stop 1) to the end of the route (Stop N), as indicated in Figure 1. Buses are dispatched at the terminal according to a given schedule or headway. The dispatching functions are assumed by a dispatch center, which is equipped with a computer-aided dispatching system and an automatic vehicle location system. At any point of time, a service vehicle could be in one of the following three possible states: moving in-between stops, dwelling at a stop for passenger alighting and boarding, or waiting to be dispatched at the terminal. The problem this paper focuses on arises whenever a vehicle is in the last state—that is, it is waiting to be dispatched at the terminal and requiring instructions on which stops to make on its route. This problem is commonly referred to as the dynamic transit scheduling problem or the dynamic stop-skipping problem.

The common approach to the dynamic scheduling problem has been to apply stop skipping controls to every vehicle dispatched from the terminal. This approach, seemingly optimal in a theoretical sense, suffers a critical limitation. That is, transit users cannot be guaranteed a minimum level of service as the model could produce solutions that allow several consecutive buses to skip the same stops. In this study, a new stop-skipping strategy is proposed, in which only every other bus will be allowed to express and skip stops. In other words, the stop-skipping control will be applied to alternate buses. One immediate consequence of this strategy is that the schedule headway at any stop will be no more than twice the dispatch headway. For example, if the scheduled dispatch headway of a route is 5 min, the maximum scheduled headway at any stop on the route will be 10 min. In addition, the skipping pattern set at the terminal is fixed once the bus departs from the terminal. This control strategy has two advantages. First, a minimum level of service is guaranteed for all transit passengers, regardless of their O-D locations. Second, this strategy is easy to implement, as the public can be informed of the service policy and what they should expect before using the service. Furthermore, stop-skipping information can be displayed at a bus stop if an electronic device is installed at the stop or it can be announced by bus drivers so that passengers whose destinations will be skipped do not board the wrong bus. This operating method therefore would cause much less confusion than the general stop-skipping strategy.

With this assumed operating strategy, the route model can be formally defined as follows. Let the bus that is currently waiting at the terminal for a decision on its stop-skipping pattern for its immediate trip be denoted Bus 1, as indicated in Figure 1. Because of the assumed operating strategy, the preceding bus (Bus 0) and the following bus (Bus 2) will have a nonexpressing trip—that is, they will not be allowed to skip any stops. Because the performance of the current bus (Bus 1) and the following bus (Bus 2) will depend on the state of the preceding bus (Bus 0) as well as the decision on the skipping pattern of Bus 1, one needs to consider all three buses in formulating the problem.

The following notations are used in describing the proposed model:

- i = index of service vehicles, $i = 0, 1, 2$;
- j = index for identifying the stations or stops on the transit route, $j = 1, 2, \ldots, N$;
- r_j = running time between Stop $j-1$ and j, assumed to be constant, $j = 2, \ldots, N$;
- $D_{i,j}$ = departure time of Bus i at Stop j, $\forall i, j$;
- $A_{i,j}$ = arrival time of Bus i at Stop j, $\forall i, j$;
- $\tau_{i,j}$ = dwell time of Bus i at Stop j, $\forall i, j$;
- $H_{i,j}$ = departure headway between Bus $i-1$ and Bus i at Stop j, $i = 1, 2$, $\forall j$;
- $W_{i,jk}$ = number of passengers waiting for Bus i and traveling from Stop j to Stop k, $i = 0, 1, 2$; $1 \le j < k \le N$;
- $L_{i,jk}$ = number of passengers traveling from Stop j to Stop k skipped by Bus i, $i = 0, 1, 2$; $1 \le j < k \le N$;
- $L_{i,j}$ = number of passengers at Stop j skipped by Bus i, $i = 0, 1, 2$; $j = 1, \ldots, N-1$ (note: $L_{i,j} = \sum_{k=j+1}^{N} L_{i,jk}$);
- $U_{i,j}$ = number of passengers boarding Bus i at Stop j, $i = 0, 1, 2$, $j = 1, \ldots, N-1$;

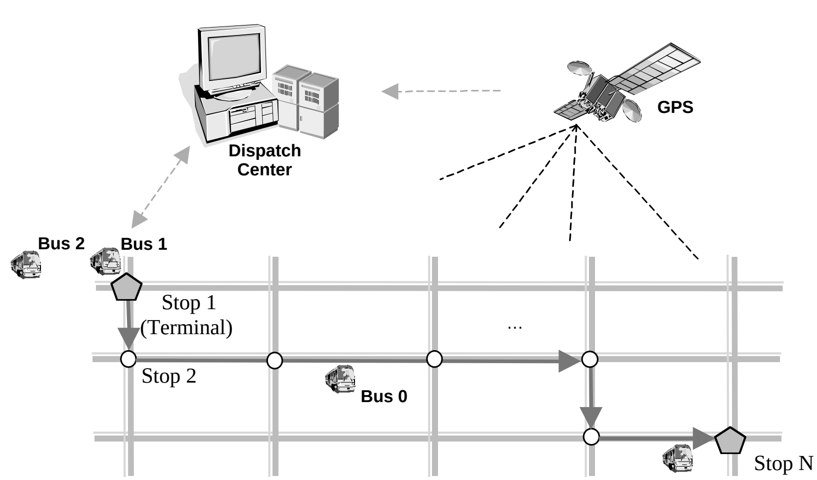

FIGURE 1 Route model. (GPS = Global Positioning System.)

$V_{i,j}$ = number of passengers alighting Bus i at Stop j, $i = 0, 1, 2$, $j = 2, \ldots, N$;
b = average boarding time per passenger, a constant;
a = average alighting time per passenger, a constant;
δ = average bus acceleration plus deceleration time, a constant;
$\lambda_{j,k}$ = average passenger arrival rate at stop j whose destination is Stop k, $1 \le j < k \le N$;
λ_j = average passenger arrival rate at stop j (note: $\lambda_j = \sum_{k-1}^{N} \lambda_{j,k}$);
c_1 = unit time value associated with passenger waiting time ($/h);
c_2 = unit time value associated with passenger in-vehicle time ($/h);
c_3 = unit time value associated with vehicle operation time ($/h); and
$y_{i,j}$ = decision variables to indicate stop status of Bus i at Stop j. $y_{i,j}$ takes two values: $y_{i,j} = 1$ if Bus i makes Stop j; $y_{i,j} = 0$, otherwise.

System State Equations

Transit vehicles operating on a given route follow an almost identical process: they arrive at a stop, dwell at the stop for passengers boarding and alighting, and then depart for the next stop. This process starts at Stop 1 and ends at Stop N. The following equations can be obtained for the relationships between the states of the three vehicles at individual stops:

$$A_{i,j} = D_{i,j-1} + r_j + \frac{\delta}{2} \times y_{i,j-1} + \frac{\delta}{2} \times y_{i,j} \qquad i = 1, 2, j = 2, \ldots, N \tag{1}$$

$$D_{i,j} = A_{i,j} + \tau_{i,j} \qquad i = 1, 2, j = 1, \ldots, N \tag{2}$$

$$H_{i,j} = D_{i,j} - D_{i-1,j} \qquad i = 1, 2, j = 2, \ldots, N \tag{3}$$

$$\tau_{i,j} = b \times U_{i,j} + a \times V_{i,j} \qquad i = 1, 2, j = 2, \ldots, N \tag{4}$$

Equation 1 indicates that the arrival time of vehicle i at stop j ($A_{i,j}$) is equal to its departure time at stop $j - 1$ ($D_{i,j-1}$) plus the running time between the two stops plus time lost in acceleration and deceleration. Equation 2 relates departure time to arrival time and dwell time. Equation 3 states that the departure headway of Bus i at a stop is the difference in departure times between itself and the preceding Bus $i - 1$ at the stop, assuming passing is not allowed or at least will not occur for the three buses under consideration. Equation 4 estimates the bus dwell time at each stop based on the number of passengers who will board and alight at the stop, denoted by $U_{i,j}$ and $V_{i,j}$, respectively.

It should be noted that the preceding dynamic equations assume that vehicles will not pass each other over the planning horizon. This assumption has been used in all existing models and its implication requires future research. However, it should be pointed out that the issue of passing and its consequence are less a problem in this model than in some existing models. This is because this model is expected to be implemented in a rolling-horizon optimization framework in which optimal decisions are applied only to those buses that need a decision at the time the optimization process is invoked. If the optimal stop-skipping decision does lead to passing, it should be reflected in the next round of optimization.

Two initial conditions need to be provided with Equations 1 to 4: the departure times of Bus 0 at all stops, $D_{0,j}$, for $j = 1, \ldots, N$, and the departure times of all buses at Stop 1, $D_{i,1}$ for $i = 0$, 1, and 2. The former is either known (when Bus 0 has already passed the stop at the time of dispatching Bus 1) or can be predicted by the control center based on Bus 0's current location and traffic conditions. In this study, a simple model was used to predict the estimated time of arrival based on current vehicle location (which could be obtained with automatic vehicle location technology), average travel speed, and average passenger arrival rates. It is also assumed that buses' departure times at the terminal follow scheduled dispatch headway and their earliest available times at the terminal (after finishing their previous trips and having a minimum amount of layover time).

The number of passengers boarding and alighting a bus can be estimated with the following recursive equations:

$$U_{i,j} = y_{i,j} \sum_{k=j+I}^{N} W_{i,jk} y_{i,k} \qquad i = 1, 2, j = 1, \ldots, N - 1 \tag{5}$$

$$V_{i,j} = y_{i,j} \sum_{k=1}^{j-1} W_{i,kj} y_{i,k} \qquad i = 1, 2, j = 2, \ldots, N \tag{6}$$

$$W_{i,jk} = L_{i-1,jk} + \lambda_{j,k} H_{i,j} \qquad i = 1, 2, j = 1, \ldots, N \tag{7}$$

$$L_{i,jk} = W_{i,jk} - W_{i,jk} y_{i,k} y_{i,j} \qquad i = 1, 2, j = 1, \ldots, N \tag{8}$$

Equation 5 indicates that the expected number of passengers who will board Bus i at Stop j (assuming Bus i stops at Stop j) depends on the number of passengers traveling between Stops j and k ($k > j$) and whether the bus will stop at Stop k. Similarly, Equation 6 indicates that the expected number of alighting passengers for Bus i at Stop j (assuming Bus i stops at Stop j) depends on the number of passengers traveling between Stops k and j ($k < j$) and whether the bus will make Stop k. The number of passengers waiting for Bus i at Stop j whose destination is Stop k depends on the number of passengers skipped by Bus $i - 1$ at Stop j, $L_{i-1,jk}$, and the average number of passengers who arrive at Stop j after Bus $i - 1$ leaves Stop j. Equation 8 specifies that the number of passengers destined for Stop k who are stranded by Bus $i - 1$ at Stop j, $L_{i-1,jk}$, will be 0 if Bus $i - 1$ stops at Stops j and k but otherwise will equal the number of passengers waiting for Bus $i - 1$ at Stop j who have Stop k as their destination.

Application of Equations 7 and 8 requires initial conditions for the number of passengers skipped by Bus 0 at Stop j—that is, $L_{0,jk}$. Because Bus 0 is not allowed to skip any stops and because capacity is assumed not to be a restrictive factor, $L_{0,jk} = 0$ for $j, k = 1, \ldots, N$. Furthermore, it is assumed that there will be no passengers boarding at Stop N and alighting at Stop 1—that is, $U_{i,N} = 0$ and $V_{i,1} = 0$.

Optimization Model

Transit operations control problems can be formulated in many different ways, depending on the choice of performance criteria and operating constraints considered. The most commonly used objective function aims to minimize the total passenger waiting time plus a discounted amount of the delay to onboard passengers. Because one of the benefits that are expected from the skipping control proposed in this study is a reduction in bus trip time, bus trip time is included in the objective function. In particular, this model is formulated to minimize the equivalent total cost of passenger waiting time and passenger in-vehicle time as well as vehicle travel time subject to the previously formulated system state equations, recursive relationships, initial conditions, and variable restrictions. Stated mathematically this yields the following model:

$$\min Z = c_1 \sum_{i=1}^{2} \sum_{j=1}^{N} \left[(U_{i,j} - L_{i-1,j}) \cdot \left(\frac{H_{i,j}}{2 + L_{i-1,j}} \right) \cdot \left(\frac{H_{i-1,j}}{2 + H_{i,j}} \right) \right]$$
$$+ c_2 \sum_{i=1}^{2} \sum_{j=1}^{N-1} \sum_{j+1}^{N} \left\{ U_{i,jk} \cdot \sum_{f=j+1}^{k} [r_f + (\tau_{i,f} + \delta) \cdot y_{i,f}] \right\}$$
$$+ c_3 \sum_{i=1}^{2} \sum_{j=2}^{N} [r_j + (\tau_{i,f} + \delta) \cdot y_{i,j}] \qquad (9)$$

subject to Equations 1 to 8.

$$y_{i,1} = y_{i,N} = 1, \qquad i = 1, 2 \qquad (10)$$

$$y_{2,j} = 1, \qquad j = 2, \ldots, N-1 \qquad (11)$$

$$y_{1,j} \in \{0, 1\} \qquad j = 2, \ldots, N-1 \qquad (12)$$

The first term of the objective function includes two components. The first component, $(U_{i,j} - L_{i-1}, j)H_{i,j}/2$, computes the total waiting time of the passengers who arrive after the departure (or passing) of Bus $i - 1$ at Stop j, assuming random arrival with an average passenger waiting time equal to half the headway. The second component represents the total waiting time of those passengers who have been stranded by Bus $i - 1$ (L_{i-1}, j) and have to wait for an average amount of time equal to ($H_{i-1,} j/2 + H_{i,j}$).

The second term in the objective function calculates the total in-vehicle time of passengers summed over all O-D pairs. The final term computes the total bus trip time. Because time values associated with passengers and transit vehicles are not of equal importance, these terms are converted to common units of cost in dollars with the weighting factors $c1$, $c2$, and $c3$ for passenger waiting time, in-vehicle time, and bus trip time, respectively. Constraint 10 specifies that stops 1 and N are not to be skipped by Bus 1 or 2, and Constraint 11 imposes a further no-stop-skipping policy on Bus 2.

Solution Method

The problem formulated in the previous section is a nonlinear 0, 1 programming problem, which can be solved with nonlinear optimization techniques. In this study, because it is necessary only to decide the stop-skipping pattern of one bus, the problem scale is relatively small. As a result, it was decided to use an exhaustive search method for optimally solving the problem. The algorithm complexity is exponential—that is, on the order of 2^{N-2}, where $N - 2$ is the number of intermediate stops on the route. Experiments on a set of realistic cases suggested that this complexity was acceptable for the simulation analysis. Cases with one-way 14 stops (see Sensitivity Analysis section) were simulated on a 1.5-GHz personal computer and it was found that the simulation speed was three to five times faster than real-time speed. This algorithm was integrated into a simulation model for evaluating the effectiveness of the proposed model, as detailed in the following section.

SENSITIVITY ANALYSIS

The stop-skipping optimization model discussed in the previous section was established on the basis of a number of assumptions such as deterministic travel time and constant headway. From a theoretical point of view, systems with this stop-skipping control strategy should always outperform those without this control. However, it is unclear what magnitude of benefits could be expected from application of this strategy and under what conditions this strategy is most beneficial. The objective of this section is to shed some light on these two issues through a sensitivity analysis with results from a simulation model.

The simulation model used is called SimTransit, which was developed specifically for modeling bus operations under a variety of operating conditions and dispatch controls (*17, 18*). The model includes three main components: a dispatch module, a traffic simulator, and a geographic information systems-based animator. The dispatch module is a representation of a transit dispatch center, integrating functions such as service monitoring, state prediction, and dispatching. The module was modified in this study to include the skipping optimization model, which determines the optimal skipping pattern based on the estimated times of arrival of individual buses under consideration and the expected passenger demand.

The sensitivity analysis was performed on a real-life bus route (Route 7D) operated by Grand River Transit (GRT), Regional Municipality of Waterloo, Ontario, Canada. Route 7D is located in the twin cities of Kitchener and Waterloo, which have a combined population of 293,800 within an area of 203 km^2. Route 7D includes 28 major stops, starting from the Transportation Centre (TC) located in downtown Kitchener, via the University of Waterloo (a major O-D), and back to the TC terminal, as indicated in Figure 2. The original headway is 7.5 min. The afternoon peak period for this route was used (the base demand profile for this route is presented in Figure 3). These data were partially provided by GRT and partially collected by University of Waterloo students around May 2001. Average passenger boarding and alighting times were assumed to be 4 and 2 s per passenger, respectively. A total of 15 buses were used and each bus was dispatched from the TC terminal at a headway of 5 min. A uniform deviation of ±90 s from the scheduled headway was introduced to model the inherent variation in dispatching headway. To model bus travel time variation, a normal distribution with a coefficient of variation (COV) of 0.20 was assumed for all links along the route. The assumed travel time COV (0.20) represents the typical variation observed in the field. Two stops were selected as the control points: the TC terminal (Stop 1) and the University of Waterloo station (Stop 14). Values of \$20/h, \$10/h, and \$50/h were used for the objective function weighting factors $c1$, $c2$, and $c3$, respectively.

Each simulation run generates statistics on the following four measures of effectiveness: (*a*) passenger waiting time, (*b*) passenger in-vehicle time, (*c*) bus travel time, and (*d*) total weighted cost. These measures of effectiveness are used in the following sensitivity analysis on three model parameters: passenger demand, headway, and travel time. For each parameter setting, bus operations with and without skipping controls were simulated and relative reductions in these four performance measures were used for comparison. In the simulation, the first hour was treated as a warm-up period for the total 6-h simulation run.

Sensitivity to Passenger Demand

To determine how the effectiveness of the proposed control strategy depends on the level of passenger demand or how the proposed strategy would perform during different times of day, cases with four levels of O-D demand, including base demand, 1.5 × base demand, 1.8 × base demand, and 2.0 × base demand, were simulated. The simulation results are presented in Figure 4. As expected, the total weighted cost, which was to be minimized explicitly in the underlying optimization process, improved under all demand scenarios. The magnitude of the improvement, however, depended on the level of passenger demand. At low passenger demand (base case), the total

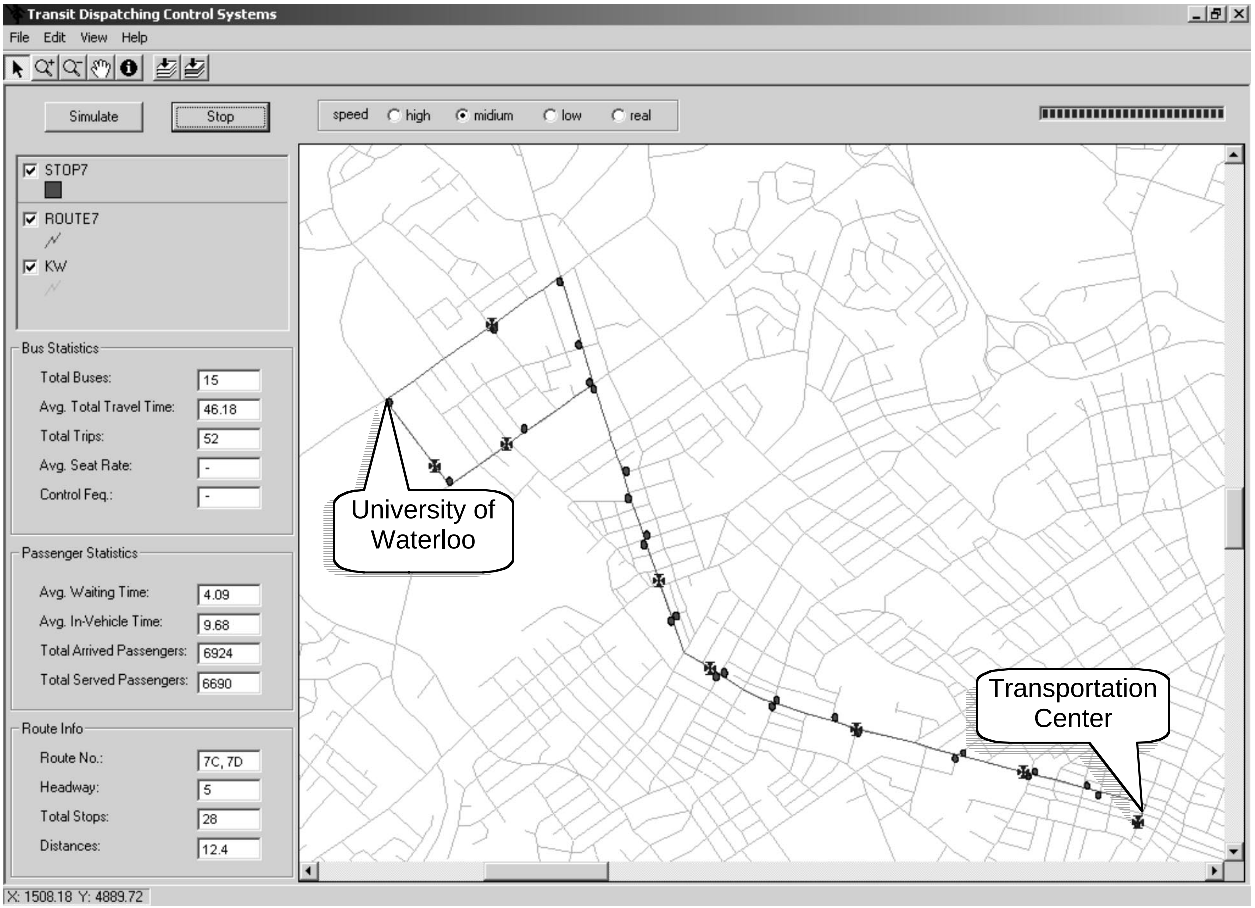

FIGURE 2 Simulation model interface with an example route.

combined benefit was relatively small (<1%). In fact, the implementation of skipping control had the negative effect of increasing passenger waiting time. This is because of the two opposing effects the stop-skipping controls have on passengers. On the one hand, stop skipping will increase the waiting time of those passengers whose O-D stops are skipped. On the other hand, allowing buses to skip some stops can prevent or mitigate bus bunching and thus reduce passenger waiting time. At a low level of passenger demand, bus bunching is less likely to develop, and therefore the effect of increasing passenger waiting time is more likely to be dominant.

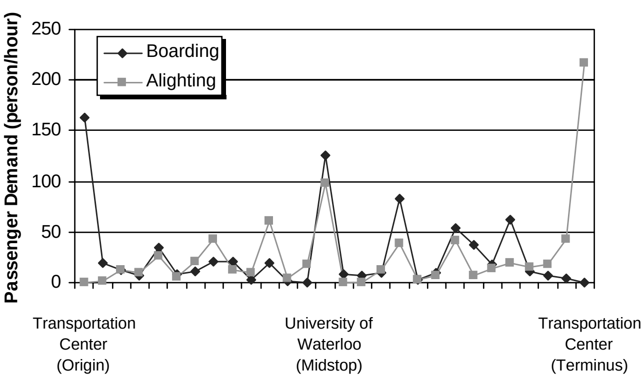

FIGURE 3 Passenger demand profile.

As passenger demand increases, the reduction in passenger waiting time, in-vehicle time, and bus trip time also increases. This trend continues until the demand reaches a certain level, beyond which both passenger waiting time and in-vehicle time start to decrease, compared with bus travel time. As a result, there is an optimal level of demand at which the total benefit or reduction in total weighted cost is maximized. This peaking phenomenon can also be attributed to the two conflicting effects of the stop-skipping strategy discussed previously.

Sensitivity to Bus Travel Time Variation

Variation in bus travel time is another factor that causes buses to deviate from their scheduled headways. This variation can cause buses to bunch and to run behind schedule. The degree of variation in travel time therefore should have some impact on the effectiveness of the proposed stop-skipping control strategy, which was intended to prevent bunching and reduce lateness. Intuitively, the higher the travel time variability, the higher the bus headway variability, and thus the more opportunities there are for applying stop-skipping controls. However, higher travel time variation also means larger errors in the estimated times of arrival used as input to the optimization model. This could result in suboptimal scheduling solutions. This intuitive observation is supported by the simulation results presented in Figure 5, where the curves represent the relationship

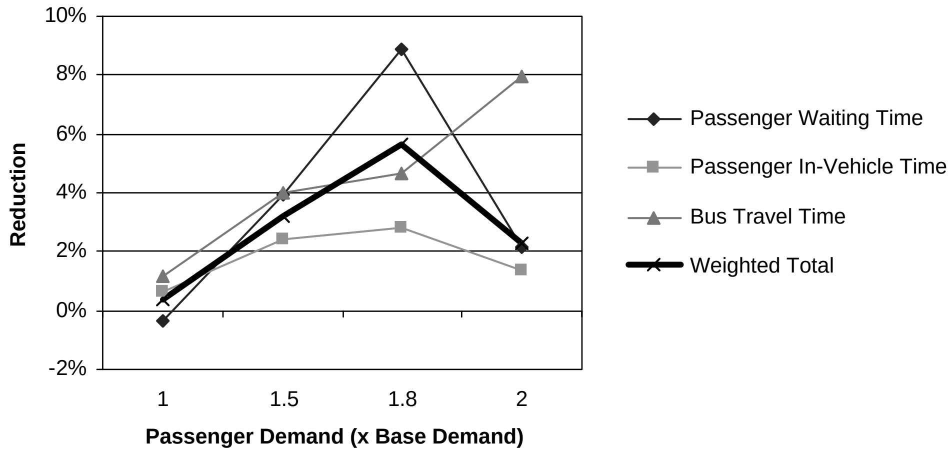

FIGURE 4 Control effectiveness versus passenger demand.

between the four measures of effectiveness and the variability of link travel time. The high-demand scenario of 1.8 × base demand was used in this analysis and the variability of link travel time was defined by the COV or the ratio of standard deviation to mean.

For the case simulated, the stop-skipping control is most effective when the coefficient of variation of travel time is close to a critical value of 0.20. When the travel time variability is smaller than the critical value, the control strategy is still beneficial but with reduced benefits. The benefits decrease quickly when the variability goes beyond the critical value, suggesting that cautions must be taken when applying the control model to highly varied traffic conditions, as it could have no effect at all or, even worse, be countereffective.

Sensitivity to Headway

When a stop is skipped, the passengers who are waiting at the stop must wait for the next bus and consequently incur the additional waiting time of one headway. Intuitively, the smaller the operating headway is, the smaller the effect skipping controls will have on these passengers. To gauge the magnitude of this effect, the 1.8 × base demand scenario was simulated with bus operating headway varied at five levels—namely, 3, 5, 7, 10, and 15 min. The demand corresponding to the 5-min headway was set at 1.8 × base demand, which was then used as a base case to determine the level of demand for other headways using the scaling factor 5/*h*, where *h* is the headway under consideration. For example, for the case with a headway of 10 min, a scaling factor of 0.5 (= 5/10) was used to calculate the demand. The logic behind this is that headways for transit routes in high-demand corridors usually are determined on the basis of passenger demand. The higher the travel demand is, the lower the headway usually will be. The simulation results for this part of the study are presented in Figure 6.

As expected, the benefits of the stop-skipping control decrease monotonically as the operating headway increases. This pattern suggests that the proposed control is more appropriate for routes with a short headway than for those with a long headway. The total benefits approached 0 when the bus headway was increased to 10 min or more.

Combined Control

Past studies have suggested that stop-skipping control may be applied as a complement to another more popular bus control strategy—

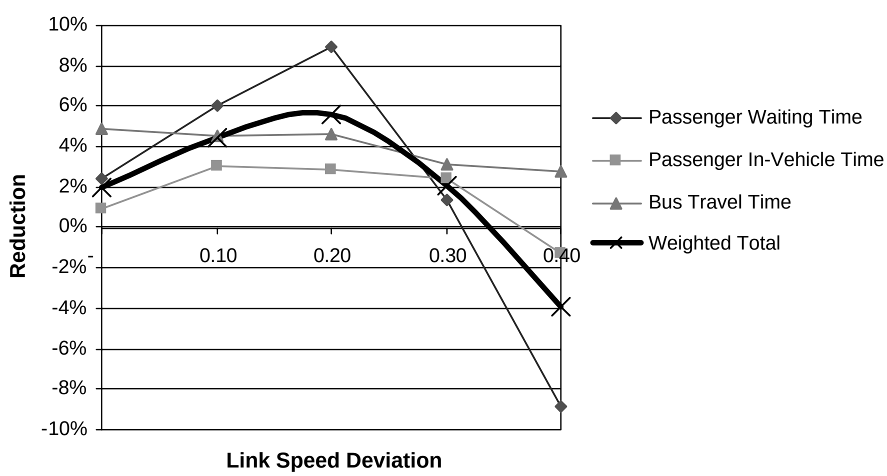

FIGURE 5 Control effectiveness versus link speed variations.

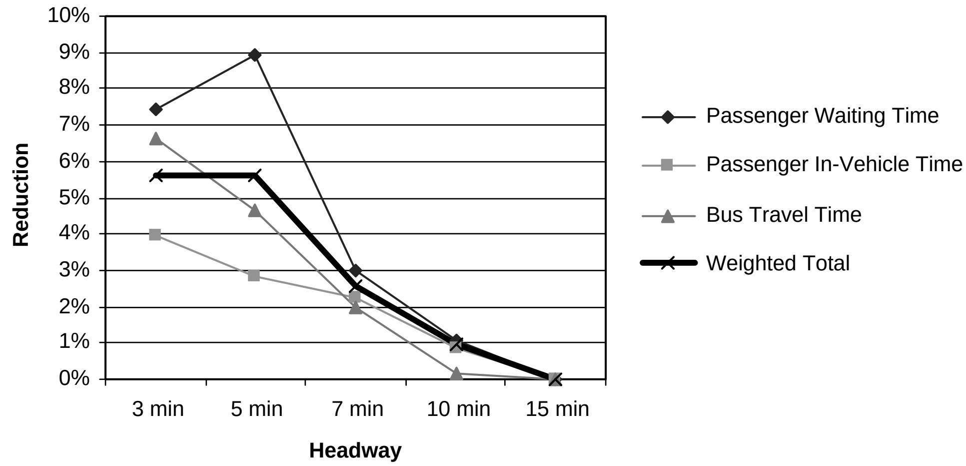

FIGURE 6 Control effectiveness versus headway.

namely, holding control (*8, 14*). The objective of holding control is to purposely delay those buses that are ahead of their planned schedule or too close to preceding buses. Conversely, stop-skipping control is used to speed up those buses that are late or too far from the preceding buses. To study this effect, a simulation was run that combined holding and skipping control strategies and compared the results with those in which only holding or skipping strategies were applied. The results are presented in Table 1. It can be observed that the holding plus skipping control has indeed improved the system performance compared with the skipping or holding control applied in isolation. This suggests that the negative effect on bus travel time caused by the holding control has been compensated by incorporating the stop-skipping strategy.

CONCLUSIONS AND FUTURE RESEARCH

In this paper, a new bus operations control strategy has been proposed in which stop skipping is applied to every other bus dispatched from the terminal. The novelty of this control strategy is that a minimum service frequency can be ensured at all stops while both passengers and transit agencies can still enjoy the benefit of reduced travel time and operating costs. The underlying control problem was formulated as a nonlinear 0, 1 programming problem and solved through an exhaustive search process. A simulation model was used in a sensitivity study to investigate the impacts of changes in various operating conditions such as demand, travel time, and headway on the effectiveness of the control strategy. The analysis has provided the following insights:

1. Stop-skipping control is an effective strategy to improve transit service quality and operating efficiency. Passenger in-vehicle time, waiting time, and operation vehicle trip time can be reduced in a wide range of operating conditions.
2. Stop-skipping controls are most effective on those bus routes with high passenger demand and short headway.
3. Stop-skipping controls should be used only on those routes with an appropriate range of travel time variation. Routes with travel time variation that is too low or too high may not benefit from this strategy.
4. A stop-skipping strategy can be applied in combination with other controls such as a holding control for further improvement in system performance.

The work presented in this paper is by no means complete and further research is needed in the following directions. First, faster algorithms need to be developed to replace the currently implemented enumeration method if large-sized problems are to be solved in real time. Second, more accurate prediction models should be developed to truly take advantage of real-time information on bus location, travel time, and passenger counts. Lastly, integration with other controls such as real-time deadheading and short turning should be explored to maximize the potential of the proposed strategy.

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TABLE 1 Effects of Holding and Stop-Skipping Controls

Control Strategy	Reduction in Average In-Vehicle Time	Reduction in Average Waiting Time	Reduction in Average Bus Trip Time	Reduction in Weighted Total
Skipping	2.84%	8.91%	4.66%	5.62%
Holding	6.46%	30.72%	-0.67%	16.17%
Holding + Skipping	8.63%	32.56%	0.58%	18.11%

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