Project No. NCHRP 20-07/Task 306

Revision of Strut-and-Tie Provisions in the AASHTO LRFD Bridge Design Specifications

Final Report

Prepared for:

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1. Introduction

This report describes the conclusions of Project 20-07/Task 306 - Revision of Strut-and-Tie Provisions in the AASHTO LRFD Bridge Design Specifications. The first phase of the project involved a comprehensive review of other codes and design guidelines, developing proposed revisions to the AASHTO Specifications and Commentary and the presentation of the proposed changes to the AASHTO SCOBS Technical Committee T-10.
A preliminary version of the proposed changes was presented to the AASHTO T-10 Committee in Salt Lake City on October 22, 2011.

The second phase of Project 20-07/Task 306 involved the development of design examples to illustrate the proposed changes to the Strut-and-Tie Provisions of the AASHTO LRFD Bridge Design Specifications.

2. Review of Codes and Guidelines

2.1 Documents Reviewed

A comprehensive review has been completed on modeling and design using strut-and-tie models. The documents reviewed include:

- TxDOT Project 0-5253 (Birrcher et al. 2008)
- NCHRP 20-07/Task 217 (Martin and Sanders 2007)
- Eurocode 2 (EN 2004)
- ACI 318-11 (ACI 318 2011)
- CAN/CSA A23.3-04 (CSA 2004)
- CEB-FIP Model Code 1990 (CEB 1990)
- 2010 fib Model Code (FIB 2010)
- 1999 FIP Recommendations on the Practical Design of Structural Concrete (FIP 1999)

Some comments on the review of other codes and research reports:

- ACI 318-11 has simple factors for the compressive strength of struts but do not directly account for tensile straining of reinforcement passing through a strut and hence need to restrict the angle between struts and ties to be not less than 25 degrees. The influence of tensile straining on the strength of struts has been retained in the proposed specifications to account for struts forming at small angles to the ties (e.g., foundation element subjected to uniform loading) and to allow the designer to take advantage of excess tie reinforcement or the benefits of using prestressed ties. These features are not accounted for in ACI 318-11.
- The FIP Recommendations allow an increase in the bearing stresses for local confining effects that is similar to that proposed.
- The current AASHTO strut-and-tie specifications are based on the 1984 CSA Standard.
- The NCHRP 20-07, Task 217 Report discusses several issues that have been considered. These include adjusting the compressive stress limit for struts to account for high-strength concrete and the anchorage of ties at bearing areas.
- The TxDOT Project 0-5253 report provides a review of many different test series as well as test results from an extensive research program carried out at the University of Texas. They

recommended accounting for the beneficial confinement effects at bearings, similar to that proposed in this project. It was also proposed that provided the main tension tie at a bearing location was properly anchored, it is not necessary to check the nodal zone stress limit on the back face of the anchorage zone. They also proposed keeping the crack control reinforcement ratio as 0.003 times the effective area of the strut.

2.2 Recent Research Findings

The authors have also included experience gained from a large number of experiments on disturbed regions carried out at McGill University and the University of Toronto. A paper accepted for publication in the ACI Structural Journal titled "A Two Parameter Kinematic Theory for the Shear Behavior of Deep Beams" (Mihaylov, Bentz and Collins 2011) has provided insight into a number of behavioral aspects. The theory was built on observing in great detail the deformation patterns of a large number of deep beams (Mihaylov, Bentz and Collins 2010). The theory is based on a kinematic model which describes the deformed shape of diagonally cracked deep beams by means of just two input parameters. In addition to the kinematic conditions the theory includes equations for equilibrium and stress-strain relationships for the materials. It has identified the key components of shear resistance in deep beams (see Fig. 1), is capable of predicting crack widths (see Fig. 2) and has demonstrated that there is a size effect in deep beams (see Fig. 3).

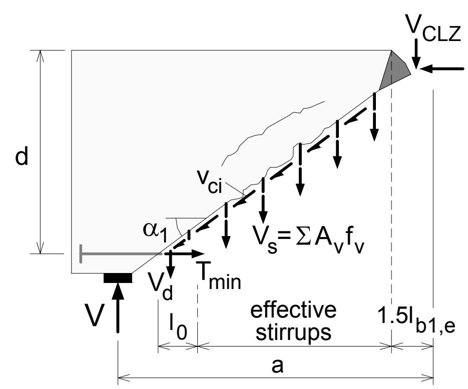


Figure 1: Components of shear strength in deep beam (Mihaylov, Bentz and Collins 2011)

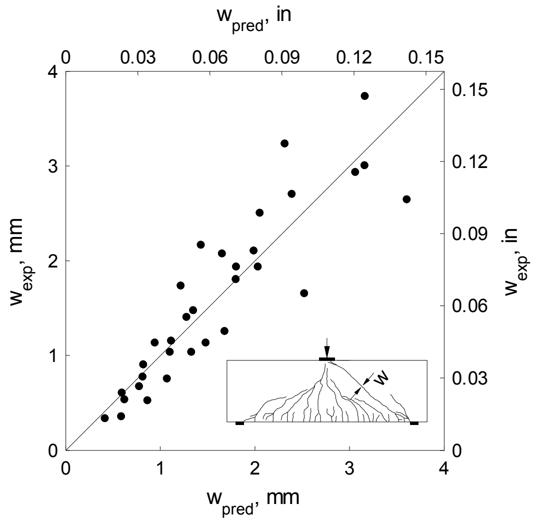


Figure 2: Predictions of the kinematic model for crack widths (Mihaylov, Bentz and Collins 2011)

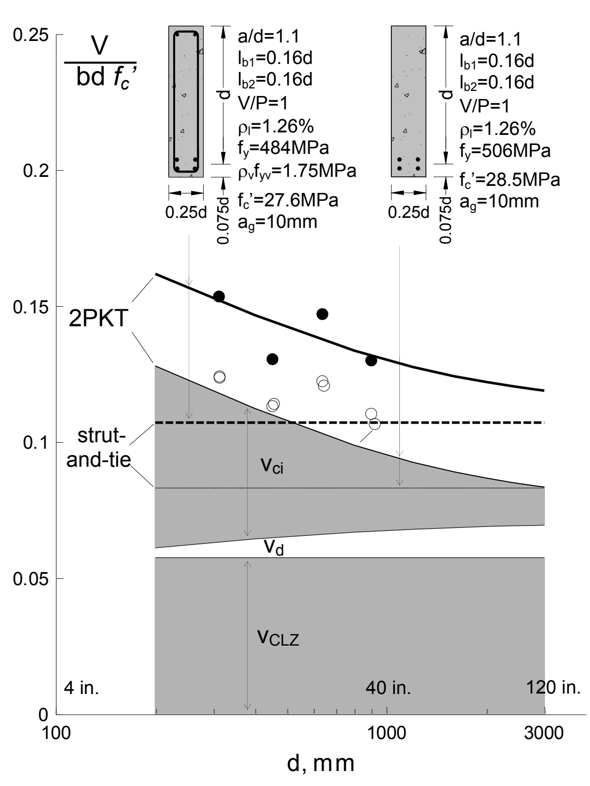
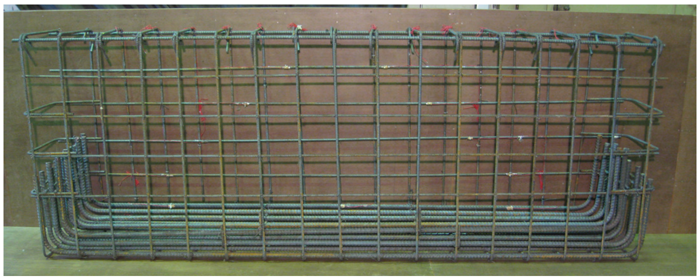
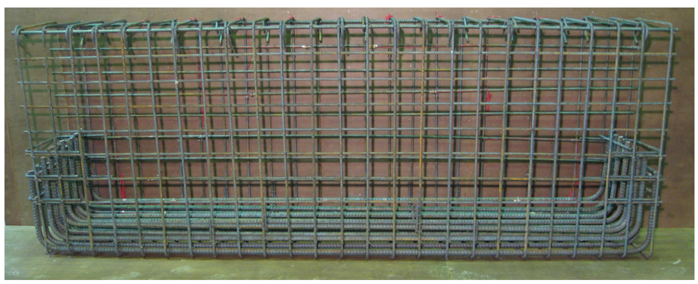


Figure 3: Size effect in deep beams with and without transverse reinforcement - theoretical predictions with 2 Parameter Kinematic Theory (2PKT) (Mihaylov, Bentz and Collins 2011), AASHTO strut-and-tie model and experimental results (Zhang and Tan (2007)).

Experimental research at McGill University (DiTommaso 2012) involves the testing of pairs of deep beams with 0.2% and 0.3% crack control reinforcement. Three pairs of beams were constructed with specified concrete compressive strengths of 30, 50 and 70 MPa (4.35, 7.25 and 10 ksi) concrete. The beams were 1200 mm (47 in.) deep, 550 mm (21.7 in.) thick and had a realistic concrete cover of 50 mm (2 in.). The main tension tie reinforcement consisted of 15-25M (1 in. diameter) bars placed in 3 layers. Figure 4 shows the reinforcing cages. The experimental results provide a comparison of the level of crack control reinforcement and the influence of concrete strength.



(a) Beam N-0.2 containing 0.2% uniformly distributed reinforcement



(b) Beam N-0.3 containing 0.3% uniformly distributed reinforcement

Figure 4: Deep beam tests at McGill University (DiTommaso 2012)

Beam N-0.3, which contained crack control reinforcement having a reinforcement ratio of 0.003, provided considerably improved response than a companion specimen with a reinforcement ratio of 0.002 (Beam N-0.2). Both beams had a concrete compressive strength of 38.2 MPa (5.5 ksi). The beams had a small bearing plate at the load point such that crushing of the concrete at this bearing would be critical. The beam with a reinforcement ratio of 0.003 was able to redistribute the stresses such that a higher failure load was achieved and the ductility was improved compared to the beam with a reinforcement ratio of 0.002 (see Fig. 5). The measured maximum crack widths at service load level were 0.3 mm (0.012 in.) and 0.15 mm (0.006 in.) in Beams N-0.2 and N-0.3, respectively.

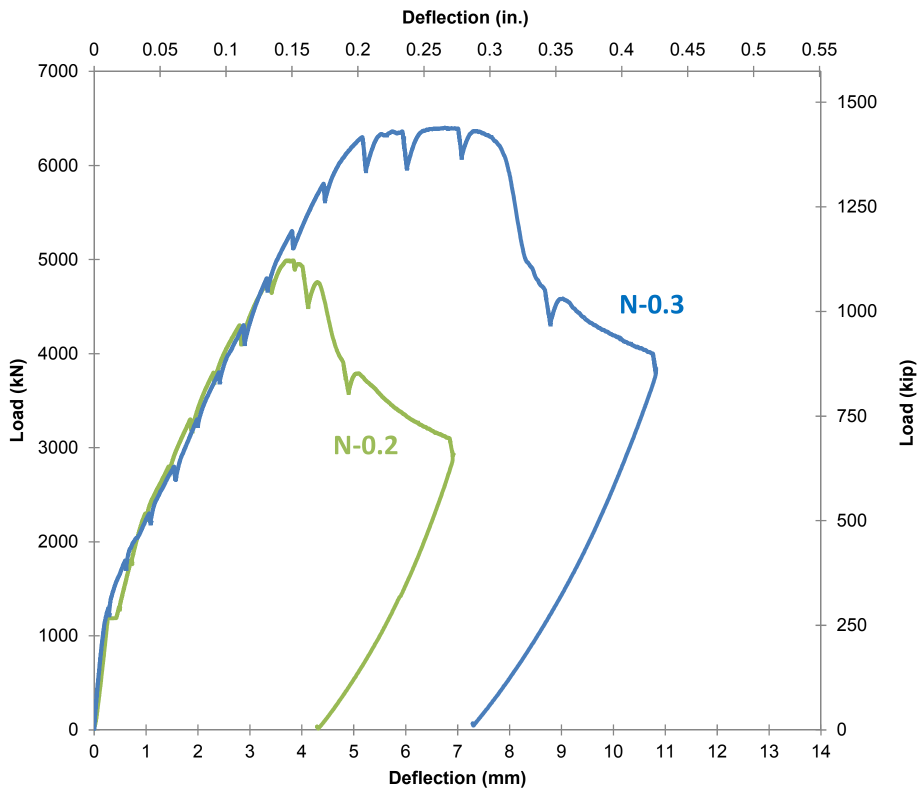


Figure 5: Comparison of load-deflection responses of deep beams N-0.2 and N-0.3 (DiTommaso, 2012).

3. Design Recommendations

It is noted that the development of the proposed AASHTO Specifications and Commentary was carried out in Task 3 involving the development of the proposed changes to the Specifications in codified form. Task 2 involved developing the key aspects leading to the proposed changes.

The issues that were considered in developing the proposed revisions are given below and have been separated into three headings "Reducing Conservatism", "Increasing Safety" and "Simplifying Design".

3.1 Reducing Conservatism

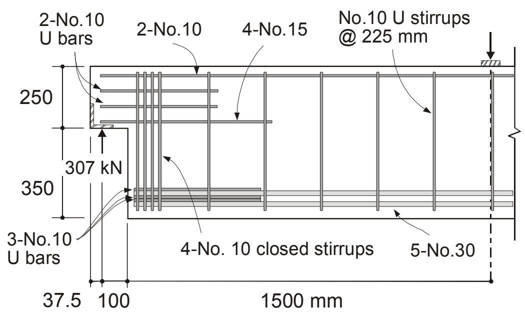
The issues that were considered that relate to reducing the conservatism of the current strut-and-tie design approach are:

3.1.1 Effective Width of Struts

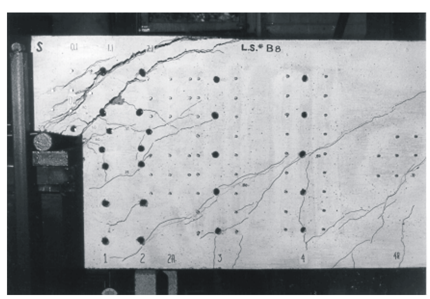
- Current Specifications (5.6.3.3.2): "When a strut is anchored by reinforcement, the effective concrete area may be considered to extend a distance of up to six bar diameters from the anchored bar".

- Proposed Change: Test results on full-scale dapped ended beams (see Fig. 6) (Mitchell et al. 2002) were designed such that the crushing of the struts occurred in a region where the primary strut was anchored by reinforcement. These tests indicated that the effective concrete area extended a distance of eight bar diameters from the anchored bar. It is proposed to change the distance "six bar diameters" to "eight bar diameters" in Article 5.6.3.3.2 and change Figure 5.6.3.3.2-1 in the Commentary.

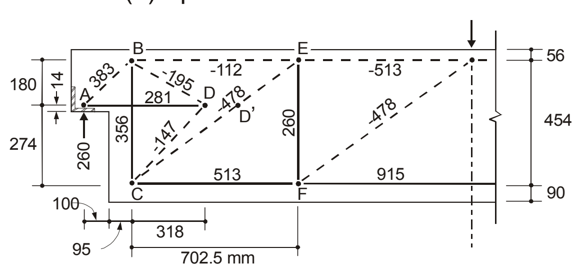
Effect of Proposed Change: This will reduce conservatism in assessing the strength of struts anchored by reinforcement.



(a) Details



(b) Specimen after failure



(c) Strut-and-tie model (forces in kN)

Figure 6: Beam with dapped ends (Mitchell et al. 2002).
(1 kN = 0.2248 kips, 1 mm = 0.0394 in.)

3.1.2 Limiting Stresses in Bearing Zones

- Current Specifications (5.6.3.5): The limiting stresses in the current specifications do not account for the beneficial effects of confinement.

- Proposed Change: Introduce a factor to account for confinement for cases where “the supporting surface is wider on all sides than the loaded area”. For simplicity and consistency the “modification factor, m”, used in Article 5.7.5 – “Bearing” could be used in Article 5.6.3.5 to modify the permissible bearing stresses where appropriate.

Effect of Proposed Change: This would reduce conservatism and maintain consistency with the specifications in Section 5.7.5.

3.1.3 Refined Strut-and-Tie Models

- Current Specifications (5.6.3.2): The structural modeling in this section does not distinguish between simple strut-and-tie models and refined strut-and-tie models.

- Proposed Change: Refined strut-and-tie models provide more realistic flow of stresses in disturbed regions. Add a sentence that permits the designer to use a simple strut-and-tie model using only the main tie reinforcement or a refined strut-and-tie model where the influence of crack control reinforcement is accounted for in the strength calculations.

For example, the tie force T_v shown in Fig. 7 is taken as the resultant yield force of the vertical “crack control” reinforcement, having a reinforcement ratio, ρ_v , over an effective length of $a_{cl}/2$ and located in the centre of the clear shear span, a_{cl} .

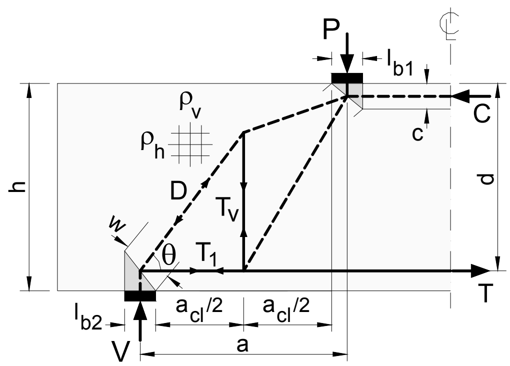


Figure 7: Refined strut-and-tie model takes account of crack control reinforcement and curving compressive stresses in deep beam (Mihaylov, Bentz and Collins 2011).

Effect of Proposed Change: The proposed change would permit the designer to choose between a simple strut-and-tie model and a refined strut-and-tie model. A refined strut-and-tie model will often involve accounting for the presence of “crack control” reinforcement that is typically neglected in the simple strut-and-tie model. This will

reduce conservatism, and would lead to more realistic strength predictions, particularly for cases with significant amounts of "crack control" reinforcement.

3.1.4 Crack Control Reinforcement Requirements

- Current Specifications (5.6.3.6): The amount of crack control reinforcement is specified by the following statement:

  "The ratio of reinforcement area to gross concrete area shall not be less than 0.003 in each direction."

- An attempt was made to develop an expression for the amount of crack control reinforcement that was a function of the concrete compressive strength, f'_c. This followed a similar philosophy as the requirements for minimum amount of shear reinforcement in beams, with the amount of reinforcement being a function of $\sqrt{f'_c}$.
- The following proposed wording was developed and was presented at the T-10 Committee meeting in Salt Lake City on October 22, 2011:

  > "The ratio of reinforcement area to gross concrete area in each direction shall not be less than $0.002\left(0.4+1.1\sqrt{\frac{f'_c}{10}}\right)$ but need not be taken greater than 0.003."

- During the T-10 Committee meeting the difficulties of relating laboratory measured crack widths on experimental specimens to actual crack widths occurring in bridges in service was discussed. It was decided that a reduction in the required amount of crack control reinforcement would not be prudent at this time. Large scale tests at the University of Texas indicated that the minimum reinforcement ratio of 0.003 was necessary to control crack widths at service load level and large-scale deep beam tests at McGill University illustrated the improved ductility when the crack control reinforcement ratio was increased from 0.002 to 0.003.

  It was decided not to recommend changes to the crack control reinforcement requirements at this time. Additional research in this area, particularly with respect to width of diagonal cracks in disturbed regions at service loads, may provide evidence to justify reducing the currently required amounts.

3.2 Increasing Safety

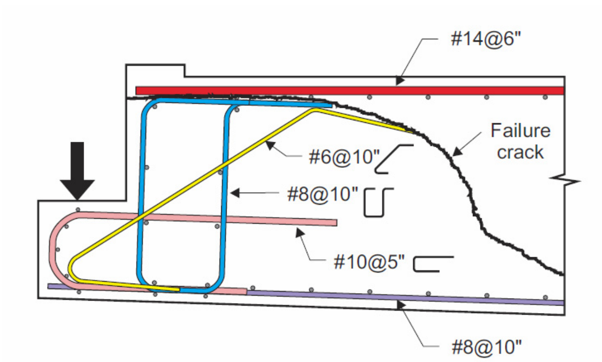
3.2.1 Accounting for Effects of High-Strength Concrete

- Current Specifications: The current specifications do not account for the more brittle compressive failures of high-strength concrete.

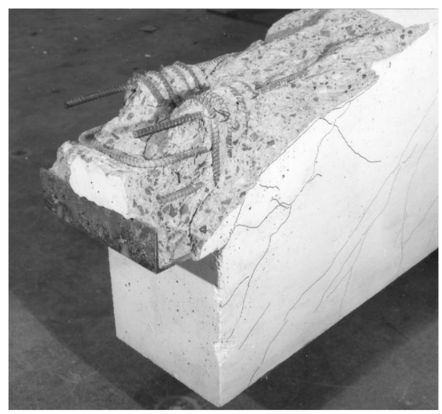
- While it is recognized that applying the current provisions to very high-strength concretes may result in some reduction in safety, it is proposed that no change be made to the compressive stress factors for struts and bearings in the current specifications. More experimental data is required in this area and furthermore such a change would have to be accompanied by changes to the flexural stress block factors in the rest of Section 5. This aspect partially influenced the decision not to liberalize the current crack control reinforcement requirements in the specifications.

3.2.2 Anchorage of Tension Ties

- Current Specifications: Section 5.6.3.4.2 on "Anchorage of Tie" requires that "The tension tie reinforcement shall be anchored to transfer the tension force therein to the node regions of the truss in accordance with requirements for development of reinforcement as specified in Article 5.11." There have been several examples of major distress in service and some failures of structures (see Fig. 8) due to improper anchorage of the ties.



(a) Inadequate anchorage of vertical tension tie in beam seat region of Concorde Overpass that collapsed in 2006 (Mitchell et al. 2011)



(b) Breakout of compressive strut between hooks of U-stirrups (closed stirrups should be used) (Mitchell et al. 2010)

Figure 8: Examples of inadequate anchorage details

- Proposed Change to Specifications: Requirements have been added on the proper anchorage of ties. The issues that have been addressed for the anchorage of tension ties include: (i) the need to provide anchorage in accordance with Article 5.11 using bar embedment or standard hooks, (ii) the addition of headed bars and prestressing anchorages as appropriate means of anchoring tension ties, (iii) the need for closed stirrups anchored around longitudinal reinforcing bars for tension ties anchoring compressive struts in regions away from bearing areas, and (iv) requirements for anchorage of headed bar reinforcement.

Effect of Proposed Change: Proposed changes will result in safer structures.

3.2.3 Structural Modeling of Uniform Loading

- Current Specifications (Section 5.6.3.2): The current Commentary does not give any examples of strut-and-tie models involving uniform loads.

- Background: More strut-and-tie modeling examples need to be given in the Specifications. Recent full scale tests (Perkins 2011) of footing-type specimens not containing crack control reinforcement have indicated that strut-and-tie models assuming two-point loads at the quarter points of the span, used to simulate uniform loading, can give unsafe predictions(see Fig. 9). It was found that a strut-and-tie model with fanning compressive struts gave more realistic predictions. Guidance for designers must be provided in the specifications and the commentary to avoid these unsafe predictions.

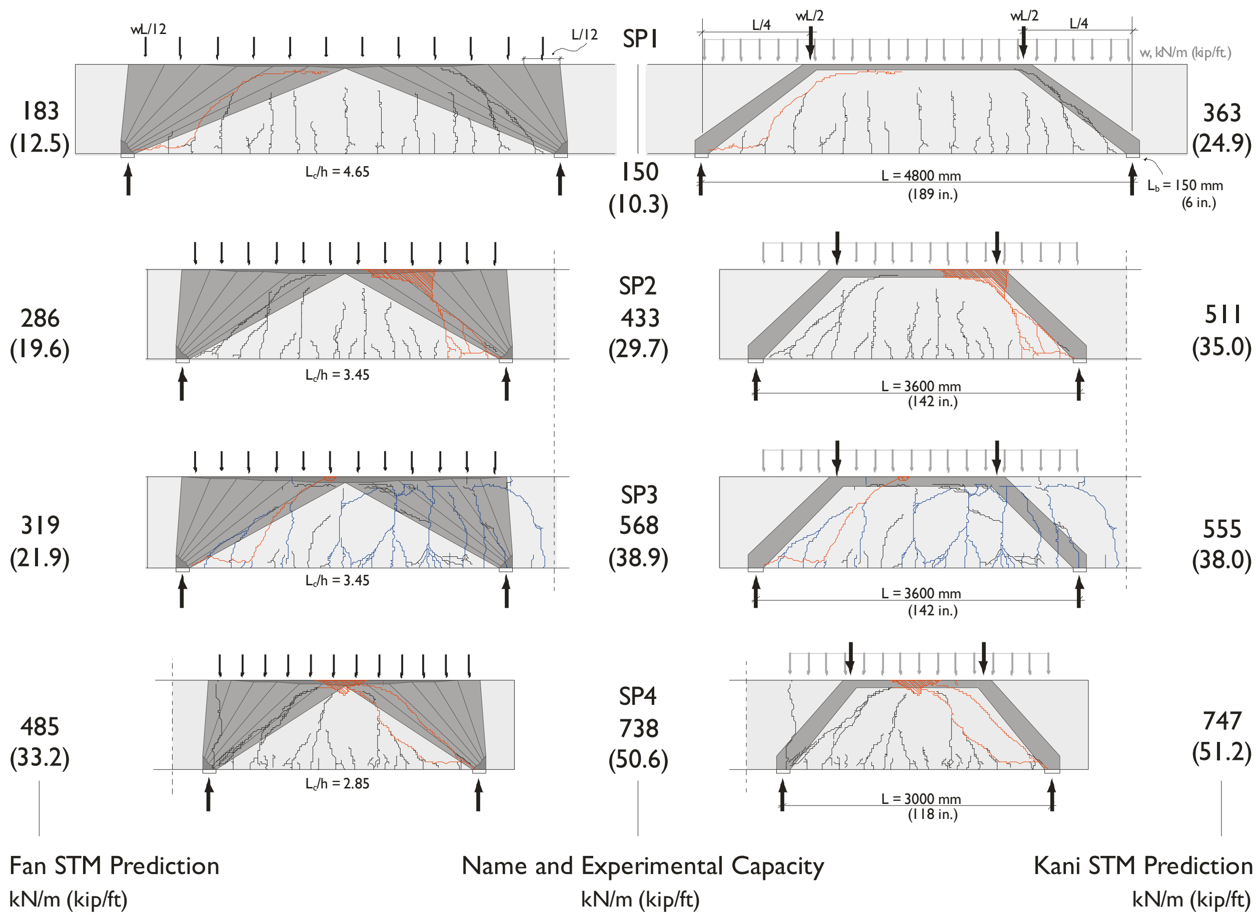


Figure 9: Comparisons of strut-and-tie models for footing-type elements subjected to uniform loading (Perkins 2011).

- Proposed Change: A new Article 5.6.3.2.2 - "Modeling Members Subjected to Uniform Loads" has been added with figures illustrating the modelling of uniform loading. The modeling of regions of fanning compression for cases with the load applied either on the flexural compression face or on the flexural tension face are illustrated. An addition to Article 5.6.3.3.2 – "Effective Cross-Sectional Area of Strut" explains how the

compressive stress in the struts representing the fanning compression should be determined.

- Additional prposed change: The experiments described in Figure 9 have shown that using the simplified procedure of 5.8.3.4.1 with $\beta = 2.0$ can be unconservative for concrete footings where the distance from the point of zero shear to the face of the column is less than $3d_v$. It is proposed that the limit for the use of this simplified method be reduced to $2d_v$.

Effect of Proposed Changes: The proposed changes will lead to safer designs for elements such as footings and thick slabs that do not contain crack control reinforcement and are subjected to uniform loading.

3.2.4 Accounting for Size Effect

- Current Specifications: The current specifications do not take account of the "size effect" when determining the strength of disturbed regions. Experimental studies by Zhang and Tan (2007) and analytical studies by Mihaylov, Bentz and Collins (2011) have shown that there is a size effect in deep beams (see Fig. 3).

No Change Proposed: There is no change proposed to account for the size effect. Although the size effect exists, the current strut-and-tie model gives typically conservative strength predictions as shown in Fig. 10.

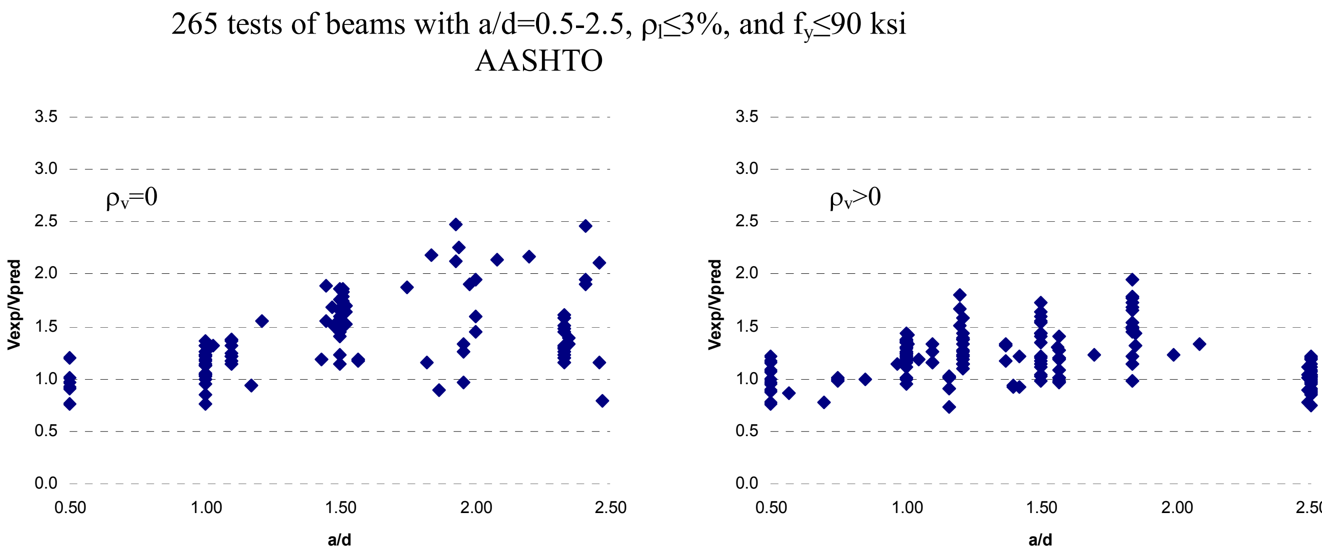


Figure 10: Comparison V_{exp}/V_{pred} as a function of a/d (Mihaylov, Bentz and Collins 2011).

It is noted that the predictions made for the strength for the members with transverse reinforcement used the refined strut-and-tie model shown in Fig. 7.

3.3 Simplifying Design

3.3.1 Checking Compressive Stresses in Struts

- Current Specifications: The current Specifications in Article 5.6.3.3.3 "Limiting Compressive Stress in Strut" provide equations for determining the limiting compressive stress in the struts as a function of f_c', the angle, α_s, between the strut and the tie as well as the strain in the tie, ε_s.

- Proposed Change: A simplified equation (5.6.3.3.3-3) has been introduced for determining the limiting compressive stress, f_{cu}, that is a function of only f_c' and the angle α_s.

Effect of Proposed Change: This addition will simplify the design process for typical designs where the yield strength of the reinforcing steel does not exceed 60 ksi.

3.3.2 Checking Compressive Stresses in Single Strut Representing a Fanning Region of Compressive Stresses

- Current Specifications: The current Specifications in Article 5.6.3.3.3 "Limiting Compressive Stress in Strut" do not give requirements for the determination of the limiting compressive stress in a strut representing a region of fanning compressive stress.

Effect of Proposed Change: This addition will simplify design of a fanning region using a single strut and will increase the safety of the design procedures for this important situation.

4. Proposed Changes to AASHTO Specifications and Commentary (Additions Shown in Red)

5.6.3 Strut-and-Tie Model

5.6.3.1 General

Strut-and-tie models may be used to determine internal force effects near supports and the points of application of concentrated loads at strength and extreme event limit states.

The strut-and-tie model should be considered for the design of deep footings and pile caps or other situations in which the distance between the centers of applied load and the supporting reactions is less than about twice the member ~~thickness~~ depth.

If the strut-and-tie model is selected for structural analysis, Articles 5.6.3.2 through 5.6.3.6 shall apply.

C5.6.3.1

Where the conventional methods of strength of materials are not applicable because of nonlinear strain distribution, the strut-and-tie modeling may provide a convenient way of approximating load paths and force effects in the structure. In fact, the load paths may be visualized and the geometry of concrete and steel selected to implement the load path.

~~The strut-and-tie model is new to these Specifications.~~ More detailed information on this method is given by Schlaich et al. (1987) and Collins and Mitchell (1991).

Traditional section-by-section design is based on the assumption that the reinforcement required at a particular section depends only on the ~~separated~~ values of the factored section force effects V_u, M_u, and T_u and does not consider the ~~mechanical interaction among these force effects as the strut-and-tie model does~~ manner in which the loads and reactions are applied which generated these sectional forces. The traditional method further assumes that the shear stress distribution ~~remains~~ is essentially uniform over the depth and that the longitudinal strains will vary linearly over the depth of the beam.

For members such as the deep beam shown in Figure C5.6.3.2-1, these assumptions are not valid. For example, the shear stresses on a section just to the right of the left support will be concentrated near the bottom face. The behavior of a component, such as the deep beam, can be predicted more accurately if the flow of forces through the complete structure is studied. Instead of determining V_u and M_u at different sections along the span, the flow of compressive stresses going from the loads P to the supports and the required tension force to be developed between the supports should be established.

For additional applications of the strut-and-tie model see Articles 5.10.9.4, 5.13.2.3, and 5.13.2.4.1.

5.6.3.2 Structural Modeling

5.6.3.2.1 General

The structure and a component or region, thereof, may be modeled as an assembly of steel tension ties and concrete compressive struts interconnected at nodes to form a truss capable of carrying all the applied loads to the supports. The required widths of compression struts and tension ties shall be considered in determining the geometry of the truss.

The factored resistance, P_r, of struts and ties shall be taken as that of axially loaded components:

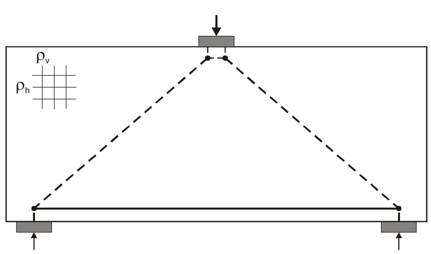
$$P_r = \phi P_n \qquad (5.6.3.2\text{-}1)$$

where:

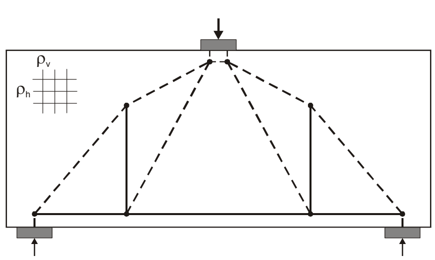
P_n = nominal resistance of strut or tie (kip)

ϕ = resistance factor for tension or compression specified in Article 5.5.4.2, as appropriate

The crack control reinforcement may also be used as tension ties in the strut-and-tie model provided this reinforcement is well anchored in accordance with Article 5.6.3.4.2 (see Figure 1).



(a) Simplified Strut-and-Tie Model



(b) Refined Strut-and Tie Model Using Crack Control Reinforcement as Additional Tension Ties

Figure 5.6.3.2-1 Simplified and Refined Strut-and-Tie Models for Deep Beam

C5.6.3.2

Cracked reinforced concrete carries load principally by compressive stresses in the concrete and tensile stresses in the reinforcement. After significant cracking has occurred, the principal compressive stress trajectories in the concrete tend toward straight lines and hence can be approximated by straight compressive struts. Tension ties are used to model the principal reinforcement.

A strut-and-tie truss model is shown in Figures C5.6.3.2-1 and C5.6.3.2-2. The zones of high unidirectional compressive stress in the concrete are represented by compressive struts. The regions of the concrete subjected to multidirectional stresses, where the struts and ties meet the joints of the truss, are represented by nodal zones.

Because of the significant transverse dimensions of the struts and ties, a "truss joint" becomes a "nodal zone" with finite dimensions. Establishing the geometry of the truss usually involves trial and error in which member sizes are assumed, the truss geometry is established, member forces are determined, and the assumed member sizes are verified.

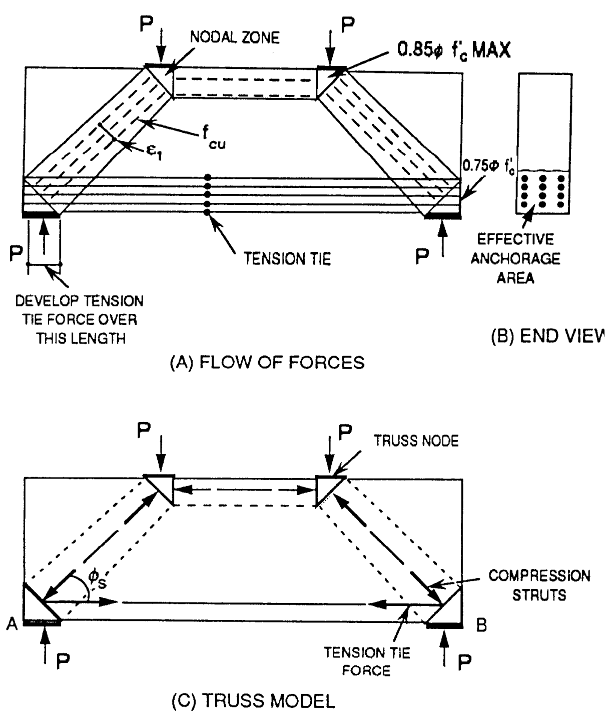


Figure C5.6.3.2-1 - Strut-and-Tie Model for a Deep Beam

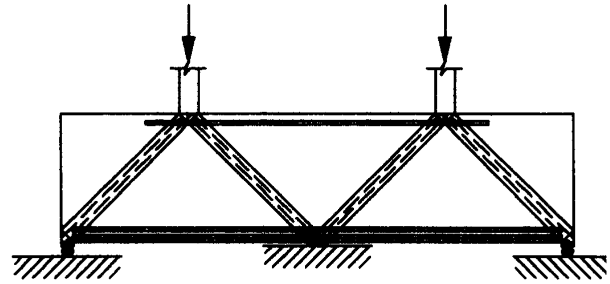
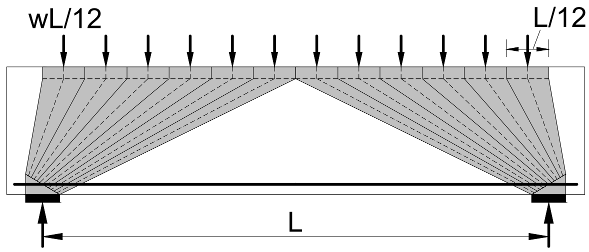


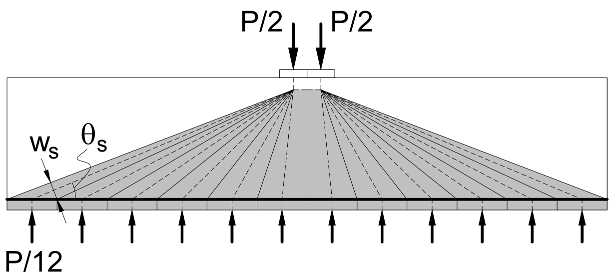
Figure C5.6.3.2-2 - Strut-and-Tie Model for Continuous Deep Beam

5.6.3.2.2 Modeling Members Subjected to Uniform Loads

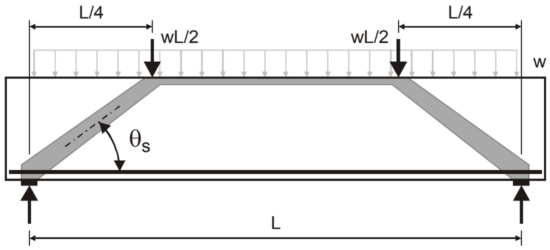
In modeling of members subjected to uniform loads, consideration shall be given to the fanning out of the compressive stresses from the bearing areas. A series of struts can be used to represent the fanning areas (see Figure 1(a) and (b)) or in cases where the uniform load is applied on the flexural compression face and the member contains crack control reinforcement the fanning area can be modeled by a single strut located at the resultant of the load (see Figure 1(c)).



(a) Modeling of Fanning Compression with a Series of Struts – Tension Tie at Narrow Part of Fan



(b) Modeling of Fanning Compression with a Series of Struts – Tension Tie at Wide Part of Fan



(c) Modeling each Fanning Area with a Single Strut for members containing crack control reinforcement.

Figure 5.6.3.2.2-1 – Modeling Fanning Regions in Slabs and Footings Subjected to Uniform Loads

5.6.3.3 Proportioning of Compressive Struts

5.6.3.3.1 Strength of Unreinforced Strut

The nominal resistance of an unreinforced compressive strut shall be taken as:

$$P_n = f_{cu} A_{cs} \qquad (5.6.3.3.1\text{-}1)$$

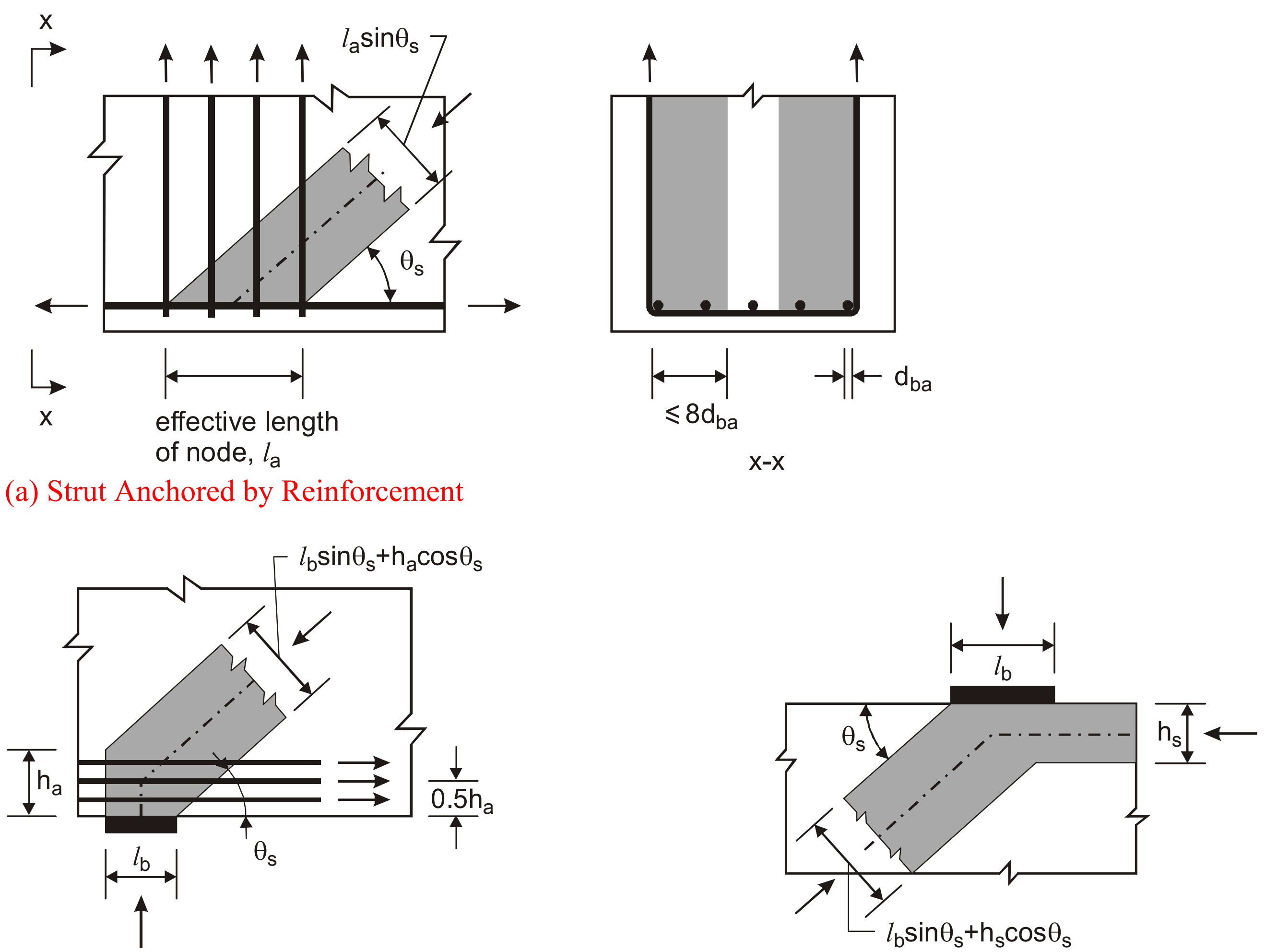
where:

P_n = nominal resistance of a compressive strut (kip)
f_{cu} = limiting compressive stress as specified in Article 5.6.3.3.3 (ksi)
A_{cs} = effective cross-sectional area of strut as specified in Article 5.6.3.3.2 (in.^{2})

5.6.3.3.2 Effective Cross-Sectional Area of Strut

The value of A_{cs} shall be determined by considering both the available concrete area and the anchorage conditions at the ends of the strut, as shown in Figure 5.6.3.3.2-1.

When a strut is anchored only by reinforcement, the effective concrete area may be considered to extend a distance of up to ~~six~~ eight bar diameters from the ~~anchored~~ bar anchoring the closed stirrups and the concrete cover should be neglected as shown in Figure 5.6.3.3.2-1 (a).



(a) Strut Anchored by Reinforcement

(b) Strut Anchored by Bearing and Reinforcement

(c) Strut Anchored by Bearing and Strut

Figure 5.6.3.3.2-1 - Influence of Anchorage Conditions on Effective Cross-Sectional Area of Strut

When a fanning region, with the tension tie at the narrow part of fan, is modeled by a series of struts the area A_{cs} shall be taken as being equal for each strut and shall be determined at the location where the strut connects to the nodal region as shown in Figure 5.6.3.3.2-2.

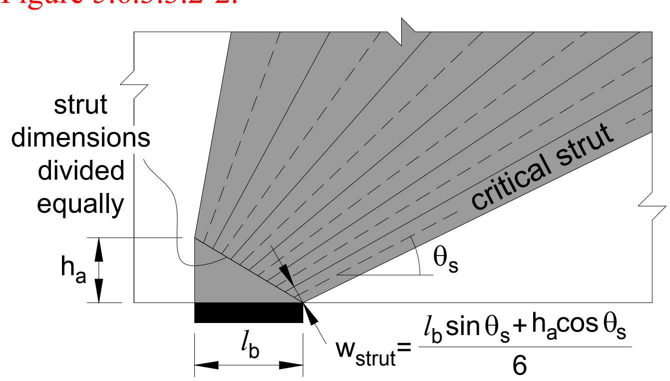


Figure 5.6.3.3.2-2 – Determining Strut Area A_{sc} for critical strut in fan region - Tension Tie at Narrow Part of Fan

5.6.3.3.3 Limiting Compressive Stress in Strut

The limiting compressive stress, f_{cu}, shall be taken as:

$$f_{cu} = \frac{f'_c}{0.8 + 170\ \varepsilon_1} \leq 0.85\ f_{c'} \qquad (5.6.3.3.3\text{-}1)$$

for which:

$$\varepsilon_1 = \varepsilon_s + (\varepsilon_s + 0.002) cot^2\ \alpha_s \qquad (5.6.3.3.3\text{-}2)$$

where:

α_s = the smallest angle between the compressive strut and adjoining tension ties (degrees)

ε_s = the tensile strain in the concrete in the direction of the tension tie (in./in.)

f'_c = specified compressive strength (ksi)

In lieu of using Eq. (5.6.3.3.3-1), the limiting compressive stress, f_{cu}, when the nominal yield strength of reinforcing steel does not exceed 60 ksi, may be taken as:

$$f_{cu} = \frac{f'_c}{1.15 + 0.69\, cot^2\ \alpha_s} \leq 0.85 f'_c \qquad (5.6.3.3.3\text{-}3)$$

C5.6.3.3.3

If the concrete is not subjected to principal tensile strains greater than about 0.002, it can resist a compressive stress of $0.85 f'_c$. This will be the limit for regions of the struts not crossed by or joined to tension ties. The reinforcing bars of a tension tie are bonded to the surrounding concrete. If the reinforcing bars are to yield in tension, there should be significant tensile strains imposed on the concrete. As these tensile strains increase, f_{cu} decreases.

The expression for ε_1 is based on the assumption that the principal compressive strain ε_2 in the direction of the strut equals 0.002 and that the tensile strain in the direction of the tension tie equals ε_s. As the angle between the strut-and-tie decreases, ε_1 increases and hence f_{cu} decreases. In the limit, no compressive stresses would be permitted in a strut that is superimposed on a tension tie, i.e., $\alpha_s = 0$, a situation that violates compatibility.

For a tension tie consisting of reinforcing bars, ε_s can be taken as the tensile strain due to factored loads in the reinforcing bars. For a tension tie consisting of prestressing, ε_s can be taken as 0.0 until the precompression of the concrete is overcome. For higher stresses, ε_s would equal $(f_{ps} - f_{pe}) / E_p$.

Eq. (5.6.3.3.3-3) has been derived from Eq. (5.6.3.3.3-1) by assuming that the strain in the tension tie is equal to the yield strain of Grade 60 reinforcement (i.e., 0.00207).

5.6.3.3.4 Reinforced Strut

If the compressive strut contains reinforcement that is parallel to the strut and detailed to develop its yield stress in compression, the nominal resistance of the strut shall be taken as:

$$P_n = f_{cu} A_{cs} + f_y A_{ss} \qquad (5.6.3.3.4\text{-}1)$$

where:

A_{ss} = area of reinforcement in the strut (in.^{2})

5.6.3.4 Proportioning of Tension Ties

5.6.3.4.1 Strength of Tie

Tension tie reinforcement shall be anchored to the nodal zones by specified embedment lengths, hooks, or mechanical anchorages. The tension force shall be developed at the inner face of the nodal zone.

The nominal resistance of a tension tie in kips shall be taken as:

~~$P_n = f_y A_{st} + A_{ps} [f_{pe} + f_y]$~~

$$P_n = f_y A_{st} + A_{ps}\left(f_{po} + f_y\right) \qquad (5.6.3.4.1\text{-}1)$$

where:

A_{st} = total area of longitudinal mild steel reinforcement in the tie (in.^{2})

A_{ps} = area of prestressing steel (in.^{2})

f_y = yield strength of mild steel longitudinal reinforcement (ksi)

~~f_{pe}~~ - f_{po} = stress in prestressing steel as defined in Article 5.8.3.4

5.6.3.4.2 Anchorage of Tie

~~The tension tie reinforcement shall be anchored to transfer the tension force therein to the node regions of the truss in accordance with the requirements for development of reinforcement as specified in Article 5.11.~~ Tension ties shall be

C5.6.3.4.1

The second term of the equation for P_n is intended to ensure that the prestressing steel does not reach its yield point, thus a measure of control over unlimited cracking is maintained. It does, however, acknowledge that the stress in the prestressing elements will be increased due to the strain that will cause the concrete to crack. The increase in stress corresponding to this action is arbitrarily limited to the same increase in stress that the mild steel will undergo. If there is no mild steel, f_y may be taken as 60.0 ksi for the second term of the equation.

anchored to the nodal zones by straight bar embedment or standard hooks in accordance with the requirements for development of reinforcement as specified in Article 5.11, by headed bars or by prestressing anchorages.

Tension ties anchoring compressive struts in regions away from bearing regions shall be detailed as closed stirrups with each bend enclosing a longitudinal bar.

Crack control reinforcement used as tension ties shall consist of closed stirrups with each bend enclosing a longitudinal bar.

Headed bars with a head of an area equal to ten times the bar area shall be deemed capable of developing the tensile strength of the bar without crushing of the concrete under the head provided that the specified concrete compressive strength is equal to or greater than 3 ksi and the yield strength of the reinforcement does not exceed 75 ksi.

5.6.3.5 Proportioning of Node Regions

Unless confining reinforcement is provided and its effect is supported by analysis or experimentation, the concrete compressive stress in the node regions of the strut shall not exceed:

- For node regions bounded by compressive struts and bearing areas: ~~0.85 φ f′c~~ $0.85\phi m f_c'$

where:

m = confinement modification factor taken as $\sqrt{\frac{A_2}{A_1}}$ but not more than 2.0 as defined in Article 5.7.5.

- For node regions anchoring a one-direction tension tie: ~~0.75 φ f′c~~ $0.75\phi m f_c'$

- For node regions anchoring tension ties in more than one direction: ~~0.65 φ f′c~~ $0.65\phi f_c'$

where:

ϕ = the resistance factor for bearing on concrete as specified in Article 5.5.4.2.

The tension tie reinforcement shall be uniformly distributed over an effective area of concrete at least equal to the tension tie force divided by the stress limits specified herein.

~~In addition to satisfying strength criteria compression struts and tension ties, the node ions shall be designed to comply with the ss and anchorage limits specified in Articles 3.4.1 and 5.6.3.4.2.~~

The bearing stress on the node region produced by concentrated loads or reaction forces shall satisfy the requirements specified in Article 5.7.5.

C5.6.3.5

The limits in concrete compressive stresses in nodal zones are related to the degree of expected confinement in these zones provided by the concrete in compression.

The stresses in the nodal zones can be reduced by increasing the:

- Size of the bearing plates,
- Dimensions of the compressive struts, and
- Dimensions of the tension ties.

The reduced stress limits on nodes anchoring tension ties are based on the detrimental effect of the tensile straining caused by these ties. If the ties consist of post-tensioned tendons and the stress in the ~~concrete~~ tendons does not need to be above ~~f_{pc}~~ f_{po}, no tensile straining of the nodal zone will be required. For this case, the ~~0.85 φ f′c~~ $0.85\phi m f_c'$ limit is appropriate. For node regions anchoring tension ties in more that one direction the confinement modification factor is taken as 1.0.

5.6.3.6 Crack Control Reinforcement

Structures and components or regions thereof, except for slabs and footings, which have been designed in accordance with the provisions of Article 5.6.3, shall contain orthogonal grids of reinforcing bars. The spacing of the bars in these grids shall not exceed the smaller of d/4 and 12.0 in.

The ratio of reinforcement area to gross concrete area shall not be less than 0.003 in each direction.

The reinforcement in the vertical and horizontal direction shall satisfy the following:

$$\frac{A_v}{b_w s_v} \geq 0.003 \qquad (5.6.3.6\text{-}1)$$

$$\frac{A_h}{b_w s_h} \geq 0.003 \qquad (5.6.3.6\text{-}2)$$

Where:

A_h = total area of horizontal crack control reinforcement within spacing s_h, respectively (in.^{2})

A_v = total area of vertical crack control reinforcement within spacing s_v, respectively (in.^{2})

b_w = width of member's web

s_v, s_h = spacing of vertical and horizontal crack control reinforcement, respectively (in.)

Crack control reinforcement shall be distributed evenly within the strut area.

C5.6.3.6

This reinforcement is intended to control the width of cracks and to ensure a minimum ductility for the member so that, if required, significant redistribution of internal stresses is possible.

The total horizontal reinforcement can be calculated as 0.003 times the effective area of the strut denoted by the shaded portion of the cross-section in Figure C5.6.3.6-1. For thinner members, this crack control reinforcement will consist of two grids of reinforcing bars, one near each face. For thicker members, multiple grids of reinforcement through the thickness may be required in order to achieve a practical layout.

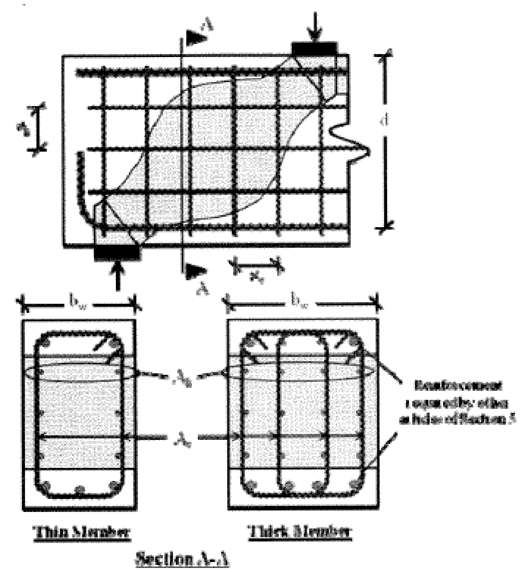


Figure C5.6.3.6-1—Distribution of Crack Control Reinforcement in Compression Strut

5.8.3.4.1 –Simplified Procedure for Nonprestressed Sections

For concrete footings in which the distance from the point of zero shear to the face of the column, pier, or wall is less than $3d_v$ for footings with transverse reinforcement or $2d_v$ without transverse reinforcement, and for other nonprestressed concrete sections not subjected to axial tension and

C5.8.3.4.1

With β taken as 2.0 and θ as 45°, the expressions for shear strength become essentially identical to those traditionally used for evaluating shear resistance. ~~Recent l~~Large-scale experiments (Shioya et al., 1989 and Uzel et al. 2010), however, have demonstrated that these traditional expressions can

containing at least the minimum amount of transverse reinforcement specified in Article 5.8.2.5, or having an overall depth of less than 16 in., the following values may be used:

$\beta = 2.0$

$\theta = 45^{\circ}$

be seriously unconservative for large members not containing transverse reinforcement.

Reference to add:

Uzel, A., Podgorniak, B., Bentz, E.C. and Collins, M.P., "Design of Large Footings for One-Way Shear", ACI Structural Journal, 108-S3, March/April, 2011, pp. 131-138.

5. STRUT-AND-TIE DESIGN PROCEDURE

The main steps for design using strut-and-tie models, with reference to the deep beam shown in Fig. C5.6.3.2-1, are given below. Article numbers in the AASHTO LRFD Specifications are preceded by the symbol "§".

1. Sketch an Idealized Strut-and-Tie Model

Visualize the flow of the compressive stresses and idealize this flow with straight-line struts (see Fig. C5.6.3.2-1(a)). Determine the locations of tension ties required for equilibrium. In sketching the idealized truss, made up of struts and ties, make suitable assumptions for the positions of the centroids of the ties, allowing sufficient space for placement of the required reinforcement. At this stage it is often advantageous to be slightly conservative in choosing the centroids of the ties so that a revision of the idealized truss will not be necessary once the tension tie reinforcement has been selected.

It is noted that the strut-and-tie model is a "lower bound approach". It is possible to develop a number of different strut-and-tie models which will all provide safe designs. However, the model that can carry the loads with the least internal energy (the stiffest model) involving the most direct load path and minimizing the amount of tie reinforcement, will provide the most realistic and the most efficient design.

2. Solve for Truss Member Forces

For the idealized truss subjected to factored loads, solve for the forces in the struts and the ties.

3. Select Area of Ties

Choose the required area of reinforcement in the tension ties to ensure that the factored resistance in the ties equals or exceeds the required factored force in the ties (§5.6.3.2 and §5.6.3.4.1). Choose a practical layout of the reinforcement making up the ties. Revise the geometry of the idealized truss if necessary and repeat steps 1 and 2.

4. Check Nodal Zone Stresses

Compare the nodal zone stresses with the nodal zone stress limits. Because of the dimensions of the struts and the reinforcement making up the ties, the truss joint, or node, represents a nodal zone with finite dimensions (see Fig. 5.6.3.3.2-1). The nodal zones serve to transfer the forces between the ties, the struts, the support reaction areas and the loaded bearing surfaces. The nodal zones occur at the intersections of the truss elements and at the loading points and support reaction areas. It is necessary to allow for the transfer of forces without overstressing the concrete in the nodal zones. In many practical situations it will be necessary to spread the tie reinforcement into several layers so that the nodal zone stress limit is not exceeded in the effective anchorage area (see Fig. 5.6.3.2-1(b)). The integrity of the nodal zone is checked by comparing the compressive stresses applied to the boundaries of the nodal zone with the specified nodal zone stress limits (§5.6.3.5). The compressive strength of the nodal zone depends on the tensile straining from intersecting tension ties and on the confinement conditions of support reaction areas and load bearing surfaces.

The nodal zone stress limits in the AASHTO Specifications (§5.6.3.5) depend on the number of ties that are being anchored in the nodal zone. For example, the nodal zone in Fig. 5.6.3.3.2-1(b), that anchors a one-direction tension tie has a stress limit of $0.75\phi m f_c'$. The nodal zone in Fig. 5.6.3.3.2-1(c), that is bounded by compressive struts and a bearing area, is an example where the stress limit is $0.85\phi m f_c'$. For cases where the supporting surface is wider on all sides than the bearing support area or the load bearing area, a confinement modification factor, *m*, may be used to increase the bearing stress (§5.6.3.5 and §5.7.5)).

5. Check Strength of Struts

Compare the factored resistance of the struts with the calculated factored loads in the strut members (§5.6.3.2 and §5.6.3.3). The nominal resistance of the strut is determined by multiplying the limiting compressive stress, f_{cu}, by the effective cross-sectional area of the strut, A_{cs}. As given by Eq. 5.6.3.3.3-1, the limiting compressive stress, f_{cu}, depends on the smallest angle, α_s, between the compressive strut and the tension tie and the tensile strain, ε_s, in the tie where it crosses the strut. For a tension tie consisting of reinforcing bars, ε_s, can be taken as the tensile strain due to factored loads in the reinforcing bars. It is noted that the limiting compressive stress, f_{cu}, reduces significantly as the angle, α_s, becomes smaller. For a tension tie consisting of prestressed steel, ε_s can be taken as zero until the pre-compression in the concrete due to the prestress is overcome. Equation 5.6.3.3.3-3 provides a simplification to Eq. 5.6.3.3.3-1 by assuming that the tie crossing the strut is at its yield strain and hence is somewhat more conservative than Eq. 5.6.3.3.3-1.

If the strut is anchored by a bearing area, the cross-sectional dimensions of the strut will be influenced by the length of the bearing area, the dimensions of the adjacent ties or struts and the inclination of the strut (see Fig. 5.6.3.3.2-1). If the strut is anchored only by reinforcement, the effective dimensions of the strut are related to the reinforcement details as shown in Fig. 5.6.3.3.2-1(a). The strut bears against the longitudinal reinforcing bars which in turn are anchored by the stirrups. It is assumed that the effective width of the strut across the thickness of the member can extend a distance of up to eight times the diameter of longitudinal bar anchored by the stirrups (i.e., $8d_{ba}$).

6. Provide Adequate Anchorage for the Ties

Provide sufficient anchorage for the ties (§5.6.3.4.2) so that they can develop the required tie force. The tie reinforcement is anchored by appropriate development length, hooks, headed bars or other mechanical anchorage so that it is capable of resisting the calculated tension in the reinforcement. At support reaction areas, a conservative approach is to provide enough embedment or mechanical anchorage so that the required tension force can be developed at the inner edge of the bearing.

7. Provide Crack Control Reinforcement
Distributed reinforcement, in the form orthogonal grids of reinforcing bars shall be provided to control the width of cracks and to ensure a minimum ductility of the member. The minimum reinforcement ratio of 0.003 must be provided in each direction. The maximum spacing of this reinforcement is the smaller of d/4 or 12 in. Slabs and footings designed in accordance with §5.6.3 are not required to contain crack control reinforcement.

7. Refined Strut-and-Tie Model
A refined strut-and-tie model, using the crack control reinforcement as additional tension ties can be used (5.6.3.2.1). For the strut-and-tie models shown in Fig. 5.6.3.2-1 the refined strut-and-tie model has a reduced strut force and reduced tension in the tie at the bearing supports than the simplified strut-and-tie model. Hence the use of a refined strut-and-tie model may avoid problems in these critical areas.

6. EXAMPLE 1 - DESIGN OF DEEP TRANSFER BEAM

The deep beam shown in Fig. 1.1 is 3 ft wide, 6 ft deep, 25 ft 6 in. long and spans 21 ft between the centers of the two supporting columns. The function of this beam is to transfer the high load from the central column over a priority lane below the beam. All three columns have cross-sectional dimensions of 2 ft 6 in. by 3 ft. The 822 kip applied factored column load, P_u , shown in Fig. 1.1, acting on the top surface of the beam, includes the effects of dead loads, lane and truck loading, including an allowance for impact. The specified concrete compressive strength, f_c', is 5 ksi and the specified yield strength of the reinforcing steel is 60 ksi.

Design the deep transfer beam using the AASHTO LRFD Specifications.

For this beam, the distance between the center of the applied load and the center of each of the supporting columns is 10 ft 6 in., which is less than twice the overall depth of the beam. Hence the entire beam is a D-Region and will be designed using the strut-and-tie model (§5.6.3.1).

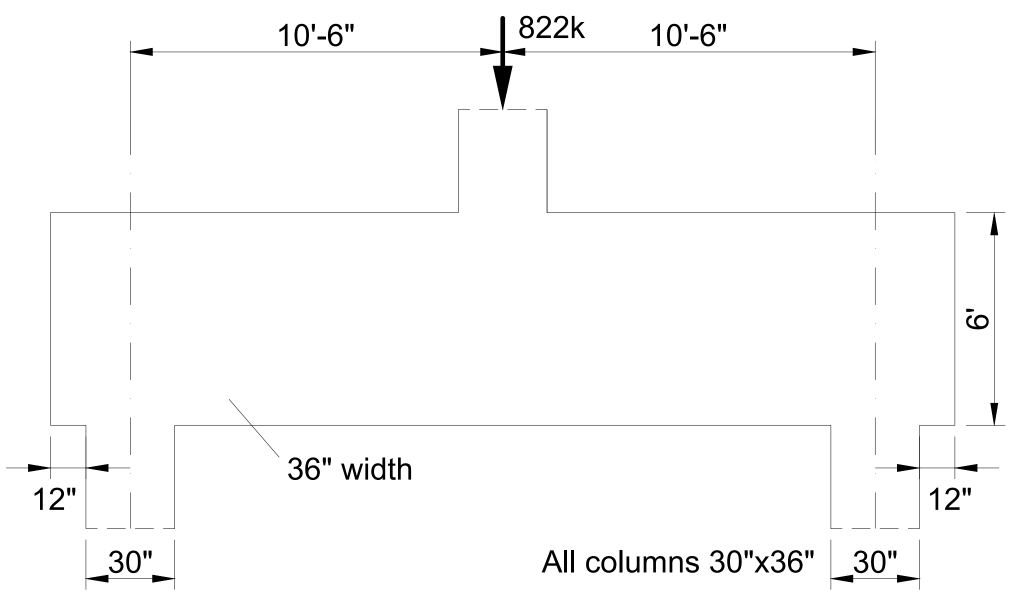


Figure 1.1. Details of deep transfer beam.

Step 1 - Draw Idealized Truss Model and Solve for Member Forces

The simple truss model shown in Fig. 1.2 represents the flow of forces in the cap beam. The three dashed lines represent compressive struts and the solid line represents the tension tie. The applied central column load has been divided into two applied loads to represent the portions of the column load transferred to the left and the right supporting columns. These two point loads are centered upon the left half and the right half of the central column. Hence these two equal point loads are located at the quarter points of the column as shown in Fig. 1.2. These applied point loads have been increased to account for the factored self-weight of the transfer beam. For simplicity it is conservatively assumed that half of the total self-weight of the beam is applied at node B and at node C. Therefore, the factored load at node B and at node C is:

$$0.5 \times 822 + 0.5 \times 1.25 \times 25.5 \times 3 \times 6 \times 0.150 = 454 \text{ kips}$$

For simplicity the moments in the supporting columns will be neglected for the design of the transfer beam, that is the axial stresses will be considered uniform at the top of the supporting columns.

In order to allow for the placement of the tension reinforcement and to account for the depth of the concrete compressive struts it has been assumed that the centroids of the top and bottom chords of the truss are located 4 in. from the top concrete surface and 6 in. from the bottom concrete surface, respectively. Hence the vertical distance between the top nodes (B and C) and the bottom nodes (A and D) of the truss is 62 in. Both nodes A and D are located at the intersection of the centerline of the supporting column and the centerline of the tension tie.

Figure 1.2 shows the truss idealization and the resulting member forces. Hence the main tension tie, AD, must be capable of resisting a force of 868 kips. The short horizontal compressive strut BC must also resist a force of 868 kips, while the diagonal struts AB and CD resist a compressive force of 980 kips.

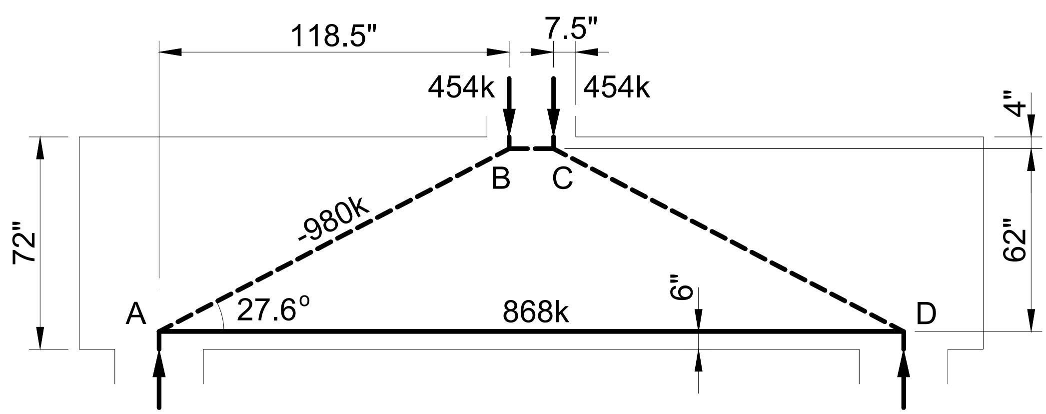


Figure 1.2. Truss idealization and member forces.

Step 2 – Check Size of Bearing Areas

The limiting concrete stresses under the bearings depend on the conditions at the nodal zone (§5.6.3.5). Nodes B and C are bounded by compressive struts and a bearing area (see Fig. 5.6.3.3.2-1(c)) with a limiting stress of $0.85\phi m f_c^{'}$ and Nodes A and D anchor a tension tie in one direction and hence have a limiting stress of $0.75\phi m f_c^{'}$.

As the width of the columns equals the 36 in. width of the beam the confinement modification factor, m, applied to the bearing areas is one.

Ignoring the beneficial effects of the column vertical reinforcing bars which will extend into the beam, the upper column at nodes B and C must transfer a load of 908 kips to the top surface of the beam. Hence:

$$\text{bearing area required} = \frac{P_u}{0.85\phi m f_c^{'}} = \frac{908}{0.85 \times 0.70 \times 1.0 \times 4} = 305\,\text{in.}^2$$

With dimensions of 30 in. by 36 in., the column has sufficient bearing area on the beam ($1080\,\text{in.}^2$).

Each of the lower two columns must resist an axial load of 454 kips. Hence for the lower columns the minimum required bearing areas are:

$$\text{bearing area required} = \frac{P_u}{0.75\phi m f_c^{'}} = \frac{454}{0.75 \times 0.70 \times 1.0 \times 5} = 173\,\text{in.}^2$$

Hence the bearing areas provided are adequate.

Step 3 – Determine Required Amount of Tension Tie Reinforcement

The minimum required area of tension tie reinforcement, A_{st}, in Tie AB is:

$$A_{st} = \frac{P_u}{\phi f_y} = \frac{868}{0.9 \times 60} = 16.1\,\text{in.}^2$$ §5.6.3.4.1

Step 4 –Choose Layout of Tension Tie Reinforcement

The layout of the longitudinal reinforcing bars must be such that the effective anchorage area for these bars is large enough to satisfy the nodal zone stress limit at the ends of the tie (§5.6.3.2). For this node anchoring one tension tie, the nodal zone stress limit is $0.75\phi m f_c'$ which means that to anchor the tension force of 868 kips the effective anchorage area (see Fig. 1.3 and Fig. C5.6.3.2-1(a)) must be at least:

$$\frac{868}{0.75\phi m f_c'} = \frac{868}{0.75 \times 0.70 \times 1.0 \times 5} = 331\,\text{in.}^2$$

As the beam is 36 in. wide the effective anchorage area must have a height of at least $331/36 = 9.2\,\text{in.}$

Figure 1.3 shows the chosen layout of longitudinal reinforcement using 21 - No. 8 bars, $A_s = 16.6\,\text{in.}^2$ resulting in an effective anchorage area of $36 \times 10.25 = 369\,\text{in.}^2$.

Step 5 –Check Assumed Geometry of Truss

In determining the layout of the reinforcement the nominal bar diameters and a maximum aggregate size of ¾ in. have been assumed. Note that the centroid of the 21 bars shown in Fig. 1.3 is located at a distance of 5.125 in. from the bottom face of the beam. Additional horizontal and vertical reinforcing bars used to position the No. 8 longitudinal bars are not shown.

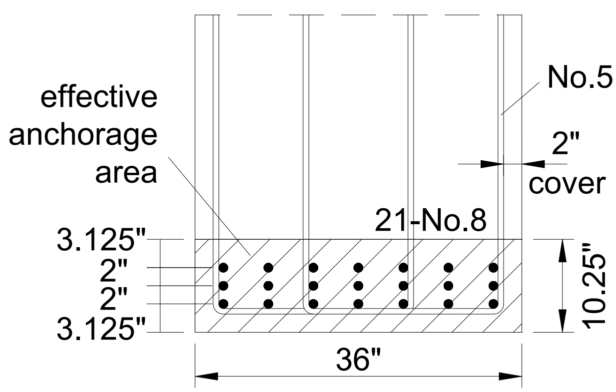


Figure 1.3. Effective anchorage area of tension tie reinforcement.

Horizontal strut BC has a calculated compressive force of 868 kips and is not crossed by any tension ties. Hence the strut can carry a stress of $0.85\phi m f_c'$ and would need to have a minimum area of:

$$\frac{868}{0.85\phi m f_c'} = \frac{868}{0.85 \times 0.70 \times 1.0 \times 5} = 292 \text{ in.}^2$$

As the width of the beam is 36 in. the strut must have a minimum depth of 8.1 in. Thus the centroid of strut BC will be 4.05 in. below the top face of the beam.

In determining the truss forces it was assumed that the vertical distance between the centroids of members BC and AD was 62 in. Based on the calculations above it can be seen that a more accurate estimate of this distance is $72 - 4.05 - 5.125 = 62.8$ in. Therefore the initial assumed geometry of the truss is slightly conservative and it is not necessary to modify the truss geometry.

Step 6 –Check Capacity of Struts

Struts AB and CD each carry a compression force of 980 kips (see Fig. 1.2). These struts are anchored at the joints which also anchor tension tie AD. From the geometry of the truss idealization, the angle between tension tie AD and the struts AB and CD is 27.6^{o}. As the specified yield strength of the reinforcing steel does not exceed 60 ksi, the limiting compressive stress in the struts can be obtained from simplified Equation 5.6.3.3.3-3 as:

$$f_{cu} = \frac{f_c'}{1.15 + 0.69\, cot^2\, \alpha_s} = \frac{5}{1.15 + 0.69\, cot^2\, 27.6^\circ} = 1.36 \text{ ksi}$$

From Fig. 5.6.3.3.2-1(b) the dimension of the strut in the plane of the truss (see Fig. 1.4) is:

$$\ell_b \, sin\, \theta_s + h_a \, cos\, \theta_s = 30 \times sin\, 27.6^\circ + 10.25\, cos\, 27.6^\circ = 23.0 \text{ in.}$$

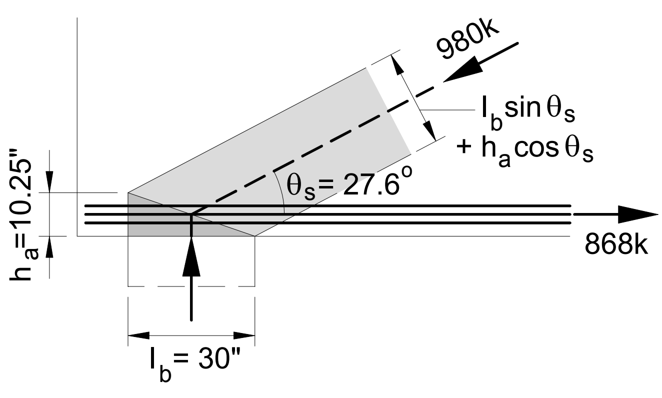


Figure 1.4. Strut dimensions

The nominal resistance of the strut is (§5.6.3.3.1) based on the limiting stress, f_{cu}, and the strut dimensions. Hence the nominal resistance of the strut, P_n, (Eq 5.6.3.3.1-1) is:

$$P_n = f_{cu} A_{cs} = 1.36 \times 23.0 \times 36 = 1126 \text{ kips}$$

The factored resistance of the strut is:

$$P_r = \phi P_n = 0.70 \times 1126 = 788 \text{ kips} < 980 \text{ kips required}$$

Therefore, based on these calculations, the strut capacity is not adequate.

It is noted that if the more detailed Eq. 5.6.3.3.3-1 were used to calculate f_{cu} this would increase f_{cu} from 1.36 ksi to 1.44 ksi resulting in a value of P_r of 834 kips. Hence these calculations which have used the simplified strut-and-tie model indicate that changes such as increasing the concrete strength, increasing the beam width and/or increasing the amount of reinforcement in the tension tie and spreading it over a larger depth seem to be necessary.

The use of the refined strut-and-tie model shown in Fig. 5.6.3.2-1(b) to investigate if such changes are required is illustrated below.

Step 7 – Provide Crack Control Reinforcement

The refined strut-and-tie model accounts for beneficial effects of the crack control reinforcement in assessing the flow of the forces in the idealized truss model. Hence it is necessary to choose the amount and distribution of the crack control reinforcement before developing the refined strut-and-tie model.

For “disturbed regions” (D-regions) the AASHTO LRFD Specifications require (§5.6.3.6) that reasonably closely spaced longitudinal and horizontal reinforcing bars for crack control and minimum ductility be provided. The spacing of this reinforcement must not exceed 12 in. and the minimum ratio of reinforcement to gross concrete area must be at least 0.003 in each direction.

(a) Horizontal crack control reinforcement

If No. 6 bars are provided near each face of the beam are provided, then the required spacing of these bars to provide the required reinforcement ratio of 0.003 is:

$$s_h = \frac{A_h}{0.003 b_w} = \frac{2 \times 0.44}{0.003 \times 36} = 8.1 \text{ in.}$$

Providing 8 – No. 6 bars near each vertical face (see Fig. 1.5) in addition to the 3 layers of the No. 8 bars in the main tension tie results in an average spacing of 7.75 in. In determining the layout of the reinforcement the nominal bar diameters have been assumed. Additional horizontal and vertical reinforcing bars used to position the No. 8 longitudinal bars are not shown.

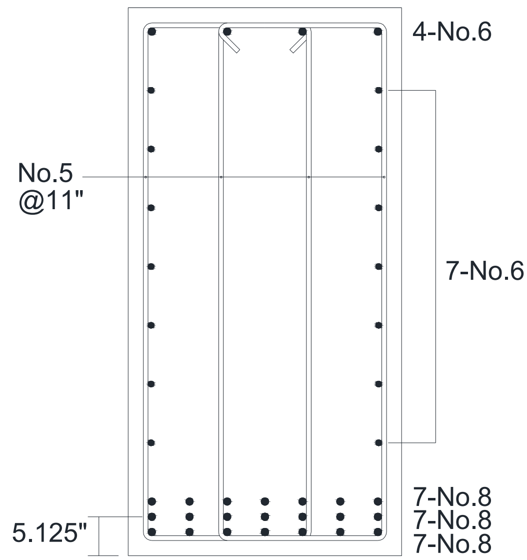


Figure 1.5. Layout of reinforcement.

(b) Vertical crack control reinforcement

While it would be possible to satisfy the crack control requirements by using No. 6 two-legged stirrups spaced at 8 in. along the length of the beam, this arrangement would limit the effective width of diagonal struts to about $2\times 8\times 1.0 = 16$ in. as illustrated in Fig. 5.6.3.3.2-1(a). Such a diagonal strut anchored by reinforcement is used in the refined model. If transverse reinforcement consisting of No. 5 four legged stirrups spaced at 11 in. is used then the full width inside the centerline of the outer legs would be effective to anchor the diagonal strut. This effective width is $36 - 2\times 2 - 0.625 = 31.4$ in. For this arrangement the reinforcement ratio in the vertical direction is:

$$\frac{A_v}{s_v b_w} = \frac{4\times 0.31}{11\times 36} = 0.0031 \geq 0.003$$

Hence the crack control requirements are satisfied.

The chosen reinforcement details are shown in Fig. 1.5.

Step 8 – Formulate and Solve Refined Strut-and-Tie Model

Figure 1.6 shows the refined strut-and-tie model which accounts for the beneficial effects of the vertical No. 5 bars used as crack control reinforcement. The vertical tension ties EF and GH represent the distributed four legged No. 5 stirrups that are considered to contribute to the vertical tension tie. Calling the distance between the face of the loading column and the face of the supporting column the clear shear span, the vertical tension ties are located at the center of the clear shear span. In the actual beam stirrups close to the face of the loading column or close to the faces of the supporting columns will not develop their full yield strength prior to failure of the beam and hence to avoid overestimating the contribution of the stirrups to the strength of the beam only those stirrups in the central half of clear shear span will be considered to be effective. Hence the factored resistance of ties EF and GH will be:

$$P_r = \phi f_y A_{st} = 0.9 \times 60 \times 4 \times 0.31 \times \frac{48}{11} = 292\,\text{kips}$$

While the refined strut-and-tie model results in new estimates for the flow of forces near the supports the forces in the members near midspan remain unchanged. Thus the compression in BC and the tension in FH are both 868 kips.

Looking at the vertical equilibrium of node F, the vertical downwards component of strut FB must balance the 292 kip tensile force in tie EF. Hence the horizontal component of FB equals $(55.5/62) \times 292 = 261$ kips and hence the compression in strut FB is 392 kips and the tension in AF is 607 kips. Considering the equilibrium of node A it can be seen that the vertical component of strut AE must equal 454 kips while the horizontal component must equal 607 kips. Hence strut AE has a compression force of 758 kips and is at an angle 36.8 degrees.

One the important advantages of the refined strut-and-tie model is that it provides a more accurate estimate of the tension demand on the longitudinal reinforcement near the supports. In this case it can be seen that the calculated tension in the longitudinal reinforcement near the supports has reduced from 868 kips to 607 kips, a 30% reduction.

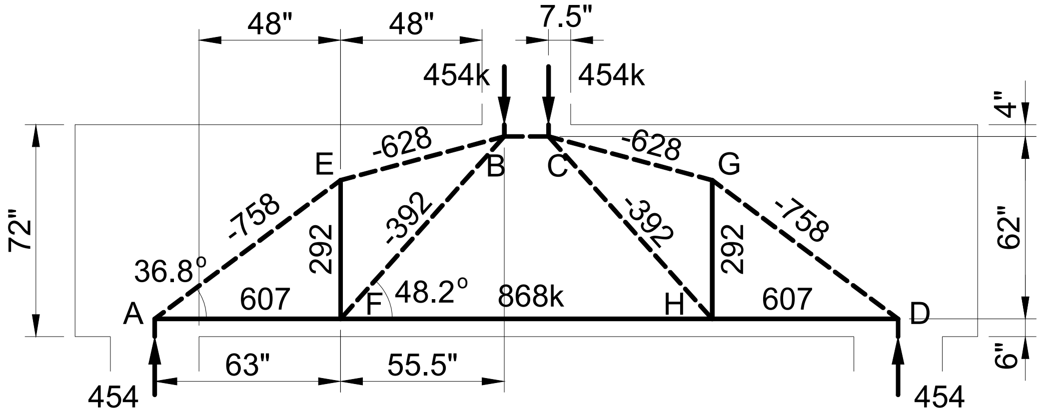


Figure 1.6. Refined strut-and-tie model.

Step 9 –Check Capacity of Struts in Refined Model

(a) Struts AE and GD

Struts AE and GD each carry a compression force of 758 kips (see Fig. 1.6). These struts are anchored at joints which also anchor tension ties. As the specified yield strength of the reinforcing steel does not exceed 60 ksi, the limiting compressive stress in the struts can be obtained from simplified Equation 5.6.3.3.3-3 as:

$$f_{cu} = \frac{f'_c}{1.15 + 0.69\,cot^2\,\alpha_s} = \frac{5}{1.15 + 0.69\,cot^2\,36.8^\circ} = 2.10\,\text{ksi}$$

From Fig. 5.6.3.3.2-1(b) the dimension of the strut in the plane of the truss is:

$$\ell_b\,sin\,\theta_s + h_a\,cos\,\theta_s = 30 \times sin\,36.8^\circ + 10.25\,cos\,36.8^\circ = 26.2\,\text{in.}$$

The factored resistance of the strut, P_r, (Eq 5.6.3.3.1-1) is:

$$P_r = \phi f_{cu} A_{cs} = 0.70 \times 2.10 \times 26.2 \times 36 = 1387\,\text{kips} > 758\,\text{kips}$$

Hence using this refined model demonstrates that the strut capacity is sufficient.

Note: It is apparent by comparing the truss models in Figs 1.2 and 1.6 that the simplified truss model overestimates the diagonal compression stress demand near the supports and also overestimates the required tension in the main tension tie reinforcement near the supports.

(b) Struts BF and CH

Struts BF and CH each carry a compression force of 392 kips (see Fig. 1.6). It is important to realize that BF and CH in the model represent a series of struts fanning out from B and C towards the bottom face of the beam where they will be anchored by the three layers of No. 8 bars which in turn are held up by the four legs of the No. 5 stirrups. Only the stirrups in the center half of the clear shear span were assumed to be effective and hence this length will be used as the effective length of the node, ℓ_a.

These struts are anchored at joints which also anchor both horizontal and vertical tension ties as shown in Fig. 1.7. As the specified yield strength of the reinforcing steel does not exceed 60 ksi, the limiting compressive stress in the struts can be obtained from simplified Equation 5.6.3.3.3-3. In this equation α_s is the smallest angle between a tension tie and the strut which in this case is 41.8 degrees. Therefore the limiting compressive stress for struts BF and CH is:

$$f_{cu} = \frac{f_c'}{1.15 + 0.69\,cot^2\,\alpha_s} = \frac{5}{1.15 + 0.69\,cot^2\,41.8^\circ} = 2.48\ \text{ksi}$$

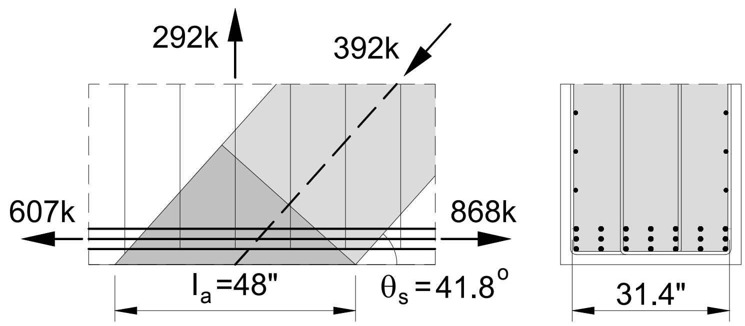


Figure 1.7. Checking capacity of struts BF and CH.

From Fig. 5.6.3.3.2-1(a) the dimension of the strut in the plane of the truss is:

$$\ell_a\, sin\,\theta_s = 48 \times sin\,48.2^\circ = 35.8\ \text{in.}$$

The factored resistance of the strut, P_r, (Eq 5.6.3.3.1-1) is:

$$P_r = \phi f_{cu} A_{cs} = 0.70 \times 2.48 \times 35.8 \times 31.4 = 1951\ \text{kips} > 392\ \text{kips}$$

Hence the strut capacity is sufficient.

Step 10 – Check Anchorage of Main Tension Tie

The No. 8 longitudinal bars making up the main tension tie must be capable of developing a tensile force of 607 kips at the inner edge of the columns at nodes A and D. An embedment length of $30 + 12 - 2\,(\text{cover}) = 40$ in. is available to develop this tensile force in bars (see Fig. 1.1). The basic development length for a straight No. 8 bar is (§5.11.2.1):

$$\ell_{db} = \frac{1.25 A_b f_y}{\sqrt{f'_c}} = \frac{1.25 \times 0.79 \times 60}{\sqrt{5}} = 26.5\,\text{in.}$$

But not less than:

$\ell_{db} = 0.4 d_b f_y = 0.4 \times 1.0 \times 60 = 24$ in. or 12.0 in.

The area of steel required to carry the required force of 607 kips is 607/(0.9 x 60) = 11.24 in.^{2}
The area of steel provided is 21 x 0.79 = 16.59 in.^{2}.

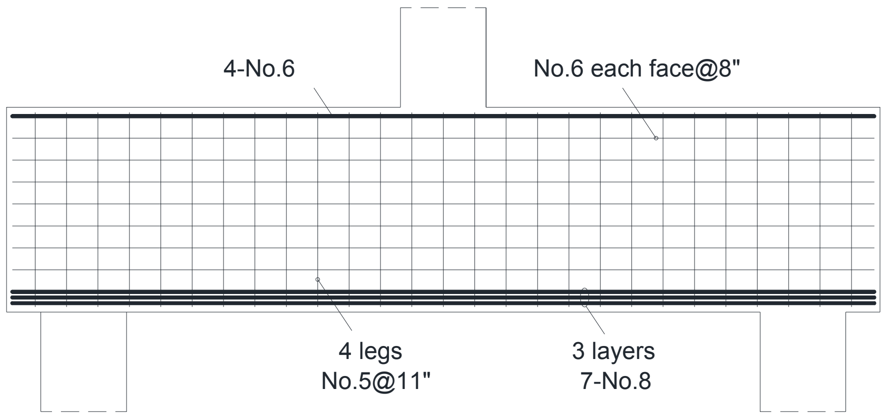
Therefore the required development length is (§5.11.2.1):

$$\ell_d = \ell_{db} \times \frac{(A_s required)}{(A_s provided)} = 26.5 \times \frac{11.24}{16.59} = 18.0 \text{ in.}$$

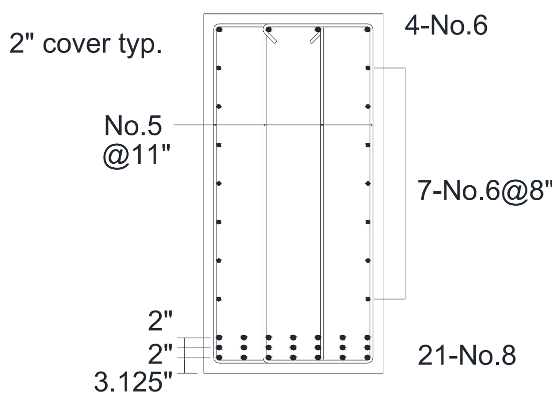
Hence the anchorage length is sufficient for the No. 8 tension tie reinforcement.

Step 11 – Sketch the Required Reinforcement

The resulting reinforcement of the deep beam is shown in Fig. 1.8.



(a) Elevation view



(b) Cross section

Figure 1.8. Reinforcement details of deep beam.

7. EXAMPLE 2 – FOOTING DESIGN

The footing shown in Fig. 2.1(a) supports two 30 in. thick reinforced concrete walls. The center-to-center distance between the two walls is 26 ft while as a first estimate the cross section of the footing is chosen 54 in. thick and 9 ft wide. The factored axial load, P_u, at the base of the wall is 797 kips. The specified concrete compressive strength, f_c', is 5 ksi and the specified yield strength of the reinforcing steel is 60 ksi. The nominal maximum size of coarse aggregate is ¾ in. The footing has been designed for flexure and contains 24 -No. 9 top headed bars (3.5 in. diameter circular heads) and 9 - No. 6 headed bottom bars (1.75 diameter circular heads). The clear cover on the heads of the headed bars is 2 in.

Design the footing to resist the factored forces without transverse reinforcement using the AASHTO LRFD Specifications.

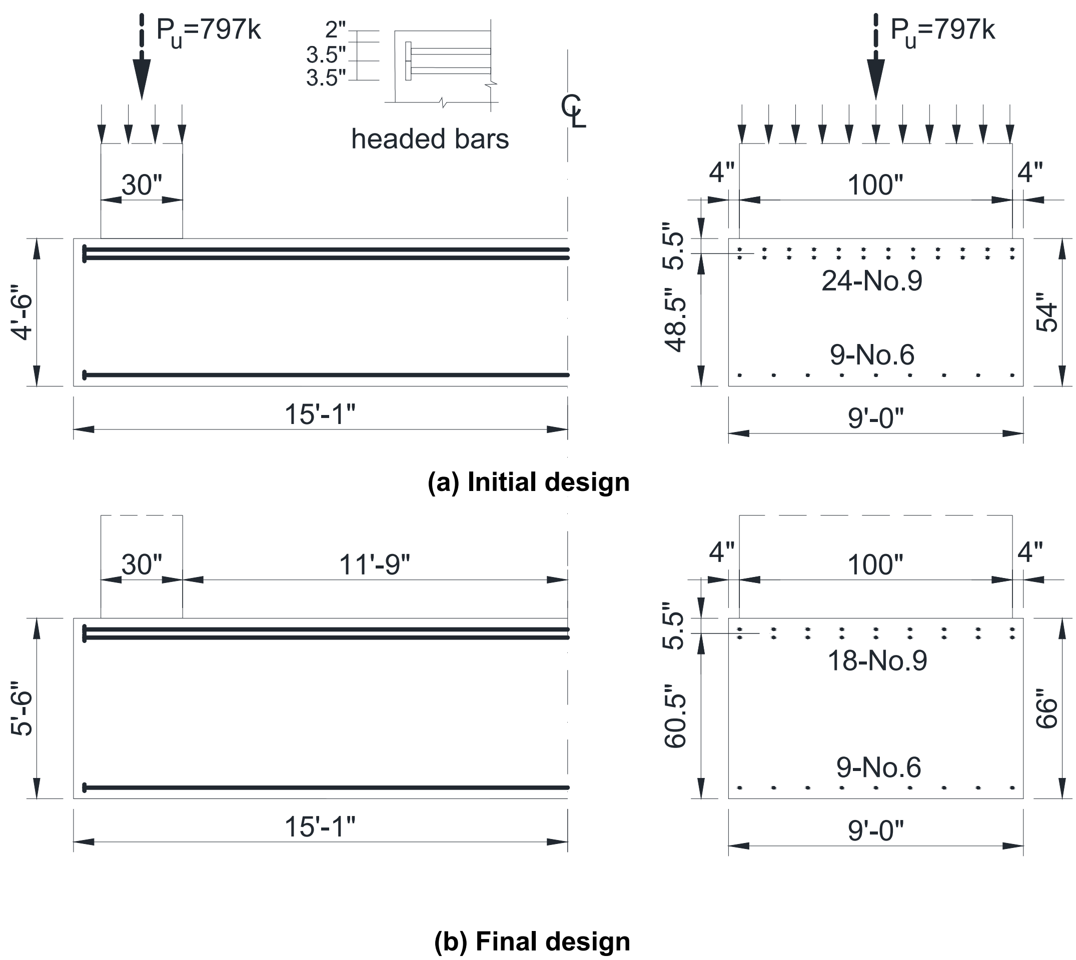


Figure 2.1. Details of footing.

The shear capacity of the footing can be calculated by using either the AASHTO sectional design method or the AASHTO strut-and-tie method. The former corresponds to the initiation of slip on the critical shear crack while the latter corresponds to a state of redistributed internal forces after significant cracking has occurred.

Step 1 – Calculate Sectional Shear Capacity of 54 in. Thick Footing

The sectional design procedure of §5.8.3 will first be used to calculate the shear resistance. It is assumed that the soil reaction is uniformly distributed across the bottom face of the footing, and thus the upward load per unit length along the footing is:

$$w_u = \frac{797}{15.08} = 52.85 \text{ kip/ft}$$

The critical section is located at distance $d_v = 0.9d = 0.9\times48.5 = 43.6$ in. from the inner face of the column or 58.6 in. (4.88 ft) from the center of the column. The distance from the end of the footing to the critical section is 10 + 30 + 43.6 = 83.6 in. (6.97 ft). The factored bending moment and the shear at this section are:

$$M_u = 797\times4.88 - 52.85\times6.97\times6.97/2 = 2606 \text{ kip-ft}$$

$$V_u = 797 - 52.85\times6.97 = 429 \text{ kips}$$

For this footing without transverse reinforcement the nominal shear resistance, V_n will be equal to V_c given from §5.8.3.3-3 as:

$$V_n = V_c = 0.0316\beta\sqrt{f_c'}b_v d_v$$

While β is given from Eq. 5.8.3.4.2-2:

$$\beta = \frac{4.8}{(1+750\varepsilon_s)}\frac{51}{(39+s_{xe})}$$

The distance, ℓ_0, from the point of zero shear to the face of the wall is 11.75 ft. Note that because ℓ_0 / d_v= 11.75 x 12/43.6=3.23 is greater than 2.0, the Simplified Procedure of §5.8.3.4.1 cannot be used for this footing without transverse reinforcement.

The longitudinal strain at the centroid of the top reinforcement at the critical section from Eq. 5.8.3.4.2-4 is:

$$\varepsilon_s = \frac{|M_u|/d_v + V_u}{E_s A_s} = \frac{2606\times12/43.6+429}{29000\times24} = 1.647\times10^{-3}$$

The crack spacing parameters are calculated as follows:

$$s_x = d_v = 43.6 \text{ in.}$$

$$s_{xe} = s_x \frac{1.38}{a_g + 0.63} = 43.6 \frac{1.38}{0.75 + 0.63} = 43.6 \text{ in.} < 80 \text{ in.}$$

$$\beta = \frac{4.8}{(1 + 750\varepsilon_s)} \frac{51}{(39 + s_{xe})} = \frac{4.8}{(1 + 750 \times 1.647 \times 10^{-3})} \frac{51}{(39 + 43.6)} = 1.326$$

The nominal shear sectional capacity is:

$$V_n = V_c = 0.0316 \beta \sqrt{f_c'} b_v d_v = 0.0316 \times 1.326 \times \sqrt{5} \times 108 \times 43.6 = 441 \text{ kips}$$

The factored shear capacity is:

$$\phi V_n = 0.9 \times 441 = 397 \text{ kip} < V_u = 429 \text{ kip}$$

And thus the sectional shear capacity of the footing is insufficient.
The next step is to check whether the strut-and-tie approach will produce higher factored shear resistance than the sectional approach.

Step 2 – Strut-and-Tie Shear Capacity of 54 in. Thick Footing

For members subjected to uniform loads the revised strut-and-tie provisions of the AASHTO LRFD Specifications permit the use of the strut-and-tie model shown in Fig. 2.2(a). The dashed lines represent compressive struts and the solid line represents the tension tie. In this model the lever arm of the internal longitudinal forces at the midspan section is assumed to be d_v = 43.6 in. The uniform soil reaction on the bottom face of the footing is represented by a statically equivalent system of 6 point loads of 126 kip and a load of 41 kip on the short cantilevering part of the footing. The 41 kip load is relatively small and located very close to the column, and hence it is not included in the strut-and-tie model.

The factored bending moment at midspan is:

$$M_u = 797 \times 13 - 52.85 \times 15.083 = 4350 \text{ kip-ft}$$

The factored tension force in the top reinforcement is:

$$P_f = M_u / d_v = 4350 \times 12 / 43.6 = 1197 \text{ kips}$$

The dimensions of the triangular nodal region under the column are determined based on the width of the column and the distance from the top face of the footing to the centroid of the top reinforcement (tie). These dimensions are used to determine the effective

anchorage area. The maximum stress in the nodal region occurs on vertical planes and equals 1197/(11.0×100) = 1.09 ksi which is less than the code limit of $0.75\phi f_c'$ = 0.75×0.70×5 = 2.63 ksi for node regions anchoring a one-direction tie.

The inclined face of the node region is divided into six equal segments which correspond to the six tributary areas on the bottom face of the footing. It is assumed that the point loads of 126 kip are transmitted to the nodal zone by direct struts that form a compression fan. The strut closest to the midspan of the footing is critical as it carries the largest compressive force of 360 kips and has the smallest angle with respect to the tie. The dashed centerline of this strut intersects the inclined nodal face 1/12 of h_a below the top face. The horizontal projection of this strut centerline can be determined as:

$$H = 6 \times 28.5 - 28.5/2 - (11/12) \times 30 = 129.25 \text{ in.}$$

The vertical projection of this strut is

$$V = 43.6 + 5.5 - (1/12) \times 11 = 48.18 \text{ in.}$$

Hence the angle between the strut and the tie, α_s, is 20.5°.

The nominal strength of the strut is obtained as follows:

Where the strut cross-sectional area is:

$$A_{cs} = \frac{l_b \times sin\,\alpha_s + h_a \times cos\,\alpha_s}{6} b = \frac{30 \times sin\,20.5^\circ + 11.0 \times cos\,20.5^\circ}{6} 100 = 347 \text{ in.}^2$$

The strain in the tie near the strut:

$$\varepsilon_s = \frac{T}{E_s A_s} = \frac{1197}{29000 \times 24 \times 1.0} = 1.720 \times 10^{-3}$$

The strain perpendicular to the strut is:

$$\varepsilon_1 = \varepsilon_s + (\varepsilon_s + 0.002)\cot^2 \alpha_s =$$
$$= 1.720 \times 10^{-3} + (1.720 \times 10^{-3} + 2 \times 10^{-3}) cot^2\, 20.5^\circ = 28.3 \times 10^{-3}$$

The strut compression strength is:

$$f_{cu} = \frac{f_c'}{0.8 + 170\varepsilon_1} = \frac{5}{0.8 + 170 \times 28.3 \times 10^{-3}} = 0.891 \text{ ksi} < 0.85 f_c' = 4.25 \text{ ksi}$$

The simplified expression from eq. 5.6.3.3.3-3 assumes that the tension reinforcement yields and hence gives a lower value of 0.822 ksi for f_{cu}. The higher value will be used.

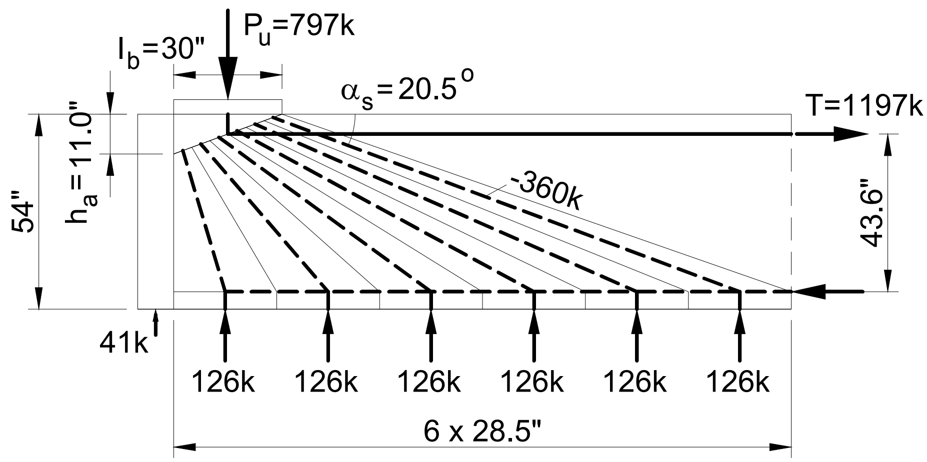
$$P_n = f_{cu} A_{cs} = 0.891 \times 347 = 309 \text{ kip}$$

The factored strut capacity is:

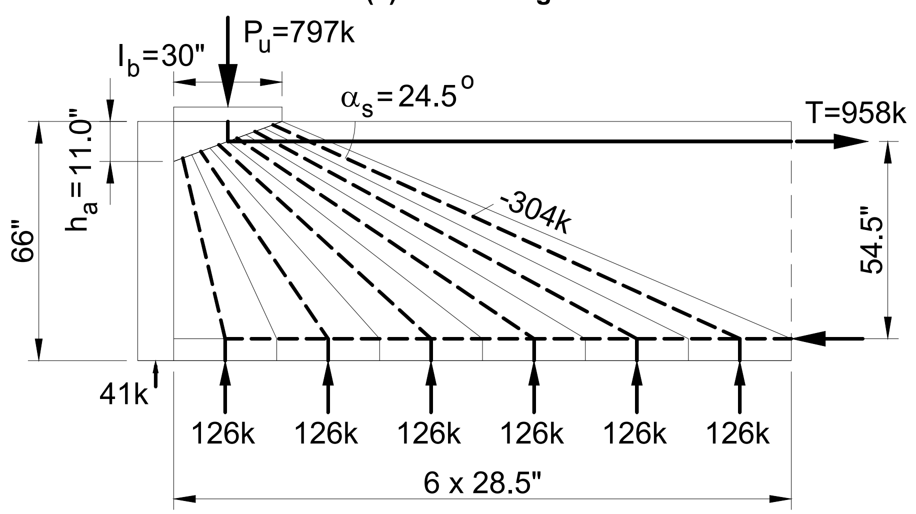
$\phi P_n = 0.7 \times 309 = 216 \text{ kip} < 360 \text{ kip}$

And thus the strut capacity is insufficient.

To increase the capacity, it is decided to increase the thickness of the footing from 54 in. (Fig. 2.1(a)) to 66 in. (see Fig. 2.1(b)) and to repeat the strut-and-tie calculations.



(a) Initial design



(b) Final design

Figure 2.2. Strut-and-tie model

Step 3 – Strut-and-Tie Design of 66 in. Thick Footing

The effective depth to the centroid of the top reinforcement is 66 - 5.5 = 60.5 in. and hence d_v is 0.9 x 60.5 = 54.5 in. (see Fig. 2.2(b)).

The required tension force in the top reinforcing can be found from the moment at midspan which equals 4350 kip-ft. Hence the required tension force in the top steel is 4350 x 12/54.5 = 958 kips. The required reinforcement is thus 958/(0.9x60) = 17.7 in.^{2} Hence use 18 No. 9 bars. Note that two layers of tension reinforcement are used to increase the dimensions of the nodal zone.

As the depth of the footing is increased, the compressive force in the critical strut decreases to 304 kips and the angle between the strut and the tie increases to 24.5°, see Fig. 2.2(b). The nominal strength of the critical strut is obtained as follows:

$$P_n = f_{cu} A_{cs}$$

where the strut cross-sectional area is:

$$A_{cs} = \frac{l_b \times sin\,\alpha_s + h_a \times cos\,\alpha_s}{6} b = \frac{30 \times sin\,24.5^\circ + 11 \times cos\,24.5^\circ}{6} 100 = 374 \text{ in.}^2$$

Strain in the tie near the strut:

$$\varepsilon_s = \frac{T}{E_s A_s} = \frac{958}{29000 \times 18 \times 1.0} = 1.835 \times 10^{-3}$$

Strain perpendicular to the strut:

$$\varepsilon_1 = \varepsilon_s + (\varepsilon_s + 0.002)\cot^2 \alpha_s =$$
$$= 1.835 \times 10^{-3} + \left(1.835 \times 10^{-3} + 2 \times 10^{-3}\right) cot^2\, 24.5^\circ = 20.3 \times 10^{-3}$$

Strut compression strength:

$$f_{cu} = \frac{f_c'}{0.8 + 170\varepsilon_1} = \frac{5}{0.8 + 170 \times 20.3 \times 10^{-3}} = 1.176 \text{ ksi} < 0.85 f_c' = 4.25 \text{ ksi}$$

The nominal strut capacity is:

$$P_n = f_{cu} A_{cs} = 1.176 \times 374 = 440 \text{ kips}$$

The factored strut capacity is:

$$\phi P_n = 0.7 \times 440 = 308 \text{ kips}$$

It is of interest to note that the simplified expression from Eq. 5.6.3.3.3-3 assumes that the tension reinforcement yields and hence gives a lower value of 1.118 ksi for f_{cu}. If this lower value were used then $P_n = 418$ kips and $\phi P_n = 293$ kips and hence gives insufficient capacity. Therefore the more accurate expression of Eq. 5.6.3.3.3-1 will be used.

Note that the strut capacity is just sufficient and so the design of the footing is adequate. Hence, use a footing thickness of 66 in.

8. EXAMPLE 3 - DESIGN OF WALL WITH CONCENTRATED LOAD

The wall shown in Fig. 3.1 is 12 ft wide, 16 ft high and 3 ft thick. The function of this wall is to transfer the high concentrated load from the centrally located bearing on top of the wall down to the footing. The 3000 kip applied factored column load, P_u , shown in Fig. 3.1, acting on the top surface of the wall, includes the effects of dead loads, lane and truck loading, including an allowance for impact. The specified concrete compressive strength, f_c' , is 4 ksi and the specified yield strength of the reinforcing steel is 60 ksi.

Design the wall for the concentrated load using the AASHTO LRFD Specifications.

For this wall, the concentrated load at the top of the wall causes a highly localized compressed region immediately below the bearing. These highly localized compressive stresses will spread out with increasing distance from the bearing until they become uniform over the full cross section of the wall. Using St. Venant's Principle the top portion of the wall over a height equal to the 12 ft width of the wall is a D-Region and will be designed using the strut-and-tie model (§5.6.3).

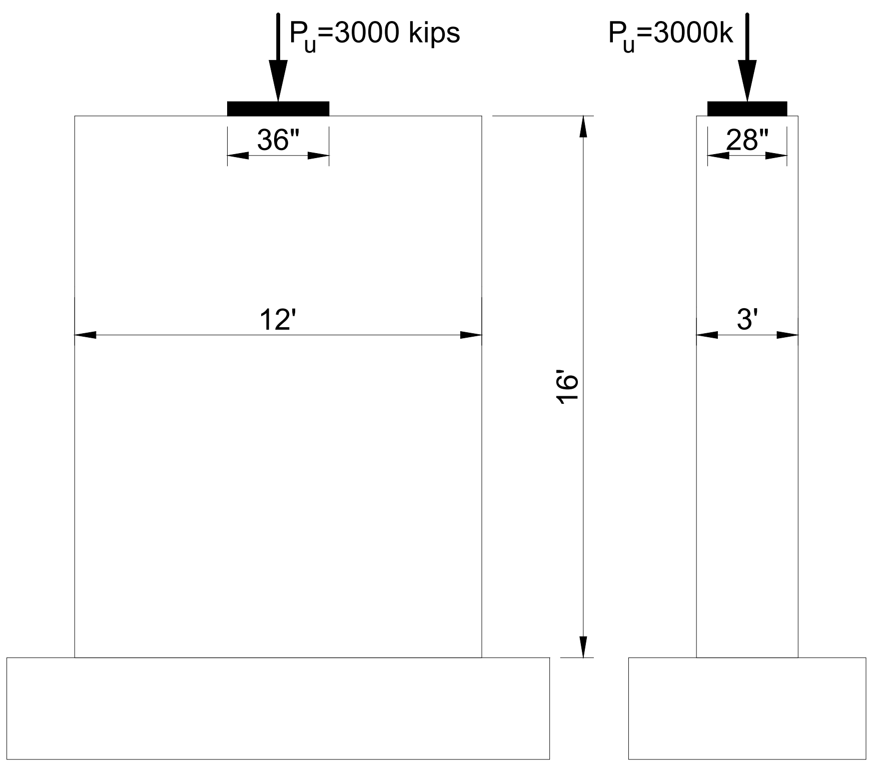


Figure 3. Details of wall subjected to a concentrated load.

Step 1 – Check Size of Bearing Area

The limiting concrete stress under the bearings depends on the conditions at the nodal zone (§5.6.3.5). Nodes A and B, shown in Fig. 3.2, are bounded by compressive struts and a bearing area (see Fig. 5.6.3.3.2-1(c)) with a limiting stress of $0.85\phi m f_c'$.

The confinement modification factor, m, applied to the bearing area is greater than 1.0 because the supporting surface is wider on all sides than the loaded area A_1. In order to determine the extent of spreading, the area A_2 is defined as the "area of the lower base of the largest frustrum of a pyramid contained wholly within the support and having for its upper base the loaded area and having side slopes of 1 vertical to 2 horizontal". Hence, in the direction of the wall thickness the compression can spread 4 in. on each side of the 28 in. bearing. This results in an area A_2 of:

$$A_2 = (28 + 2 \times 4) \times (36 + 2 \times 4) = 36 \times 44 = 1584\,\text{in.}^2$$

And the confinement factor m is (§5.7.5):

$$m = \sqrt{\frac{A_2}{A_1}} = \sqrt{\frac{1584}{28 \times 36}} = 1.254 \le 2.0$$

The required bearing area to resist the 3000 kip applied load is:

$$\text{bearing area required} = \frac{P_u}{0.85\phi m f_c'} = \frac{3000}{0.85 \times 0.70 \times 1.254 \times 4} = 1006\,\text{in.}^2$$

With dimensions of 28 in. by 36 in., the bearing size is sufficient (area = $1008\,\text{in.}^2$).

If the beneficial effects of confinement had been neglected then the required bearing area would have been 1260 in.^{2} which would have required the bearing area to be increased.

Step 2 - Draw Idealized Truss Models and Solve for Member Forces

The concentrated compressive stresses under the bearing will spread out across the 12 ft wall dimension and will also spread out over the wall thickness. Separate strut-and-tie models will be developed for the spreading in these two directions as described below:
,

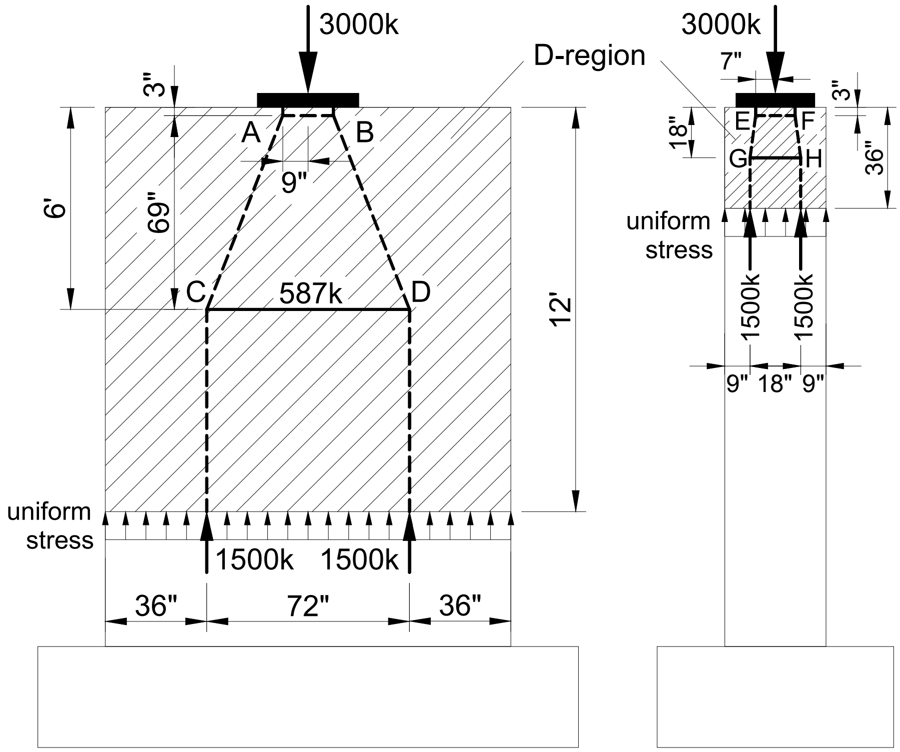
(a) Spreading of compressive stresses in 12 ft wall dimension

The simple truss model shown in Fig. 3.2a represents the flow of forces in the wall. The dashed lines represent compressive struts and the solid line represents the tension tie. The applied central bearing load has been divided into two applied loads to represent the portions of the bearing load transferred to the left and the right halves of the wall width. These two point loads are centered upon the left half and the right half of the bearing. Hence these two equal point loads are resisted by struts at the quarter points of the bearing as shown in Fig. 3.2a.

In order to account for the depth of the concrete compressive struts it has been assumed that top nodes A and B are located 3 in. from the top concrete surface. The D-region occurs over a height of 12 ft as shown and it has been assumed that the tension tie, CD, represents the horizontal reinforcement over the 12 ft height and is located 6 ft from the top of the wall. At a distance of 12 ft from the top of the wall the compressive stresses are assumed to be uniform and hence the two 1500 kip compressive forces are located at the quarter points of the 12 ft width to represent this uniform stress.

Figure 3.2a shows the truss idealization and the resulting member forces. The horizontal distance between nodes A and C is 36 – 9 = 27 in. Hence the main tension tie, CD, must be capable of resisting a force of:

$$P_u = 1500\frac{27}{69} = 587\,\text{kips}$$



(a) Spreading over 12 ft width (b) Spreading over 3 ft thickness

Figure 3.2. Truss idealization and member forces.

(b) Spreading of compressive stresses in 3 ft thick wall dimension

Fig. 3.2(b) shows the flow of forces through the thickness of the wall. The applied central bearing load has been divided into two applied loads to represent the portions of the bearing load transferred to the left and the right of the wall thickness. These two point loads are centered upon the left half and the right half of the bearing. Hence these two equal point loads are located at the quarter points of the bearing as shown in Fig. 3.2b.

In order to account for the depth of the concrete compressive struts it has been assumed that top nodes E and F are located 3 in. from the top concrete surface. The D-region occurs over a height of 3 ft as shown and it has been assumed that the tension tie, GH, represents the horizontal reinforcement over the 3 ft height and is located 18 in. from the top of the wall. At a distance of 3 ft from the top of the wall the compressive stresses are assumed to be uniform and hence the two 1500 kip compressive forces are located at the quarter points of the 3 ft thickness to represent this uniform stress.

Figure 3.2b shows the truss idealization and the resulting member forces. The horizontal distance between nodes E and G is 9 – 7 = 2 in. Hence the main tension tie, GH, must be capable of resisting a force of:

$$P_u = 1500\frac{2}{15} = 200 \text{ kips}$$

Step 3 – Determine Required Amount of Tension Tie Reinforcement (§5.6.3.4.1)

(a) Spreading of compressive stresses in 12 ft wall dimension

The minimum area of horizontal tension tie reinforcement, A_{st}, in Tie CD required to resist the 587 kip force is:

$$A_{st} = \frac{P_u}{\phi f_y} = \frac{587}{0.9 \times 60} = 10.9 \text{ in.}^2$$

This amount of reinforcement is required for strength in the 12 ft high disturbed region as determined from the strut-and-tie model. If pairs of No. 6 bars are used then the number of pairs required would be $10.9/(2 \times 0.44) = 12.4$ or 13 pairs. Hence for strength considerations 2 – No. 6 bars at a spacing of 11 in. would be required over the height of the D-region.

(b) Spreading of compressive stresses in 3 ft thick wall dimension

The minimum area of horizontal tension tie reinforcement, A_{st}, in Tie GH required to resist the 200 kip force is:

$$A_{st} = \frac{P_u}{\phi f_y} = \frac{200}{0.9 \times 60} = 3.70 \text{ in.}^2$$

This amount of reinforcement is required for strength in the 3 ft high disturbed region as determined from the strut-and-tie model. If pairs of No. 6 bars are used then the required number of pairs required would be $3.70/(2 \times 0.44) = 4.2$ or 5 pairs. Hence for strength considerations provide No. 6 hoops over the 3 ft height as shown in Fig. 3.3.

Step 4 – Provide Crack Control Reinforcement

For “disturbed regions” (D-regions) the AASHTO LRFD Specifications require that reasonably closely spaced longitudinal and horizontal reinforcing bars for crack control and minimum ductility be provided (§5.6.3.6). The spacing of this reinforcement must not exceed 12 in. and the minimum ratio of reinforcement to gross concrete area must be at least 0.003 in each direction.

(a) Horizontal crack control reinforcement

If No. 6 bars are provided near each face of the beam, then the required spacing of these bars to provide the required reinforcement ratio of 0.003 is:

$$s_h = \frac{A_h}{0.003b_w} = \frac{2\times 0.44}{0.003\times 36} = 8.1\,\text{in.}$$

The amount of crack control reinforcement exceeds the amount required for strength. Hence provide pairs of No. 6 bars at a spacing of 8 in. in the disturbed region (see Fig. 3.3). The No. 6 horizontal bars will be hooked around the corner reinforcement at the edges of the wall.

(b) Vertical crack control reinforcement

Provide pairs of No. 6 bars at a spacing of 8 in. in the disturbed region (see Fig. 3.3). The No. 6 vertical bars will be hooked around the corner reinforcement at the top of the wall.

(c) Reinforcement required in local zone below the bearing

The local zone below the bearing requires 5 pairs of No. 6 bars as calculated in Step 3(b). This "bursting" reinforcement is provided by 5 closed stirrups at a spacing of 7 in. and also serves as confinement reinforcement in the region below the bearing (see §5.10.9.3.2).

In addition, there is a tendency for cracks to form on the top surface and on the sides of the wall near the top surface due to "spalling" forces. The minimum spalling force for design is usually taken as 2 percent of the concentrated force (§C5.10.9.3.2) and therefore the minimum area of spalling reinforcement is:

$$A_{spalling} = \frac{0.02\times P_u}{\phi f_y} = \frac{0.02\times 3000}{0.9\times 60} = 1.11\ \text{in}^2$$

Hence provide 2 – No. 7 bars near the top and side surfaces (area of 1.20 in.^{2} provided).

Step 5 – Sketch the Required Reinforcement

The resulting reinforcement of the wall is shown in Fig. 3.3.

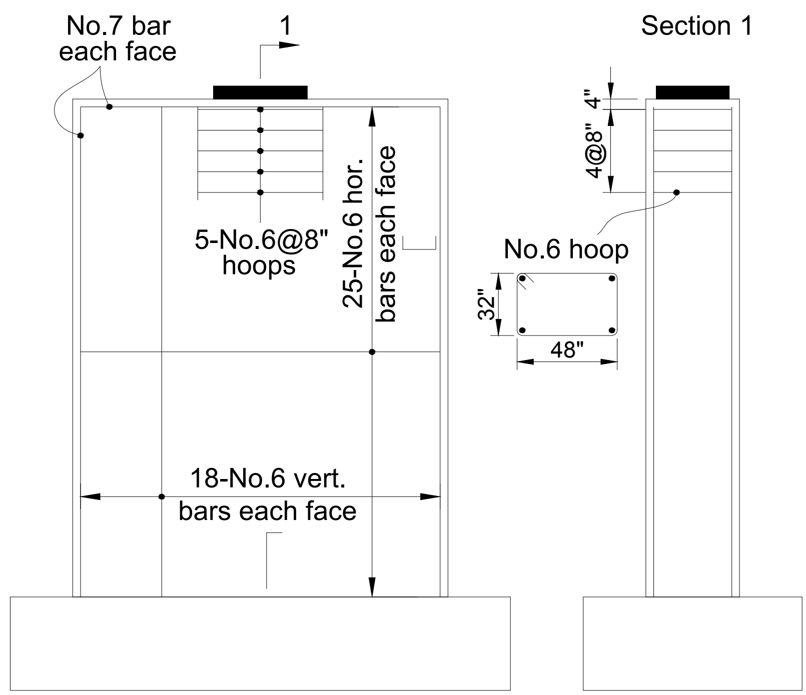


Figure 3.3. Reinforcement details of wall (footing reinforcement not shown).

9. EXAMPLE 4 - DESIGN OF WALL WITH TWO CONCENTRATED LOADS

The wall shown in Fig. 4.1 is 12 ft wide, 16 ft high and 3 ft thick. The function of this wall is to carry two concentrated loads from the bearings located on top of the wall. The two 1500 kip applied factored column loads, P_u , shown in Fig. 4.1, include the effects of dead loads, lane and truck loading, including an allowance for impact. This loading arrangement, using two bearings, is being considered as an alternative to the single concentrated load case given in Example 3. The specified concrete compressive strength, $f_c^{'}$, is 4 ksi and the specified yield strength of the reinforcing steel is 60 ksi.

Design the wall for the concentrated load using the AASHTO LRFD Specifications.

The highly localized compressive stresses due to the concentrated loads at the top of the wall will spread out with increasing distance from the bearings until they become uniform over the full cross section of the wall. Using St. Venant's Principle the top portion of the wall over a height equal to one-half of the 12 ft width of the wall is a D-Region and will be designed using the strut-and-tie model (§5.6.3.1).

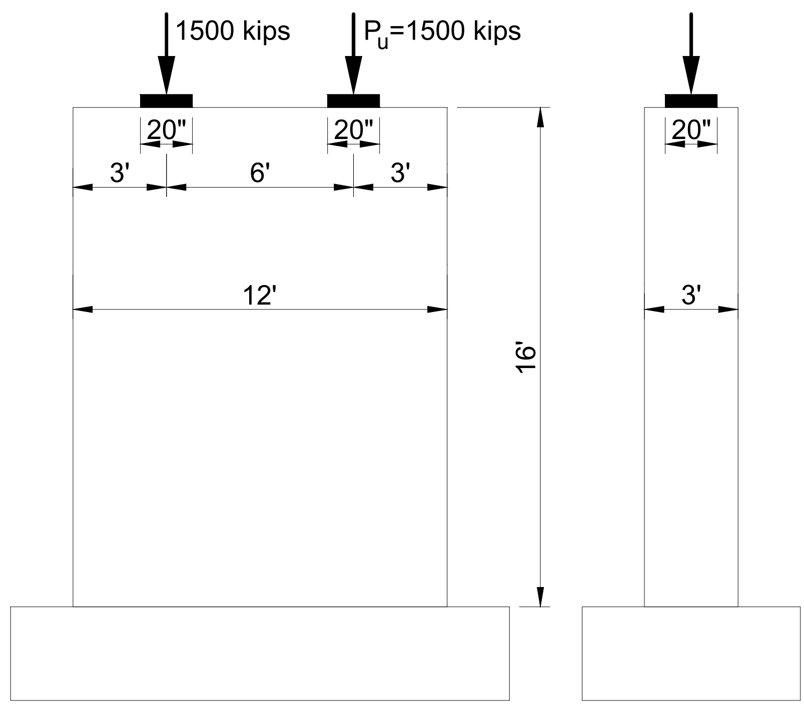


Figure 4.1. Details of wall subjected to two concentrated loads.

Step 1 – Check Size of Bearing Area

The limiting concrete stress under the bearings depends on the conditions at the nodal zone (§5.6.3.5). Nodes A and B (see Fig. 4.2) are bounded by compressive struts and a bearing area (see Fig. 5.6.3.3.2-1(c)) with a limiting compressive stress of $0.85\phi m f_c^{'}$.

The confinement modification factor, m, applied to the bearing area is greater than 1.0 because the supporting surface is wider on all sides than the loaded area A_1. In order to determine the extent of spreading, the area A_2 is defined as the "area of the lower base of the largest frustrum of a pyramid contained wholly within the support and having for its upper base the loaded area and having side slopes of 1 vertical to 2 horizontal". Hence, in the direction of the wall thickness the compression can spread from the 20 in. width of the bearing to the 36 in. wall thickness, or 8 in. on each side of the 20 in. bearing. This results in an area A_2 of:

$$A_2 = (20 + 2\times 8)\times(20 + 2\times 8) = 36\times 36 = 1296 \text{ in.}^2$$

And the confinement factor m is (§5.7.5):

$$m = \sqrt{\frac{A_2}{A_1}} = \sqrt{\frac{1296}{20\times 20}} = 1.80 \le 2.0$$

The required bearing area to resist the 3000 kip applied load is:

$$\text{bearing area required} = \frac{P_u}{0.85\phi m f_c^{'}} = \frac{1500}{0.85\times 0.70\times 1.80\times 4} = 350 \text{ in.}^2$$

With dimensions of 20 in. by 20 in., the bearing size is sufficient (area = 400 in.^2).

If the beneficial effects of confinement had been neglected then the required bearing area would have been 630 in.^{2} which would have required the bearing area to be increased.

Step 2 - Draw Idealized Truss Models and Solve for Member Forces

The concentrated compressive stresses under the two bearings will spread out across the 12 ft wall dimension and will also spread out over the wall thickness. Separate strut-and-tie models will be developed for the spreading in these two directions as described below:

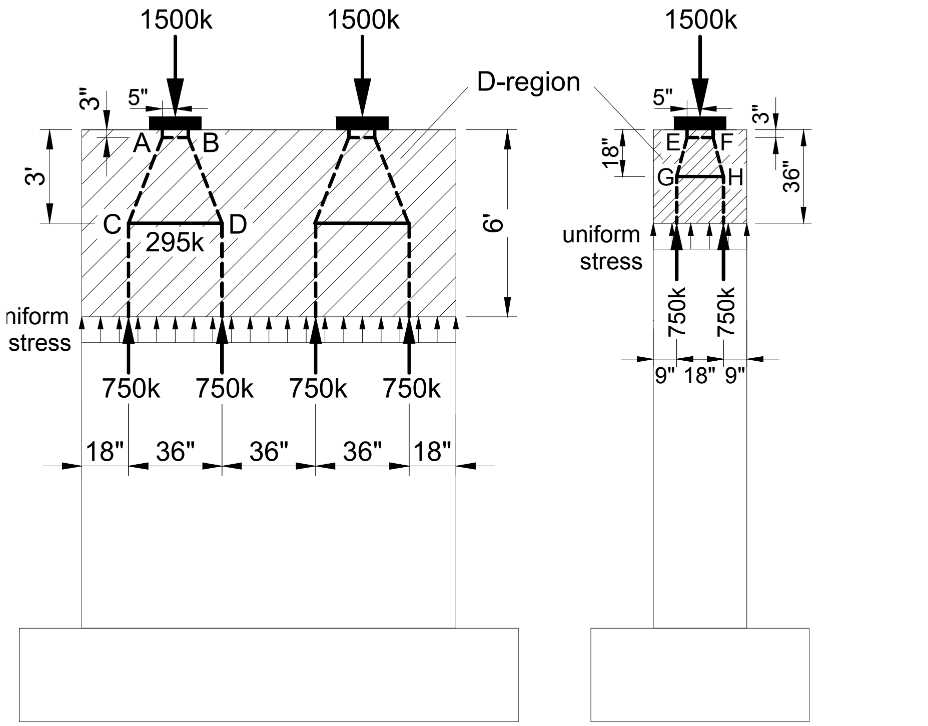
(a) Spreading of compressive stresses in 12 ft wall dimension

The simple truss model shown in Fig. 4.2a represents the flow of forces across the 12 ft width of the wall. The dashed lines represent compressive struts and the solid lines represent the tension ties. Each bearing load has been divided into two applied loads to represent the portions of the bearing load transferred to the left and the right of the bearing. These two applied loads are centered upon the left half and the right half of the bearing. Hence these two equal point loads are located at the quarter points of the bearing as shown in Fig. 4.2a.

In order to account for the depth of the concrete compressive struts it has been assumed that top nodes A and B are located 3 in. from the top concrete surface. The D-region occurs over a height of 6 ft as shown and it has been assumed that the tension tie, CD, represents the horizontal reinforcement over the 6 ft height and is located 3 ft from the top of the wall. At a distance of 6 ft from the top of the wall the compressive stresses are assumed to be uniform and hence the 750 kip compressive forces are located such that they act as resultants of the uniform stress over a width of 36 in.

Figure 4.2a shows the truss idealization and the resulting member forces. The horizontal distance between nodes A and C is 18 - 5 = 13 in. From equilibrium of the strut-and-tie model the tension tie, CD, must be capable of resisting a force of:

$$P_u = 750 \times \frac{13}{33} = 295 \text{ kips}$$



(a) Spreading over 12 ft width

(b) Spreading over 3 ft thickness

Figure 4.2. Truss idealization and member forces.

(b) Spreading of compressive stresses in 3 ft thick wall dimension

Fig. 4.2(b) shows the flow of forces through the thickness of the wall. The applied central bearing load has been divided into two applied loads to represent the portions of the bearing load transferred to the left and the right of the wall thickness. These two point loads are centered upon the left half and the right half of the bearing and are located at the quarter points of the bearing as shown in Fig. 4.2b.

In order to account for the depth of the concrete compressive struts it has been assumed that top nodes E and F are located 3 in. from the top concrete surface. The D-region occurs over a height of 3 ft as shown and it has been assumed that the tension tie, GH, represents the horizontal reinforcement over the 3 ft height and is located 18 in. from the top of the wall. At a distance of 3 ft from the top of the wall the compressive stresses are assumed to be uniform and hence the two 750 kip compressive forces are located at the quarter points of the 3 ft thickness to represent this uniform stress.

Figure 4.2b shows the truss idealization and the resulting member forces. The horizontal distance between nodes E and G is 9 – 5 = 4 in. Hence the main tension tie, GH, must be capable of resisting a force of:

$$P_u = 750 \times \frac{4}{15} = 200 \text{ kips}$$

Step 3 – Determine Required Amount of Tension Tie Reinforcement (§5.6.3.4.1)

(a) Spreading of compressive stresses in 12 ft wall dimension

The minimum area of horizontal tension tie reinforcement, A_{st} , in Tie CD required to resist the 295 kip force is:

$$A_{st} = \frac{P_u}{\phi f_y} = \frac{295}{0.9 \times 60} = 5.46 \text{ in.}^2$$

This amount of reinforcement is required for strength in the 6 ft high disturbed region as determined from the strut-and-tie model. If pairs of No. 6 bars are used then the number of pairs required would be $5.46/(2 \times 0.44) = 6.2$ or 7 pairs. Hence for strength considerations 2 – No. 6 bars at a spacing of 10 in. would be required over the 6 ft height of the D-region.

(b) Spreading of compressive stresses in 3 ft thick wall dimension

The minimum area of horizontal tension tie reinforcement, A_{st} , in Tie GH required to resist the 200 kip force is:

$$A_{st} = \frac{P_u}{\phi f_y} = \frac{200}{0.9 \times 60} = 3.70 \text{ in.}^2$$

This amount of reinforcement is required for strength in the 3 ft high disturbed region as determined from the strut-and-tie model. If pairs of No. 6 bars are used then the required number of pairs required would be $3.70/(2 \times 0.44) = 4.2$ or 5 pairs. Hence for strength considerations provide No. 6 hoops at a spacing of 7 in. over the 3 ft height as shown in Fig. 4.3.

Step 4 – Provide Crack Control Reinforcement

For “disturbed regions” (D-regions) the AASHTO LRFD Specifications require that reasonably closely spaced longitudinal and horizontal reinforcing bars for crack control and minimum ductility be provided (§5.6.3.6). The spacing of this reinforcement must not exceed 12 in. and the minimum ratio of reinforcement to gross concrete area must be at least 0.003 in each direction.

(a) Horizontal crack control reinforcement

If No. 6 bars are provided near each face of the beam, then the required spacing of these bars to provide the required reinforcement ratio of 0.003 is:

$$s_h = \frac{A_h}{0.003 b_w} = \frac{2 \times 0.44}{0.003 \times 36} = 8.1 \text{ in.}$$

The amount of crack control reinforcement exceeds the amount required for strength. Hence provide pairs of No. 6 bars at a spacing of 8 in. in the disturbed region (see Fig. 4.3). The No. 6 horizontal bars will be hooked around the corner reinforcement at the edges of the wall.

(b) Vertical crack control reinforcement

Provide pairs of No. 6 bars at a spacing of 8 in. in the disturbed region (see Fig. 4.3). The No. 6 vertical bars will be hooked around the corner reinforcement at the top of the wall.

(c) Reinforcement required in local zone below the bearing

The local zone below the bearing requires 5 pairs of No. 6 bars as determined in Step 3(b). This "bursting" reinforcement is provided by 5 closed stirrups at a spacing of 7 in. and also serves as confinement reinforcement in the region below the bearing (see §C5.10.9.3.2).

In addition, there is a tendency for cracks to form on the top surface and on the sides of the wall near the top surface due to "spalling" forces. The minimum spalling force for design is usually taken as 2 percent of the concentrated force (§5.10.9.3.2) and therefore the minimum area of spalling reinforcement is:

$$A_{spalling} = \frac{0.02 \times P_u}{\phi f_y} = \frac{0.02 \times 1500}{0.9 \times 60} = 0.56 \text{ in}^2$$

Hence provide 2 – No. 6 bars near the top and side surfaces (area of 0.88 in.^{2} provided).

Step 5 – Sketch the Required Reinforcement

The resulting reinforcement of the wall is shown in Fig. 4.3.

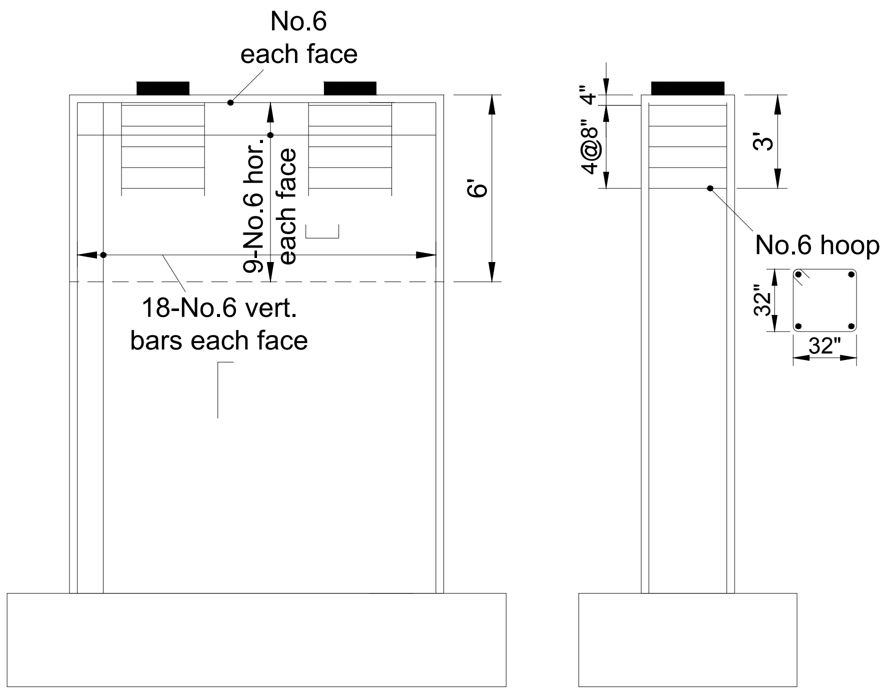


Figure 4.3. Reinforcement details of wall (reinforcement outside of D-region and footing reinforcement not shown).

10. EXAMPLE 5 - DESIGN OF INVERTED TEE CAP BEAM

The inverted tee beam shown in Fig. 5.1 is 4 ft wide and spans 30 ft between the centers of the two supporting columns. The function of this beam is to support the ends of six precast pretensioned girders and the cast-in-place deck slab. As can be seen in Fig. 5.1 the beam is subjected six 175 kip applied factored loads, P_u , acting on the top surface of the bottom ledge. These loads include the effects of dead loads (including an allowance for the self-weight of the inverted tee beam), lane and truck loading and an allowance for impact. The specified concrete compressive strength, $f_c^{'}$, is 5 ksi and the specified yield strength of the reinforcing steel is 60 ksi. The concrete clear cover is 2 in.

Design the inverted tee beam using the AASHTO LRFD Specifications. Note that the factored loads shown in Fig. 5.1 are appropriate for the design of the finished bridge (i.e., the cast-in-place deck slab is effective).

The entire inverted tee beam will be treated as a disturbed region (D-Region) and will be designed using strut-and-tie models because:

1. Because the loads are applied to the bottom ledges and not to the top face of the girder, there are disturbed regions in the beam centered upon each loading location.

2. The distance between the center of the exterior applied loads and the center of the reactions in the supporting columns is 5 ft, which is less than twice the overall depth of the beam (§5.6.3.1).

The design will be carried out in two stages: first, the disturbed regions around the applied loading locations will be designed and then the additional reinforcement required to ensure that the overall member behaviour is satisfactory will be determined.

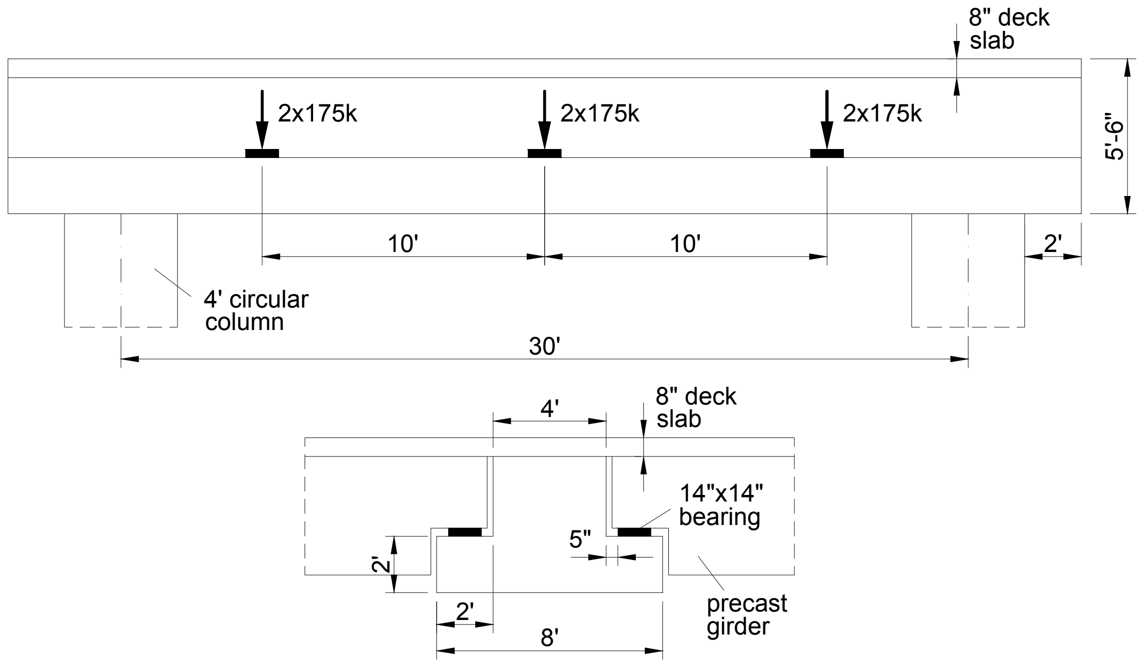


Figure 5.1. Details of inverted tee beam.

Step 1 – Design of Disturbed Regions Around Ledge Loads

The truss models for the design of the local forces around the ledges are shown in Fig. 5.2. The dashed lines represent compressive struts and the solid lines represent tension ties. Fig. 5.2(a) shows the flow of forces in the bottom ledge.

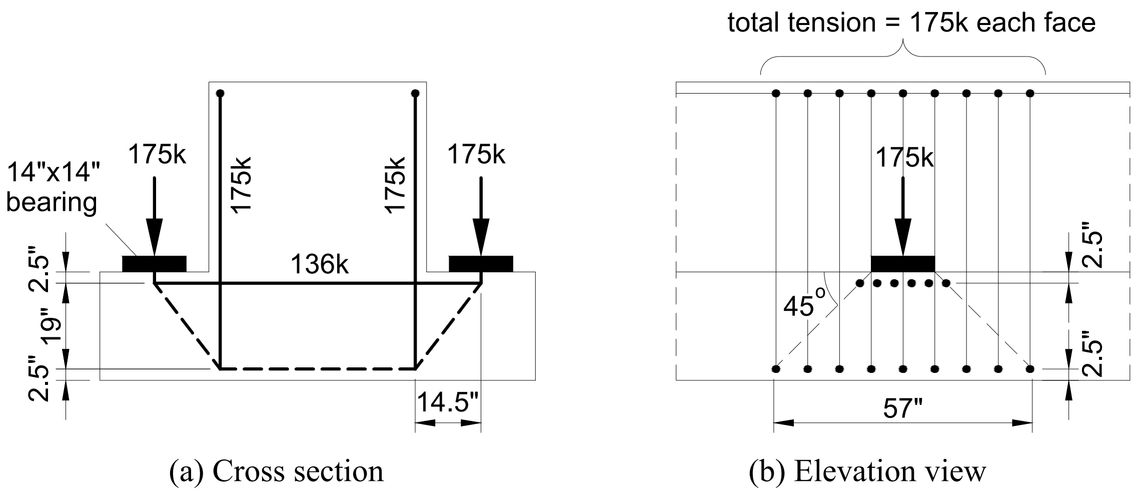


Figure 5.2. Truss idealization for disturbed regions around ledges.

A vertical tension tie is required near each face of the inverted tee beam to lift the two 175 kip reactions to the top of the beam. This requires an area of tension tie reinforcement near each face of:

$$A_{st} = \frac{P_u}{\phi f_y} = \frac{175}{0.9 \times 60} = 3.24 \text{ in.}^2$$

To meet this requirement using No. 6 reinforcing bars would require 3.24/0.44 = 7.4 bars near each face. As can be seen in Fig. 5.2(b) the load under the bearing pad is assumed to fan out at an angle of 45 degrees. Hence the closed stirrups need to be placed within a length of about 57 inches. Thus use 8 No. 6 closed stirrups at a spacing of 8 in. for a total length of 56 in. These closed stirrups will provide the 8 vertical bars needed near each face to equilibrate the fanning diagonal compressive stresses coming from the applied bearing loads.

With the assumed geometry shown in Fig. 5.2(a), the horizontal tension tie must resist 134 kips. This requires an area of tension tie reinforcement of:

$$A_{st} = \frac{P_u}{\phi f_y} = \frac{134}{0.9 \times 60} = 2.48 \text{ in.}^2$$

This requires 6 - No. 6 bars, having an area of 2.64 in.^{2}. This reinforcement will spread over a distance of 14 + 2 x 2.5 = 19 in., resulting in a spacing of 3.2 in. Because these

bars must resist significant tension beneath the bearings, a closed hoop will be used and it is necessary to check the development length of these hooked bars.

The basic development length required for a hooked No. 6 bar is (§5.11.2.4):

$$\ell_{hb} = \frac{38.0 d_b}{\sqrt{f_c'}} = \frac{38.0 \times 0.75}{\sqrt{5}} = 12.75 \text{ in.}$$

Although the basic hook development length can be reduced by applying modification factors (§5.11.2.4) to give ℓ_{dh}, the ℓ_{dh} provided to the inner edge of the bearing is 14 +5 - 2 = 17 in. Hence sufficient anchorage is provided to develop the yield stress at the inner edge of the bearing. Furthermore, this horizontal reinforcement is provided in the form of a closed hoop with corner bars which improves the anchorage (see Fig. 5.4).

The sizes of the 14 in. square bearings were chosen to limit the stresses in the neoprene bearing pads. The compression can spread from the 14 in. width of the bearing to the 24 in. ledge length, or 5 in. on each side of the 14 in. bearing. This results in an area A_2 of 24 x 24 = 576 in.^{2} The confinement factor m is (§5.7.5) is:

$$m = \sqrt{\frac{A_2}{A_1}} = \sqrt{\frac{576}{14 \times 14}} = 1.71 \le 2.0$$

Hence the confinement factor, m, is 1.71.

The required bearing area to resist the 175 kip applied load for this nodal zone anchoring a one-direction tension tie is:

$$\text{bearing area required} = \frac{P_u}{0.75 \phi m f_c'} = \frac{175}{0.75 \times 0.70 \times 1.71 \times 5} = 39 \text{ in.}^2$$

With dimensions of 14 in. by 14 in., the bearing size is sufficient (area = 196in.^{2}) the concrete stress limit for this node region of $0.75\phi m f_c'$ is easily satisfied.

Step 2 – Design of Overall Member

The assumed truss model for the design of the overall member is shown in Fig. 5.3 along with the member forces determined from equilibrium. For the 2 in. clear cover and the use of No. 6 stirrups it has been conservatively assumed that the centroid of the bottom longitudinal tension reinforcement is located 3.5 in. from the bottom face of the member and that the top chord of the truss is horizontal with its centroid located 2.5 in. from the top face of the member.

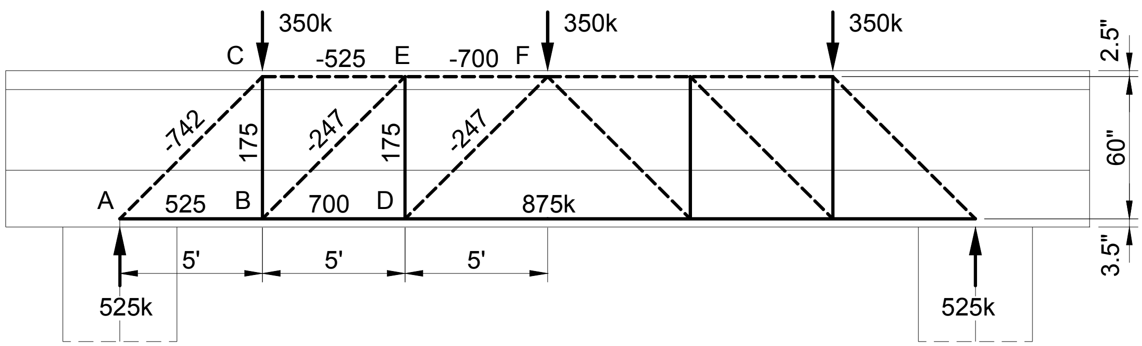


Figure 5.3. Strut and tie model for overall member.

The highest tension demand on the bottom longitudinal reinforcement is equal to 875 kips and hence the minimum area of longitudinal bottom reinforcement is:

$$A_{st} = \frac{P_u}{\phi f_y} = \frac{875}{0.9 \times 60} = 16.2 \text{ in.}^2$$

To meet this requirement use 17 - No. 9 reinforcing bars (area of 17.0 in^2) in one layer across the 8 ft width of the beam.

Check development of these bottom longitudinal bars:

The No. 9 longitudinal bars making up the bottom tension tie must be capable of developing a tensile force of 525 kips near the inner edge of the columns. Because the columns are circular it is appropriate to replace the circular bearing area by a square bearing having the same area. The equivalent square column has side dimensions of 3.54 ft. The embedment length from the inside face of this equivalent area to the end of the beam is $24 + 3.54 \times 12 - 2\,(\text{cover}) = 64.5$ in. is available to develop this tensile force in bars (see Fig. 5.1). The basic development length for a straight No. 9 bar is (§5.11.2.1):

$$\ell_{db} = \frac{1.25 A_b f_y}{\sqrt{f_c'}} = \frac{1.25 \times 1.0 \times 60}{\sqrt{5}} = 33.5 \text{ in.}$$

But not less than:

$\ell_{db} = 0.4 d_b f_y = 0.4 \times 1.125 \times 60 = 27.0$ in. or 12.0 in.

The area of steel required to carry the required force of 525 kips is 525/(0.9 x 60) = 9.72 in.^{2}

The area of steel provided is 17 x 1.00 = 17.0 in.^{2}.

Therefore the required development length is (§5.11.2.1):

$$\ell_d = \ell_{db} \times \frac{(A_s required)}{(A_s provided)} = 33.5 \times \frac{9.72}{17.0} = 19.2 \text{ in.}$$

Hence the anchorage length is sufficient for the No. 9 tension tie reinforcement.

In order to resist the 175 kips of tension in the vertical truss members the minimum area of vertical reinforcement required is:

$$A_{st} = \frac{P_u}{\phi f_y} = \frac{175}{0.9 \times 60} = 3.24\ \text{in.}^2$$

The vertical tension tie member in the truss represents the stirrups distributed over a length of 5 ft along the beam (see Fig. 5.3). Using No. 6 double legged stirrups, the number of stirrups required in this distance is 3.24/(2 x 0.44) = 3.68. Hence double legged stirrups at a spacing of 16 in. will satisfy this requirement.

The truss model shown in Fig. 5.3 is for the situation where the 350 kip forces are applied near the top face of the beam. This is a reasonable assumption if the hanger reinforcement designed in Step 1 is provided. Hence the hanger reinforcement determined in Step 1 needs to be added to the required vertical tie reinforcement from the overall member design shown in Fig. 5.3.

Step 3 – Choose Reinforcement to Satisfy Strength Requirements

The amounts of reinforcement required to resist the tension forces calculated from the strut-and-tie models have been determined in Steps 1 and 2. This reinforcement is shown in Fig. 5.4 for the ledge-load regions and for the zones outside of these regions.

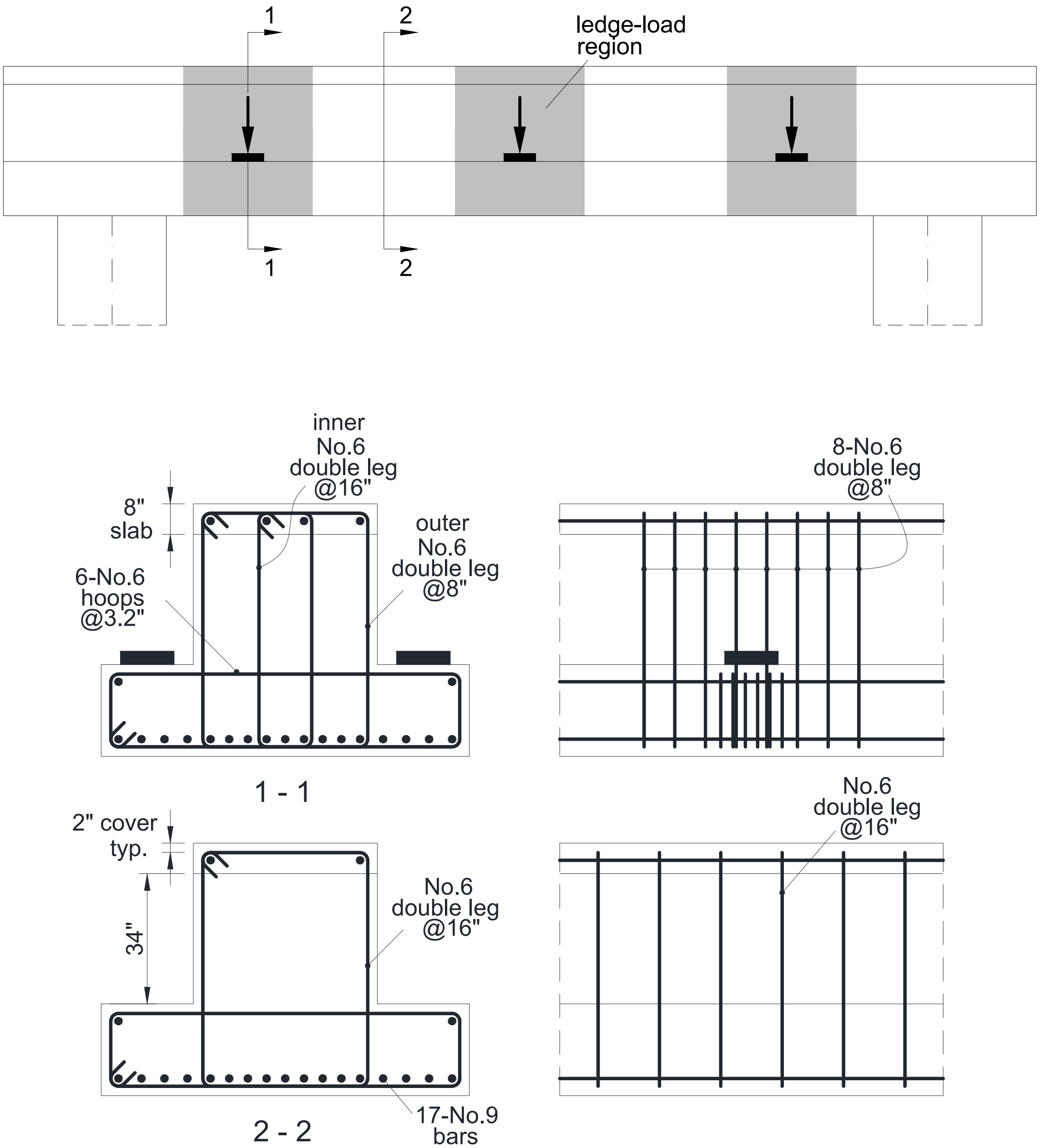


Figure 5.4. Reinforcement required to satisfy strength requirements.

Step 4 – Provide Crack Control Reinforcement

Because the entire inverted tee beam is being treated as a disturbed region, reasonably closely spaced longitudinal and horizontal reinforcing bars for crack control and minimum ductility need to be provided (§5.6.3.6). The spacing of this reinforcement must not exceed 12 in. and the minimum ratio of reinforcement to gross concrete needs to be at least 0.003 in each direction to satisfy this requirement.

(a) In 4 ft wide section above ledge

Vertical reinforcement outside ledge-load regions:
If 4 legged No. 6 bars are provided at 12 in. spacing, then the reinforcement ratio would be:

$$\rho = \frac{A_v}{s_v b_w} = \frac{4 \times 0.44}{12 \times 48} = 0.0031$$

which satisfies the maximum spacing and the minimum reinforcement ratio requirements.

Vertical reinforcement in ledge-load regions:
If a combination of No. 6 double legged outer stirrups at 8 in. spacing and No. 6 double legged inner stirrups at 12 in. spacing are used then the reinforcement ratio, calculated for a length of 24 in. would be:

$$\rho = \frac{\left(2 \times \frac{24}{8} \times 0.44\right) + \left(2 \times \frac{24}{12} \times 0.44\right)}{24 \times 48} = 0.0038$$

This arrangement satisfies the following:

a. the need for the hanger steel near the face
b. the need for the vertical tension tie reinforcement for the overall strut-and-tie model
c. the 12 in. maximum spacing requirement, and
d. the minimum reinforcement ratio requirement of 0.003.

Horizontal reinforcement:
In the 34 in. high portion of the 4 ft wide section below the slab, the provision of 3 layers, each with 4 - No. 6 bars, will result in a reinforcement ratio of:

$$\rho = \frac{3 \times 4 \times 0.44}{34 \times 48} = 0.0032$$

Hence the crack control requirements are satisfied.

(b) In 8 ft wide section

Vertical reinforcement:
Outside of the ledge load regions, with the provision of No. 6 closed hoops at 6 in. spacing in the ledge and 4 legs of No. 6 stirrups at 12 in. spacing (see Step 4(a)) results in a reinforcement ratio in a 12 in. length of:

$$\rho = \frac{\left(2 \times \frac{12}{6} \times 0.44\right) + \left(4 \times 0.44\right)}{12 \times 96} = 0.0031$$

Therefore the crack control requirements are satisfied.

Horizontal reinforcement:
The combination of 6 – No. 6 longitudinal bars in the upper portion of the ledge plus 17 – No. 9 tension tie reinforcing bars gives a reinforcement ratio of:

$$\rho = \frac{\left(6 \times 0.44\right) + \left(17 \times 1.00\right)}{24 \times 96} = 0.0085$$

Hence there is sufficient reinforcement in the 8 ft wide bottom region of the beam to provide good crack control.

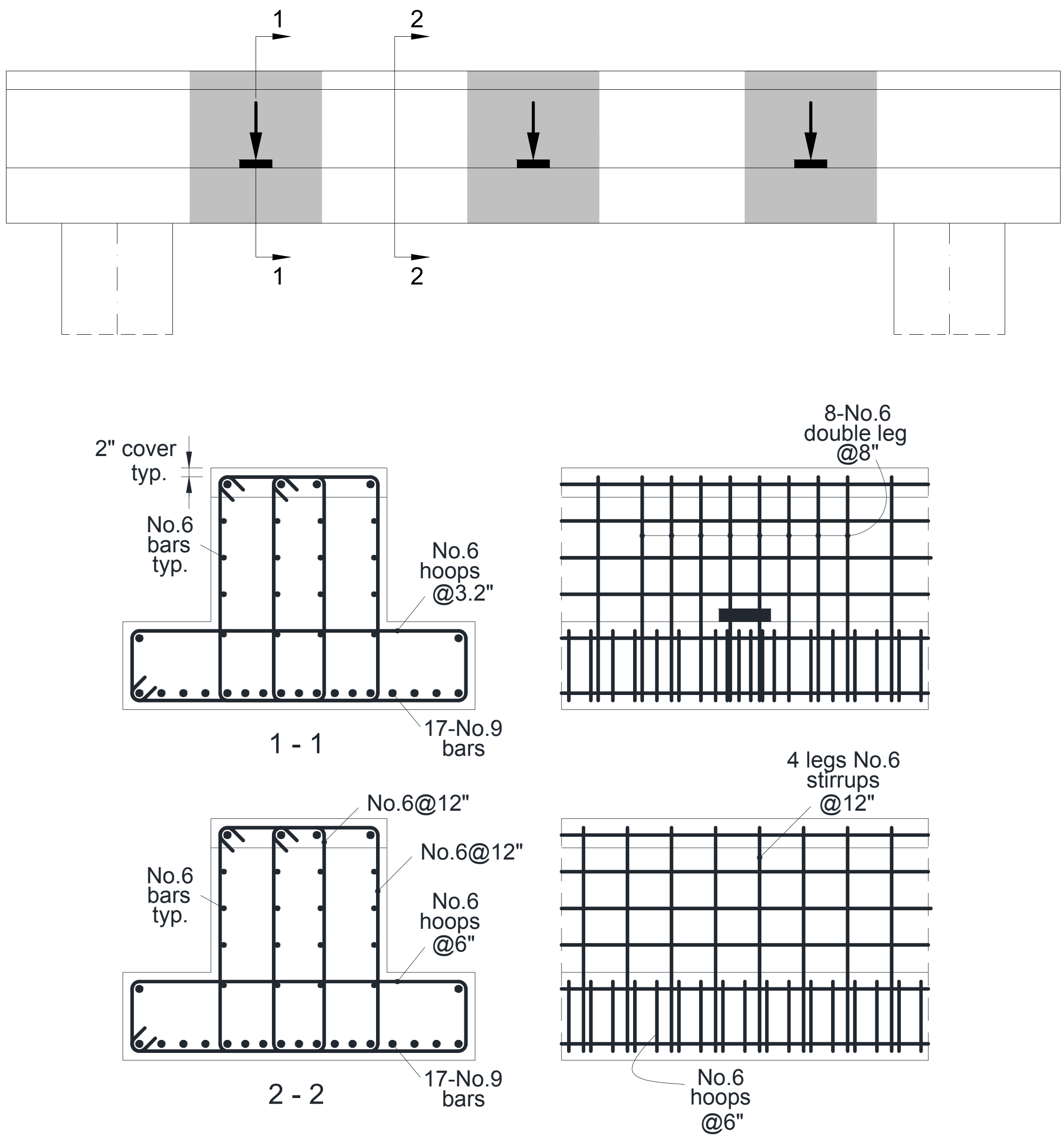


Figure 5.5. Reinforcement required to satisfy strength and crack control requirements.

11. EXAMPLE 6 – CONTINUOUS CAP BEAM

The 44 in. deep by 24 in. wide continuous cap beam shown in Fig. 6.1 has two spans of 13 ft and two cantilevers of 6.5 ft each. The forces on the girder are factored loads and include an allowance for the factored self-weight of the member. These forces are applied through 18 in. by 18 in. bearing pads placed on top of the girder. The specified concrete compressive strength, $f_c^{'}$, is 6 ksi and the specified yield strength of the reinforcing steel is 60 ksi. The bending moment and shear diagrams obtained from a linear elastic analysis are also shown in Fig. 6.1.

Design the reinforcement of the girder based on the strut-and-tie provisions of the AASHTO LRFD Specifications.

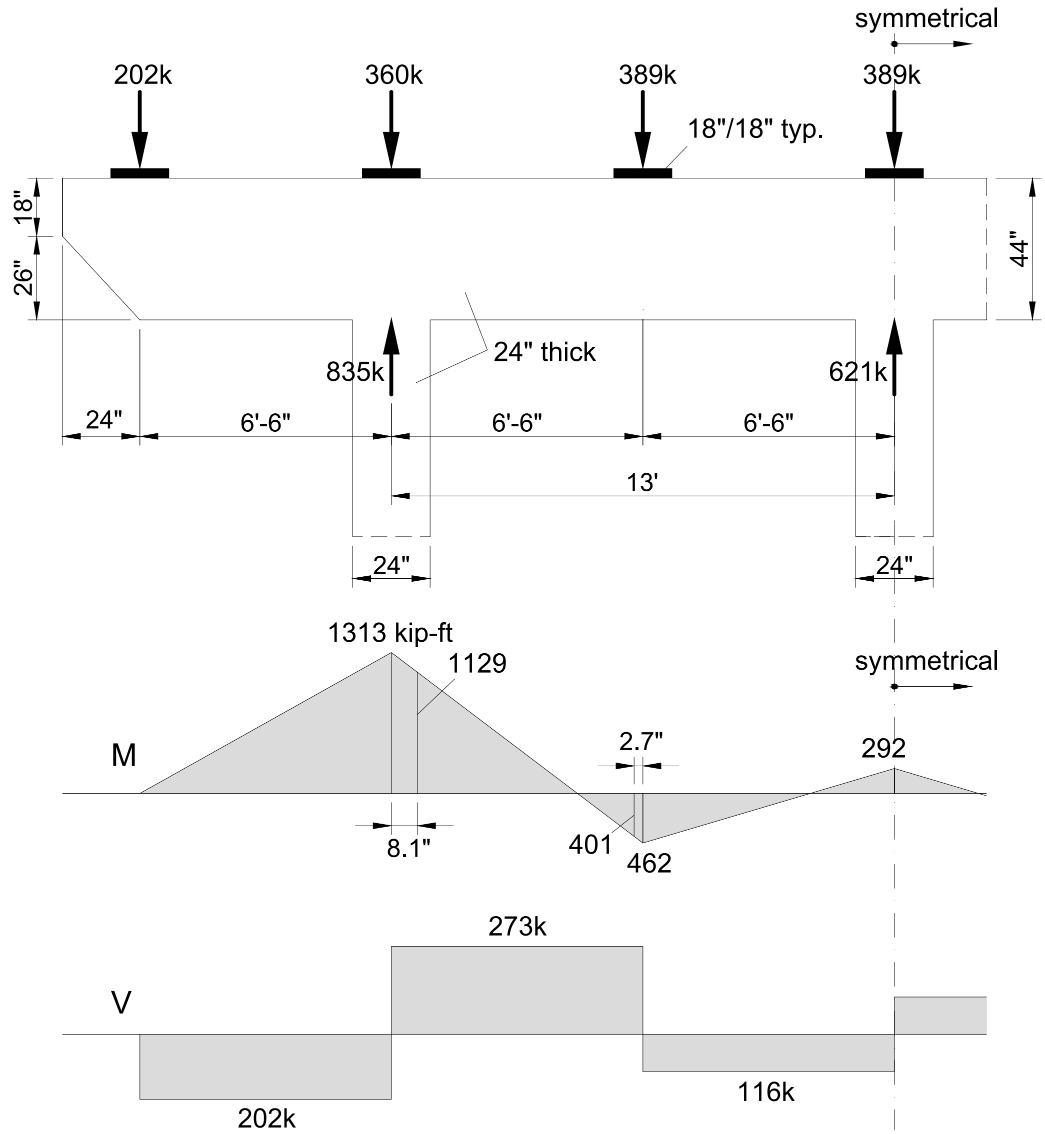


Figure 6.1. Details of cap beam.

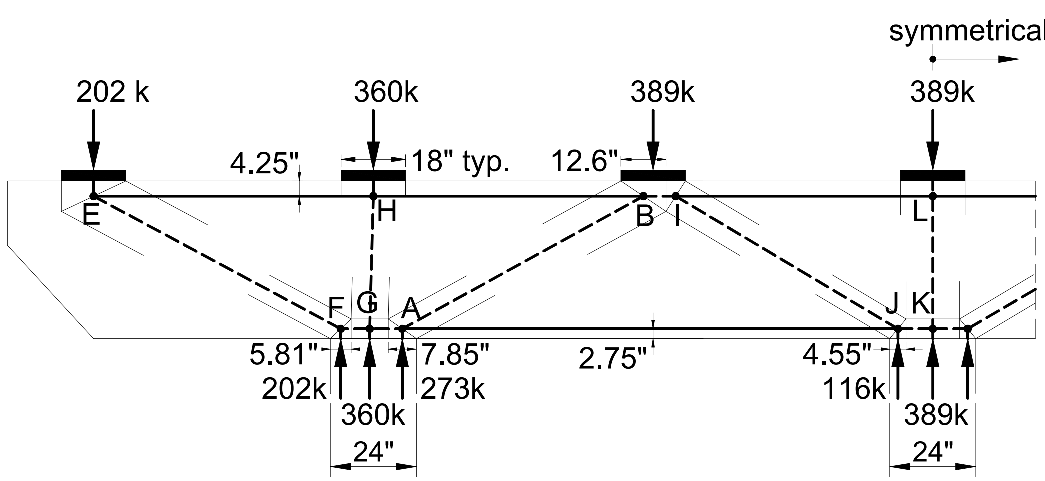
Step 1 – Draw Idealized Truss Model and Solve for Member Forces

Figure 6.2(a) shows a strut-and-tie model of the cap beam. The dashed lines represent compressive struts and the solid lines represent the tension ties. The required concrete cover for the No. 6 stirrups is 1.5 in. and it is assumed that two layers of No. 8 top bars, with a vertical centre-to center spacing of 3 in., and one layer of No. 8 bottom bars will be used. In determining the layout of the reinforcement the nominal bar diameters and a maximum aggregate size of ¾ in. have been assumed. The centroid of the top reinforcement is located 4.25 in. from the top face of the girder while the bottom is at 2.75 in. from the face. It will be assumed that the locations of the top and bottom reinforcement represent the top and bottom chords of the truss model. The distance between the chords is 44 – 4.25 - 2.75 = 37.0 in.

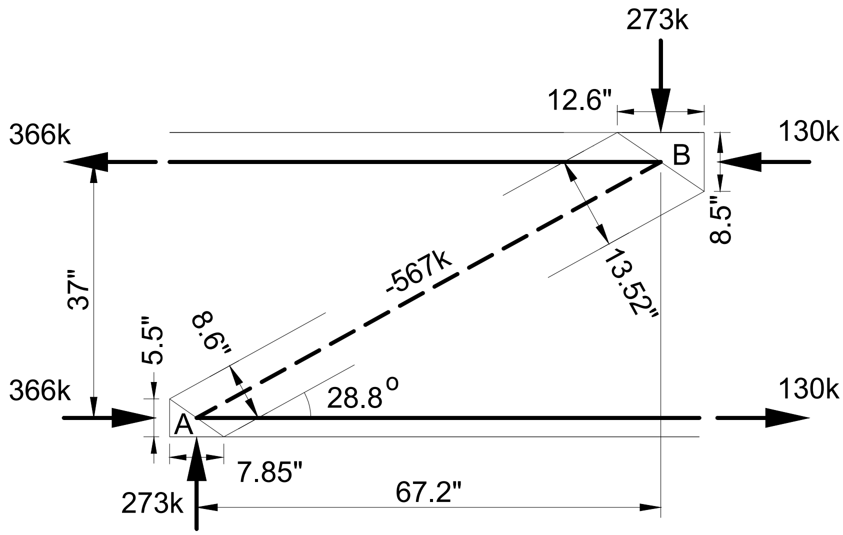
It is appropriate to subdivide each column bearing area into 3 tributary parts in proportion to the vertical component of the struts meeting at the column at nodes F, G and A (see Fig. 6.2(a)). For example at node A the width of the nodal region is:

$$\frac{273}{835} \times 24 = 7.85 \text{ in.}$$

Figure 6.2(b) shows in more detail the strut-and-tie model of the critical shear span of the girder between the exterior column and the first point load of 389 kips. The tension forces in the ties of this model are obtained from equilibrium. It is usual in design to check the flexural demand at the face of a column however when using truss models it is more convenient to calculate the tension in the reinforcement based on the moments at the locations of the nodes of the truss. For example, at node A the moment is 1129 kip-ft (see Fig. 6.1) and hence the tension in tie HB is 1129 x 12/37 = 366 kips. The compression force in the diagonal strut is calculated as 273/sin 28.8º = 567 kips.



a) Entire cap beam



b) Critical shear span

Figure 6.2. Strut-and-tie model without transverse ties.

Step 2 – Check Size of Bearing Areas

The vertical forces at nodal regions A and B require the following minimum bearing areas:

At node A:

bearing area required $= \dfrac{V_u}{0.75\phi m f_c'} = \dfrac{273}{0.75 \times 0.70 \times 1.0 \times 6} = 86.7 \text{ in}^2 < 24 \times 7.85 = 188 \text{ in}^2$

At node B:

bearing area required $= \dfrac{V_u}{0.75\phi m f_c'} = \dfrac{273}{0.75 \times 0.70 \times 1.33 \times 6} = 65 \text{ in}^2 < 18 \times 12.6 = 227 \text{ in}^2$

Hence, the available top and bottom bearing areas are adequate.
The confinement modification factor *m* is larger than 1 in the second calculation because the 18 in. by 18 in. bearing pad is narrower than the cross section of the girder:

$$m = \sqrt{\frac{24 \times 24}{18 \times 18}} = 1.33 < 2.0$$

Step 3 – Determine Required Amount of Tension Tie Reinforcement

The area of reinforcement required to resist the tension in the top tie is:

$$A_{st} = \frac{366}{0.9 \times 60} = 6.8 \text{ in}^2$$

While for the bottom tie this area is:

$$A_{sb} = \frac{130}{0.9 \times 60} = 2.41 \text{ in}^2$$

Step 4 –Choose Layout of Tension Tie Reinforcement

The reinforcement in the longitudinal ties is arranged as shown in Fig. 6.3. The top reinforcement consists of 10 - No.8 bars placed in two layers of 5 bars (A_{st} = 7.9 in.^{2}) while 4 - No.8 bars are provided at the bottom of the section (A_{sb} = 3.16 in.^{2}). This layout may require adjustments if the areas of the nodal zones perpendicular to the ties are insufficient. The checks for the required areas of the nodal regions perpendicular to the longitudinal reinforcement are similar to those for the size of the bearing areas performed in Step 2:

At node A:

$$\text{area required } = \frac{366 + 130}{0.75 \times 0.70 \times 1.0 \times 6} = 157 \text{ in.}^2 > 24 \times 5.5 = 132 \text{ in.}^2$$

At node B:

$$\text{area required } = \frac{366 + 130}{0.75 \times 0.70 \times 1.0 \times 6} = 157 \text{ in.}^2 > 18 \times 8.5 = 153 \text{ in.}^2$$

Hence, both the top and bottom nodal regions have insufficient area perpendicular to the ties.

One way to lower the horizontal stresses in the nodal regions is to either distribute the longitudinal reinforcement in more layers or increase the distance between layers. This would result in a decreased lever arm and therefore higher forces in the ties. A better solution would be to use a refined strut-and-tie model by accounting for the beneficial effects of the transverse reinforcement in the member. To satisfy the AASHTO requirement for crack control reinforcement (Eq. 5.6.3.6-1), double-leg No.6 stirrups at 12 in. (ρ_v = 0.306%) are provided (see Fig. 6.3). Place 2-No. 6 horizontal bars at a spacing of 12 in. on each face to satisfy the crack control reinforcement requirements (see Fig. 6.3). Additional reinforcing bars used to position the No. 8 longitudinal bars are not shown.

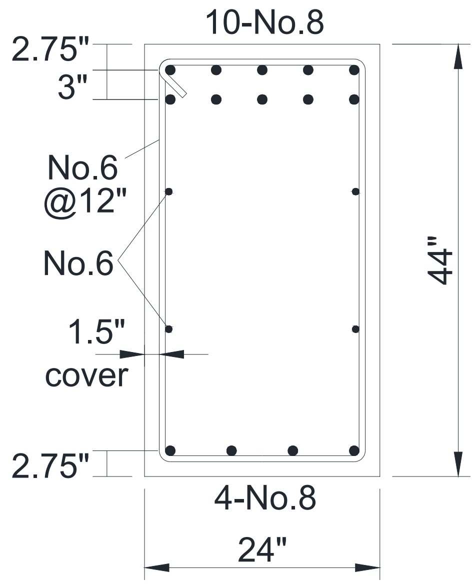


Figure 6.3. Longitudinal and transverse reinforcement.

Step 5 – Formulate and Solve Refined Strut-and-Tie Model

A refined strut-and-tie model of the critical shear span is shown in Fig. 6.4. The vertical tie in the middle of the shear span represents the stirrups located within the middle ½ of the clear span of 57.0 in. from the inner edge of the support to the inner edge of the loading pad. It is assumed that the vertical tie yields, and thus the factored tensile force in this tie is:

$$P_u = \phi P_n = \phi f_y A_{st} = 0.9 \times 60 \times \left(2 \times 0.44 \frac{(57/12)}{2} \right) = 113 \text{ kips}$$

Nodes C and D are located halfway between nodes A and B. The new load path BDCA provided by the vertical tie results in reduced tension forces in the top and bottom ties near nodes B and A. The reduced tension forces shown in Fig. 6.4 are calculated by considering the equilibrium of nodes C and D.

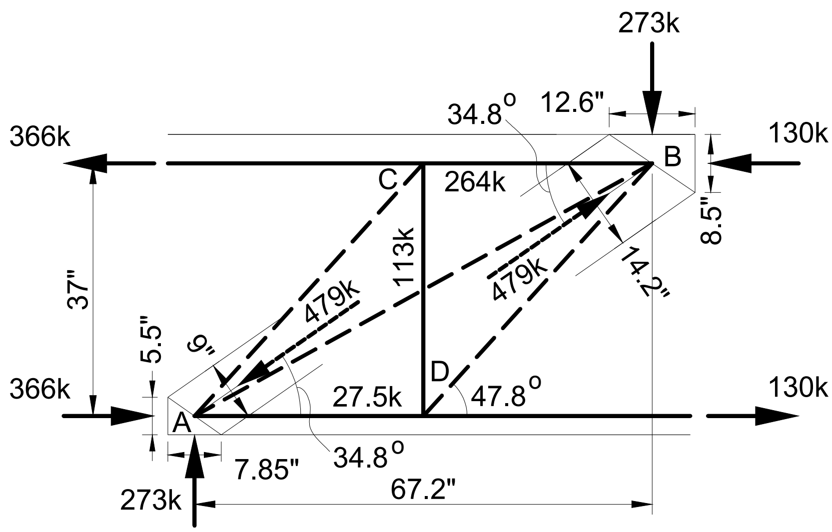


Figure 6.4. Strut-and-tie model with transverse ties.

Step 6 – Check Area of Nodal Regions Perpendicular to Horizontal Ties

The areas of the nodal regions perpendicular to the horizontal ties are checked as in Step 4 but with the reduced forces in the ties at nodes A and B:

At node A:

$$\text{area required } = \frac{366+27.5}{0.75\times 0.70\times 1.0\times 6} = 125 \text{ in.}^2 < 24\times5.5 = 132 \text{ in.}^2$$

At node B:

$$\text{area required } = \frac{264+130}{0.75\times 0.70\times 1.0\times 6} = 125 \text{ in.}^2 < 18\times8.5 = 153 \text{ in.}^2$$

Hence, the stresses in nodal regions A and B now satisfy the AASHTO limit of $0.75\phi m f_c'$.

Step 7 – Check Capacity of Struts in Refined Model

The critical locations for strut crushing are those near nodes A and B where large diagonal compressive forces are transferred across relatively small areas. The magnitude and the angle of these forces with respect to the horizontal ties are obtained by considering the equilibrium of nodes A and B. For example, the 479 kip compressive force at an angle of 34.8° is the resultant of the compressive force in AC and the compressive force in AB.

In order to calculate the capacity of the diagonal struts, it is first necessary to calculate the strains in the top and bottom ties near nodal regions B and A, respectively:

Top tie:

$$\varepsilon_{st} = \frac{264}{29000 \times 10 \times 0.79} = 1.152 \times 10^{-3}$$

Bottom tie:

$$\varepsilon_{sb} = \frac{27.5}{29000 \times 4 \times 0.79} = 0.300 \times 10^{-3}$$

The strains perpendicular to the diagonal struts in the vicinity of the top and bottom nodal regions are then obtained from compatibility as (Eq. 5.6.3.3.3-2):

$$\varepsilon_{1t} = 1.152 \times 10^{-3} + \left(1.152 \times 10^{-3} + 2 \times 10^{-3}\right) cot^2 34.8^\circ = 7.67 \times 10^{-3}$$
$$\varepsilon_{1b} = 0.300 \times 10^{-3} + \left(0.300 \times 10^{-3} + 2 \times 10^{-3}\right) cot^2 34.8^\circ = 5.06 \times 10^{-3}$$

These strains result in reduced compressive strength of the diagonal struts calculated from Eq. 5.6.3.3.3-1 as:

$$f_{cu,t} = \frac{f_c'}{0.8 + 170\varepsilon_1} = \frac{6}{0.8 + 170 \times 7.67 \times 10^{-3}} = 0.475 \times 6 = 2.85 \text{ ksi}$$
$$f_{cu,b} = \frac{f_c'}{0.8 + 170\varepsilon_1} = \frac{6}{0.8 + 170 \times 5.06 \times 10^{-3}} = 0.602 \times 6 = 3.61 \text{ ksi}$$

In order to calculate the nominal force capacity of the struts, the above stresses need to be multiplied by the cross sectional area of the struts. The width of the struts in the plane of the girder is a function of the horizontal and vertical dimensions of the nodal regions, as well as the angle of the strut (see Fig. 5.6.3.3.2-1(b)):

$$w_t = l_b \sin\theta + h_a \cos\theta = 12.6 \times sin 34.8^\circ + 8.5 \times cos 34.8^\circ = 14.2 \text{ in.}$$
$$w_b = l_b \sin\theta + h_a \cos\theta = 7.85 \times sin 34.8^\circ + 5.5 \times cos 34.8^\circ = 9.0 \text{ in.}$$

And thus the nominal capacities of the struts are:

$$P_{nt} = f_{cu} A_{cs} = 2.85 \times (14.2 \times 18) = 728 \text{ kips}$$
$$P_{nb} = f_{cu} A_{cs} = 3.61 \times (9.0 \times 24) = 780 \text{ kips}$$

Finally, the factored capacities of the struts are compared to the factored demand on the strut:

$$P_r = \phi P_{nt} = 0.7 \times 728 = 510 \text{ kips} > 479 \text{ kips (OK)}$$
$$P_r = \phi P_{nb} = 0.7 \times 780 = 546 \text{ kips} > 479 \text{ kips (OK)}$$

Hence, the capacities of the struts are adequate.

It is noted that in these calculations for the limiting compressive stress in the struts, Eq. 5.6.3.3.3-1 was used with the appropriate value of ε_s in the tension tie crossing the strut. If the limiting compressive stress in the struts were calculated using the simplified Eq. 5.6.3.3.3-3, which assumes that the tension ties yield, then the following values of limiting stresses and strut capacities would result:

$$f_{cu,t} = f_{cu,b} = \frac{f_c'}{1.15 + 0.69\,cot^2\,\alpha_s} = \frac{6}{1.15 + 0.69\,cot^2\,34.8^\circ} = 2.33 \text{ ksi}$$

$$P_{nt} = f_{cu}A_{cs} = 2.33 \times (14.2 \times 18) = 596 \text{ kips}$$

$$P_{nb} = f_{cu}A_{cs} = 2.33 \times (9.0 \times 24) = 503 \text{ kips}$$

$$P_{rt} = \phi P_{nt} = 0.7 \times 596 = 417 \text{ kips} < 479 \text{ kips (NG)}$$

$$P_{rb} = \phi P_{nb} = 0.7 \times 503 = 352 \text{ kips} < 479 \text{ kips (NG)}$$

Hence, the use of the simplified expression of Eq. 5.6.3.3.3-3 does not result in sufficient strut capacities. The strut capacities are however adequate, using the more accurate expression of Eq. 5.6.3.3.3-1.

Step 10 – Check Anchorage of Tension Ties

It is necessary to check the anchorage of the top tension ties.

(a) Anchorage near Node E

The tension force in the tie EH can be conservatively taken as 366 kips and this force must be anchored at the inner edge of the bearing near node E.

The basic development length for a straight No. 8 bar is (§5.11.2.1):

$$\ell_{db} = \frac{1.25A_b f_y}{\sqrt{f_c'}} = \frac{1.25 \times 0.79 \times 60}{\sqrt{6}} = 24.2 \text{ in.}$$

but not less than:

$$\ell_{db} = 0.4 d_b f_y = 0.4 \times 1.0 \times 60 = 24 \text{ in. or 12.0 in.}$$

The modification factor for "top reinforcement" is 1.4.

The area of steel required to carry the required force of 366 kips is 366/(0.9 x 60) = 6.78 in.^{2}
The area of steel provided is 10 x 0.79 = 7.9 in.^{2} and hence a modification factor of 6.78/7.9 = 0.858 can be used.

Hence the development length required is 24.2 x 1.4 x 0.858 = 29.1 in. The embedment length provided is 24 + 9 – 2 (cover on No. 8 bars) = 31 in. Hence sufficient development length is provided.

The required development length would be reduced if a refined strut-and-tie model were employed to represent the flow of forces in the cantilever. The beneficial effects of accounting for the presence of the vertical crack control reinforcement would reduce the calculated tension force in the top reinforcement at node E.

(b) Anchorage near Node B

It is possible to terminate half of the top longitudinal reinforcement to the right of node B. It has been determined from the refined strut-and-tie model shown in Fig. 6.4 that the 366 kip tension reduces to 264 kips near Node B. The development length for these bars can be determined as calculated above except that the ratio of the area required to the area provided is 4.89/(10 x 0.79) = 0.619. Hence the embedment length required beyond the left edge of the bearing for the lower layer of terminated bars is 24.2 x 1.4 x 0.619 = 21 in. Hence the bars must extend at least 3 in. from the right face of the bearing.

Step 11 – Sketch the Required Reinforcement

The final reinforcement layout is shown in Fig. 6.5.

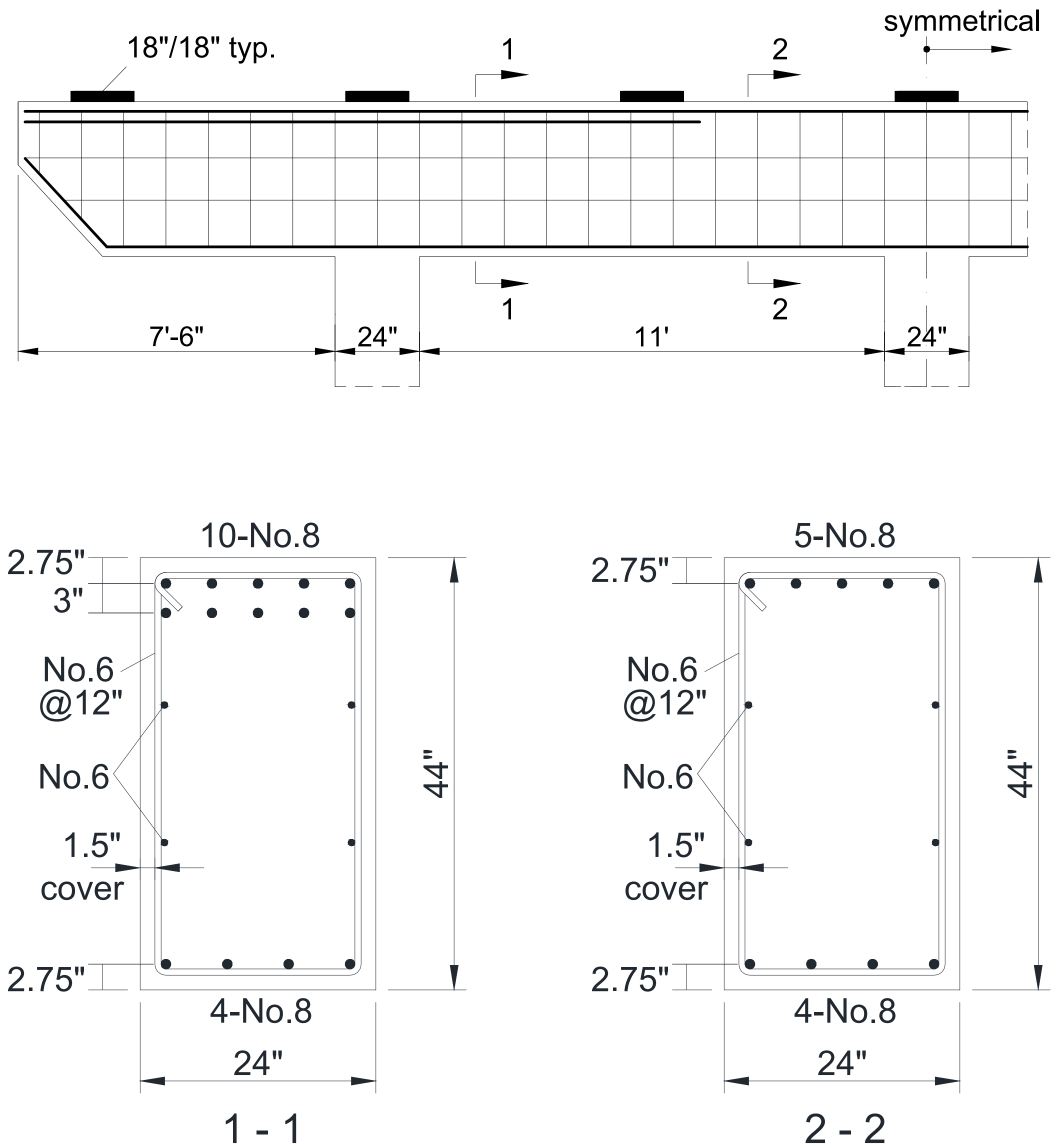


Figure 6.5. Reinforcement details of continuous cap beam.

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