NATIONAL COOPERATIVE HIGHWAY RESEARCH PROGRAM
REPORT **3**

IMPROVED CRITERIA FOR TRAFFIC SIGNALS AT INDIVIDUAL INTERSECTIONS INTERIM REPORT

HIGHWAY RESEARCH BOARD NATIONAL ACADEMY OF SCIENCES - NATIONAL RESEARCH COUNCIL

NATIONAL COOPERATIVE HIGHWAY RESEARCH PROGRAM REPORT 3

IMPROVED CRITERIA FOR TRAFFIC SIGNALS AT INDIVIDUAL INTERSECTIONS INTERIM REPORT

BY D. L. GERLOUGH AND F. A. WAGNER, PLANNING RESEARCH CORPORATION
LOS ANGELES, CALIF. WASHINGTON, D. C.

HIGHWAY RESEARCH BOARD OF THE DIVISION OF ENGINEERING AND INDUSTRIAL RESEARCH
NATIONAL ACADEMY OF SCIENCES - NATIONAL RESEARCH COUNCIL 1964

NATIONAL COOPERATIVE HIGHWAY RESEARCH PROGRAM

Systematic, well-designed research provides the most effective approach to the solution of many problems facing highway administrators and engineers. Often, highway problems are of local interest and can best be studied by highway departments individually or in cooperation with their state universities and others. However, the accelerat-in growth of highway transportation develops increasingly complex problems of wide interest to highway authorities. These problems are best studied through a coordinated program of cooperative research.

In recognition of these needs, the highway administrators of the American Association of State Highway Officials initiated in 1962 an objective national highway research program employing modern scientific techniques. This program is supported on a continuing basis by Highway Planning and Research funds from participating member states of the Association and it receives the full cooperation tion and support of the Bureau of Public Roads, United States Department of Commerce.

The Highway Research Board of the National Academy of Sciences-National Research Council was requested by the Association to administer the research program because of the Board's recognized objectivity and understanding of modern research practices. The Board is uniquely suited for this purpose as: it maintains an extensive committee structure from which authorities on any highway transportation subject may be drawn; it possesses avenues of communications and cooperation with federal, state, and local governmental agencies, universities, and industry; its relationship to its parent organization, the National Academy of Sciences, a private, non-profit institution, is an insurance of objectivity; it maintains a full-time research correlation staff of specialists in highway transportation matters to bring the findings of research directly to those who are in a position to use them.

The program is developed on the basis of research needs identified by chief administrators of the highway departments and by committees of AASHO. Each year, specific areas of research needs to be included in the program are proposed to the Academy and the Board by the American Association of State Highway Officials. Research projects to fulfill these needs are defined by the Board, and qualified research agencies are selected from those that have submitted proposals. Administration and surveillance of research contracts are responsibilities of the Academy and its Highway Research Board.

The needs for highway research are many, and the National Cooperative Highway Research Program can make significant contributions to the solution of highway transportation problems of mutual concern to many responsible groups. The program, however, is intended to complement rather than to substitute for or duplicate other highway research programs.

This report is one of a series of reports issuing from a continuing research program conducted under a three-way agreement entered into in June 1962 by and among the National Academy of Sciences-National Research Council, the American Association of State Highway Officials, and the U. S. Bureau of Public Roads. Individual fiscal agreements are executed annually by the Academy-Research Council, the Bureau of Public Roads, and participating state highway departments, members of the American Association of State Highway Officials.

This report was prepared by the contracting research agency. It has been reviewed by the appropriate Advisory Panel for clarity, documentation, and fulfillment of the contract. It has been accepted by the Highway Research Board and published in the interest of an effectual dissemination of findings and their application in the formulation of policies, procedures, and practices in the subject problem area.

NCHRP Project 3-5, FY '63
NAS-NRC Publication 1202
Library of Congress Catalog Card Number: 64-60080

FOREWORD

By Staff
Highway Research Board

To the traffic engineer interested in signal control development and to the researcher interested in the application of traffic-flow theories, this report should have immediate as well as reference value. It incorporates a state-of-the-art summary and a digital computer simulation model formulated to describe traffic performance at isolated intersections. This model permits the study of new techniques in signal control and an evaluation of their effectiveness.

In deliberations concerning priority among the many important and urgent problems suggested for the first year of work under the National Cooperative Highway Research Program, one of the problem areas chosen was that of "Traffic Capacity and Safety." Further considerations by an Advisory Panel regarding the most feasible and promising avenues of research within the area led to the selection of studies on functional design and timing of traffic signal systems. For, despite the advent of signal-free, limited-access expressways, much travel will continue on signalized traffic ways. There is, therefore, a great need to utilize these existing facilities with maximum possible efficiency.

Traffic signals provide effective control in the allocation of the right-of-way through an intersection, first to one group and then to another. Yet in pretimed signalization whether by fixed-time or by periodic programming there is some degree of inefficiency. This is substantial in some cases and during certain periods of time. Traffic-responsive signals can also be developed and utilized with greater flexibility and efficiency. A better understanding of behavior of traffic at intersections and in the adjacent area and a better knowledge of how to measure its behavior are necessary for more responsive and efficient traffic signals.

The concept of the traffic signal is simple; it allocates time and space according to demand. Carrying the concept into the real world and making it work is not simple. The variety in intersectional geometry, the variety in behavior among different types of vehicles in turning movements, the randomness in composition and arrival time, and the infinite number of possible combinations of these variables make the allocation of time and space a very complex problem.

The individual signal, distant one-half mile or more from the nearest other controlled intersection, lends itself admirably to the initial research task, but not because it is the simplest task. It is not the simplest task to provide an understanding and a model of the fundamental space-time relationships involved in signal operation. In some aspects the isolated intersection is the most difficult to model, but it is of fundamental importance because there are so many individually signalized intersections and there are so many individual signals that could perform more efficiently. The model designed for the individual intersection also becomes a basic component in the signal system development for arterial routes and street networks.

If the behavioral pattern of vehicular movements at individual or isolated intersections were better understood and mathematically modeled and if the functional specifications of the traffic signal fit the model in all of its features, a basis for more efficiency would be provided.

This project, planned for exploring the whole field of signalization for the individually signalized intersection through to a network of intersections began

with a simple intersection of four lanes crossing four lanes. This phase of the study will be completed later this year at which time it is planned to publish a full report covering findings for the individual intersection with field-tested techniques.

At the end of the first year of research, however, an interim report was submitted consisting of a narrative on the techniques being developed. This report also included an extensive selected bibliography and abstracts of the most pertinent selections. Because of the value of this interim report on the state-of-knowledge of the subject and on the techniques thus far developed to provide more effective signals, it was deemed worthy of immediate publication.

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IMPROVED CRITERIA FOR TRAFFIC SIGNALS AT INDIVIDUAL INTERSECTIONS — INTERIM REPORT

SUMMARY In street and highway systems incorporating signalized intersections, the intersection continues to be the point of minimum capacity. To improve system capacity without sacrificing safety, it is necessary to bring traffic signal operation to the highest possible level of efficiency. NCHRP Project Statement 3-5, September 10, 1962, states the following: "The problem is not confined to complicated street networks. In the case of isolated intersections, the use of traffic-responsive signals has not always given the desired efficiency in actual operation. This suggests that present methods and equipment could be improved. Research is needed to describe the behavior of traffic in different situations and the effects of different control systems in different situations such as a network of streets, a single arterial street or highway, and an isolated intersection. The research should produce new methods of designing and pretesting signal systems and the functional specifications of signal equipment. . . ."

For these reasons, Project 3-5, "New Criteria for Designing and Timing Traffic Signals," was established under the National Cooperative Highway Research Program with the stipulation that the first year be devoted to individual intersections.

The first year of research has been completed and exhaustively documented in a final technical report. The research tasks performed during this period were (1) review of prior research; (2) formulation of a preliminary model; (3) testing and refining the model; and (4) examination of controllers and control techniques.

All known past and present research having a direct bearing on the project was reviewed, and a comprehensive 282-entry bibliography was assembled. Approximately 10 percent of these entries were judged to bear a particularly close relationship to the work at hand and were therefore studied in greater depth. Abstracts of these papers, averaging some 400 words each, were prepared.

As a result of the review of intersection performance literature, it appeared that the inclusion of all influential factors resulted in a complexity which precluded the use of an analytic model. Consequently, a digital computer simulation model was formulated to describe traffic performance at isolated intersections. This model permits the study of new control techniques and measures of effectiveness.

Various portions of the preliminary model were tested separately and in combinations with other portions, using special programs prepared specifically for this task. Results of the testing were compared with empirical data obtained from several sources. Wherever serious lack of agreement existed, the model was modified in order to give a more refined presentation.

The refined model is considered unique and valuable in several respects: (a) it conforms closely to reality in that a mixture of vehicle types and sizes with realistically varying characteristics of desired velocity, acceleration, deceleration and driver decisions and responses flow through the system; (b) it is sufficiently detailed to permit measurement, at any time or place in the system, of a full range of traffic characteristics, either for control purposes or to gage performance; and (c) it is

characterized by flexibility which permits many alternative control schemes to be submitted for testing and evaluation.

During the first year a preliminary examination of existing traffic signals was conducted. Initial consideration was also given to new control techniques which are to be investigated in greater detail in the continuing phase of the project.

The second phase in the study of isolated intersections is now in progress. Applying the simulation model developed, laboratory experimentation will be performed (1) to study various measures of effectiveness under existing modes of traffic signal control to provide a basis for selection of the best measure of intersection performance, as well as procedures for obtaining better use of existing controllers, and (2) to conceive and to test comprehensively new control modes for isolated intersections. Functional specifications of the new control modes will be prepared, and field implementation of the mode showing greatest promise of increased effectiveness will be undertaken.

CHAPTER ONE

INTRODUCTION

BACKGROUND

In street and highway systems incorporating signalized intersections, the intersection continues to be the point of minimum capacity. To improve system capacity without sacrificing safety, it is necessary to bring traffic signal operation to the highest possible level of efficiency. However, NCHRP Project Statement 3-5, September 10, 1962, stated: "Experience with conventional methods of timing traffic signals has repeatedly shown that they do not take into account all of the variables. Thus, the final timing must be made in the field after the signals have been installed. Such methods give no assurance that the final result is the most efficient system, or even that the system is operating at peak efficiency. The problem is not confined to complicated street networks. In the case of isolated intersections, the use of traffic responsive signals has not always given the desired efficiency in actual operation. All of this suggests that present methods and equipment could be improved. Research is needed to describe the behavior of traffic in different situations and the effect of different control systems in different situations such as a network of streets, a single arterial street or highway and an isolated intersection. The research should produce new methods of designing and pretesting signal systems and the functional specifications of signal equipment. . . ." For these reasons, Project 3-5, "New Criteria for Designing and Timing Traffic Signals," was established under the National Cooperative Highway Research Program with the stipulation that the first year be devoted to individual intersections.

The first year of research has been completed and has been documented in a highly technical and lengthy final report. This summary report was prepared to make available to highway traffic engineers and administrators a concise review of the more important aspects of the work.

TECHNICAL APPROACH

Review of Prior Research

All known past and present research having a direct bearing on this project was reviewed, and a comprehensive 282-entry bibliography was assembled (Appendix A). Approximately 10 percent of these entries were judged to bear a particularly close relationship to the work at hand and were therefore studied in greater depth. Technical abstracts of these papers, averaging some 400 words each, were prepared (Appendix B).

Formulation of Preliminary Model

One of the principal objectives in the study of the individual intersection was the formulation of a model to describe intersection behavior, and which would permit the study of new control techniques and measures of effectiveness. It was desired that the model contain all those factors thought to influence intersection efficiency, as well as all measures of effectivness which might appear to be appropriate. As a result of a review of previous research on models of traffic flow, it appeared that the inclusion of the factors considered important resulted in a complexity that precluded the use of an analytic model. Therefore, the decision was made to develop a digital computer simulation model.

Testing and Refining the Model

Various portions of the preliminary model were tested separately and in combination with other portions, using special computer programs prepared specifically for this task. Results of the testing were compared with empirical data obtained from several sources. Wherever serious lack of agreement existed, the model was modified in order to give a more refined representation.

Preliminary Examination of Controllers and Control Techniques

There has been a preliminary examination of various existing traffic signal controllers. Initial consideration was also given to new control techniques which may be investigated in greater detail in the continuing phase of the project.

CHAPTER TWO

THE MODEL

A high level of priority was assigned to the formulation of a multipurpose model of traffic performance at an isolated, signalized intersection. On the basis of the extensive review of pertinent research, it was concluded that completion of this was necessary to provide a tool with which to attack the broad overall objectives of the applied research program. It was also determined that there was in existence no complete operational model which satisfied all the requirements, but that certain portions of complete models developed by others could be applied directly or with minor modifications. Wherever appropriate, full utilization was made of such previous work.

PURPOSE OF THE MODEL

The initial purposes or applications planned for the model were as follows:

1. To study a variety of measures of effectiveness of traffic performance at an isolated, signalized intersection.
2. To compare and correlate measures over broad ranges of traffic variables to determine the single measure or combination of measures that most faithfully represents effectiveness of traffic performance.
3. To perform controlled experimentation with new and/or improved methods of traffic signal operation at such intersections. The new methods of control would be traffic responsive and would doubtless stem from traffic measurements and decision logic not currently practiced.

SOME BASIC REQUIREMENTS OF THE MODEL

In order to meet the stated purposes effectively, it was necessary for the model to conform to certain basic requirements.

First and foremost, realism of vehicular movement in the single intersection system was stressed. This meant that both the central tendency and the dispersion of characteristics of behavior should be realistically represented. It was considered important to avoid deterministic "rules of the road." It was further desired that, insofar as possible, homogeneity of the physical and performance characteristics of vehicles be avoided.

Second, in order to produce a satisfactory degree of refinement in measurement, both for purposes of evaluating performance and for making control decisions, the model should be "microscopic," with individual vehicles distinguished from the stream.

To test alternative control techniques objectively, the model should take into account those factors that significantly affect intersection performance, and it must have the ability to reproduce identical samples of vehicles using the system.

Finally, the model should have the inherent flexibility to represent a variety of traffic signal operation modes, some of which were already specified or outlined and others that were yet to be formulated.

SELECTION OF THE DIGITAL SIMULATION APPROACH

At a relatively early stage of this project a decision was made to utilize a digital simulation approach to the problem. It was expected it would be possible to formulate a simulation model which would satisfy the basic requirements and which, in addition, could be applied to the study of a broad range of generally-related research problems. An analytical model approach, on the other hand, was considered unacceptable—either it would be necessary to omit certain requirements such as the interest in individual vehicles, the desire for nonhomogeneity, and the flexibility to represent a diverse group of control methods, or to achieve a model with intractable mathematics.

Consideration was also given to the utilization of a strictly empirical approach. In this way, the requirements for realism would certainly be satisfied. However, for reasons relating to cost of study, time required, quality of study (including precision of control over variables, reproducibility, degree of detail of measurements, and statistical reliability) and safety, it was believed that simulation held significant advantages over empirical study in the early phases of development of improved control techniques.

ROADWAY AND VEHICLE REPRESENTATION IN THE MODEL

A modified memorandum notation was selected for use because it was more amenable to the requirements of individuality and nonhomogeneity of vehicular characteristics and performance. This technique involves storing pertinent data about individual vehicles in several computer words which can be fetched and operated on by appropriate computer routines. Vehicles operate in coordinate systems representative of the intersection area. To provide for greatest flexibility in the application of the model, to facilitate measurements for evaluation and for control purposes, and to facilitate the programming, a periodic scanning method was selected. In periodic scanning, the entire system of vehicles is reviewed at regular intervals of time (the scanning cycle), and equations of motion are utilized to update a vehicle's position and velocity based on its position, velocity, and acceleration during the previous cycle.

Roadway Representation

In order to make meaningful the stored vehicle position data, the coordinate system in which vehicles operate was devised, as illustrated by the sketch of a basic intersection in Figure 1. An orthogonal intersection of two 4-lane, bidirectional roadways is shown. Four basic coordinate systems are shown, one for each direction of traffic (NB, SB, EB, and WB). Each of the four has its origin at the point where its coordinate axis crosses the intersection entry line (*i.e.*, the projection of the curb line on the near side of the cross street), and is positive in the direction of the traffic flow (leaving the intersection) and negative in the direction opposing traffic flow.

Turns in either direction are assumed to follow 90° circular arcs, the radii of which are specified as input data. The end-of-turn point (*i.e.*, the point of tangency in the receiving lane) is considered the discharge boundary where the vehicle must be moved from one coordinated system to the other. The appropriate position transformations necessary are computed internally based on the geometric data supplied. Different radii are used by free-flowing left turners and delayed left turners.

Vehicle Representation

A vehicle is represented in the model by a series of 11 computer words which contain data pertinent to the vehicle's behavior in the system. These data are as follows: (a) position, (b) velocity, (c) acceleration, (d) flag word, (e) actual length, (f) effective length, (g) target velocity, (h) maximum desired acceleration, (i) desired negative acceleration, (j) system entry time, and (k) stopped position.

Items a through d are dynamic; they are subject to change during each simulation scanning cycle. Items e

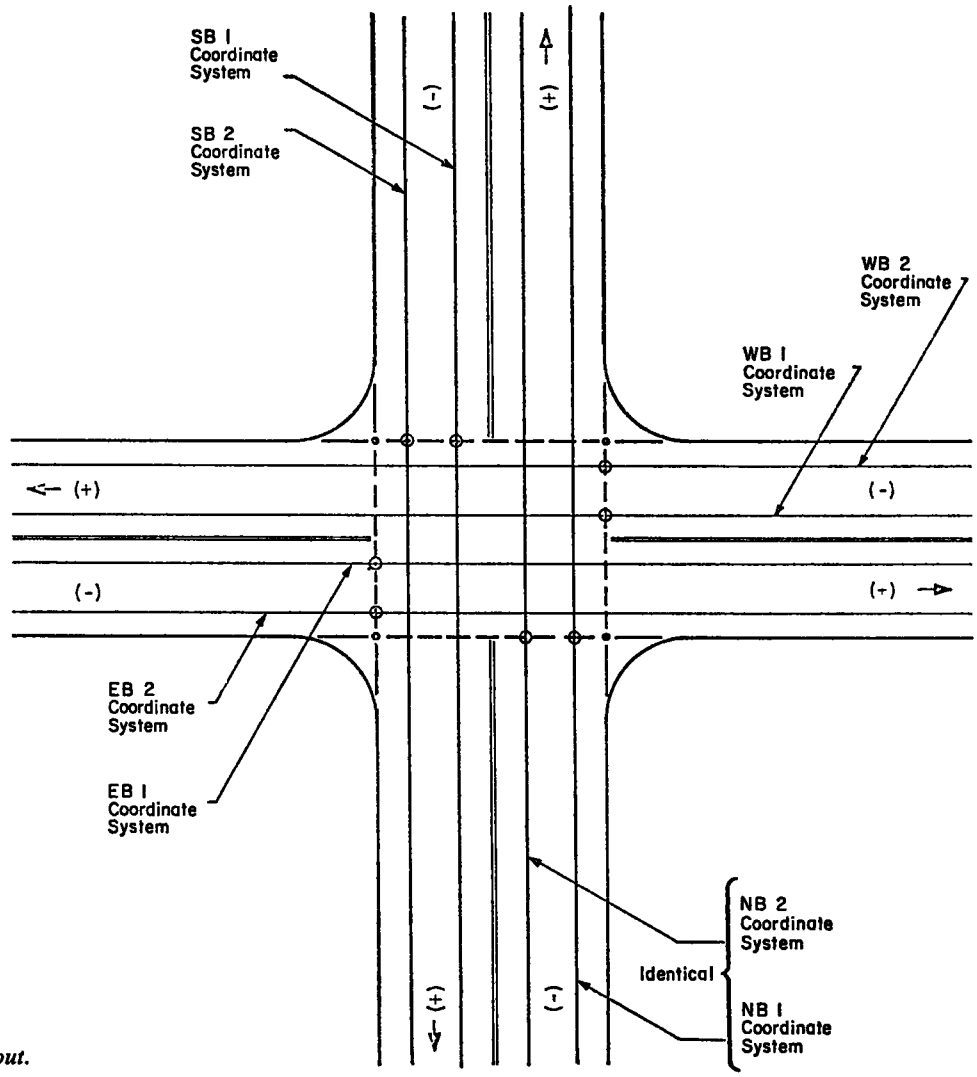


Figure 1. Coordinate system layout.

through i are constant for an individual vehicle during its passage through the system, but are randomly selected from distributions, the parameters of which are supplied as input data. Item j, system entry time, is the value of the clock at the time the vehicle is generated at the entrance boundary. Each time the vehicle prepares to stop, a stopped-position value is determined. Consequently, item k may assume one or more values during the vehicle's passage through the system, or if the vehicle never needs to stop, k is never determined. Several pertinent items of data are packed into the flags word as illustrated by Figure 2. Data may be extracted, utilized, and repacked, as appropriate.

Traffic within the system is represented by lists of vehicle words. A single list contains all vehicles currently in a given lane of the system. For the intersection of two 4-lane, bidirectional streets previously illustrated, all traffic in the system is represented by eight lists. An individual vehicle can appear on only one of the eight lists at a given instant, but turning vehicles must be moved from one list to another as they complete their turning maneuvers.

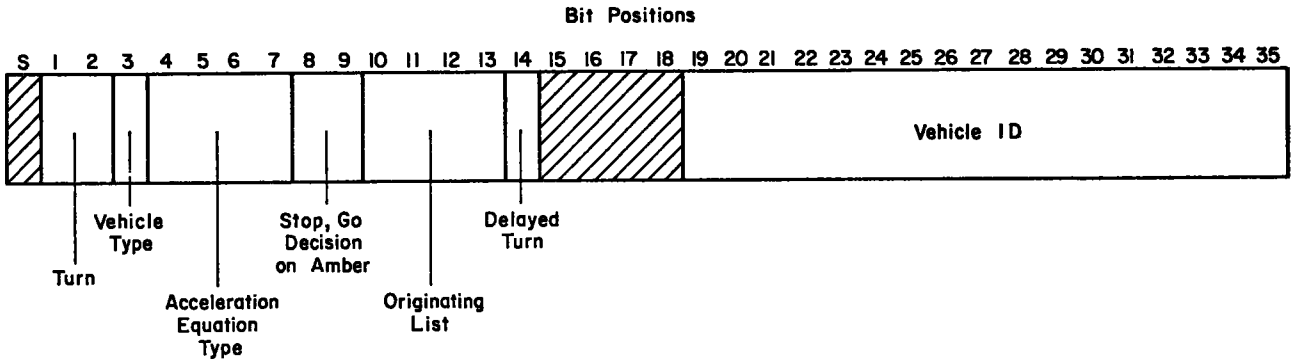
A further refinement introduced was that the lists have a preset maximum size. When the last vehicle on the list has an index equal to this maximum, the next vehicle generated is placed at the top of the list. Thus, the list is in effect a circular array rather than being of infinite length.

VEHICLE/DRIVER PERFORMANCE IN THE MODEL

Much of the full spectrum of behavior at intersections involves tracking or following processes. Stimulus-response equations of car following theory which have developed in recent years, particularly due to Herman and his associates (*107, 108,* see Appendix A), appeared highly valuable and directly applicable to simulating vehicle/driver performance. With two basic exceptions (turning movements and stopping performance), two stimulus-response relations were used to model behavior.

Car Following

A vehicle's following behavior is dictated by the *reciprocal spacing model.* Under this model the follower responds (*i.e.,* accelerates or decelerates after a time lag):



Turn: 0 = straight, 1 = left, 2 = right

Vehicle type: 0 = auto, 1 = truck

Acceleration equations: 0 = stopped, 1 = car following, 2 = free behavior, 3 = stopping, 4 = slowing for turn, 5 = stopping for left turn, 6 = stopped for left turn, 7 = going in free left turn, 8 = going in delayed left turn, 9 = going in right turn

Stop, go decision on amber: 0 = no decision (vehicle not affected by amber), 1 = stop, 2 = go

Originating list: Number of list (or lane) for which vehicle was generated. Possible values are 1 through maximum number of lists; examples 1–8.

Delayed left turn: 1 = vehicle is delayed in making a left turn, 0 = vehicle is not delayed in making a left turn or is not making left turn.

Vehicle ID: Sequence number starting at 1 for the first car generated on each list.

Figure 2. Content of the flags word.

$$\text{Acceleration} = \frac{\text{Characteristic Speed} \times \text{Relative Velocity}}{\text{Spacing}}$$

Free Behavior

It is known, however, that not all vehicles in a real system act as followers—some behave independently of the cars which precede them. In such a case, behavior can be described by the equation termed the *free behavior model:*

$$\text{Acceleration} = \text{Constant (Target Velocity} - \text{Current Velocity)}$$

As before, the acceleration is initiated after a time lag.

When spacing is greater than a specified value, behavior is dictated by the free behavior model. Otherwise, the vehicle will behave according to the reciprocal spacing model.

Queue Discharge

One important fact that has been noted is the compatibility of the reciprocal spacing model of individual vehicle behavior and the steady-state relationship of the traffic stream. The characteristic speed in the steady-state relationship is the same one used in the car following equation. Consequently, for any intersection it would only be necessary to measure maximum flow past the entry point (for a series of straight-through movements from a long queue); then characteristic speed could be computed and used in the car following equation. Applying the reciprocal spacing model to queue members and the free behavior model to queue leaders results in realistic intersection entry line characteristics for a discharging queue.

It should be understood that characteristic speed does not correspond to the target or free-flow speed around which vehicles enter, but is the typical speed past the entry point (or point of maximum flow) of vehicles which started as part of a long stationary queue once maximum flow has been attained. Generally speaking, passage through the green of platoons of undelayed, free flowing vehicles will *not* result in maximum flow.

Stopping Performance

Two types of stops occur in the model: (1) stopping first in line at the intersection, and (2) stopping behind another stopped vehicle.

Empirical work at Ohio State University *(174)* indicated that the use of a constant deceleration stopping model would be quite realistic. They found that, when given the choice, drivers tended to decelerate to a stop at an approximately constant rate. Further, they determined the mean and variance of this rate for a sample of drivers.

In the stopping model used in the current study, the parameters of a desired negative acceleration rate are supplied as input data, and a value from the distribution is randomly selected for each vehicle. When not forced into using a more rapid deceleration rate, such as in response to an amber signal, vehicles begin stopping at a constant rate required to bring them to a stop at a precisely defined stopped position.

Each scanning cycle, the required stopping rate for the first vehicle on an approach facing a red signal, is computed. When the required rate becomes algebraically less than or equal to the vehicle's desired rate (*i.e.*, required deceleration equal to or greater than the desired rate), the vehicle begins stopping. Otherwise, the vehicle continues to behave under the appropriate stimulus-response equation, either car following or free behavior.

A similar model is employed for stopping behind another vehicle, the principal difference being found in the computation of a target stopped position. In both types of stopping a random normal stopping error term is introduced in determining target stopped position.

Turning Performance

Vehicles that desire to turn left or right at the intersection must at some point cease operating under the stimulus-response model and undertake an independent fixed turning schedule. The principal requirement is that vehicles must not exceed a certain maximum speed during the turn.

As a turning car approaches the intersection it is scanned each simulation cycle, and if current velocity is greater than maximum turning velocity, the deceleration rate required to reach maximum turning velocity exactly at the start-turn point is computed. As in the case of stopping, vehicles begin slowing to maximum turning velocity only after the rate required becomes less than or equal to their desired negative acceleration rate. Once begun, the constant slowing continues until the vehicle reaches the start-turn position, unless affected by more stringent conditions. Maximum turning speed is then maintained throughout the turn, whereupon the vehicle leaves the approach leg system and joins the exit leg system. From this point on, the vehicle resumes behavior under one of the stimulus-response models (car following or free behavior) whichever is more stringent.

The discussion above does not imply that all vehicles will make their turns at maximum speed. Some will be affected by other vehicles in queue such that their turns will be made at speeds considerably lower. However, none will exceed maximum turning speed. Left turns are subject to further complications because they must be made across the path of opposing traffic.

Driver Decisions

RESPONSE TO THE AMBER SIGNAL

The decision to stop or go on amber is based on probability distributions developed from data contained in the literature. Data were taken from the work of Olson and Rothery *(178)* and Blackman and Crawford *(14)*. In this earlier work, probability of stopping was considered as a function of either distance or travel time from the intersection at the beginning of amber. Olson and Rothery studied low-, medium-, and high-speed approaches, and found three distinct probability distributions as a function of distance from the intersection.

The method used for this project was to compute, from the data, deceleration rates required to stop at the stop line. In making this computation, a 1-sec reaction time was assumed. Probability of stopping was plotted as a function of negative acceleration rate required to stop. The composite data from both sources and several intersections with varying approach speeds appeared to form a single distribution. Data were plotted on normal probability paper and an excellent linear regression fit was determined. The amber decision probability table, with values selected from the graph, was used in the model.

LEFT TURNS

The left-turning driver must decide if gaps and lags in opposing approach traffic are sufficiently large to be accepted. In this model, when a left-turning driver crosses the start-free-left-turn position, a decision is made to either accept or reject a time lag (*i.e.*, the time required for the first straight-through vehicle on an opposing approach to reach the center-of-intersection conflict point). The decision is made probabilistically by reference to a left turn lag acceptance table. If the leg is acceptable, the vehicle continues through the turn, never exceeding maximum free turning velocity. If the lag is rejected, the vehicle immediately begins stopping at the constant-rate required to stop at the left turn waiting point, one-quarter of the way through the turn. Thereafter, the driver views each time gap appearing in the opposing traffic and determines its acceptability by reference to a left turn gap acceptance probability table. If a gap is acceptable, the vehicle negotiates a delayed left turn, never exceeding maximum delayed turning velocity.

Log normal probability distributions for gap and lag acceptance were developed from data supplied by Kell in a private communication. These data were modified to meet the conditions set forth in this model.

The exact logic employed in processing left-turning vehicles is considerably more complex than would be inferred from the above discussion, principally due to the fact that other behavior conditions (such as stopping in queue) may supersede the standard left turn schedule and decisions.

LANE CHANGING

A limited amount of lane changing is permitted in the model. The only vehicles that become potential lane changers are those straight-through vehicles in the left approach lanes that follow vehicles that are either stopping for a left turn or waiting to make left turns. The potential lane changer first checks to see if a vehicle in the adjacent right lane is already located in the zone into which he must move. In the absence of such conflict, the potential lane changer determines the expected travel time to the conflict zone of the next vehicle in the adjacent lane, based on its current position and velocity. This travel time is defined as the lane change lag. The decision to change or not to change lanes is then made probabilistically by reference to a lane change lag acceptance table. For the lack of directly pertinent empirical data on lane changing, Midwest Research Institute data *(78)* relating to the analogous situation of merging from a stopped position were utilized.

Vehicle Generation

Vehicles are generated at the system entrance boundaries on a per-lane basis through the use of a composite time

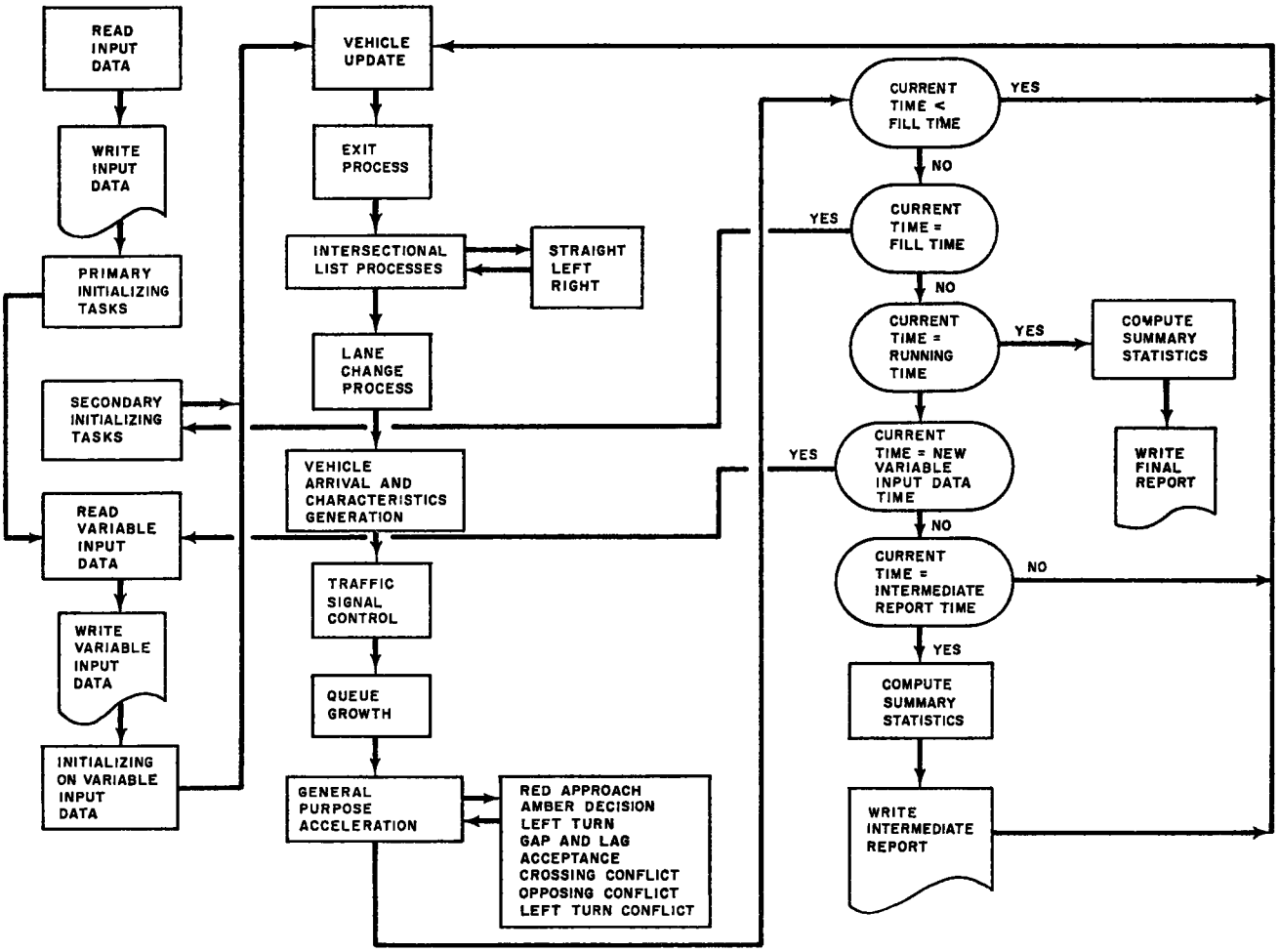


Figure 3. Main program chain.

headway distribution refined by Kell *(128)*. The distribution is composed of the headway distributions of two classes of vehicles which have in the past been somewhat inappropriately termed "free moving" and "restrained." As each vehicle is generated, its pertinent characteristics are determined by a random process. Included are vehicle type (truck or car), turning movement to be made, target velocity, maximum acceleration, desired negative acceleration, actual length, stopped spacing error, and effective length.

Constraints

The principal constraints imposed on the model of behavior are the maximum acceleration and minimum acceleration (*i.e.*, maximum deceleration) values. As noted before, a unique maximum acceleration is randomly selected for each vehicle. The parameters of two maximum accleration distributions, one for passenger cars and one for trucks, are supplied as input data. A minimum acceleration constraint of -20 ft per sec^2 is imposed on all vehicles.

It was determined during the testing and refining of the model that imposition of a minimum safe spacing constraint improved stability. Spacing of all vehicles in the system is checked each scanning cycle, and no vehicle is permitted to follow so close as to force a deceleration exceeding 20 ft per sec^2 to avoid conflict with the leading vehicle which initiates a stop at this maximum rate. It is implicit, of course, that this constraint renders the model "accident-free."

TRAFFIC MEASUREMENTS IN THE MODEL

Two distinct types of traffic measurements are necessary in the model: (1) those which form the input to the traffic signal controller subroutines; and (2) those which form the basis for computation of summary statistics and measures of effectiveness. It is probable that certain measurements will be utilized for both purposes. In any case, is it believed that any traffic measurement known today can be determined in the model by applying certain list-scanning routines. At any point or space along the roadway, presence, speed, spacing, density, time headway and queue length can be made in a straightforward manner.

COMPUTER PROGRAM DESCRIPTION

The computer program is written in FORTRAN II, except for four short routines coded in FAP (FORTRAN Assembly

Program) for the IBM 7090. It uses the FORTRAN Monitor System for compiling and execution.

The program requires approximately 30,000 words in core memory, which includes approximately 12,000 instructions, 14,000 storage locations, and space for the FORTRAN Monitor System and the subroutines it needs. The program has been compiled and executed only on the IBM 7090, but is compatible with the IBM 709 and 7094 without revision.

Currently, the simulation program is general for orthogonal intersections with either one or two lanes on a given approach. Minor modifications will permit the simulation of 3-lane approaches. Scanning cycle times permitted range from 0.25 to 0.50 sec. Times shorter than 0.25 sec would require too much computer memory; times greater than 0.50 sec would be too coarse, in that reaction time would become too long (the minimum reaction time usable is twice the scanning cycle). The maximum number of vehicles on any lane at a given time is 50.

In Figure 3, the main program chain is presented, schematically illustrating the operating relationships of the various routines of the intersection simulation program.

COMPUTER OUTPUT

Measurements and statistical calculations made during a simulation run result in a comprehensive summary-statistics output report. The format utilized in the first four pages of the output report prints statistics for individual lanes, approach directions, two directions (*i.e.*, streets), and the entire intersection.

1. Page 1 of the output report contains detailed data relating to input volume (by vehicle type), output volume (by vehicle type), intersection discharge volume (by turning movement), lane change volume, and initial and final system populations.
2. Pages 2 and 3 of the output report contain statistics (mean, standard and deviation, maximum and minimum) of the following performance measurements: undelayed travel time, actual travel time, system delay (including proportion delayed), stopped delay (including proportion stopped), queue lengths, undelayed travel speed, actual travel speed, and spot speeds at system entrance, exit and intersection discharge points.
3. Page 4 of the output report contains the behavior profile summary which is the proportion of time that vehicles spend operating under the following classifications of behavior: free behavior, car following, stopping, slowing for turn, stopping for turn, stopped for left turn, going in free left turn, going in delayed left turn, going in right turn and stopped.
4. Remaining pages of the output report contain frequency tables of the following performance measurements compiled for the entire intersection system: system delay, actual travel time, stopped time, actual travel speed and card in queue.

CHAPTER THREE

INITIAL TESTS AND APPLICATIONS OF THE MODEL

QUEUE DISCHARGE TESTS

The queue discharge portion of the model was considered to be the phase of behavior in which the greatest degree of realism was required. Delays at an intersection are very strongly dependent on the number of vehicles which can be discharged from a queue during a given green period. If the model is sluggish in discharging vehicles, a small number of vehicles, which in real life would normally make it through on green, will be forced to stop and wait through another full red period. Conversely, if queue discharge in the model is unrealistically rapid, vehicles which in real life would be delayed for another red period will consistently be discharged near the end of green. Clearly, the realism of intersection capacity and measures of effectiveness would suffer if the queue discharge model is not effectively validated. To this end, a special purpose detailed queue discharge test program was prepared for the IBM 7090.

Several past intersection simulations have had built-in entry line headways, either as fixed values or as distributions. In the present model, however, cars enter the intersection while obeying appropriate equations of dynamic behavior with individual vehicle characteristics selected randomly from distributions. Thus, one test to be performed on the model was a comparison of the entry line headways produced by the model with entry line headways from field observations. Table 1 compares the results of a set of model tests with the entry line headway data reported by Greenshields *(90)* and Fiske *(57)*, and is indicative of excellent agreement in the central tendencies of model results and empirical data.

The second test program was a modification of the first program, designed to permit comprehensive investigation of the dispersion of queue discharge entry line characteristics caused by variations in vehicle characteristics.

In this case, rather than specifying the pertinent characteristics of individual vehicles in the queue as input, the parameters of the distributions of these characteristics were supplied, and many successive queues were in turn generated and discharged. Using this program variables affecting

TABLE 1

COMPARISON OF QUEUE DISCHARGE ENTRY LINE CHARACTERISTICS

VEHICLE NO.	PRC SIMULATION		FISKE FIELD DATA		GREENSHIELDS FIELD DATA	
	ENTRY TIME	HEADWAY	ENTRY TIME	HEADWAY	ENTRY TIME	HEADWAY
1	3.81	3.81	2.14	2.14	3.8	3.8
2	6.57	2.76	5.20	3.06	6.9	3.1
3	9.00	2.43	7.47	2.27	9.6	2.7
4	11.31	2.31	9.89	2.42	12.0	2.4
5	13.57	2.26	12.33	2.44	14.2	2.2
6	15.80	2.23	14.72	2.39	16.3	2.1
7	18.02	2.22	17.16	2.44	18.4	2.1
8	20.24	2.21	19.53	2.37	20.5	2.1
9	22.44	2.21	21.95	2.42	22.6	2.1
10	24.64	2.20	24.07	2.12	24.7	2.1
11	26.85	2.20	26.23	2.16	26.8	2.1
12	29.05	2.20	28.02	1.79	28.9	2.1
13	31.25	2.19	29.96	1.94	31.0	2.1
14	33.44	2.20	32.20	2.24	33.0	2.1
15	35.64	2.20	34.78	2.58	35.2	2.1

NOTES: Field data and simulation represent average starting performance of straight-through passenger vehicles. PRC test run shown utilized $\alpha_0 = 30$ ft per sec and reaction time = 1 sec.

queue discharge may be studied singly or in combinations, thereby permitting a broad variety of experimental designs to be implemented. Figure 4 shows the results of a test in which 50 queues of mixed-traffic composition (10 percent trucks) were generated and discharged. Excellent agreement between simulated and empirical entry line headway data is noted, both in central tendency and dispersion.

INTERSECTION DELAY TEST

To begin the investigation of the effects of various forms of traffic signal timing and concurrently to perform validation tests on the complete model, operation at an orthogonal, single-lane-approach intersection controlled by a fixed-time traffic signal was simulated. Traffic volumes and turning proportions corresponded to a field test reported by Kell *(129)*. Table 2 summarizes the stopped delay experienced in the simulation run compared with Kell's field observations.

TABLE 2

COMPARISON OF MEAN STOPPED DELAY VALUES

DIRECTION	MEAN STOPPED DELAY (SEC)	
	BY SIMULATION	BY FIELD OBSERVATION*
Northbound	18.6	18.5
Southbound	17.4	17.8
Eastbound	3.6	5.8
Westbound	3.0	5.5

* Data from Kell.

TRAFFIC SIGNAL CONTROLLER SIMULATION

To permit the study of existing types of vehicle-actuated controllers and to provide for easy switching back and forth between fixed-cycle and actuated operation, a signal controller computer routine has been designed for use with the intersection simulation program. In this routine, switches set up during input control the selection of paths through the routine to cause fixed-cycle, semi-actuated, or fully actuated operation. The control of the green phase may terminate because of the expiration of the fixed portion, if pretimed, or by expiration of unit extension or maximum, if actuated. Yellow extension can be provided with both types of actuated operation. The time and manner in which each green and yellow phase terminates is recorded. Thus a complete record of traffic signal alternation is made available for analysis.

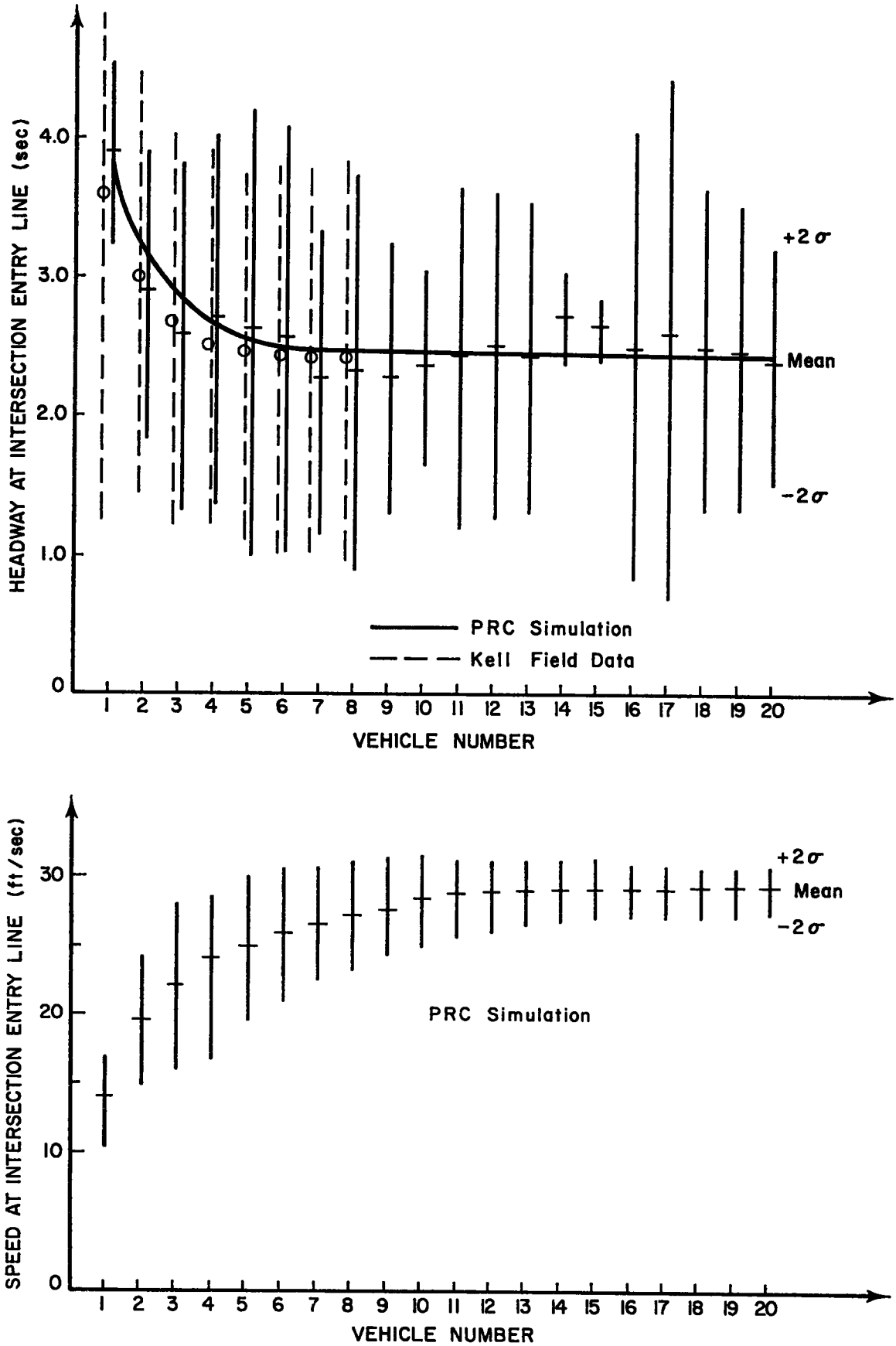


Figure 4. Comparison of PRC simulation with field data.

CHAPTER FOUR

CONTINUING STUDY

The second phase of the study of individual intersections is now in progress. It is believed that by the end of this second year, the research objectives with respect to individual intersections will have been satisfied, and thereafter attention should be centered on the problems of signalized arterials and networks. The research program now under way is outlined in the following.

LABORATORY EXPERIMENTATION

Using the simulation model developed in Phase 1, the following laboratory experimentation will be performed.

1. Study of various measures of effectiveness on existing modes of traffic signal control. Inputs will be based on field conditions reported in various studies across the country. This work will be an extensive continuation of work completed in Phase 1. This task will provide a basis for selection of the best measures of effectiveness of intersection performance, as well as procedures for obtaining the best settings on existing controllers.
2. Conception of and experimentation with new control modes for isolated intersections. New modes to be examined will include measurements of traffic not now performed. Control measurements that will be candidates for consideration will include, but not necessarily be limited to, such phenomena as queue length and expected time of gap arrival. Experimentation will again employ the model.
3. Statement of functional specifications of improved control modes. The modes selected will be the ones that resulted in most effective performance when tested experimentally by the simulation model.

FIELD TEST OF IMPROVED CONTROL MODEL

The objective of this task will be to undertake field implementation of the most promising new control mode earlier specified in functional form. The procedure for the field test will be as follows:

1. Select site.
2. Select and install appropriate equipment.
3. Perform field experiment. Field measurement of traffic and signal performance data, from which the measures of effectiveness will be derived, will be taken from (a) detector inputs and signal control outputs of the controller mockup, and (b) other data as needed, recorded by suitable instruments and/or time-lapse photography. The variables and measures of effectiveness will be those selected during the laboratory experimentation as most faithfully representing intersection performance. An equipment and procedure shakedown period will precede the final field testing to insure the smooth conduct of this phase of the work.
4. Analyze data and interpret results. The analyses will serve to accomplish the following: (a) verify the effectiveness of the new control mode under actual operating conditions (performance will be compared with existing operation at the test site; operational inadequacies will be isolated and necessary revisions in the specifications of the improved traffic signal control mode will be recommended); and (b) further validate the simulation model (by operating the model with inputs measured during the field test, the ability of the model to represent intersection performance and signal controller operations effectively will be verified).

APPENDIX A

BIBLIOGRAPHY

1. BARKER, J. L., "A Traffic Actuated Cycle and Offset Selector System." *Proceedings,* Institute of Traffic Engineers, 1949, pp. 70-76.
2. BARNES, H. A., "Use of Electronics in Traffic Control, World Traffic Engineering Conference, General Report—Theme V." *Traffic Engineering,* Vol. 32, No. 4, pp. 25-30 (1962).
3. BARNES, H. A., "Denver Installs a Modern Signal System." *Proceedings,* Institute of Traffic Engineers, 1951, pp. 69-74.
4. BARTLE, R. M., "Effects of Parked Vehicle on Traffic Capacity of Signalized Intersection." Bulletin 112, Highway Research Board, 1956, pp. 42-52.
5. BARTLE, R. M., SKORO, V., and GERLOUGH, D. L., "Starting Delay Time and Time Spacing of Vehicles Entering Signalized Intersections." Bulletin 112, Highway Research Board, 1956, pp. 33-41.
6. BAYLEY, J. M., "Intersection Capacity." *Traffic Engineering,* Vol. 29, No. 6, pp. 11-16, 24 (1959).
7. BEAUBIEN, W. H., "The GE Volume-Density Controller." *Traffic Engineering,* Vol. 25, No. 3, pp. 106-114 (1954).
8. BELLIS, W. R., "Capacity of Traffic Signals and Traffic Signal Timing." Bulletin 271, Highway Research Board, 1960, pp. 45-67.
9. BENHARD, F. G., "Simulation of a Traffic Intersection on a Digital Computer." M.S. thesis, University of California, Los Angeles, June 1959.
10. BENSON, J. C., "A Programme to Simulate on a Pegasus II Computer the Behavior of Traffic at a Single Intersection Controlled by Vehicle-Actuated Traffic Signals." Unpublished laboratory note LN/182/JCB, Department of Scientific and Industrial Research, Road Research Laboratory, Sept. 1962.
11. BERMAN, J., and CARTER, A. A., "Increasing the Traffic-Carrying Capability of Urban Arterial Streets." Bulletin 271, Highway Research Board, 1960, pp. 1-13.
12. BERRY, D. S., "Field Measurement of Delay at Signalized Intersections." *Proceedings,* Highway Research Board, 1956, pp. 505-522.
13. BERRY, D. S., and VANTIL, C. J., "A Comparison of Three Methods for Measuring Delay at Intersections." *Traffic Engineering,* Vol. 25, No. 3, pp. 93-99 (1954).
14. BLACKMAN, A. R., and CRAWFORD, A., "Driver Behavior During the Amber Period of Traffic Lights—(6) Road Observations." Unpublished research note LN/106/ARB, AC, Department of Scientific and Industrial Research, Road Research Laboratory, May 1962.
15. BLUNDEN, W. R., "On the Theory of Road Traffic." Paper presented at First Biennial Conference, Australian Road Research Board, Sept. 1962.
16. BLUNDEN, W. R., CLISSOLD, C. M., and FISHER, R. B., "Distribution of Acceptance Gaps for Crossing and Turning Manœuvres." *Proceedings,* Australian Road Research Board, Vol. 1 (in press), (1962).
17. BONE, A. J., MARTIN, B. V., and HARVEY, T. N., "The Selection of a Cycle Length for Fixed Time Traffic Signals." Joint Highway Research Project, Department of Civil Engineering, Massachusetts Institute of Technology and Department of Public Works, Nov. 1962.
18. BOWERS, D. A., "Progressive Timing for Traffic Signals." *Proceedings,* Institute of Traffic Engineers, 1947, pp. 93-100.
19. BREENKOTTER, G., "Hamburg's Traffic Signal Control." *Traffic Engineering and Control,* Vol. 4, pp. 438-443 (1962).
20. BREUNING, S. M., "Intersection Control Through Coordination of Approach Speeds." Bulletin 338, Highway Research Board, 1962, pp. 69-80.
21. BRUENING, M. E., "Traffic Control Signals." *Proceedings,* Institute of Traffic Engineers, 1952, pp. 20-22.
22. BRUENING, M. E., "How Are Progressive Signals Timed." *Proceedings,* Institute of Traffic Engineers, 1940.
23. BRUENING, M. E., "The Traffic-Actuated Signal Controller." *Proceedings,* Institute of Traffic Engineers, 1947, pp. 101-109.
24. BUCKLEY, D. J., "Road Traffic Headway Distributions." *Proceedings,* Australian Road Research Board, Vol. 1 (in press) (1962).
25. BUCKLEY, D. J., and BLUNDEN, W. R., "Some Delay-Flow Characteristics for Conflicting Traffic Streams." Paper presented at the Second International Symposium on the Theory of Road Traffic Flow, London, June 1963.
26. Bureau of Street Traffic, Chicago, Illinois, "The Treatment of the Intersection of Archer and Ashland Avenues." *Traffic Engineering,* Vol. 30, No. 1, pp. 15-19 (1959).
27. CASCIATO, L., "The Control of Traffic Signals with an Electronic Computer—A New Application of Real Time Data Processing." Presented at the meeting of the International Federation of Information Processing Societies, Munich, Germany, Aug. 27 to Sept. 1, 1962.
28. CASCIATO, L., "The Control of Traffic Signals in Metropolitan Toronto with an Electronic Computer." Traffic Research Corporation, Ltd., Toronto, 1961.
29. CASCIATO, L., and CASS, S., "Pilot Study of the Automatic Control of Signals by a General Purpose Electronic Computer." Bulletin 338, Highway Research Board, 1962, pp. 28-39.

30. CASS, S., and CASCIATO, L., "Centralized Traffic Signal Control by a General Purpose Computer." *Proceedings,* Institute of Traffic Engineering, Vol. 30, pp. 203-211 (1960).

31. CASS, S., and CASCIATO, L., "Toronto Successfully Pioneers Automatic Control of Traffic Signals by General Purpose Electronic Computer." *Traffic Engineering and Control,* Vol. 4, No. 2 (1962).

32. CASTLE, F. L., and HORLER, F., "Street Traffic Signals." *Proceedings,* Institution of Railway Signal Engineers (Part I) (British), 1930, pp. 179.

33. CASTOLDI, LUIGI, "Queue Alternance and Traffic Flow Control at a Crossroad." *Bollettino del Centro per La Ricerca Operative,* Vols. 5 and 6, pp. 1-13 (1957).

34. CEIR, Inc., "Final Report on Intersection Traffic Flow." Prepared for U.S. Bureau of Public Roads, under contract, Dec. 1960.

35. CHARLESWORTH, G., and TANNER, J. C., "Road Junctions in Rural Areas." Road Research Technical Paper No. 47, Department of Scientific and Industrial Research, Road Research Laboratory, London, 1960.

36. CHARLESWORTH, G., and WEBSTER, F. V., "Some Factors Affecting the Capacity of Intersections Controlled by Traffic Signals." Paper presented at the Fourth International Study Week in Traffic Engineering, Copenhagen, Denmark, Sept. 1958.

37. CHRISTOFFERS, C., "Practical Experience in the Design of High Capacity Urban Intersections." International Study Week in Traffic Engineering, Salzburg, Austria, 1962.

38. Citizens Traffic Safety Board of Metropolitan Chicago, "Chicago Controls Traffic Signals with Radio." *Traffic Engineering,* Vol. 26, No. 5, pp. 209-211 (1956).

39. CLARKE, J. L., and SAGI, G., "Circuit Design for Railway Actuated Signals at Three Intersections." *Traffic Engineering,* Vol. 32, No. 6, pp. 25-29 (1962).

40. CLAYTON, A. J. H., "Road Traffic Calculations." *Journal, Institution of Civil Engineers,* Vol. 16, No. 7, pp. 247-284 (1941).

41. CLAYTON, A. J. H., "Road Traffic Calculations." *Journal, Institution of Civil Engineers,* Vol. 16, No. 8, pp. 588-594 (1941).

42. COBBE, B. M., "Development of Traffic Signals for Area Traffic Control." Paper presented at the Second International Symposium on the Theory of Road Traffic Flow, London, June 1963.

43. Committee on Highway Capacity, Department of Traffic and Operations, Highway Research Board, *Highway Capacity Manual.* U.S. Department of Commerce, Bureau of Public Roads Publication, U.S. Government Printing Office, Washington, 1950.

44. CREIGHTON, R. L., "Speed Volume Relationships on Signalized Roads." *CATS Research News,* Vol. 1, No. 11, pp. 6-11 (1957).

45. CRAWFORD, A., and TAYLOR, D. H., "Critical Amber Period—Driver Behavior at Traffic Lights." *Traffic Engineering and Control,* Vol. 3, No. 8, pp. 473-478, 482 (1961).

46. DAVIDSON, B. M., "Traffic Signal Timing Utilizing Probability Curves." *Traffic Engineering,* Vol. 32, No. 2, pp. 48, 49 (1961).

47. DELANO, W. B., "Storage Control: A New Concept." *Traffic Engineering,* Vol. 28, No. 2, pp. 18-19, 50 (1957).

48. DICK, A. C., "The Effect of Gradient on Saturation Flow." Unpublished laboratory note No. LN/190/ACD, Department of Scientific and Industrial Research, Road Research Laboratory, Oct. 1962.

49. DIER, R. D., "Pedestrian Scramble Control." *Traffic Engineering,* Vol. 24, No. 11, pp. 413-414 (1954).

50. DOWNES, F. A., "Automatic Control of Road Traffic with Particular Reference to the Avotoflex System." *G.E.C. Journal,* Vol. 7, pp. 202, 208 (1936).

51. DUNCAN, K. L., "Traffic Engineering Group, Institution of Civil Engineers—Traffic Signals." *Traffic Engineering and Control,* Vol. 4, No. 10, pp. 564-567 (1963).

52. DYKES, J. O., "Development in Traffic Control." *I.E.E. Students' Quarterly Journal,* Vol. 10, p. 65 (1939).

53. EAMES, E. H., "Traffic-Actuated Progressive Systems." *Proceedings,* Institute of Traffic Engineers, 1936.

54. EDIE, L. C., "Discussion of Traffic Stream Measurements and Definitions." Paper presented at Second International Symposium on the Theory of Road Traffic Flow, London, June 1963.

55. EDWARDS, F., "Traffic Actuated Road Signals." *Proceedings,* Institution of Railway Signal Engineers (Part I), 1935, p. 87.

56. EVANS, H. K., Editor, *Traffic Engineering Handbook.* Second Edition, Institute of Traffic Engineers Publication, New Haven, Conn., 1950.

57. FISKE, J. R., "Pressurized Versus Normal Headways at a Signalized Intersection." Student thesis, Bureau of Highway Traffic, Yale University, 1957.

58. FLYE, W. D., "Traffic Signal Systems for Small Cities." *Traffic Engineering,* Vol. 32, No. 9, pp. 11-15 (1962).

59. FORCHHAMMER, N., and ERICSSON, L. M., "Single Lane Saturated Flow—Starting Performance." Paper presented at the Second International Symposium on the Theory of Road Traffic Flow, London, June 1963.

60. FRANCIS, J. G. F., and LOTT, R. S., "A Simulation Program for Linked Traffic Signals." Paper presented at the Second International Symposium on the Theory of Road Traffic Flow, London, June 1963.

61. FRANKLIN, R. E., "On the Flow-Concentration Relationship for Traffic." Paper presented at the Second International Symposium on the Theory of Road Traffic Flow, London, June 1963.

62. FREER, J. A., "Los Angeles Installs America's First Computer System for Traffic Control." *Traffic Engineering and Control,* Vol. 4, No. 3, pp. 143-147 (1962).
63. FRENCH, A., "Capacities of One-Way and Two-Way Streets with Signals and Stop Signs." Bulletin 112, Highway Research Board, 1956, pp. 16-32.
64. GALLOWAY, W. G., "Expanded Signal Volume Warrants." *Traffic Engineering,* Vol. 28, No. 12, pp. 9-10, 35-36 (1958).
65. GARWOOD, F., "An Application of the Theory of Probability to the Operation of Vehicle-Controlled Traffic Signals." *Journal,* Royal Statistical Society, Supplement, Vol. 7, No. 1, pp. 65-77 (1940).
66. GAZIS, D., HERMAN, R., and MARADUDIN, A., "The Problem of the Amber Signal Light in Traffic Flow." *Traffic Engineering,* Vol. 30, No. 10, pp. 19-26 (1960).
67. GAZIS, D. C., and POTTS, R. B., "The Oversaturated Intersection." Paper presented at the Second International Symposium on the Theory of Road Traffic Flow, London, June 1963.
68. GERLOUGH, D. L., "Control of Automotive Traffic on an Area Basis." Unpublished technical memorandum, Institute of Transportation and Traffic Engineering, 1954.
69. GERLOUGH, D. L., "Analogs and Simulators for the Study of Traffic Problems." *Proceedings,* Sixth California Street and Highway Conference, 1954, pp. 82-83.
70. GERLOUGH, D. L., "The Use of the Poisson Distribution in Problems of Street and Highway Traffic." *Poisson and Traffic,* The Eno Foundation, 1955.
71. GERLOUGH, D. L., "A Comparison of Techniques for Simulating the Flow of Discrete Objects." Paper presented at the National Simulation Conference, Dallas, Texas, Oct. 23-25, 1958.
72. GERLOUGH, D. L., "Traffic Inputs for Simulation on a Digital Computer." *Proceedings,* Highway Research Board, Vol. 38, pp. 480-492 (1959).
73. GERLOUGH, D. L., "Notes on Recent Advances in Traffic Signalling." Unpublished notes, Thompson Ramo Wooldridge Inc., May 1960.
74. GERLOUGH, D. L., "Some Problems in Intersection Traffic Control." *Theory of Traffic Flow,* edited by Robert Herman, Elsevier Publishing Company, 1961, pp. 10-27.
75. GERLOUGH, D. L., "Notes on the Use of the Digital Computer for Traffic Signal Control." Unpublished notes, Thompson Ramo Wooldridge Inc., 1962.
76. GERLOUGH, D. L., and MATHEWSON, J. H., "Approaches to Operational Problems in Street and Highway Traffic." *Operations Research,* Vol. 4, No. 1, pp. 32-41 (1956).
77. GERLOUGH, D. L., and WAGNER, F. A., "Simulation of Traffic in a Large Network of Signalized Intersections." Paper prepared for presentation at the Second International Symposium on the Theory of Road Traffic, London, June 1963.
78. GLICKSTEIN, A., FINDLEY, L. D., and LEVY, S. L., "A Study of the Application of Computer Simulation to Interchange Design Problems." Final report prepared for the U.S. Bureau of Public Roads, under contract, Midwest Research Institute, Oct. 1960.
79. GLUSS, B., "Four Streams of Traffic Converging on a Crossroad." *Annals of Mathematical Statistics,* Vol. 27, pp. 215-216 (1956).
80. GOODE, H. H., POLLMAR, C. H., and WRIGHT, J. B., "The Use of a Digital Computer to Model a Signalized Intersection." *Proceedings,* Highway Research Board, Vol. 35, pp. 548-557 (1956).
81. GOODE, H. H., "The Application of a High Speed Computer to the Definition and Solution of the Vehicular Traffic Problem." *Operations Research,* Vol. 5, No. 6, pp. 775-793 (1957).
82. GOODE, H. H., "Simulation—Its Place in System Design." *Proceedings,* Institute of Radio Engineers, Vol. 39, No. 12, pp. 1501-1506 (1951).
83. GOODE, H. H., and TRUE, W. C., "Vehicular Traffic Intersections." Paper presented at the 13th National Meeting of the Association for Computing Machinery, June 11-13, 1958.
84. GORE, A. W., "Pedestrian Signal Warrants." *Proceedings,* Institute of Traffic Engineers, 1958, pp. 97-99.
85. GRACE, M. J., and POTTS, R. B., "A Theory of the Diffusion of Traffic Platoons." University of Adelaide, South Australia, 1963.
86. GRAHAM, E. F., and CHENU, D. C., "A Study of Unrestricted Platoon Movement of Traffic." *Traffic Engineering,* Vol. 32, No. 7, pp. 11-13 (1962).
87. GREEN, F. B., "Traffic Signal Installations." *Traffic Engineering and Control,* Vol. 4, No. 3, p. 152 (1962).
88. GREENSHIELDS, B. D., "Some Time Space Relationships of Traffic in Urban Areas." *Proceedings,* Institute of Traffic Engineers, 1946, pp. 114-134.
89. GREENSHIELDS, B. D., "A Study of Traffic Capacity." *Proceedings,* Highway Research Board, Vol. 14, p. 468 (1935).
90. GREENSHIELDS, B. D., SCHAPIRO, D., and ERICKSON, E. L., "Traffic Performance at Urban Intersections." Yale University, Bureau of Highway Traffic, Technical Report 1, New Haven, Conn., 1947.
91. GULSTAD, E., "Design of Intersections in Urban Areas from the Capacity Point of View." International Study Week in Traffic Engineering, Salzburg, Austria, 1962.
92. HAIGHT, F. A., "Overflow at a Traffic Light." *Biometrika,* Vol. 46, Parts 3 and 4, pp. 420-424 (1959).
93. HAIGHT, F. A., "The Future of Traffic Flow Theory." Paper presented at the Second International Symposium on the Theory of Road Traffic Flow, London, June 1963.
94. HAIGHT, F. A., *Mathematical Theories of Traffic Flow.* Academic Press Inc., 1963.

95. HALL, E. M., "A Comparison of Delay to Vehicles Crossing Urban Intersections, Four-Way Stop vs. Semi-Traffic-Actuated Signal Control." Student research report No. 4, Institute of Transportation and Traffic Engineering, University of California, Jan. 1952.
96. HALL, E. M., "Intersection Delay—Signal vs. Four Way Stop." *Proceedings,* Institute of Traffic Engineers, 1952, pp. 60-64.
97. HARRISON, H. H., and PRIEST, T. P., *Automatic Street Traffic Signalling.* Sir Isaac Pittman and Sons Ltd., London, 1934.
98. HAUGH, H. A., "The Electromatic Dispatching System." *Transactions,* American Institute of Electrical Engineers, Vol. 51, p. 790 (1932).
99. HAUGH, H. A., "The Field of Modern Interconnected Traffic Signal Controls." *Traffic Engineering,* Vol. 24, No. 7, pp. 236-238, 245, 248 (1954).
100. HAWKES, A. G., "Queueing at Traffic Intersections." Paper presented at the Second International Symposium on the Theory of Road Traffic Flow, London, June 1963.
101. HAYES, A. T., "Easing the Headaches of Multi-Dial Traffic Signal Operation." *Traffic Engineering,* Vol. 26, No. 1, p. 30 (1955).
102. Open Discussion of Hayes, A. T., and Schwanhausser, W. E., "Multi Dial Signal Operation." *Traffic Engineering,* Vol. 26, No. 5, pp. 212, 213 (1956).
103. HELLY, W., "Simulation of Bottlenecks in Single Lane Traffic Flow." *Proceedings,* Symposium on Theory of Traffic Flow, 1959, pp. 205-238.
104. HELLY, W., "Efficiency in Road Traffic Flow." Paper presented at the Second International Symposium on the Theory of Road Traffic Flow, London, June 1963.
105. HELM, B., "Saturation Flow at Light-Controlled Intersections." *Traffic Engineering,* Vol. 32, No. 5, pp. 22-27 (1962).
106. HERWALD, N., "Some Theoretical Aspects of Control of Road Traffic." *Journal of Scientific Instruments,* Vol. 14, No. 12 (1937).
107. HERMAN, R., and POTTS, R. B., "Single Lane Traffic Theory and Experiment." *Theory of Traffic Flow,* Elsevier Publishing Company, 1961, pp. 120-146.
108. HERMAN, R., and ROTHERY, R., "Car Following and Steady State Flow." Paper presented at the Second International Symposium on the Theory of Road Traffic Flow, London, June 1963.
109. HERMAN, R., and WEISS, G., "Comments on the Highway Crossing Problem." *Operations Research,* Vol. 9, pp. 828-840 (1961).
110. HERZOG, A., "Design of Intersections in Urban Areas from the Capacity Point of View." International Study Week in Traffic Engineering, Salzburg, Austria, 1962.
111. HEWTON, J. T., "Modification of Fixed Time Signal Controller to Give Semi-Actuated Operation." *Traffic Engineering,* Vol. 27, No. 6, pp. 268-271 (1957).
112. HILLER, S. H., "The Tampa Signal System." *Traffic Engineering,* Vol. 32, No. 10, pp. 24-26 (1962).
113. HILLIER, J. A., "Instrumentation for Traffic Studies." *Traffic Engineering and Control,* Vol. 3, No. 1, pp. 49-51 (1961).
114. HILLIER, J. A., "A Review of the Developments in Area Traffic Control." Paper presented at the 1st Biennial Conference of the Australian Road Research Board, 1962.
115. HILLIER, J. A., "Regulation and Automatic Control." Unpublished laboratory note LN/256/JAH, Department of Scientific and Industrial Research, Road Research Laboratory, Feb. 1963.
116. HOPKINS, R. C., "Vehicle Detection for Traffic Analysis and Control." *Traffic Engineering,* Vol. 31, No. 10, pp. 14-16 (1961).
117. HOOSE, H. J., "Direct Time Cycle Control by Traffic Density in Charlotte, North Carolina." *Traffic Engineering,* Vol. 26, No. 7, pp. 304-306 (1956).
118. HOWIE, G. W., "New Signal Standards." *Proceedings,* Institute of Traffic Engineers, 1960, pp. 130-134.
119. HUNNICUTT, J. M., "A Capacity Relationship Between Four-Way Stop Intersection Control and Fixed-Time Traffic Signal." Student thesis, Bureau of Highway Traffic, Yale University, 1954.
120. INGLIS, B. G., "Traffic Signal Systems." *Journal,* Institution of Municipal Engineers, Vol. 80, pp. 369-392 (1954).
121. ITE Committee 7-A, "Pre-Timed, Fixed Cycle Traffic Signal Controllers." *Traffic Engineering,* Vol. 28, No. 12, pp. 38-44 (1958).
122. ITE Technical Committee 7A, "Traffic Actuated Traffic Signal Controllers and Detectors." *Traffic Engineering,* Vol. 29, No. 1, pp. 49-58, 62 (1958).
123. ITE Standards and Specifications Committee, "Standards of the Institute of Traffic Engineers—Pre-Timed, Fixed Cycle Traffic Signal Controllers." *Proceedings,* Institute of Traffic Engineers, 1941, pp. 143-160.
124. ITE Committee on Standards and Specifications, "Pre-Timed, Fixed Cycle Traffic Signal Controllers." *Proceedings,* Institute of Traffic Engineers, 1942, pp. 37-57.
125. JANET, A., "Evolution of Light Signalling in Paris." *Bulletin de la Société Française des Electriciens,* Vol. 6, p. 637 (1936).
126. JEWELL, W. S., "Multiple Entries in Traffic." Research Report 29, Operations Research Center, University of California, Berkeley, 1962.
127. JØRGENSEN, N. O., "Determination of the Capacity of Road Intersections by Model Testing." *Ingeniøren* (International Edition), Vol. 5, No. 3, pp. 99-101 (1961).
128. KELL, J. H., "Analyzing Vehicular Delay at Intersections Through Simulation." Bulletin 356, Highway Research Board, 1962, pp. 28-39.
129. KELL, J. H., "Results of Computer Simulation Studies as Related to Traffic Signal Operation." *Proceedings,* Institute of Traffic Engineers, 1963, pp. 70-107.

130. KELL, J. H., "Intersection Delay Obtained by Simulating Traffic on a Computer." Highway Research Record No. 15, 1963, pp. 73-97.

131. KENDALL, D. G., "Some Problems in the Theory of Queues." *Journal,* Royal Statistical Society, Vol. 13, pp. 151-185 (1951).

132. KOEFOED, K. V. M., "The Capacity of Urban Streets and Intersections." *Traffic Engineering,* Vol. 25, No. 11, pp. 456-458 (1955).

133. KOESTER, E. F., "Speed and Accident Control Through Signalization." *Traffic Engineering,* Jan. 1946.

134. LA VALLEE, R. S., "Scheduling of Traffic Signals by Linear Programming." *Proceedings,* Highway Research Board, Vol. 35, pp. 534-542 (1956).

135. LEMA, J. E., "Presence Detector for Left-Turning Vehicles." *Proceedings,* Institute of Traffic Engineers, 1960, pp. 122-127.

136. LEROY, H. M., and Clinton, J. W., "Signalization of Diamond Interchanges." *Proceedings,* Institute of Traffic Engineers, 1960, pp. 196-202.

137. LEVY, S. L., CARTER, M., and GLICKSTEIN, A., "Traffic and Simulation." Paper presented at the Second International Symposium on the Theory of Road Traffic Flow, London, June 1963.

138. LEWARCH, E. E., "Seattle's Radar Signal Installation." *Traffic Engineering,* Vol. 26, No. 2, pp. 59-61 (1955).

139. LEWIS, R. M., and MICHAEL, H. L., "The Simulation of Traffic Flow to Obtain Volume Warrants for Intersection Control." Highway Research Record No. 15, 1963, pp. 1-43.

140. LUCKENBACH, R. B., "Traffic-Actuated Signal System in San Diego Solves Congestion Hazards." *California Highway and Public Works,* March and April 1943.

141. LUCKENBACH, R. B., "Automatic Signal Controls 3,000 Cars an Hour at Four-Way Intersection." *California Highways and Public Works,* March 1942.

142. MACKALL, K. W., "The History and Development of Modern Traffic Control." *Municipal Signal Engineer,* Nov. and Dec. 1940.

143. MAITLAND, C. E. A., "Street Traffic Control Centralized in the Telephone Exchanges in Amsterdam." *Post Office Electrical Engineers' Journal,* Vol. 30, p. 301 (1937).

144. MAJOR, N. G., and BUCKLEY, D. J., "Entry to a Traffic Stream." *Proceedings,* Australian Road Research Board, Vol. 1 (in press) (1962).

145. MARCONI, W., "The Relative Efficiency of Various Types of Traffic Signal Controls." *Traffic Engineering,* April 1963, pp. 13-17.

146. MARSH, B. W., and CARSON, W. L., "Experience with Traffic Signals." *Traffic Engineering and Control,* Vol. 2, No. 9, p. 537 (1961).

147. MASSEY, S. A., "Mathematical Determination of Warrants for Pedestrian Crossings." *Traffic Engineering,* Vol. 32, No. 12, pp. 19-21 (1962).

148. MATHEWSON, J. H., TRAUTMAN, D. L., and GERLOUGH, D. L., "Study of Traffic Flow by Simulation." *Proceedings,* Highway Research Board, Vol. 34, pp. 522-530 (1955).

149. MATSON, T. M., "Principles of Traffic Signal Timing." *Transactions of the National Safety Council,* 1929.

150. MATSON, T. M., and MCGRATH, W. R., "Cycle Length and Signal Delay." *Traffic Quarterly,* Vol. 8, No. 2, pp. 161-178 (1954).

151. MATSON, T. M., SMITH, W. S., and HURD, F. W., *Traffic Engineering.* McGraw-Hill Book Company, Inc., 1955.

152. MAYR, R., "Cologne's Automatic Traffic Control System." *Traffic Engineering and Control,* Vol. 4, No. 5, pp. 256-259 (1962).

153. MCNEIL, R. A., "New Techniques in Traffic Signal Coordination." *Proceedings,* Institute of Traffic Engineers, pp. 83-85 (1961).

154. MICHAELS, R. M., "Perceptual Factors in Car Following." Paper presented at the Second International Symposium on the Theory of Road Traffic Flow, London, June 1963.

155. MICHALSKI, C. S., "Manhattan Traffic Signal Plan." *Proceedings,* Institute of Traffic Engineers, pp. 86-87 (1950).

156. MICHEL, R. F., "Radio Coordinated Traffic Control Signals." *Proceedings,* Institute of Traffic Engineers, pp. 104-113 (1955).

157. MILLER, A. J., "A Queueing Model Road Traffic." *Journal of the Royal Statistical Society,* Series B, Vol. 23, pp. 64-75 (1961).

158. MILLER, A. J., "Settings for Fixed Cycle Traffic Signals." Unpublished paper, University of Birmingham, 1963.

159. MILLER, A. J., "A Traffic Control System for Urban Networks." Unpublished paper, University of Birmingham, 1963.

160. Ministry of Transport, *Design and Layout of Roads in Built-up Areas.* His Majesty's Stationery Office, London, 1946.

161. MITCHELL, R. A., "Traffic Signal Warrants." *Proceedings,* Institute of Traffic Engineers, pp. 81-88 (1944).

162. MITCHELL, R. A., "Traffic Signalization Plan for the Central Business District." *Proceedings,* Institute of Traffic Engineers, pp. 85-86 (1961).

163. MORRISON, H. M., UNDERWOOD, A. F., and BIERLEY, R. L., "Traffic Pacer." Bulletin 338, Highway Research Board, pp. 40-68 (1962).

164. MOSKOWITZ, K., "Waiting for a Gap in a Traffic Stream." *Proceedings,* Highway Research Board, Vol. 33, pp. 385-395 (1954).

165. MULROY, T. M., "An Investigation into Delay at Controlled Intersections." Student thesis, Cornell University, 1960.

166. National Joint Committee on Uniform Traffic Control Devices, *Manual on Uniform Traffic Control Devices for Streets and Highways.* U.S. Department of Commerce, Bureau of Public Roads, Washington, D. C., 1961.

167. NERURKAR, M. S., "Effect of Traffic Signal on Free Flow of Traffic." Student thesis, Yale Bureau of Highway Traffic, 1952.

168. NEWELL, G. F., "Statistical Analysis of the Flow of Highway Traffic Through a Signalized Intersection." *Quarterly of Applied Mathematics,* Vol. 13, No. 4, pp. 353-369 (1956).

169. NEWELL, G. F., "The Effect of Left Turns on the Capacity of a Traffic Intersection." *Quarterly of Applied Mathematics,* Vol. 17, No. 1, pp. 67-76 (1959).

170. NEWELL, G. F., "The Flow of Highway Traffic Through a Sequence of Synchronized Traffic Signals." *Operations Research,* Vol. 8, pp. 390-405 (1960).

171. NEWELL, G. F., "Synchronization of Lights for High Flow." Unpublished paper, research supported by General Motors, Brown University, 1963.

172. NORMANN, O. K., "Report of Committee on Highway Capacity." Bulletin 271, Highway Research Board, p. 68 (1960).

173. NORMANN, O. K., "Research to Improve Tomorrow's Traffic." *Traffic Engineering,* Vol. 29, No. 8, pp. 11-21 (1959).

174. Ohio State University, Transportation Engineering Center, "Study of Electronic Devices as Traffic Aids." Report No. 202-1, 1962.

175. OLIVER, R. M., "Distribution of Gaps and Blocks in a Traffic Stream." *Operations Research,* Vol. 10, No. 2, pp. 197-217 (1962).

176. OLIVER, R. M., CATOIRE, B., and SKEATES, R. S., "On a Mathematical Function of Traffic Flow Theory." Paper presented at the Second International Symposium on the Theory of Road Traffic Flow, London, June 1963.

177. OLIVER, R. M., and BISBEE, E. F., "Queueing for Gaps in High Flow Traffic Streams." *Operations Research,* Vol. 10, No. 1 (1962).

178. OLSON, P. L., and ROTHERY, R. W., "Driver Response to the Amber Phase of Traffic Signals." *Traffic Engineering,* Vol. 32, No. 5, pp. 17-21, 29 (1962).

179. OVERMYER, R. A., "A New Approach to Traffic Controller Specifications." *Traffic Engineering,* Vol. 26, No. 5, pp. 207-208 (1956).

180. OVERMYER, R. A., "Philadelphia's Electronic Controlled Signal System." *Traffic Engineering and Control,* Vol. 2, No. 5, pp. 262-265, 271 (1960).

181. PACEY, G. M., "The Progress of a Bunch of Vehicles Released from a Traffic Signal." Unpublished research note RN/2665/GMP, Department of Scientific and Industrial Research, Road Research Laboratory, 1956.

182. PARKER, H., and BARTLE, R. M., "Signalization Priority in Los Angeles County." *Traffic Engineering,* Vol. 24, No. 9, pp. 319-322 (1954).

183. PARKER, E., "Trafficometry." *Traffic Engineering and Control,* Vol. 4, No. 6, pp. 318-323 (1962).

184. PASSAU, P. H., "Delays Incurred by Vehicles Stopped at Traffic Lights." *International Road Safety and Traffic Reveiw,* Vol. 11, No. 3 (1963).

185. PELEG, M., "Encounter of Vehicles at Intersections." *Bulletin,* Research Council Israel, Vol. 67, No. 1, pp. 55-60 (1959).

186. PETTERMAN, J. L., "Timing Progressive Signal Systems." *Traffic Engineering,* 1947.

187. PRIEST, T. P., "Electrical Control of Road Traffic by Vehicle Actuation." *Journal,* Institution of Electrical Engineers, Vol. 77, p. 149 (1953).

188. PRIEST, T. P., "The Interlinking of Street Traffic Signals: Some Problems and Practical Solutions." *Strowger Journal,* Vol. 3, p. 170 (1936).

189. PRIEST, T. P., "The Trafalgar Square and Piccadilly Electromatic Street Traffic Control Installations." *Strowger Journal,* Vol. 2, p. 125 (1933).

190. PRIEST, T. P., "The Electromatic System of Street Traffic Control." *Strowger Journal,* Vol. 2, p. 20 (1932).

191. RAFF, M. S., "The Distribution of Blocks in an Uncongested Stream of Automobile Traffic." *Journal,* American Statistical Association, Vol. 46, pp. 114-123 (1951).

192. RAFF, M. S., "Space-Time Relationships at Stop Intersections." *Proceedings,* Institute of Traffic Engineers, pp. 42-49 (1949).

193. RANKIN, W. W., "Report on Results from Right Turn on Red Light Questionnaire Survey." *Traffic Engineering,* Vol. 26, No. 1, pp. 30, 51 (1955).

194. RAUS, J., "A Method of Computing Offset Patterns for Multicycle Signal Systems." *Traffic Engineering,* Vol. 33, No. 8, pp. 25-28 (1963).

195. RAY, J. C., "Experience with Right Turn on Red." *Proceedings,* Institute of Traffic Engineers, pp. 11-116 (1956).

196. RAY, J. C., "Use of Pedestrian Delay in Walk-Wait Signals." *Traffic Engineering,* Vol. 27, No. 10, pp. 467-468 (1957).

197. RAY, J. C., "Experience with Right Turn on Red." *Traffic Engineering,* Vol. 28, No. 3, p. 46 (1957).

198. READING, J. E., "Traffic Provides Its Own Progression." *Proceedings,* Institute of Traffic Engineers, pp. 92-95 (1950).

199. REINER, M. J., "Actuated Coordinated Traffic Control Systems." *Municipal Signal Engineer,* 1948.

200. REXWORTHY, D. R., and KING, D. W. W., "Freeing City Traffic by Computer." *New Scientist,* Vol. 7, pp. 197-199 (1960).

201. RHEE, S. Y., "An Urban Traffic Control Simulator." Paper presented at the Second International Symposium on the Theory of Road Traffic Flow, London, 1963.

202. RICHARDSON, B. D., "Advantages of Traffic Signal Control." *Traffic Engineering and Control,* Vol. 2, No. 4, p. 218 (1960).

203. RIDDLE, H. J. N., "The Installation of Autoflex Road Traffic Signalling Equipment." *Siemens Magazine Engineering Supplement* (No. 123), 1935.

204. RIDDLE, H. J. N., "Road Traffic Signalling Systems." *Journal,* Junior Institution of Engineers, Vol. 44, p. 369 (1934).

205. RODGERS, L. M., "A New Technique for Progressive Timing of Traffic Signals." *Traffic Engineering,* Vol. 25, No. 9, pp. 357-360, 365-366 (1955).

206. RODGERS, L. M., "More Flexibility with Fixed Time Equipment." *Traffic Engineering,* Vol. 26, No. 3, pp. 110-112 (1955).

207. RODGERS, L. M., "A New Traffic Delay Measuring Device." *Traffic Engineering,* Vol. 27, No. 5, pp. 225-228 (1957).

208. RODWELL, A. T., "Street Traffic Control by Electrically Operated Equipment." *Transactions,* South African Institution of Electrical Engineers, Vol. 20, p. 91 (1929).

209. ROTH, G. J., "The Application of Some Economic Concepts to the Study of Urban Traffic Congestion." Paper presented at the Second International Symposium on the Theory of Road Traffic Flow, London, 1963.

210. ROTHROCK, C. A., and KEEFER, L. E., "Measurement of Urban Traffic Congestion." Bulletin 156, Highway Research Board, pp. 1-13 (1957).

211. RUDDEN, J. B., "Warrants for and Experiences with Pedestrian Intervals at Signalized Intersections." *Proceedings,* Institute of Traffic Engineers, pp. 181-182 (1959).

212. RUDY, B. M., "Limitations of Signal Spacing in a Coordinated System on a High Speed Dual Highway." *Traffic Engineering,* Vol. 28, No. 1, pp. 20-25 (1957).

213. SAGEN, R., "An Analytical Method to Determine Time-Space Relationships Between Coordinated Signals." Master's thesis, Northwestern University, 1962.

214. SAGI, G., "Theoretical Traffic Volume and Timing Studies." *Traffic Engineering,* Vol. 30, No. 8, pp. 11-18, 58 (1960).

215. SANSOM, F. J., "Use of Flashing Green Signal to Indicate Advance Green Phase." *Proceedings,* Institute of Traffic Engineers, pp. 212-216 (1960).

216. SCHEINESON, J., "Control of Erratic Traffic Flows," *Traffic Engineering,* Vol. 30, No. 11, pp. 33-36 (1960).

217. SCHENLER, W. W., and MICHAEL, H. L., "Urban Intersection Evaluation Utilizing Average Delay per Vehicle." Paper presented at the 41st Annual Meeting of the Highway Research Board, 1962.

218. SCHUHL, A., "The Probability Theory Applied to Distribution of Vehicles on Two-Lane Highways." *Poisson and Traffic,* The Eno Foundation, 1955.

219. SCHWANHAUSSER, W. E., "A Turn to the Left." *Traffic Engineering,* Vol. 27, No. 7, pp. 313-317 (1957).

220. SCHWARZ, H., "The Influence of the Amber Light on Starting Delay at Intersections." Masters thesis, Northwestern University, 1961.

221. S. G. E. Signals Ltd., "Signal Control, Chapel Ash, Wolverhampton." *Traffic Engineering and Control,* Vol. 2, No. 12, pp. 729-731 (1961).

222. SHOAF, R. T., "A Discussion of Warrants for Scramble Signals." *Traffic Engineering,* Vol. 25, No. 7, pp. 261-263 (1955).

223. SHOAF, R. T., and MARCONI, W., "Scramble in San Francisco." *Traffic Engineering,* Vol. 25, No. 2, pp. 53-57 (1954).

224. Siemens and General Electric Railway Signal Co., Ltd., "Autoflex Convertible Controllers." *S. G. E. Pamphlet No. TS8,* 1936.

225. Siemens and General Electric Railway Signal Co., Ltd., "Autoflex Vehicle-Actuated Signals—Coordinated Systems in London." *S. G. E. Pamphlet No. TS6,* 1935.

226. SIMMONS, A. E., "New Traffic Signal Dispatcher Possessed of Human Abilities." *California Highways and Public Works,* July 1942.

227. SMEED, R. J., "The Traffic Problem in Towns." Paper presented before Manchester Statistical Society, Manchester, England, 1961.

228. SMEED, R. J., "Traffic Flow." *Operations Research Quarterly,* Vol. 8, No. 3, pp. 115-123 (1957).

229. SOLOMON, D., "Accuracy of the Volume-Density Method of Measuring Travel Time." *Traffic Engineering,* Vol. 27, No. 6, pp. 261-262, 288 (1957).

230. SORENSON, L. J., "Chicago's Traffic Signal System." *Municipal Signal Engineer,* 1946.

231. STANLY, A. L., "A Study of the Variables Involved and the Statistical Techniques Utilized in Conducting and Analyzing a Traffic Experiment on Traffic Delays at an Overloaded Intersection." Ph.D. thesis, U.C.L.A., 1949.

232. STARK, M. C., "Computer Simulation of Nine Blocks of a City Street." Bulletin 356, Highway Research Board, pp. 40-47 (1962).

233. SUTERMEISTER, O., "Capacities of Narrow Streets with Manual Control and Signal Control." Bulletin 112, Highway Research Board, pp. 1-15 (1956).

234. TANNER, J. C., "The Delay to Pedestrians Crossing a Road." *Biometrika,* Vol. 38, pp. 383-392 (1951).

235. TANNER, J. C., "Problems in the Interference of Two Queues." *Biometrika,* Vol. 40, Nos. 1 and 2, pp. 58-69 (1953).

236. TANNER, J. C., "A Theoretical Analysis of Delays at an Uncontrolled Intersection." *Biometrika,* Vol. 49, Nos. 1 and 2, pp. 163-170 (1962).

237. TAYLOR, S. S., "General Report No. 1—Means of Improving Traffic Signal Systems." *Proceedings,* Institute of Traffic Engineers, pp. 67-79 (1962).

238. THORPE, E., "Electromatic Vehicle-Actuated Traffic Signals at the Bank Complex." *Strowger Journal,* p. 99 (1948).

239. THORPE, E., and GREEN, R. G., "Modern Vehicle-Actuated Road Signals." *Contractors Record,* Vol. 65, pp. 25-29 (1954).

240. TRAUTMAN, D. L., DAVIS, H., HEILFRON, J., HO, E. C., MATHEWSON, J. H., and ROSENBLOOM, A., "Analysis and Simulation of Vehicle Traffic Flow." Institute of Transportation and Traffic Engineering Research Report 20, 1954.

241. TURNER, H. I., "The Flexible Progressive Traffic Signal System." *Transactions,* American Institute of Electrical Engineers, Vol. 51, p. 786 (1932).

242. TURNER, J. K., and WARDROP, J. G., "The Variation in the Time Stopped at Controlled Intersections in Central London." Unpublished research note RN/2158/JKT, JGW, Department of Scientific and Industrial Research, Road Research Laboratory, 1954.

243. TWISS, F. E., "Centralized Control of a Metropolitan Signal System." *Traffic Engineering,* Vol. 32, No. 10, pp. 20-23 (1962).

244. TWISS, F. E., "Traffic Engineering in the Nation's Capital." *Traffic Engineering,* Vol. 31, No. 11, pp. 11-15 (1961).

245. TYACK, F. G., "Street Traffic Signals with Particular Reference to Vehicle-Actuation." *Journal,* Institution of Electrical Engineers, Vol. 82, pp. 125-153 (1938).

246. TYACK, F. G., "Street Traffic Signals from 1868 to 1934," and "The Autoflex Vehicle-Actuated Traffic Control System." *Siemens Magazine Engineering Supplement,* No. 107, 1934.

247. TYACK, F. G., "Autoflex Progressive Traffic Signals in London." *Siemens Magazine Engineering Supplement,* No. 126, 1935.

248. TYACK, F. G., "Electrical Traffic Control Progress Reviewed." *Electrical Industries,* Vol. 35, p. 1747 (1935).

249. TYACK, F. G., "Traffic Light Control Systems." *Electrical Industries,* Vol. 34, p. 1740 (1934).

250. UEMATU, T., "On the Traffic Control at an Intersection Controlled by Repeated Fixed-Cycle." *Annals of Institute of Statistical Mathematics,* Tokyo, Vol. 9, pp. 87-107 (1958).

251. VAUSE, A. H., "Traffic Control Signals." *Post Office Electrical Engineers' Journal,* Vol. 28, p. 301 (1936).

252. VICKERY, H. W., and LEONARD, V. W., "A Recent Development in Traffic Control." *Transactions,* American Institute of Electrical Engineers, Vol. 51, p. 796 (1932).

253. VON STEIN, W., "Traffic Flow with Pre-Signals and the Signal Funnel." Edited by Robert Herman, Elsevier Publishing Company, 1961, pp. 28-56.

254. WALKER, W. P., "Speed and Travel Time Measurement in Urban Areas." Bulletin 156, Highway Research Board, p. 43 (1957).

255. WARDROP, J. G., "Some Theoretical Aspects of Road Traffic Research." *Proceedings,* Institution of Civil Engineers, Part II, pp. 325-378 (1952).

256. WARDROP, J. G., "Electronic Control of Traffic." *Proceedings,* World Traffic Engineering Conference, Theme V (1961).

257. WARDROP, J. G., "The Capacity of a Network of Streets: Definition and Method of Measurement." Unpublished laboratory note LN/204/JGW, Department of Scientific and Industrial Research, Road Research Laboratory, 1962.

258. WARR, R. H., " 'Pen-Pointing' Traffic Signal Troubles." *Traffic Engineering,* Vol. 26, No. 8, pp. 335-337, 340 (1956).

259. WEBB, G. M., "The Relation Between Accidents and Traffic Volumes at Signalized Intersections." *Proceedings,* Institute of Traffic Engineers, pp. 149-160 (1955).

260. WEBB, G. M., "Prunedale Signals." *Traffic Engineering,* Vol. 32, No. 8, pp. 14-15 (1962).

261. WEBB, G. M., and MOSKOWITZ, K., "Intersection Capacity." *Traffic Engineering,* Vol. 26, No. 4, pp. 147-152 (1956).

262. WEBSTER, F. V., "Traffic Signal Settings and Expected Delay." International Study Week in Traffic Engineering, Theme VI, Stresa, Italy, 1956.

263. WEBSTER, F. V., "Road Traffic Signals." *Journal,* Institution of Highway Engineers, Vol. 4, No. 3, pp. 48-65 (1957).

264. WEBSTER, F. V., "Traffic Signal Settings." Road Research Technical Paper No. 39, Her Majesty's Stationery Office, London, 1958.

265. WEBSTER, F. V., "Greenford Roundabout Experiment." *Traffic Engineering and Control,* Vol. 2, No. 5, pp. 266-271 (1960).

266. WEBSTER, F. V., "Traffic Signals." Lecture given at conference on Traffic Engineering, Trondheim, Norway, 1961.

267. WEBSTER, F. V., "Future Developments in Traffic Signals." *Traffic Engineering and Control,* Vol. 3, No. 2, pp. 108-111 (1961).

268. WEBSTER, F. V., "Possible Developments in Traffic Signal Techniques." Paper presented at First Meeting of the Australian Road Research Congress (1962).

269. WEBSTER, F. V., "Intersection Control." Unpublished laboratory note LN/255/FVW, Department of Scientific and Industrial Research, Road Research Laboratory, 1963.

270. WEBSTER, F. V., "Practical Aspects of Traffic Signals," Road Research Technical Paper No. 56, Department of Scientific and Industrial Research, Road Research Laboratory (in press).

271. WEBSTER, F. V., and EATON, J. E., "A Method for Measuring Saturation Flow at Traffic Signals." Unpublished laboratory note RN/2559/FVW, Department of Scientific and Industrial Research, Road Research Laboratory, 1955.

272. WEBSTER, F. V., and NEWBY, R. F., "Research into the Design of Roundabouts and Traffic-Signal Controlled Intersections." Institution of Civil Engineers (to be published).

273. WEBSTER, F. V., and WARDROP, J. G., "Capacity of Urban Intersections." Paper presented at the Sixth International Study Week Traffic Engineering, Salzburg, 1962.

274. WEISS, H. G., and MARADUDIN, A. A., "Some Problems in Traffic Delay." *Operations Research*, Vol. 10, No. 1, pp. 74-104 (1962).

275. WELDING, P. I., "Time Series Analyses as Applied to Traffic Flow." Paper presented at the Second International Symposium on the Theory of Road Traffic Flow, London, June 1963.

276. WILDERMUTH, B. R., "Average Vehicle Headways at Signalized Intersections." *Traffic Engineering*, Vol. 33, No. 2, pp. 17-20 (1962).

277. WOHL, M., "Simulation—Its Application to Traffic Engineering. Part II." *Traffic Engineering*, Vol. 31, No. 1, pp. 19-25, 56 (1960).

278. WOHL, M., "Simulation—Its Application to Traffic Engineering. Part I." *Traffic Engineering*, Vol. 30, No. 11, pp. 13-17, 29 (1960).

279. WONG, S. Y., "Traffic Simulator with a Digital Computer." *Proceedings*, Western Joint Computer Conference, p. 92 (1956).

280. WOODS, K. B., Editor, *Highway Engineering Handbook*, McGraw-Hill Book Company, Inc., Section 7, pp. 38-44 (1960).

281. WOOLDRIDGE, D. E., "The Expanding Role of Electronics." Unpublished speech, Thompson Ramo Wooldridge, Inc., 1959.

282. YEO, G. F., "Single Server Queues with Modified Service Mechanisms." *Journal*, Australian Mathematical Society (Vol. II), pp. 499-507.

APPENDIX B

ABSTRACTS

BARTLE, R. M., "Effects of Parked Vehicle on Traffic Capacity of Signalized Intersection," Bulletin 112, Highway Research Board, 1956, pp. 42-52.

To examine the effect of a single parked car on intersection capacity, data were taken at an intersection approach operating under capacity conditions (continual backlog of vehicles) without parking and with a single car parked at two different distances from the intersection. Mean volumes entering under different parking conditions during short time intervals after the beginning of green were compared and tested statistically for significant differences.

The intersection approach studied was 26 ft from centerline to curb, with one 11-ft inside lane marked, controlled by fixed-time traffic signal with a 60-sec cycle: 27-sec green and 3-sec amber. Normally, parking was prohibited on the approach. Vehicles occasionally formed three lanes with the majority of traffic in the outside lane turning right. The other two conditions studied were created by parking a car 55 ft and 100 ft back from the stop line. Under these conditions, traffic still flowed past the parked car in two lanes, and between the parked car and the intersection, occasional use was made of the extreme right edge of the pavement.

The results of the study were summarized as follows:

1. The effect of a parked car on entering traffic volumes under capacity conditions was found to be significant, but the absolute difference in volumes was small (about 5 percent).
2. The effect of parking was not noted in the first half (15 sec) of the green, but it was pronounced in the last half.
3. The center lane of traffic (inside lane) was not affected by the car parked at the curb.
4. Although approximately 100 cycles were recorded for each of two parking positions used (100 ft and 55 ft), no significant differences in volumes were noted between them.

BARTLE, R. M., SKORO, VAL, and GERLOUGH, D. L., "Starting Delay Time and Time Spacing of Vehicles Entering Signalized Intersections," Bulletin 112, Highway Research Board, 1956, p. 33-41.

Two important parameters of traffic performance at intersections were investigated, starting delay and time spacing:

1. Starting delay, D, is the time, in sec, required for the first vehicle to enter the intersection after display of green signal. Corresponds to "entrance time for first vehicle in lane" used by Greenshields, *et al.*
2. Time spacing, S, is the average time headway in seconds between successive vehicles in an entering platoon. In measuring S, disregard number of traffic lanes.

$$S = \frac{\text{Total time in signal cycle used for platoon movement}}{\text{Number of cars in platoon} - 1}$$

Intersection capacity is expressed in terms of S and D. Assuming that a time S follows entrance of last vehicle entering in the cycle, then time used during each cycle for actual traffic movement equals NS, where N is the number of vehicles. Making allowance for starting delay, the total time, T, required for N cars to enter intersection is

$$T = D + NS \qquad \text{or} \qquad N = \frac{T - D}{S}$$

If green, G, and amber, A, times are considered available for movement,

$$N = \frac{M(G + A - D)}{S} \tag{1}$$

where M = number of cycles per hour.

Eq. 1 is for fixed-time control, and expresses maximum hourly capacity.

For any type of signal control,

$$N = \frac{G + A - MD}{S}$$

where G and A are total hourly green and amber times, respectively.

This approach differs from highway capacity technique in that the computation of capacity is based on values of S and D for an individual intersection approach, thus taking into account the effect of all traffic conditions peculiar to that intersection.

Regarding traffic signal timing, the authors point out that formulas have been proposed for computing optimum timing of a signal which utilize S and D. They indicate that the usefulness of these parameters in practical traffic problems appears to be dependent on the variability of the S and D values—both at any one intersection approach and among different intersections and approaches.

Conclusions regarding D are as follows:

1. There is significant difference in D among approaches at different intersections and among different approaches to the same intersection.
2. D on any one weekday can be considered equal to D for all weekdays.
3. Factors that influence D were not isolated; however, cross traffic (vehicle and pedestrian), gradient, and percent of right turns are believed to have some effect.
4. D's are in most cases normally distributed, with positive skewing (long tail of high D-values). Standard deviations of individual D's were found to be of the order of 1¼ sec, or about 30 percent of the mean.

Conclusions regarding S are as follows:

1. S is significantly different for different intersections, but is not usually significantly different for weekdays at same intersection approach.

2. S appears to be normally distributed. $S + D$ dev. $\cong$ 0.23 sec and does not vary greatly from intersection to intersection.

3. S appears to be a function of street width and parking conditions. For streets with parking and without parking, S is approximately a linear function of street width. Range of street widths studied was from 50 to 76 ft.

BELLIS, W. R., "Capacity of Traffic Signals and Traffic Signal Timing," Bulletin 271, Highway Research Board, 1960, pp. 45-67.

This paper presents a formula describing the behavior of vehicles responding to a green signal after waiting during the red signal and families of curves generated from the formulas used in determining signal cycles and splits. Examples of signal timing problem solutions are given to illustrate the use of the curves.

The paper makes no reference to the methodology utilized to derive the equation, except that it was a "trial and success" technique in which different mathematical equations were tried for fit with field data. The author recognizes that the units in the equation are incongruous, and states that it makes little difference if the equation fits the data and is of practical value. ". . . [V]ery few, if any, individuals will really care how it was derived."

$$T = PN + \frac{K}{S}\sqrt{\left\{D + C(N-1)\right\}\left\{D + C(N-1) + \frac{S^2}{4}\right\}}$$

in which

- T = time, in sec, after the beginning of the green signal to arrive at the distance D (it is believed that this is for the Nth vehicle in the queue).
- P = perception and reaction time, in sec. (For the first vehicle, P is the time between the beginning of green and the beginning of the vehicle's forward motion. For other vehicles it is the time between the beginning of its motion. This value is assumed the same for the first and Nth cars.)
- N = Nth vehicle stopped in line.
- K = constant of acceleration, the units of which cannot be explained, but which varies with the acceleration capabilities of the vehicle.
- S = speed limit (real), in mph. If speed limit is based on 85th percentile, speed limit is used. Otherwise, 85th percentile is used.
- D = distance, in ft, measured from the stop line of the first car.
- C = spacing, in ft, measured front to front of standing vehicles.

The values of the variables used in developing the signal timing charts are summarized:

VEHICLES	S	P	K	C
Passenger cars	50	1.2	0.95	25
Passenger cars	40	1.6	0.95	25
Passenger cars	30	2.0	0.95	25
Passenger cars	20	2.4	0.95	25
Heavy trucks	50	2.25	1.32	50

Assuming that vehicle arrivals follow the Poisson distribution, the maximum number of vehicles expected to arrive per cycle during a 1-hr period was computed. A family of curves (hourly volume) versus maximum number arriving per cycle) for a range of cycle lengths is presented. It is not clear what is meant by "maximum," for to be explicit a probability level must be stated. In any case, it is theorized that signal timing should be such that the maximum number of vehicles per cycle occurs only once during the design hour.

On the probability chart, then, the time required for each maximum number of vehicles expected per cycle to arrive at the 50-ft point beyond the stop line after the beginning of green was computed from the equation, and these times were placed along the ordinate beside the number of vehicles. The times are interpreted as the minimum required green time. Charts are presented for passenger vehicles with four speed ranges (*i.e.*, 45-55, 35-45, 25-35, and 15-25) and for trucks in the 45-55 speed range. Given the design hour volume of the maximum lane of an approach and its speed, the required green time for any cycle length can be found on the charts. (Evidently for establishing the complete intersection timing, a cut and try method is used to find individual green times, the sum of which does not exceed the cycle length.)

The applications of the charts presented are limited to cases in which there are no turning movements and the composition is either all trucks or all cars. It is implied that a short-cut method could be easily developed to determine timing given a certain percentage of trucks.

BENSON, J. C., "A Programme to Simulate on a Pegasus II Computer the Behavior of Traffic at a Single Intersection Controlled by Vehicle-Actuated Traffic Signals," Laboratory Note LN/182/JCB, Department of Scientific and Industrial Research, Road Research Laboratory, Sept. 1962.

The program described simulates the arrival, queuing, and discharge of vehicles, and accumulates delays to vehicles by integrating the queue in each interval of time at a 4-arm, 2-phase intersection having a vehicle-actuated traffic controller. The program can simulate the actuated controller, either with the "density" control or without this feature. The simulation cycle time is taken as ½ sec. For any phase, time is divided into effective green and effective red. The timing is related to events that occur at the detector.

Vehicle arrivals to and departures from the system are assumed to take place at the detector pad; vehicles queuing up are assumed to occupy zero space. If there is no queue, a vehicle arriving in the system (at the detector) during the effective green period is assumed to have free passage through the intersection. Arrival of a car on the stopped

phase registers a demand for change of right-of-way. Thereafter, if the vehicle interval on the green phase is exceeded, right-of-way is changed, and any vehicle arriving during amber must stop (amber is considered part of effective red). In the event vehicle interval is not exceeded, right-of-way is changed upon expiration of the maximum interval.

When green commences, the first vehicle in queue departs after a random interval. This interval plus the travel time to the stop line, which would occur in practice, are considered effectively red. The discharge intervals of succeeding vehicles are computed by means of:

$$d = M + 2\,Z\,(p - \overline{p})$$

in which

- p = a pseudorandom number lying between 0 and 31 with mean $\overline{p}$;
- Z = a scaling factor which multiplies the standard deviation of the distribution of p (an integer from 0 to 7); and
- M = mean discharge interval.

Vehicles are generated on the four approaches in a pseudorandom manner. A vehicle is generated if

$$\alpha + B > 256$$

in which

- α's = integers 1 to 255, each stored in 10 random addresses in a block of 7,200; the remaining locations contain zero.
- B = flow in vph/10 (given a ½-sec simulation cycle).

During each cycle, an α is read and added to B. For example, the number 255 will be read 10 times in 7,200, each time yielding a sum of 256, if B had been set equal to 1. Therefore, 10 vehicles are generated.

An example of simulation results is presented. Average delay per vehicle is shown as a function of maximum period for vehicle intervals equal to 10 sec and 2.5 sec. Using such a procedure, optimum actuated controller settings can be determined for given volume levels.

BERRY, D. S., "Field Measurement of Delay at Signalized Intersections," Highway Research Board *Proceedings*, 1956.

Three methods for measuring stopped time delay were compared:

1. ITTE delay meter—which uses electrically operated counters that accumulate vehicle seconds stopped at a rate proportional to the number of vehicles standing at any instant. Rate is controlled by a manual rotary switch which is adjusted to position corresponding to number of vehicles standing.
2. Sampling method—at periodic intervals (such as every 15 sec), observer records the number of cars stopped in approach, recording this number and exact time of observation.
3. Spaced aerial photo method—photographs taken at intervals of ½, 1, or 2 sec, allows determination of travel time and stopped time for each vehicle.

All three methods were satisfactory for obtaining stopped-time delay at signalized intersections. Photo method required greater manpower for data reduction, but studying travel time delay is possible. Sampling method gave unreliable results in certain instances, indicating need for care in selecting time interval for sampling.

Some definitions of types of delay were presented:

1. Travel time delay—difference between the actual time required to traverse some fixed distance at the approach to an intersection, and the travel time that would have been required had the vehicle been able to continue at the average speed of approaching traffic. This type of delay has the greatest significance in comparing effectiveness of different traffic control devices.
2. Stopped-time delay—the time a vehicle is standing still while waiting in lane in the approach to a signalized intersection.
3. Percent stopped—ratio of number of vehicles that came to a stop in approaching an intersection to the total number entering, expressed as a percentage.
4. Unnecessary stopped time delay—that portion of stopped time delay which occurs where there is no vehicle entering or approaching on opposing legs of the intersection.

Curves were drawn illustrating the distribution of travel-time delay and stopped-time-delay and comparing the two measures. For this comparison, the no-delay condition was taken as the 67th percentile of the nonstopping vehicles.

Comparison of stopped-time delay at three-phased signalized intersection, traffic-actuated versus fixed-time control: actuated timing was better able to accept short-term fluctuations. The percent of delay classed as "unnecessary" was substantially lower with actuated timing, for tests made during off-peak hours at several intersections.

Semi-actuated versus full-actuated at two intersections: stopped-time delays substantially lower for full actuated control.

Two signalized intersections 1,885 ft apart: with coordinated timing, delay to main street traffic was less than one-half that experienced when using actuated signals that were not coordinated.

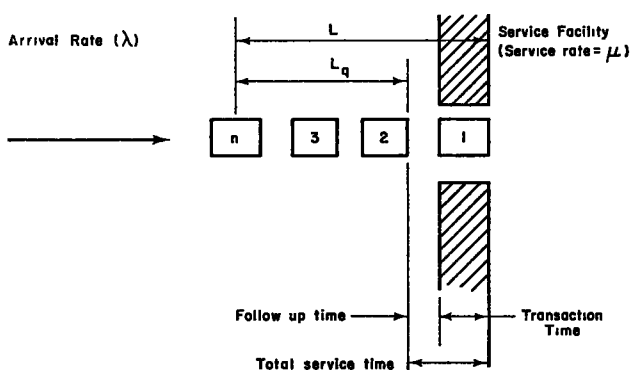
Berry presented the equation for stopped-time delay formulated by F. V. Webster of the Road Research Laboratory in England. Field studies were made at several intersections in the San Francisco area to compare actual with computed delay. The computed values for off-peak hours generally checked observed values, within 25 percent. Differences for peak hour conditions sometimes exceeded 25 percent. He notes that the formula points out that saturation flow (which is comparable to basic capacity) remains constant, whereas actually it may vary according to time of day or changed traffic conditions.

BLUNDEN, W. R., "On the Theory of Traffic Flow," First Biennial Conference, Australian Road Research Board, Sept. 1962.

This paper discusses the applicability of the queuing model to the problem of traffic flow. Reference is made to many related studies carried out by graduate students of the author.

The intersection between the input to a traffic facility (*i.e.*, demand for service, volume) and its inherent flow capability (*i.e.*, saturation flow rate or capacity) gives rise to delay, the principal measure of performance of the facility. The saturation flow rate is derived from the distribution of service times, and data are given on rates of toll gates, bus stops, intersections, retailing channels, and air strips. Discussion is included on the stability of traffic flow, its operating efficiency under varying traffic loads, and the principles underlying the analysis and synthesis of flow in traffic networks.

The elements of the single-channel queuing model are illustrated:



Given:

The arrivals are exponential and the mean arrival rate is constant and equal to λ per second.

The service times are exponential with mean $T_s = 1/\mu$ seconds.

The ratio of mean arrival rate to mean service rate is denoted by $\rho = \lambda/\mu$

P_0 (the probability that no unit is in the system) $= (1 - \rho)$

Pn (the probability that n are in the system) $= (1 - \rho)\rho^n$

$1 - P_0$ (the probability that there is one or more in the system) $= \rho$

ρ is thus the *utilization factor* of the system.

Provided that $\lambda < \mu$:

Total delay in the system, $W = \lambda \dfrac{\rho}{(1 - \rho)}$ seconds per vehicle

Delay in the queue, $W_q - \lambda \dfrac{\rho^2}{(1 - \rho)}$ seconds per vehicle

Probability of delay in the *system* greater than T,

$$P(W > T) = e^{-(1-\rho)\mu T}$$

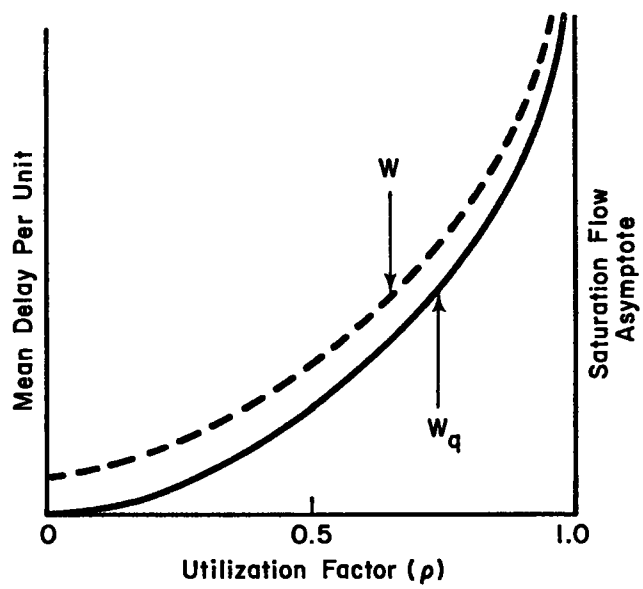
Probability of delay in the *queue* greater than T,

$$P(W_q > T) = \rho\, e^{-(1-\rho)\mu T}$$

Similar expressions are developed for system with M parallel service channels.

Working with the queuing model equations, familiar delay-flow relationships are generated as illustrated for the single-channel case.

Traffic systems are inherently stable. Instability is a condition in which the delay gets out of hand. The stability of traffic systems is due to the feedback to the demand, and the steepness of the approach to saturation makes the response rapid. An example is described wherein a construction project which required lane closure generated a traffic jam of major proportion the first morning, but by the next morning demand had adjusted and operation was back to normal even though the lane closure still existed.



It is stated that when the utilization factor (ρ) is above 70 percent, the steady-state efficiency is low, and unpredictable minor reductions of saturation flow rate can cause violent instabilities.

The delay-flow characteristics of a system are naturally dependent on the form of the distributions of arrivals and service times. A formula for delay is given, attributed to Kendall, which is useful in a practical sense because it can be used to give an estimate of delay under almost any situation even when the distributions are highly complex and defy analytical description.

$$W = 1/\mu + \frac{\rho}{2\mu(1 - \rho)} \left\{ 1 + \mu^2 \sigma_s^2 \right\}$$

where T_s and σ_s are the mean and variance of the service time distribution, $\mu = 1/T_s$ is the mean rate of exponential arrivals.

The Highway Capacity Manual definitions are linked with queuing theory terms. Possible capacity is more or less synonymous with saturation flow. Basic capacity is

seen as a somewhat hypothetical reference standard. Practical capacity is an arbitrary measure of a safe and efficient level of flow which is, in fact, a function of delay.

It is noted that although one might attack the problem of defining a particular service time distribution by microscopically studying the mechanical and psychological aspects of the service, it is generally a much more effective approach to obtain experimentally the probability distributions of service time. Such distributions often exhibit a remarkable amount of consistency in their means and variances.

Three general categories of service time distributions are listed and discussed:

1. The negative exponential form, where there are many independent factors operating and the service is *not* repeated regularly (example: supermarket cash register channel).
2. Cumulative distribution (greater than) form, where the service operation is of a regularly repeated or "practical" kind (examples: toll gates, bus stops, airport runways).
3. Service operations that are in a large measure dependent on skilled machine-operator control tasks (examples: highway and intersection flow, intervehicle headways, intersection departure headways).

The delay characteristics of barrier elements (intersections, toll gates, etc.) can be readily determined from queuing theory. Formulas already given can be applied to single channels with any form of service time distributions and exponential arrival distributions. Traffic systems are generally simple series and/or parallel arrangements of single-channel elements.

In the case of series elements, total delay is the sum of individual delays obtained for the single-channel elements as long as

1. The output of the nth element has a chance to randomize before reaching the $(n + 1)$th element.
2. The queue of the $(n + 1)$th element does not back up into the nth element.

In the case of parallel element, the *saturation flow rate* increases in direct proportion to the number of channels. Delay, however, depends on the method of feeding the system and the queuing discipline.

A number of approximations based on queuing theory can be used to develop intersection delay formulas. The author has so developed a modification of Webster's formula.

$$W = \delta + \frac{C(1-\alpha)^2}{2} + \frac{\rho^2}{2(1-\rho)}\lambda\left\{\frac{K+1}{K}\right\}$$

in which

δ = an extra starting-up lag for the first vehicle;
C = cycle time;
α = effective green time;
$\rho = \dfrac{\lambda}{\alpha\mu}$ is the degree of saturation; and
K = the Erlang number of the starting up headway distribution, which varies with the percent of commercial vehicles in the stream.

This formula was developed using saturation flow rate obtained from the *distribution* of starting headways. The difference in the delays obtained by using such a distribution as compared with the constant headways used by Webster seems to compensate for his empirical correction term.

BONE, A. J., MARTIN, B. V., and HARVEY, T. N., "The Selection of a Cycle Length for Fixed Time Traffic Signals," Joint Highway Research Project, Department of Civil Engineering, Massachusetts Institute of Technology and Department of Public Works, Nov. 1962.

This paper describes a general method for determining the optimum cycle length for fixed-time signalized intersections. It represents an excellent example of the exploitation of theoretical findings for a practical purpose.

Three criteria were used in the selection of cycle length: (1) total delay; (2) expected queue length; and (3) probability of entering the intersection during the first green phase.

The procedure was as follows:

1. Determine optimum cycle length for minimum delay per vehicle.
2. Determine length of amber phases.
3. Select trial cycle length and distribute green time to phases 1 and 2.
4. Compute delay per vehicle for critical approach lanes.
5. Compute total delay for entire intersection.
6. Compute probability of all arriving vehicles entering intersection during first green phase.
7. Compute length of queue to be expected for chosen probability of occurrence.
8. Repeat computations 3 through 7 for a range of cycle lengths that will satisfy one or more of the three criteria for cycle selection.
9. Select cycle length.

The computation techniques utilized were principally due to Webster's work on signal timing. An equation developed by Olson and Rothery was used to determine amber phase length. The probability of arriving vehicles entering the intersection during the first green phase relied on the assumption of Poisson arrivals, and was in accordance with a method used earlier by Webster.

The application of the method is illustrated in complete detail by two examples. Also included is a description and operating instructions for a computer program used to expedite the necessary computations of vehicle delay.

The authors note that the relative importance attached to each of the three criteria may vary from one location to another; consequently, in each case the final selection of a cycle length requires an engineering judgment.

BREUNING, S. M., "Intersection Control Through Coordination of Approach Speed," Bulletin 338, Highway Research Board, 1962, pp. 69-80.

This paper develops the concept of using a two-speed, nontraffic-actuated speed sign in advance of a traffic signal for the purpose of funneling traffic into the available green phase.

The sign layout is as follows:

IF SIGNAL AHEAD IS NOW	APPROACH SIGNAL AT
GREEN	30 MPH
RED OR AMBER	45 MPH

The mathematical relationships among fast approach speed, slow approach speed, signal cycle, red duration, and distance from sign to signal are shown and example computations are presented.

The simplicity and low cost of applying this concept warrant further study to determine its practicability. No experiments have yet been performed to evaluate driver response to the sign. Because of the relatively long distances involved, it is expected that the concept would be most applicable to isolated signals in rural or suburban areas.

CASCIATO, L., and CASS, S., "Pilot Study of the Automatic Control of Traffic Signals by a General Purpose Electronic Computer," Bulletin 338, Highway Research Board, 1962, pp. 28-39.

This paper summarizes the major aspects of a pilot study of the remote operation by a general-purpose digital computer of a network of traffic signals in Metropolitan Toronto, the objectives of which were as follows:

1. To demonstrate that an electronic computer could be connected into an existing traffic signal network to provide a very flexible, reliable, and powerful coordinated signal system free of most of the limitations of existing traffic signal control equipment.
2. To obtain at least a first impression of how this powerful traffic control system could be used for improving traffic flow, for measuring the improvement, and for providing data on traffic to enable further improvements to be made.

The existing local controllers were equipped with special modification units connected to the central computer by telephone line to enable the signal switching functions to be commanded remotely. Spare contacts on the signal switches were interconnected to central in order that signal operation could be monitored. Traffic detectors located on nearly all approaches at distances ranging from 200 to 600 ft back from the crosswalks were connected to central by a third set of telephone lines. Located at central headquarters were an IBM 650 computer, a special input-output buffer, a digital clock, and auxiliary tape memory modules.

The computer controlled traffic signals by means of a stored master control program, consisting of a large number of subroutines which carried out a broad range of logical and mathematical operations relating to traffic movements and signal timing. Essentially, any existing signal control mode could be effected (*e.g.*, the computer could simulate volume-density control, PR control, etc.) as well as experimental control modes. Furthermore, different intersections in the system could be operating under different control modes at the same time.

During automatic control, the computer continuously repeats the following sequence of actions at 2-sec intervals: all current traffic data are read in—the detectors, monitors, and clock; computations are done for each intersection in turn to determine which signals should be switched; an output pulse is transmitted to command the switching, and immediate confirmation is performed by checking the monitors. Concurrently, a complete record of traffic and control data is stored on magnetic tape for later analysis of system performance.

The method of conducting the pilot tests is briefly discussed. The primary concern was to evaluate comprehensively the capabilities and shortcomings of existing control schemes and compare these with the newly developed schemes which had never been used before. The details of the new control modes are not presented.

The pilot study indicated that the automatically recorded data are sufficient to produce close estimates of travel time, delays, and congestion, with field data necessary only for corroboration.

The use of platoon arrival data, referenced against the beginning of the upstream green phase, for determining efficient offsets is described. The method used in computing intersection delay is outlined, and results are presented which show excellent agreement between measured and computed delay. A definition of congestion is set forth (*i.e.*, an approach was considered congested whenever the queue of waiting vehicles extended past the detector), and the method is explained for determining from recorded traffic data and traffic signal monitoring data whether or not congestion exists.

The pilot study compared performance under the automatic computer-controlled system with that achieved under the existing fixed-time system. The automatic system decreased delay 11 percent during evening peak and 25 percent during morning peak. Congestion was reduced by 28 percent during afternoon peak. The pilot study demonstrated that with an electronic computer as the basis of control, a powerful traffic signal system can be realized that is practical for citywide installation.

CEIR, INC., "Final Report on Intersection Traffic Flow," Prepared for U.S. Bureau of Public Roads Under Contract, Dec. 1960.

This report presents the results of a multiple regression analysis of factors affecting volume of traffic flow through heavily utilized, signalized intersections. Data used in the analysis were collected by traffic engineers in states and cities throughout the country under the guidance of the Bureau as part of the program for the development of a new Highway Capacity Manual.

The traffic flow into the intersection per hour of through green time for a given intersection approach was selected from among several alternatives as the dependent variable. Originally field observations and measurements provided 48 separate factors affecting traffic flow. Preliminary screening reduced the independent variables to 20.

A total of 994 intersection approaches passed preliminary screening. This total sample was stratified or segmented into five groups, each with a particular set of attributes.

I. All adverse weather conditions (144 observations).

II. Fair weather, central business district (164 observations).

III. Fair weather, fringe business district (211 observations).

IV. Fair weather, noncentral, with traffic street markings (359 observations).

V. Fair weather, noncentral, with no traffic street markings (116 observations).

The final multiple correlation equations were determined for each of the five groups using 13 independent variables, namely: (1) approach width; (2) speed limit; (3) population of area; (4) ratio of percent of local buses to number of moving lanes; (5) ratio of percent of commercial vehicles to number of moving lanes; (6) midblock narrowing (actually approach widening); (7) parking; (8) parking turnover; (9) freedom for left turns; (10) percent loading factor; (11) peak flow ratio; (12) percent right turns; (13) percent left turns.

The analyses yielded coefficients of determination, R^2, ranging from 0.59 for group I to 0.82 for group V. The relative standard errors of estimates ranged from 25 percent of the actual mean for group V to 36 percent of the mean for group II. In other words, even for the best R^2 of 0.82 for group V, results were obtained which yield estimates of flow differing from the true value on the average by 406 vehicles per hour of green. (For this group the mean flow value was 1,630 vehicles per hour of green.)

The equations are suitable to indicate *relative* magnitudes of changes in flow resulting from changed intersection conditions. They are not sensitive enough for precise traffic flow prediction at a given intersection with a given set of independent variables.

Certain results obtained suggested that there were several significant biases in the original data. Some factors appeared important but could not be used because of insufficient data. With the relatively large amount of variability left unexplained, it is believed that important factors exist which were not measured or observed.

DICK, A. C., "The Effect of Gradient on Saturation Flow," Laboratory Note No. LN/190/ACD, Department of Scientific and Industrial Research, Road Research Laboratory, Oct. 1962.

The number of vehicles crossing the stop line in successive 6-sec periods was measured at 21 saturated intersection approaches with various gradients, ranging from 5.2 percent downhill to 8.1 percent uphill. Saturation flow was determined by calculating average discharge rates (vehicles per hour) for all but the first and last 6-sec interval during the green period. These saturation flows were compared with those at level but otherwise similar approaches which had been studied previously. Analysis showed a significant correlation between the change in saturation flow and the gradient.

Each increase of 1 percent in gradient, throughout the range studied, produced a decrease of 3 percent in the saturation flow (and vice versa). No correlation was found between the effect of heavy vehicles (as stated in equivalent passenger car units) and the slope, nor between the lost time and the slope.

FORCHHAMMER, N., and ERICSSON, L. M., "Single-Lane Saturated Flow-Starting Performance," paper presented at the Second International Symposium on the Theory of Road Traffic Flow, London, June 1963.

Based on certain simplifying assumptions of homogeneity (i.e., vehicle speed, length, emergency deceleration rate, and driver reaction time), four policies of safe driving were defined and compared on a time-headway-vs-speed graph. The policies differed with regard to the driver's willingness to accept risk. The premise that there is a different absolute value of risk associated with each of the policies was partially confirmed by a special interpretation of accident statistics that had been correlated with traffic flow and speed. The curves representing driving policies were then considered in the light of Greenburg's law for the traffic equation of state, and it was concluded that such a description of traffic flow may be made more flexible by utilizing the parameter optimum concentration, rather than jam concentration, along with optimum speed.

Of greater direct pertinence to this project, two models of starting performance of a queue at a traffic signal were formulated and compared. The first was a simple model for harmonious starting performance which involved the driver in a series of decisions:

1. Decide when to start, and the value of acceleration;
2. Reduce acceleration to maintain safe headway;
3. Increase acceleration, following the car ahead;
4. Reduce acceleration to fit speed of road.

Under this model, the effects on capacity and speed at the stop line of variations in stopped spacing in the queue and time interval between starts of cars in the queue were described.

The second model of starting performance was based on

Herman's reciprocal-spacing-car-following equation. To avoid the development of shock waves, a maximum acceleration constraint was introduced.

Time-space diagrams of starting performance associated with each of the models were presented, and important similarities and differences in performance were noted and discussed.

FRANCIS, J. G. F., and LOTT, R. S., "A Simulation Program for Linked Traffic Signals," paper presented at the Second International Symposium on the Theory of Road Traffic Flow, London, June 1963.

A computer program is described that simulates traffic in a network of roads controlled by fixed-time, linked traffic signals.

Networks can be simulated which contain no more than 30 intersections, connected by a maximum of 80 links which carry single streams of vehicles which enter and leave the network at no more than 20 peripheral arms. Vehicle representation in the system is physical, with the presence or absence of a vehicle at a given time and place indicated by the value of the appropriate bit in a series. A fixed simulation cycle time of 1 sec was utilized in the original work.

Vehicle entries, discharge from queues, turning movements, and individual speeds along links are determined probabilistically.

Homogeneity of vehicle type is assumed, and the system does not permit the path through the network of an individual vehicle to be followed. Actual spacing of vehicles along a link is not considered, but the single-stream character of the links ensures that not more than one vehicle reaches the end of a link during a single time interval, thereby causing vehicle delays.

Input volume rates, saturation flows, turning proportions, and traffic signal timing data are included as simulation input. Output includes the flows, average delays, and average queue lengths on all links.

Using a 1-sec cycle, the program consumes approximately $n/18$ real time, where n is the number of intersections, when run on a Ferranti Pegasus computer.

GRAHAM, E. F., and CHENU, D. C., "A Study of Unrestricted Platoon Movement of Traffic," *Traffic Engineering*, April 1962, pp. 11-13.

This article describes a study conducted by the California Division of Highways, in cooperation with the Bureau of Public Roads, at rural, isolated, signalized intersections to determine the amount of dispersion of platoons at various distances downstream from their origin.

Data were collected at points 150 ft, ¼ mile, ½ mile, ¾ mile, and 1 mile downstream of a 2-phase fully actuated signal on a 4-lane rural expressway. Vehicle arrivals were detected at these points by road tubes; the first two were connected to the BPR traffic analyzer, and the last three were connected to graphical recorders. A platoon was defined as all vehicles passing through the signal during each green phase, and platoon length was defined as the difference in arrival time of the first and last vehicle in the platoon at the 150-ft point. A total of 70 platoons moving away from the intersection were observed. At locations 2, 3, 4, and 5, the average arrival time of the first vehicle in a platoon was measured, and the offset (i.e., the difference in average arrival time of the first vehicle at the downstream location and at location 1) was determined. Then, the percentage of vehicles arriving within a time period equal to platoon length was determined for each of the locations. A summary of the data follows:

LOCATION	VEHICLES REMAINING IN PLATOON
2 (¼ mile)	91 percent
3 (½ mile)	85 percent
4 (¾ mile)	80 percent
5 (1 mile)	77 percent

It is concluded that vehicles remained in well-defined platoons for distances of at least 1 mile beyond the signal. No comment is made, however, regarding the obvious spreading or dispersion of the platoon. It would seem from the data that the percentage of vehicles remaining in the platoon is significantly reduced at each successive downstream point.

LEWIS, R. M., and MICHAEL, H. L., "The Simulation of Traffic Flow to Obtain Volume Warrants for Intersection Control," Highway Research Record, No. 15, 1963, pp. 1-43.

This paper describes the development and application of a digital computer simulation model of traffic flow at an orthogonal intersection of a 4-lane major arterial street with a 2-lane minor street. Operation under two control models (two-way stop sign and semi-traffic-actuated signal) was simulated for the purpose of establishing a realistic set of volume warrants which, when applied, would result in the control mode causing lowest *delay* to be used.

Total delay encompasses any delay caused by the existence of traffic control devices and interaction with other vehicles. Vehicles that decelerate to turn and then accelerate to their desired velocities are not considered as being delayed. Straight-through vehicles pass through the intersection without slowing. Any travel time in addition to these requirements is considered delay.

Average delay per vehicle for all four approaches combined was the *figure of merit*. However, excessive unbalance of the figure of merit among the approaches was recognized as undesirable.

The Model

The mode of representation employed was a variation of the mathematical notation. The entire representation was

in algebraic format because an algebraic compiler was used. Each vehicle had several characteristics, each of which was defined in a computer word; thus each vehicle was composed of several computer words. The roadway system was represented by a three-dimensional array whose length dimension corresponded to relative position along the road, width dimension represented traffic lanes, and vertical dimension accommodated characteristics of individual vehicles. The ends of the array were mathematically connected to form a "circular array" long enough to handle all traffic in the system.

A 1-sec simulation scanning cycle was utilized. During every time increment, vehicles were processed by proceeding sequentially in each lane in a direction opposite to flow.

1. Distance traveled, Z, during the time increment was computed in accordance with restrictions relating to spacing, acceleration, stopping, or turning from basic equations of motion.
2. The critical Z was selected (the smallest of the Z's computed in step 1).
3. The new X coordinate was computed.
4. The new velocity was computed.

Vehicle generation was accomplished using a theoretical probability distribution which incorporated the platooning effect of traffic. The turning maneuvers of vehicles are randomly selected. Vehicles travel so as to minimize their delay and have a uniform free flow or maximum speed, have uniform acceleration and deceleration rates (subject to special conditions), are all 17 ft long with 22-ft minimum spacing, have a maximum velocity at the turn point (same for right and left turns), are not subjected to pedestrian interference, and never collide, travel backward, or break down. For a queue of vehicles being served by the green signal, 3.8 sec is required for the first vehicle to enter, subsequent headways are reduced to about 2.1 sec for the 4th vehicle, and speeds continue increasing and headways decreasing until the 20th vehicle passes at free flow speed with a 1.5-sec headway. Merging and crossing maneuvers are based on fixed-gap acceptance criteria (in other words a gap size is stated which is considered acceptable to all). A list of 21 rules of operation was formulated.

The simulation program is written in FORTRAN for the 7090. In its present state 15,271 words of core storage are required. A favorable ratio of real time to computer time of 45 to 1 is achieved.

Application

The application of the simulation program was rather severely limited to a set of fixed parameters which were considered to be representative of typical operation of traffic at an intersection. To begin with, the geometric design of the intersection was fixed. Then fixed values were selected for variable traffic factors: velocities, acceleration-deceleration rates, vehicle size, directional distribution, lane distribution, turning probabilities, and acceptable gap sizes. Finally the variables involved in semi-actuated signal control were set: location of detectors, main street green and amber, and side street initial, extension, maximum, and amber.

Simulation runs were made varying the traffic volumes on the cross streets over a broad range of values. Both stop-sign and semi-actuated control were imposed, and the simulation results were analyzed to determine volume-delay relationships. The relationships for the two types of control were superimposed to determine points of equal delay; these points were then plotted as a function of volumes on the two streets, and lines of equal delay were constructed.

Conclusions

1. The simulation model performed in the desired manner.
2. The volume warrants developed are directly applicable to intersections of the class studied. The findings are of general interest and contribute to general understanding of the effect of control on delay.
3. When the intersection was under stop-sign control: (a) unless an average wait of 30 sec per vehicle is acceptable on the side street, the critical factor in determining the adequacy of stop-sign control will generally be delay to side-street vehicles; and (b) average wait on the side street is quite sensitive to gap acceptance criteria.
4. Operating under semi-actuated signal control: (a) for many volumes occurring during the day, overall delay would be reduced by placing signals on flashing operation (same as stop-sign control); and (b) for normal volume distributions (heavier traffic on major street) average delay per vehicle is lowest when detectors are placed close to side-street stop lines.

MARCONI, W., "The Relative Efficiency of Various Types of Traffic Signal Controls," *Traffic Engineering*, April 1963, pp. 13-17.

The paper reports the measurement of traffic performance at two signalized intersections to compare the relative efficiency of four alternative modes of traffic signal control: (1) volume-density control; (2) fully actuated control; (3) semi-actuated control; and (4) fixed-time progressive control.

The principal performance criterion was average stopped delay per vehicle. In addition, a supplementary criterion, percentage of vehicles delayed (stopped), was utilized.

The traffic signal controller settings were selected in accordance with standard practice of the San Francisco Traffic Engineering Department. In the case of fixed-time control, settings provided excellent progression from nearby upstream signals in the major direction of flow.

Sample data were collected which represented a broad range of traffic flow levels, from 200 vehicles per lane hour to saturation levels exceeding 1,200 to 1,300 vehicles per lane hour. At one intersection, data were gathered with an electric calculator and delay meter developed by the University of California ITTE, while at the second study location the sampling method of recording stopped delays was utilized.

Among the more important conclusions of the study were the following:

1. Of the traffic-actuated controllers, the volume-density mode of signal control resulted in least delay to motorists, followed closely by the fully actuated type. The semi-actuated mode resulted in relatively poor performance, indicating that it was not as suitable for use at intersections with high peak-period demands.

2. Under circumstances where good progressive movement on the main artery can be achieved, it will prove to be considerably more effective in terms of reduced delays and percentage of vehicles stopped than isolated operation of traffic-actuated modes of control.

It was noted, however, that the overall improvement under fixed-time progressive control was at the expense of side street traffic, which suffered considerable delay.

MILLER, ALAN J., "Settings for Fixed Cycle Traffic Signals," (unpublished paper), University of Birmingham.

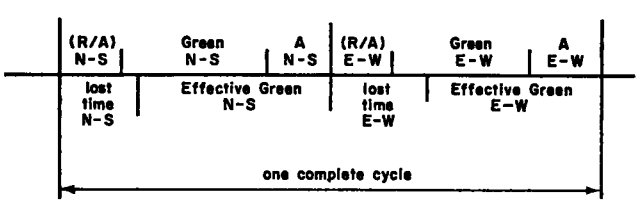
A formula is derived for average delay to vehicles at intersections controlled by fixed-cycle traffic signals. The delay to all vehicles was found by considering separately those arriving during the red phase and those arriving during the green phase. Assumptions include the following: (1) vehicle arrival distributions with any variance to mean ratio may be used; (2) departures from the queue are regular (*i.e.*, vehicles leave at regular intervals of time; (3) delay is the difference between delayed and undelayed journey times over a distance which includes acceleration and deceleration distances on each side of the junction, and delay is unaltered if instantaneous deceleration and acceleration at the stop line are assumed.

The formula for expected total delay per cycle is

$$T = \frac{q(c-g)}{z(s-g)}\left\{\frac{sI(2x-1)}{q(1-x)} + S(c-g) + I - 1 + \frac{q}{s}\right\}$$

in which

- c — cycle length;
- g — length of the effective green phase;
- q — the arrival rate of vehicles;
- s — saturation flow, *i.e.*, the average departure rate during the effective green phase;
- I — the ratio of the variance to the mean count of arrivals over the period excluding the effective green time; and
- x — the traffic intensity qc/sg



Average delay per vehicle

$$\bar{d} = \frac{T}{qc}$$

Webster's formula for average delay per vehicle is

$$\bar{d}_w = \frac{c(1-g/c)^2}{z(1-q/s)} + \frac{x^2}{2q(1-x)} - 0.65\left(\frac{c}{q^2}\right)^{1/3} x\,(2+5g/c)$$

Delays at several intersections have been observed in the field and compared with both Miller's and Webster's computed delay. When arrivals are nearly random ($I \cong 1$), both formulas yield almost the same delay as observed. Where I is appreciably greater than one, observed delay is greater than Webster's but still close to Miller's.

Formulas for computing split and cycle have then been derived. Denoting the highest intensity directions on the two streets as i and j, the proportion of total effective green time allocated to one particular green phase $[g/(c-L)]$ is

$$\pi i = \frac{a_i s_j + \frac{c}{c-L}(a_j q_i - a_i q_j)}{a_i s_j + a_j s_i}$$

where

$$a_i = \sqrt{q_i s_i I_i}$$

and the cycle is

$$c = \frac{L + \sqrt{\frac{I_i L(1-q_i/s_i)}{S_i \pi_i (1-\pi_i^2)}}}{1-(q_i/\pi_i S_i)}$$

where L = total lost time per cycle. Lost time is delay imposed on the first vehicle after the start of green to allow for its acceleration delay in practice.

The two equations can be simplified for practical use:

$$\pi i = \frac{a_i s_j + 1.4(a_j q_i - a_i q_j)}{a_i s_j + a_j s_i}$$

$$C = \frac{L + \sqrt{2I_i L/s_i}}{1-(q_i/\pi_i s_i)}$$

MILLER, ALAN J., "A Traffic Control System for Urban Network," (unpublished paper), University of Birmingham, 1963.

This paper develops a "minimum delay" method of controlling traffic signal switching by a digital computer. While the goal is the control of networks of signals, this paper's detailed discussion of control is limited to individual intersections. The control procedure developed requires two detectors on each approach, one at the stop line and one approximately 300 ft back. The computer scans the intersection every h seconds and makes a choice between two alternatives:

1. Leave the signals as they are for another h seconds.
2. Change the signals to favor the other directions of traffic.

The basis of the decision is a test quantity, T, which, given green on N-S approaches, is

T — the *saving* of delay to N-S traffic by extending the green phase h seconds minus the delay *caused* to E-W traffic by leaving the signals as they are for a further h seconds.

The elements of the equation for *saving* of delay, which either are known or must be established before computations are performed, are as follows: length of the amber phase, length of the next N-S red phase, time lost during acceleration after the end of the red phase, number of vehicles expected to pass through the intersection from N and S approaches during the next *h* seconds, arrival rates of flow per *h* seconds on the N and S approaches, and saturation rates of flow per *h* seconds through the intersection from the N and S approaches (queue discharge rates).

For computation of delay *caused* the following are needed: the number of vehicles already waiting (queue length) on the E and W approaches, the arrival rates per *h* seconds on E and W approaches, the saturation flow or departure rates per *h* seconds on E and W approaches, and the initial lost time (starting delay) on the E and W approaches.

In the procedure developed, the saturation flow and lost time values are continuously estimated using exponentially weighted moving averages. Starting values are from Webster and data are counts of vehicles crossing the stop line detector. The length of the next N-S red phase is assumed to depend linearly upon the length of the queues on the E and W approaches.

A least squares regression line, which is updated using exponentially weighted moving averages of the variables, is used to estimate the length of the next red phase. Arrival rates are determined from counts past the approach detectors. Regarding queue length, it is not clear whether count-in/count-out or count-in/assumed launch rate-out is used. The possibility of providing special right turn (same as U.S. left) phases upon detection of stationary vehicles in the right lane during the green phase is briefly discussed.

A simulation program has been written for a 30- to 40-intersection network. First use was to compare performance at a single intersection, minimum delay method versus fully actuated. Under very low volume, 200 vph on each of four approaches, the minimum delay system yields about 10 percent less delay. On a Ferranti Mercury computer, simulation of one intersection takes about two-thirds real time.

A few ideas about network control are briefly discussed, which are of interest. One idea, called quantity control, would prevent queues from spreading backward through the system by never permitting more vehicles to be discharged at an intersection than could be accommodated downstream.

MORRISON, H. M., UNDERWOOD, A. F., and BIERLEY, R. L., "Traffic Pacer," Bulletin 338, Highway Research Board, 1962, pp. 40-68.

This paper describes the design and evaluation of a "traffic pacer" system in a 4-mi section of a two-way signalized arterial near Detroit, on which there are 9 signalized intersections. The object of the traffic pacer is to form compact groups of moving vehicles timed to arrive during the green phase. The pacer system makes use of two control elements which were conceived by Von Stein in 1954 in Düsseldorf, namely, the changeable message speed sign and the pre-signal.

The purpose of the speed signal is to inform the drivers of the travel speed which will allow them to pass through the next signal without stopping. With proper driver response, vehicles are compressed or funneled into a more compact group which fits the green phase. The pre-signal is designed to give vehicles which were forced to stop a flying start through the intersection signal, thus eliminating or minimizing starting delay. It is shown that the pacer system operation maximizes intersection capacity and thus maximizes system capacity.

Most of the 9 signalized intersections in the test section were equipped with pre-signals. The exact distances of the pre-signals from the intersection signals are not given, but they were varied for experimental reasons and generally allowed acceleration to 30 mph at the intersection. Speed signals were installed at each pre-signal location and at intervals of approximately 1,500 ft back from the intersection. Thus the number of speed signals used upstream of an intersection depended on the distance back to the preceding signalized intersection. The maximum distance was 1 mile, in which case 4 speed signals and 1 pre-signal were utilized. In all, 43 speed signals and 11 pre-signals were installed and operated.

Traffic operation using the pacer system was compared with operation in a noninterconnected system and in a progressive interconnected system.

Criteria used to compare the three systems were (1) average trip time, (2) average number of stops, (3) average speed, (4) queue lengths, (5) safety, and (6) public opinion.

Significantly fewer stops were made with the pacer. Higher average speeds were maintained and trip times were less for the pacer and the progressive systems than for the noninterconnected. (The pacer and progressive systems did not differ significantly on these criteria.) The average number of cars queued under the pacer system (queue at intersection and pre-signal) was 44 percent less than under progressive operation. It is stated that the pacer system has equivalent safety, but little evidence is set forth to support this conclusion.

Results of questionnaires distributed to drivers using the test section indicated that approximately 65 percent of the people felt that the pacer system was faster, safer, and caused fewer stops than the other two systems.

Intersection capacity under the three systems was also given consideration. Several of the high-volume intersections at rush periods were compared by measuring the frequency of cycles in which some arbitrary maximum number of cars or more made it through. Results are given for one intersection which show that 30 or more cars per cycle were passed during 25.4 percent of the cycles under the pacer system, as compared with 19.4 percent and 17.1 percent under the progressive and noninterconnected systems, respectively.

National Joint Committee on Uniform Traffic Control Devices, *Manual on Uniform Traffic Control Devices for Streets and Highways,* 1961, Sections 3D-14 and 3D-15, pp. 192-193.

No information is given with regard to selection of total cycle length.

Some of the factors that must be considered in assigning green time to the intersecting streets are (1) number of traffic lanes and other physical conditions, (2) volume of traffic in the critical lanes, (3) requirements of commercial and public transit vehicles, (4) vehicle headways on the intersecting streets (departure, headways), (5) pedestrian requirements, (6) vehicle and pedestrian clearance requirements, and (7) turning movements.

With regard to splits, two different conditions are considered:

1. If, during the heaviest traffic hour, the effect of turning movements, slow-moving commercial vehicles, and other factors on the timespacing between vehicles in the critical lane leaving the intersection is equal for the two heavier traffic flows at right angles, the division of the total time cycle into two green periods will be approximately correct if the two periods are made directly proportional to the intersecting volumes of traffic per critical lane.

$$\frac{T_A}{T_B} = \frac{V_A}{V_B}$$

where $T_A + T_B$ = total cycle (A clearance interval + B clearance interval).

2. If, during the heaviest traffic hour, there is considerable difference in the time spacing between vehicles leaving in the two intersecting critical lanes, this fact should be taken into account, and the green periods should be made proportional to the products of the critical lane volumes and the time spacing on the respective streets.

$$\frac{T_A}{T_B} = \frac{V_A \times H_A}{V_B \times H_B}$$

NEWELL, G. F., "Synchronization of Lights for High Flow," (unpublished paper), Research Supported by General Motors, 1963.

The flow of traffic on a signalized arterial is the subject of theoretical treatment to determine optimal synchronization (that which minimizes total delay). Assumptions made include the following:

1. Dense traffic with intersection flow near saturation.
2. No passing is done; cars behave as described by continuum or car-following theories.
3. The duration (time) of a platoon increases as it moves between traffic signals. This spreading of the platoon is mainly due to the acceleration of the platoon as a whole rather than random differences in speeds of the first few cars. The platoon duration exceeds the downstream green phase.
4. Cross street traffic is disregarded.

Theoretical conclusions are as follows:

1. The synchronization that yields minimum total delay for unidirectional traffic is the one for which the tail car of the platoon always just makes it through the succeeding intersection, but the lead car arrives early and is delayed waiting for the green phase.
2. For heavy but unequal two-directional flow, total delay is minimized by providing optimal synchronization for the direction with *lower* traffic flow.

The problem of the lead driver not cooperating with the behavior model is discussed. It is theorized that if the lead driver slows down so he can pass through the light without stopping, the whole platoon is proportionately retarded and the last car will fail to make it through. Total delay will be increased. If all cars are slowed down enough to effect a reduction in average time headway, the whole platoon will make it through but the advantage gained will be more than offset by increased journey time between intersections. Regarding the neglect of statistical fluctuations in the model, it is concluded that the theorized flow patterns are quite stable, and, even though fluctuations may be important in describing details, they are not likely to alter the qualitative conclusions.

PASSAU, P. H., "Delays Incurred by Vehicles Stopped at Traffic Lights," *International Road Safety and Traffic Review,* Vol. XI, No. 3, Summer 1963.

This paper examines the statistical characteristics of vehicles stopped at a signalized intersection, describes a method for determining optimal fixed-cycle length, and proposes an alternative method of traffic signal operation which theoretically would reduce delays.

Basic assumptions were (1) interarrival times between vehicles are exponentially distributed, and (2) the departing time of the stopped vehicle depends linearly on its rank in the queue. Implicit were assumptions of homogeneity of vehicle starting performance and the absence of turning maneuvers.

Formulas were derived to describe characteristics of performance for an intersection of two one-way streets, as follows:

1. The mean and standard deviation of the number of vehicles coming to a full stop during a red period of given duration on each road.
2. The average delay time experienced by vehicles on each road, given fixed cycle conditions.
3. The mean and standard deviation of number of vehicles stopped and the average delay time in a system where signal operation is a Markovian process generated by relating the length of successive green periods to the number of vehicles that arrived during preceding red periods.

The performance characteristics of an intersection controlled by an "optimal" fixed-cycle timer were shown to be theoretically better than performance under arbitrarily

selected fixed cycles, particularly in the case of heavy demand.

Much greater theoretical improvement in performance was shown to result from the "Markovian process" traffic signal control.

Similar results were shown for an intersection of bidirectional streets. In this case, it was found to be very difficult to evaluate the problem theoretically, hence simulation tests using probability tables were applied to yield numerical results.

SAGEN, RAGNVALD, "An Analytical Method to Determine Time-Space Relationships Beween Coordinated Signals," (master's thesis), Northwestern University, 1962.

Through comprehensive treatment of the geometrics of time-space diagrams for two-way streets, a method for determining an "optimum" timing plan is developed. Optimization is interpreted as meaning the maximization of the ratio between bandwidth and cycle length. For a given street layout, a speed and cycle length are found which provide optimum operation. In a brief discussion it is theorized that the approach could be used successfully in the solution of closed network timing problems.

SCHWARZ, HEINZ, "The Influence of the Amber Light on Starting Delay at Intersections," (master's thesis), Northwestern University, 1961.

Starting delay data were collected at seven intersections in Chicago, both with and without the amber light preceding green. The experiment was well designed statistically, making possible the testing of two hypotheses with a known degree of significance.

The first hypothesis was that the standard deviations of starting delay, both with and without preceding amber, were equal. At none of the seven intersections was there evidence to reject this hypothesis ($\alpha = 0.05$).

The second hypothesis was that mean starting delay with preceding amber equals mean starting delay without amber. This hypothesis was rejected at all seven intersections ($\alpha = 0.01$). Starting delay averaged 2.97 sec with amber preceding green and 4.17 sec without the amber.

Accident records were reviewed, but there were insufficient data to evaluate the effect of the amber light on safety.

STARK, M. C., "Computer Simulation of Nine Blocks of a City Street," NBS Technical Note 119, U.S. Department of Commerce, National Bureau of Standards, Nov. 1961.

This paper describes the development of a microscopic digital simulation model of traffic operation along a 9-block section of an arterial street in Washington, D.C. Ten intersections were included in the system, seven of which were controlled by traffic signals, and the other three by stop signs on the side streets.

Written for the IBM 704, the program utilizes a modified memorandum notation with each lane of each street divided into 12-ft unit blocks. Two computer words were reserved for information about each unit block, including the characteristics of the vehicle contained therein, if any. Vehicles and their physical and operational characteristics were generated randomly. The unit blocks were reviewed each ¼ sec, and vehicles were moved in accordance with relatively rigid deterministic rules of safe following, acceleration rates, turning speeds, response to amber signals, lane changing, gap acceptance, etc. Individual vehicles may be tagged and their progress through the system viewed in detail.

The simulation system provided two outputs:

1. Vehicle positions during each ¼-sec increment were plotted on an oscilloscope and photographed, resulting in a symbolic aerial-view picture of simulated operation.
2. A series of tables cataloging vehicle entries, passage at key intersections, exits, running times, vehicle types, speed, and lane usage was produced.

The immediate area of application of the simulation system was to study the effectiveness of experimental signal timing changes. It was also noted, however, that to the extent that the model can faithfully represent realism, volume rates and other physical and operational characteristics of traffic can be modified and the resulting effect on system performance measured.

WEBB, G. M., "Prunedale Signals," *Traffic Engineering*, May 1962, pp. 14-15.

This article describes an experimental changeable message sign system used in advance of a volume-density-controlled signalized intersection on US 101. The signs are located 600 ft in advance of the signal and 200 ft beyond the detectors; they advise motorists who cross the detectors after the controller has yielded, but before the signal has changed to red, to "prepare to stop." This message is actuated 10 sec before the end of the green phase and remains on through the red phase. Upon the beginning of the next green phase, the sign message is changed to "slow traffic ahead," which remains illuminated for 22 sec. The sign then remains blank until the succeeding "prepare to stop" message is actuated.

The signs have not been in place long enough to evaluate their effectiveness in reducing accidents.

WEBSTER, F. V., and WARDROP, J. G., "Capacity of Urban Intersections," Sixth International Study Week Traffic Engineering," Salzburg, 1962.

This paper deals only with the capacity aspect of single-level intersection design. The analyses given are preliminary, and further work is needed to cover wider ranges on the variables.

Capacities of priority, time-sharing, and space-sharing intersections are determined and compared with one another, and the effects of certain variables are discussed. This review concerns time-sharing intersections only (*i.e.*, those controlled by traffic signals).

The capacity of signalized intersections depends on the *saturation flows* of individual approaches and on the amount of "dead time" in the cycle. The green and amber periods are replaced by an "effective green" period throughout which flow is assumed equal to saturation rate, and lost time during which no flow takes place. Capacity, then, is directly proportional to effective green time.

Saturation flow was shown to be linearly related to approach widths for widths between 17 and 60 ft, and is given by

$$S = 160\,W$$

in which

S = saturation flow in passenger car units per hour;
W = approch width in feet (curb to centerline).

The effects of different classes of vehicles are given by various passenger car unit equivalents. For example, one heavy or medium commercial vehicle is equivalent to 1¾ passenger car units.

The effect of a right-turning (U.S. left) vehicle which crosses an opposing stream was found to be equivalent to 1¾ passenger car units.

The reduction in saturation flow caused by a parked vehicle is equivalent to a loss of carriageway width as given by

$$\text{Effective loss of width} = 5.5 - \frac{0.9(Z-25)}{k}$$

in which

Z = distance from stop line to parked car, or 25 ft;
k = green time, in sec.

Capacity is dependent on total cycle time. Suppose that the ratio of flow to saturation flow is calculated for each approach to an intersection and

y = the highest ratio for each phase;
Y = the sum of y's over all phases; and
L = total lost time in the cycle (periods when all signals are red or red amber plus the lost time for individual phases).

Then the cycle time which will just pass all traffic is given by

$$cm = \frac{L}{1-Y}$$

Given the cycle length, the maximum Y value which can be accommodated is

$$Y = 1 - \frac{L}{cm}$$

The practical value of Y which produces generally acceptable delays is taken as 90 percent of the maximum.

WEBSTER, F. V., "Traffic Signals," Lecture Given at Conference on Traffic Engineering, Trondheim, Norway, Jan. 1961.

The first portion of this paper serves essentially as a primer of present-day traffic signaling in Great Britain. Included are descriptions of (1) sequence of signal indications, (2) vehicle-actuated signal operation, (3) coordinated signal systems, and (4) practical considerations of signal warrants, types of signals, phasing, clearance periods, filter signals, signaling and marking layout, and detectors.

Next the "saturation flow—lost time" determination of capacity is described, and the effects of approach with traffic composition, turning movements, and parked vehicles are shown. The saturation flow values given did not account for the effects of two-wheeled vehicles and are some 10 percent lower than the revised values shown in more recent papers.

Fixed-Time Signals

The formula for average delay per vehicle which was determined by fitting a curve to simulated intersection traffic data is given:

$$d = \frac{c(1-\lambda)^2}{2(1-\lambda x)} + \frac{x^2}{2q(1-x)} - 0.65\left(\frac{c}{q^2}\right)^{\frac{1}{3}} x^{(2+5\lambda)}$$

in which

d = average delay per vehicle,
c = cycle time,
λ = effective green time/cycle time,
q = flow,
s = saturation flow, and
x = degree of saturation (the ratio of actual flow to possible flow, or $q/\lambda s$).

An alternative form is given where the last term is dropped and a linear reduction factor applied.

$$d = \frac{9}{10}\,\frac{c(1-\lambda)^2}{2(1-x)} + \frac{x^2}{2q(1-x)}$$

Optimum cycle time (that which results in least overall delay) is given by

$$Co = \frac{1.5L + 5}{1-y_1-y_2-\cdots y_n}$$

where $y_1, y_2, \cdots y_n$ are the maximum q/s values for phase $1, 2, \cdots n$, and L is total lost time.

To optimize splits, the rule was derived to set

$$\frac{g_1}{g_2} = \frac{y_1}{y_2}$$

where g_1 and g_2 are the effective green times of phases 1 and 2. It is stated that this rule may be extended to multiphase operations, but exactly how this is accomplished is not clear, unless

$$\frac{y_1}{g_1} = \frac{y_2}{g_2} = \cdots = \frac{y_n}{g_n}$$

A formula for average queue length at the beginning of green is derived:

$$N = q\frac{r}{2} + d \qquad \text{or} \qquad N = qr$$

whichever is larger, and where r is the red time.

It is possible to compute the queue size, N, which is exceeded once in a certain number of cycles. The sizes which occur every 100 cycles are computed as functions of average arrivals per cycle, degree of saturation, and proportion effective green, and are presented in tabular form.

Vehicle-Actuated Signals

A digital computer was programmed to simulate traffic conditions at intersections controlled by vehicle-actuated signals. No details of the simulation model are given. Several results are discussed.

Average delay was determined for various combinations of vehicle-extension and maximum periods, and it was found that for minimum delay the extension period should be as short as practicable (about 4 sec) so that the signals just allow the queue to disperse before changing to the next phase. The best value for maximum did not vary much with changing extension periods, and the fixed-time formula can be used reasonably well to determine the best maximum period.

For peak delays, when the greens are running to maximum, the average delay per vehicle is calculated accurately using the fixed-time formula. During off peak, when maximums are generally not realized, the average delay occurring with a 4-sec extension period in effect may be roughly calculated by the fixed-time delay formula in which calculated values of optimum cycle and split for each particular volume level are substituted. These are not necessarily the cycle and split values occurring in the field.

In practice, vehicle-actuated signals provide lower delay under light flow than fixed-time signals whose green periods are set equal to the maximums. Under peak volumes, of course, the actuated signal operates as a fixed time and the same delay occurs. It is concluded that the real value of vehicle-actuated operation is not that it caters to cycle-to-cycle variations but that it deals satisfactorily with longer term changes in total flow and flow ratios among the arms.

WILDERMUTH, B. R., "Average Vehicle Headways at Signalized Intersections," *Traffic Engineering*, Nov. 1962, pp. 17-20.

This paper describes a study of average vehicle headways (departure headways) at signalized intersections in Zurich, Switzerland. It was the purpose of the study to evaluate the effect on average vehicle headway of th following variables: (1) length of green phase, (2) amount of heavy vehicles, (3) amount of turning traffic, and (4) direction of traffic movement.

Data were collected at free flowing, level intersections, free from parking and stopping, with lanes no narrower than 10 ft, and with insignificant motorcycle and bicycle traffic.

Average vehicle headways were determined by counting and classifying vehicle movements during green and amber intervals, measuring the duration of the green interval, and performing the necessary division. The only data used were for those green intervals which were fully utilized.

The following conclusions were made:

1. The length of the green phase does affect average headway. For a 10-sec phase, the average was found to be 2.35 sec. As the green interval increases, average headway decreases; at 35- to 45-sec green intervals, the average is just below 2.00 (which corresponds to a saturation flow rate of 1,800 vph). For longer green intervals the average increased slightly. (Actually this is probably caused by some thinning out of demand.)

2. Heavy vehicles have an effect on average headway which was found slightly smaller for short phases than for long ones. For green intervals between 10 and 30 sec, each percent of heavy vehicles increased the average headway by about 0.7 percent. For greens between 30 and 50 sec long, the increase caused by each percent of heavy vehicles was about 1 percent. (Using Webster's passenger car equivalence approach, a heavy vehicle would equal 1.70 passenger cars for the short phases and 2.0 passenger cars for the longer phases.)

3. Turning traffic, below 45 percent turns, were found to decrease average headways It was observed that when only a few cars turn off, succeeding cars tend to close up the gap created, thus increasing the discharge rate and resulting in slightly lower average headway. For phases between 35 and 45 sec with no heavy vehicles, average headway was reduced to 1.88 sec when there was 13.5 percent turning traffic. With 25 percent turns, average headway was 1.92; and for 45 percent turns headways were back to the basic level of 2.00.

4. For special turning lanes, right and left, average headways were found to be slightly larger than for the through lanes. For right turn lanes, average headway was 2.13 sec which was larger than the average headway for left turn lanes of 2.08, probably because right turns are sharper.

THE NATIONAL ACADEMY OF SCIENCES — NATIONAL RESEARCH COUNCIL is a private, nonprofit organization of scientists, dedicated to the furtherance of science and to its use for the general welfare. The Academy itself was established in 1863 under a congressional charter signed by President Lincoln. Empowered to provide for all activities appropriate to academies of science, it was also required by its charter to act as an adviser to the federal government in scientific matters. This provision accounts for the close ties that have always existed between the Academy and the government, although the Academy is not a governmental agency.

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Receiving funds from both public and private sources, by contribution, grant, or contract, the Academy and its Research Council thus work to stimulate research and its applications, to survey the broad possibilities of science, to promote effective utilization of the scientific and technical resources of the country, to serve the government, and to further the general interests of science.

The Highway Research Board was organized November 11, 1920, as an agency of the Division of Engineering and Industrial Research, one of the eight functional divisions of the National Research Council. The Board is a cooperative organization of the highway technologists of America operating under the auspices of the Academy-Council and with the support of the several highway departments, the Bureau of Public Roads, and many other organizations interested in the development of highway transportation. The purposes of the Board are to encourage research and to provide a national clearinghouse and correlation service for research activities and information on highway administration and technology.