NCHRP Project 12-71
Design Specifications and Commentary for Horizontally Curved Concrete Box-Girder Highway Bridges

Appendix A
Proposed LRFD Specifications and Commentary
4.2 DEFINITIONS (Additional)

Central Angle – The angle included between two points along the centerline of a curved bridge measured from the center of the curve as shown in Figure 4.6.1.2.3-1.

Spine Beam Model——An analytical model of a bridge in which the superstructure is represented by a single beam element or series of straight, chorded beam elements located along the centerline of the bridge.
4.6 STATIC ANALYSIS

4.6.1 Influence of Plan Geometry

4.6.1.1 Plan Aspect Ratio

If the span length of a superstructure with torsionally stiff closed cross-sections exceeds 2.5 times its width, the superstructure may be idealized as a single-spine beam. The following dimensional definitions shall be used to apply this criterion:

- **Width**—the core width of a monolithic deck or the average distance between the outside faces of exterior webs.

- **Length for rectangular simply supported bridges**—the distance between deck joints.

- **Length for continuous and/or skewed bridges**—the length of the longest side of the rectangle that can be drawn within the plan view of the width of the smallest span, as defined herein.

This restriction does not apply to cast-in-place multicell box girders or to concrete box girder bridges curved in plan.

4.6.1.2 Structures Curved in Plan

4.6.1.2.1 General

The moments, shears, and other force effects required to proportion the superstructure components shall be based on a rational analysis of the entire superstructure.

The entire superstructure, including bearings, shall be considered as an integral structural unit. Boundary conditions shall represent the articulations provided by the bearings and/or integral connections used in the design. Analyses may be based on elastic small deflection theory, unless more rigorous approaches are deemed necessary by the Engineer.

Analyses shall consider bearing orientation and restraint of bearings afforded by the substructure. These load effects shall be considered in designing bearings, cross-frames, diaphragms, bracing, and the deck.

Distortion of the cross-section need not be considered in the structural analysis.

4.6.1.2.2 Structures Curved in Plan

**C4.6.1.1**

Where transverse distortion of a superstructure is small in comparison with longitudinal deformation, the former does not significantly affect load distribution, hence, an equivalent beam idealization is appropriate. The relative transverse distortion is a function of the ratio between structural width and height, the latter, in turn, depending on the length. Hence, the limits of such idealization are determined in terms of the width-to-effective length ratio.

Simultaneous torsion, moment, shear, and reaction forces and the attendant stresses are to be superimposed as appropriate. The equivalent beam idealization does not alleviate the need to investigate warping effects in steel structures. In all equivalent beam idealizations, the eccentricity of loads should be taken with respect to the centerline of the equivalent beam.

**Requirements for concrete box girder bridges curved in plan are given in Article 4.6.1.2.3.**

4.6.1.2.3 Requirements for Concrete Box Girder Bridges Curved in Plan

**C4.6.1.2.1**

Since equilibrium of horizontally curved I-girders is developed by the transfer of load between the girders, the analysis must recognize the integrated behavior of all structural components. Equilibrium of curved box girders may be less dependent on the interaction between girders. Bracing members are considered primary members in curved bridges since they transmit forces necessary to provide equilibrium.

The deck acts in flexure, vertical shear, and horizontal shear. Torsion increases the horizontal deck shear, particularly in curved box girders. The lateral restraint of the bearings may also cause horizontal shear in the deck.

Small-deflection theory is adequate for the analysis of most curved-girder bridges. However, curved I girders are prone to deflect laterally when the girders are insufficiently braced during erection. This behavior may not be well recognized by small-deflection theory. Classical methods of analysis usually are based on strength of materials assumptions that do not
Centrifugal force effects shall be considered in accordance with Article 3.6.3.

4.6.1.2.2 Single-Girder Torsionally Stiff Superstructures

Except for concrete box girder bridges, a horizontally curved, torsionally stiff single-girder superstructure meeting the requirements of Article 4.6.1.1 may be analyzed for global force effects as a curved spine beam.

The location of the centerline of such a beam shall be taken at the center of gravity of the cross-section, and the eccentricity of dead loads shall be established by volumetric consideration.

4.6.1.2.3 Multicell Concrete Box Girders Bridges

Horizontally curved cast-in-place multicell box girders may be designed as single-spine beams with straight segments, for central angles up to 34° within one span, unless concerns about other force effects dictate otherwise.

Horizontally curved concrete box girder bridge superstructures whose maximum central angle, as shown in Figure 4.6.1.2.3-1, is less than 12° may be analyzed as a straight bridge provided substructure elements, when present, are included in the model and an allowance for prestress friction loss due to horizontal curvature or horizontal tendon deviation is included.

In order to apply the aspect ratio provisions of Article 4.6.1.1, as specified, the plan needs to be hypothetically straightened. Force effects should be calculated on the basis of the actual curved layout.

With symmetrical cross-sections, the center of gravity of permanent loads falls outside the center of gravity. Shear center of the cross-section and the resulting eccentricity need to be investigated.

A parameter study conducted by Song, Chai, and Hida (2003) indicated that the distribution factors from the LRFD formulae compared well with the distribution factors from grillage analyses when using straight segments on spans with central angles up to 34° in one span.

An analytical study included in NCHRP 12-71 determined the limits of applicability of various methods for analyzing horizontally curved concrete box girder bridges. Three basic approaches are described in these specifications. The range of applicability presented is expected to yield results within 5 percent of the most detailed type of analysis.

These methods are applicable for determining both substructure and bearing forces as well as superstructure forces. They may also be used for both cast-in-place and precast construction, and for segmental construction provided construction staging and the time dependent properties of concrete are considered in the analysis. Non-gravity loads may also be analyzed using these methods.

The first method allows the bridge to be analyzed as if it were straight because curvature has a minor effect on response. This is typically done with a plane frame analysis.
Horizontally curved concrete box girder superstructures with plan aspect ratios greater than 2.0 (as described in Article 4.6.1.1), and whose central angle is between $12^\circ$ and $46^\circ$ may be analyzed as a single spine-beam comprised of straight segments provided no segment has a central angle greater than $3.5^\circ$. An accurate three-dimensional model of the substructure, when present, shall be included. This method is illustrated in Figure 4.6.1.2.3-2.

Figure 4.6.1.2.3-2 Three-Dimensional Spine Model of Curved Concrete Box Girder Bridge

Methods for adjusting live load support shear for skew described in Article 4.6.2.2.3c may be used for curved concrete box girder bridges analyzed as plane or space frames.

For central angles exceeding $46^\circ$ within any one span or for bridges with a maximum central angle in excess of $12^\circ$ with unusual plan geometry, the bridge shall be analyzed using finite element, finite strip, folded plate, grillage analogy, or other proven three-dimensional analysis method.

The design of curved concrete box girder bridges shall account for the redistribution of forces due to the time dependant properties of concrete.

The second method requires a space frame analysis in which the superstructure is idealized as a series of straight beam chored segments of limited central angle located along the bridge centerline. NCHRP 12-71 includes guidelines for performing this type of analysis along with a comprehensive example problem. Whole-width design as described in Article 4.6.2.2.1 was found to yield conservative results when space frame analysis was used. It is acceptable to reduce the number of live load lanes applied to the whole-width model to those that can fit on the bridge when global response such as torsion or transverse bending is being considered.

This result was confirmed by analytical studies conducted in NCHRP 12-71.

Bridges with high curvatures or unusual plan geometry require a third method of analysis that utilizes sophisticated three-dimensional computer models. Unusual plan geometry includes, but is not limited to bridges with variable widths, plan aspect ratios below 2.0, or unconventional orientation of skewed supports. NCHRP 12-71 includes guidelines for performing a grillage analogy analysis of curved concrete box girder bridges and an example problem that illustrates how to apply this method. This method was shown to provide results that compared well with finite element analysis.

Curved concrete box girder bridges may experience redistribution of torsion and bending forces over time. In some cases this has resulted in bearings being unloaded or overloaded. NCHRP 12-71 includes a discussion of this issue and provides some simplified methods for accounting for the redistribution of bearing forces. Commercially available time-dependent software usually does not consider torsion creep (i.e., shear stress redistribution), but will generally yield conservative results.
5.2 DEFINITIONS (Additional)

*Duct Stack* – A vertical group of tendons in which the space between individual tendons is less than 1-1/2”.

*Local Bending* – The flexural behavior caused by curved post-tensioning tendons on the concrete cover between the internal ducts and the inside face of the curved element (usually webs).

*Local Shear* – The lateral shear caused by curved post-tensioning tendons on the concrete cover between the internal ducts and the inside face of the curved element (usually web).

*Regional Bending* – Transverse bending of a concrete box girder web due to concentrated lateral prestress forces resisted by the frame action of the box acting as a whole

*Web and Duct Ties* – Reinforcement designed to prevent local flexural and/or shear failure of curved prestress tendons cast within a web.

*Wobble* – The unintended deviation of a tendon duct or sheath from its specified profile.
5.3 NOTATION (Additional)

\(d_{\text{eff}}\) = one-half the effective length of the failure plane in shear and tension as shown in Figure 5.10.4.3.1-1 (in.)

\(h_c\) = Span of the web of concrete box girder bridges between the top and bottom slabs measured along the axis of the webs as shown in Figure C5.10.4.3.1-2.

\(h_{ds}\) = The height of a vertical group of ducts as shown in Figure C5.10.4.3.1-2.

\(M_{\text{end}}\) = The moment at the ends of a hypothetical unreinforced concrete beam consisting of the cover concrete over the inside face of a stack of horizontally curved prestress tendons (5.10.4.3.1-4).

\(M_{\text{mid}}\) = The moment at the midpoint of a hypothetical unreinforced concrete beam consisting of the cover concrete over the inside face of a stack of horizontally curved prestress tendons (5.10.4.3.1-5).

\(\sigma_{cr}\) = The design flexural cracking stress of the hypothetical unreinforced concrete beam consisting of the cover concrete over the inside face of a stack of horizontally curved prestress tendons (5.10.4.3.1-6).

\(\sigma_{n}\) = The nominal flexural cracking stress of the hypothetical unreinforced concrete beam consisting of the cover concrete over the inside face of a stack of horizontally curved prestress tendons (5.10.4.3.1-6).

\(\phi_{\text{duct}}\) = Outside diameter of prestress duct

\(\psi\) = Girder web continuity factor for evaluating regional bending (5.10.4.3.1)
5.8.1.5 Webs of Curved Post-Tensioned Box Girder Bridges

Unless a more rigorous approach is used, the webs of curved post-tensioned box girder bridges shall be designed for the combined effects of global shear resulting from vertical shear and torsion and transverse web regional bending resulting from lateral prestress force and the effects of dead load, live load and transverse post-tensioning.

C5.8.1.5

Considering global web shear and regional web transverse bending separately will tend to underestimate the amount of vertical reinforcing steel required in the webs. Combining the requirements for global shear and regional bending will assure sufficient reinforcement. More rigorous approaches that considers the interaction of these forces are presented in “Construction and Design of Prestressed Concrete Segmental Bridges” by Walter Podolny, Jr. and Jean M. Muller, 1982, and “Prestressed Concrete Bridges” by Christian Menn, 1990. NCHRP 12-71 includes example problems illustrating both of these methods.

Notice that the transverse web bending can also be effected by dead load, live load and transverse prestressing that may be significant when determining the total transverse web bending moment.
5.10.4.3 Effects of Curved Tendons

Reinforcement shall be used to confine curved tendons. The reinforcement shall be proportioned to ensure that the steel stress at service limit state does not exceed 0.6 $f_y$, and the assumed value of $f_y$ shall not exceed 60.0 ksi. Spacing of the confinement reinforcement shall not exceed either 3.0 times the outside diameter of the duct or 24.0 in.

Where tendons are located in curved webs or flanges or are curved around and close to re-entrant corners or internal voids, additional concrete cover and/or confinement reinforcement shall be provided. The distance between a re-entrant corner or void and the near edge of the duct shall not be less than 1.5 duct diameters.

When a tendon curves in two planes, the in-plane and out-of-plane forces shall be added together vectorially.

5.10.4.3.1 In-Plane Force Effects

In-plane deviation force effects due to the change in direction of tendons shall be taken as:

$$F_{u-in} = \frac{P_u}{R} \quad (5.10.4.3.1-1)$$

where:

- $F_{u-in}$ = the in-plane deviation force effect per unit length of tendon (kips/ft.)
- $P_u$ = the tendon force in kips factored by 1.2
- $R$ = the radius of curvature of the tendon at the considered location (ft.)

The maximum deviation force shall be determined on the basis that all the tendons, including provisional tendons, are stressed.

C5.10.4.3

Curved tendons induce deviation forces that are radial to the tendon in the plane of tendon curvature. Curved tendons with multiple strands or wires also induce out-of-plane forces that are perpendicular to the plane of tendon curvature.

Resistance to in-plane forces in curved girders may be provided by increasing the concrete cover over the duct, by adding confinement tie reinforcement or by a combination thereof.

It is not the purpose of this Article to encourage the use of curved tendons around re-entrant corners or voids. Where possible, this type of detail should be avoided.

C5.10.4.3.1

In-plane forces occur, for example, in anchorage blisters or curved webs, as shown in Figures C1 and C2. Without adequate reinforcement or cover, the tendon deviation forces may rip through the concrete cover on the inside of the tendon curve, or unbalanced compressive forces may push off the concrete on the outside of the curve. Small radial tensile stresses may be resisted by concrete in tension.

The load factor of 1.2 taken from Article 3.4.3 and applied to the maximum tendon jacking force results in a design load of about 96 percent of the nominal ultimate strength of the tendon. This number compares well with the maximum attainable jacking force, which is limited by the anchor efficiency factor.

When the type of prestressing system is not specified, the designer should consider the range of duct and tendon arrangements that are possible based on the commercially available prestress systems that are likely to be used.

The radius of curvature of the tendon, $R$, may vary locally at flares as shown in Figure C5.10.4.3.1-1 or due to unintended wobble of the tendon. The load factor of 1.2 and low resistance factors accounts for some variation due to construction tolerance, but when construction dimensions are not well controlled, the designer may wish to consider using a larger load factor and/or smaller resistance factors. NCHRP 12-71 briefly discusses construction tolerances and how to account for them in design.
5.10.4.3.1a **Shear Resistance to Pull-out**

The shear resistance per unit length of the concrete cover against pull-out by deviation forces, $V_c$, shall be taken as:

$$V_c > \phi V_n$$  \hspace{1cm} (5.10.4.3.1-2)
in which:

\[
V_n = 0.125d_c \sqrt{f_{ci}} \quad (5.10.4.3.1-3)
\]

\[
V_n = 1.5d_{\text{eff}} \sqrt{f_{ci}} \quad (5.10.4.3.1-3)
\]

where:

\( V_n \) = nominal shear resistance of two shear planes per unit length (kips/ft.)

\( \phi \) = resistance factor for shear specified in Article 5.5.4.2 (0.75)

\( d_{\text{eff}} \) = As shown in Figure 5.10.4.3.1-1 (in.)

\( d_c \) = minimum concrete cover over the tendon duct, plus one half of the duct diameter (in.)

\( f'_{ci} \) = specified compressive strength of concrete at time of initial loading or prestressing (ksi)

\( 0.75d_c \) = minimum concrete cover over the tendon duct, plus one half of the duct diameter (in.)

\( f'_{ci} \) = specified compressive strength of concrete at time of initial loading or prestressing (ksi)

When more than one vertical group of ducts are located side by side in a single web, all possible shear and tension failure planes should be considered in determining \( d_{\text{eff}} \).

The concrete cover over the ducts at the inside of the curve should be made as large as practicable. In evaluating \( d_c \) for design, the effect of construction tolerances should be considered.

NCHRP 12-71 has an example problem that illustrates the design of a web and duct tie system using strut and tie methods. A generic web and duct tie detail is shown in Figure C5.10.4.3.1-3. Small diameter reinforcing bars should be used for better development of these bars. There have been no reported web failures when this detail has been used.
5.10.4.3.1b Cracking of Cover Concrete

When the clear distance between ducts oriented in a vertical column is less than 1-1/2”, the ducts shall be considered stacked, and resistance to cracking shall be investigated at the ends and mid-height of the unreinforced cover concrete. No more than three ducts shall be allowed in any one stack.

Where stacked ducts are used in curved girders, the moment resistance of the concrete cover, acting in flexure, shall be investigated.

The applied local moment per unit length at the ends of the cover shall be taken as:

\[ M_{end} = - \frac{\left( \sum F_{u-in} / h_{ds} \right) h_{ds}^2}{12} \]  
(5.10.4.3.1-4)

and the applied local moment per unit length at the mid-height of the cover shall be taken as:

\[ M_{mid} = \frac{\left( \sum F_{u-in} / h_{ds} \right) h_{ds}^2}{24} \]  
(5.10.4.3.1-5)

where

\[ h_{ds} = \text{the height of the duct stack as shown in Figure C5.10.4.3.1-2} \]

Tensile stresses in the unreinforced concrete cover resulting from Equations 5.10.4.3.1-4 and 5.10.4.3.1-5 shall be combined with the tensile stresses from regional bending of the web as defined in Article 5.10.4.3.1c to evaluate the potential for cracking of the cover concrete. If combined tensile stresses exceed the cracking stresses given by Equation 5.10.4.3.1-6, ducts shall be restrained by web and duct tie reinforcement.

\[ \sigma_{cr} = \phi \sigma_n \]  
(5.10.4.3.1-6)

where:

\[ \sigma_n = 0.16 \sqrt{f_{ci}} \]

\[ \phi = 0.55 \]
5.10.4.3.1c Regional Bending

The regional flexural effects of in-plane forces may be taken as:

\[ M_u = \psi \sum F_{u-in} h_c / 4 \]  

(5.10.4.3.1-7)

where:

\[ \psi = 0.6 \]  
continuity factor for interior webs

\[ \psi = 0.7 \]  
continuity factor for exterior webs

\[ h_c \]  
span of the web between the top and bottom slabs measured along the axis of the web as shown in Figure C5.10.4.3.1-2b.

For curved girders, the global local flexural and shear effects of out-of-plane forces as described in Article 5.10.4.3.2 shall be investigated.

Where curved ducts for tendons other than those crossing at approximately 90° are located so that the direction of the radial force from one tendon is toward another, confinement of the ducts shall be provided by:

- Spacing the ducts to ensure adequate nominal shear resistance, as specified in Eq. 2 5.10.4.3.1-2:

- Providing confinement reinforcement to resist the radial force; or

- Specifying that each inner duct be grouted before the adjacent outer duct is stressed.

C5.10.4.3.1c

When determining tensile stresses for the purpose of evaluating the potential for cracking of the cover concrete as specified in Article 5.10.4.3.1b, the effect of regional bending shall be combined with bending of the local concrete cover beam. It is recommended that the effect of stirrups in resisting bending be ignored, and that the ducts be considered as voids in the transverse section of the webs. This approach is illustrated in the NCHRP 12-71 Example Problem.

The wedging action of strands within the duct due to vertical curvature of the tendon can exacerbate tendon pull out resulting from horizontal curvature of the tendon as described in Articles 5.10.4.3.1a and 5.10.4.3.1b.
5.10.4.3.2 Out-of-Plane Force Effects

Out-of-plane force effects due to the wedging action of strands against the duct wall may be estimated as:

\[ F_{\text{out}} = \frac{P_u}{\pi R} \]  
(5.10.4.3.2-1)

where:

- \( F_{\text{out}} \) = out-of-plane force effect per unit length of tendon (kips/ft.)
- \( P_u \) = tendon force, factored as specified in Article 3.4.3 (kip)
- \( R \) = radius of curvature of the tendon in a vertical plane at the considered location (ft.)

If the factored shear resistance given by Eq. 5.10.4.3.1-2 is not adequate, local confining reinforcement shall be provided throughout the curved tendon segments to resist all of the out-of-plane forces, preferably in the form of spiral reinforcement.

Out-of-plane forces in multistrand, post-tensioning tendons are caused by the spreading of the strands or wires within the duct, as shown in Figure C5.10.4.3.2-1. Small out-of-plane forces may be resisted by concrete in shear; otherwise, spiral reinforcement is most effective to resist out-of-plane forces. In horizontally curved bridges, out-of-plane forces due to the vertical curvature of tendons should be added to in-plane forces resulting from horizontal curvature of the tendons.

Figure C5.10.4.3.2-1 Effects of Out-of-Plane Forces.
5.10.5 External Tendon Supports

Unless a vibration analysis indicates otherwise, the unsupported length of external tendons shall not exceed 25.0 ft. **External Tendon Supports in curved concrete box girders shall be located as to prevent tendons from touching the interior faces of webs. When deviation saddles are required for this purpose, they shall be designed in accordance with Article 5.10.9.3.7.**

C5.10.5

**Deviation saddles in tightly curved bridges should be continuous across the soffit as recommended by University of Texas research Beaufre et. al. (1988).**
5.10.9.3.7 Deviation Saddles

Deviation saddles shall be designed using the strut-and-tie model or using methods based on test results. **A load factor of 1.7 shall be used with the maximum deviation force.** If using a method based on test results, resistance factors of 0.90 shall be used for direct tension and 0.85 shall be used for shear.

C5.10.9.3.7

Deviation saddles are disturbed regions of the structure and can be designed using the strut-and-tie model. Tests of scale model deviation saddles have provided important information on the behavior of deviation saddle regions. Design and detailing guidelines presented in Beaupre et al. (1988) should result in safe and serviceable designs.
5.13.2.2 Diaphragms

Unless otherwise specified, diaphragms shall be provided at abutments, piers, and hinge joints to resist lateral forces and transmit loads to points of support.

Intermediate diaphragms may be used between beams in curved systems or where necessary to provide torsional resistance and to support the deck at points of discontinuity or at right angle points of discontinuity or at angle points in girders.

For spread box beams and for curved box girders having an inside radius less than 800 ft., intermediate diaphragms shall be used.

Diaphragms should be designed by the strut-and-tie method, where applicable.

In bridges with post tensioned diaphragms, the diaphragm tendons must be effectively tied into the diaphragms with bonded nonprestressed reinforcement to resist tendon forces at the corners of openings in the diaphragms.

C5.13.2.2

In certain types of construction, end diaphragms may be replaced by an edge beam or a straightened strip of slab made to act as a vertical frame with the beam ends. Such types are low I-beams and double-T beams. These frames should be designed for wheel loads.

The diaphragms should be essentially solid, except for access openings and utility holes where required.

For curved bridges, the need for and the required spacing of diaphragms depends on the radius of curvature and the proportions of the webs and flanges. NCHRP 12-71 found that interior diaphragms contributed very little to the global behavior of concrete box girder bridges.