Appendix K
Design Examples
Example 1* Two-Span I-Girder Bridge Continuous for Live Loads

(a) Bridge Deck

The bridge deck reinforcement using A615 rebars is shown below.

* Based on an example prepared as part of “Training Classes on AASHTO LRFD Bridge Specifications,” ODOT, R.A. Miller
Redesign by using A1035 bars with $f_y = 100$ ksi.

$$3,483(12) = 0.90A_s(100)\left(58.25 - \frac{A_s(100)}{1.7(7.0)(26)}\right); \quad A_s = 8.36 \text{in}^2$$

Provide 28 # 5 bars (14 on the top with 2.5” cover and 14 on the bottom with 2 5/8” cover). The bars will be at 8” o.c. Note: A1035 bars are not epoxy coated; hence, the cover is 2.5”.

The centroid of the bars from the top is

$$x = \frac{14 \times (2.5 + 0.5 \times 5/8) + 14(8.5 - (2 + 5/8 + 0.5 \times 5/8))}{28} = 4.12\"$$

Crack control reinforcement:

$$s \leq \frac{700\gamma_e}{\beta_s f_s} - 2d_c$$

$$\gamma_e = 0.75$$

$$f_s = \text{Tensile stress in steel reinforcement at the service limit state.}$$

$$d = 62.5 - 4.12 = 58.4\"$$

$$\rho = \frac{A_s}{bd} = \frac{28 \times 0.31}{(26)(58.4)} = 0.0057$$

$$k = \sqrt{2\rho n + (\rho n)^2} - \rho n = \sqrt{2(0.0057)(5.718) + (0.0057 \times 5.718)^2} - 0.0057 \times 5.718 = 0.225$$

$$j = 1 - k/3 = 1 - 0.225/3 = 0.925$$

$$f_s = \frac{M_{sl}}{A_s jd} = \frac{2,141(12)}{28 \times 0.31(0.925 \times 58.4)} = 56.6\text{ksi} \leq 60\text{ksi}$$

$$\therefore f_s = 56.6\text{ksi}$$

$$d_c = 2.5 + 0.5 \times 5/8 = 2.81$$

$$\beta_s = 1 + \frac{d_c}{0.7(h - d_c)} = 1 + \frac{2.81}{0.7(62.5 - 2.81)} = 1.067$$

$$s \leq \frac{700\gamma_e}{\beta_s f_s} - 2d_c = \frac{700 \times 0.75}{1.067 \times 56.6} - 2 \times 2.81 = 3.07\"$$

This spacing is too small. Aim for 6” spacing, which is more realistic.

Try: 17 top bars: # 5 @ 12’’ alternating with # 6 @ 12’’
17 bottom bars: # 5 @ 12’’ alternating with # 6 @ 12’’
Distances to centroid of bars:

Top Bars: \[ \frac{(2.5 + 0.5 \times 5/8) + (2.5 + 0.5 \times 3/4)}{2} = 2.84" \]

Bottom Bars: \[ \frac{(8.5 - (2 + 5/8 + 0.5 \times 5/8)) + (8.5 - (2 + 5/8 + 0.5 \times 3/4))}{2} = 5.53" \]

\[ x = \frac{17 \times 2.84 + 17 \times 5.53}{34} = 4.19" \]

\[ d = 62.5 - 4.19 = 58.31" \]

\[ \rho = \frac{A_k}{bd} = \frac{17 \times 0.31 + 17 \times 0.44}{(26)(58.31)} = \frac{12.75}{(26)(58.31)} = 0.0084 \]

\[ k = \sqrt{2\rho n + (\rho n)^2} - \rho n = \sqrt{2(0.0084)(5.718) + (0.0084 \times 5.718)^2} - 0.0084 \times 5.718 = 0.266 \]

\[ j = 1 - k/3 = 1 - 0.266/3 = 0.911 \]

\[ f_s = \frac{M_{si}}{A_s j d} = \frac{2,141(12)}{12.75(0.911 \times 58.3)} = 37.9 ksi \]

\[ d_c = 2.84375 \]

\[ \beta_s = 1 + \frac{d_c}{0.7(h - d_c)} = 1 + \frac{2.84375}{0.7(62.5 - 2.84375)} = 1.068 \]

\[ s \leq \frac{700 \gamma_c}{\beta_s f_s} - 2d_c = \frac{700 \times 0.75}{1.068 \times 37.9} - 2 \times 2.84375 = 7.28" > 6" \text{ O.K.} \]

(b) **Shear Reinforcement**

If prestressing steel is ignored, #4 A615 stirrups @ 4" o.c. will be needed.

Redesign by using A1035 U shaped #4 stirrups.

\[ s = \frac{A_s f_s d_c \cot \theta}{V_s} = \frac{0.4(100)(55.63) \cot 45}{284.6} = 7.8" \text{ Controls, say 7"} \]

\[ V_s \leq 0.0316 f' c \frac{b_s s}{f_y} ; \quad 0.4 \geq 0.0316 \sqrt{7} \frac{8s}{124} ; \quad s \leq 74" \]

\[ n_u = \frac{|V_u - \Phi V_F|}{\phi b_d d_v} = \frac{323.1}{0.9 \times 8 \times 55.63} = 0.81 ksi \]

\[ n_u = 0.81 ksi < 0.125 f_c' = 0.125 \times 8 = 1 ksi \]

\[ s_{max} \leq \text{smaller of} \left\{ \begin{array}{l} 0.8d_v = 0.8 \times 55.63 = 44.5" \text{ Controls} \\ 24" \end{array} \right. \]

Provide A1035 U shaped #4 stirrups @ 7" o.c.
**Interface Shear Reinforcement**

Factored horizontal shear, $V_u = 323$ kips

$$V_n = c A_{cv} + \mu (A_{nf} f_y + P_c)$$

$c = 0.28; \quad \mu = 1.0$

Although shear reinforcement spacing was previously calculated to be 7” o.c., calculate the spacing of A1035 interface shear reinforcement with $f_y$ limited to 60 ksi.

$$V_u = \phi V_{ni} = \phi [c A_{cv} + \mu (A_{nf} f_y + P_c)]$$

$$323 = 0.9 \left[0.28(20 \times s) + 1.0(0.4 \times 100 + 0)\right]; \quad s = 57''$$

Say 55”

5.8.4.1-2 & 5.8.4.1-3 $V_n \leq \max$ of

$$0.2 f_y' A_{cv} = 0.2 \times 7(52 \times 20) = 1456 \text{kips}$$
$$0.8 A_{cv} = 0.8 \times 7(52 \times 20) = 5824 \text{kips}$$

Hence, $V_n = 323/0.9 = 359$ kips as used is o.k.

$$A_{nf} \geq \frac{0.05 b_v}{f_y} = \frac{0.05 \times 20}{60} = 0.017 \text{ in}^2 / \text{length}$$

5.8.4.10-4 **Actual** $A_{nf} = 0.40 / 57 = 0.0070 \text{ in}^2 / \text{length} \quad \therefore \text{N.G., Reduse the spacing}$

$$0.40 / s = 0.017; \quad s = 23.5'' \quad \text{say 23}''$$

Use #4 U shaped A1035 interface reinforcement at $s = 23$”.

From a durability point of view corrosion resistant A1035 interface shear reinforcement provides advantages. However, practical issues may arise from a placement point of view as the spacing of girder shear reinforcement and that for interface reinforcement are significantly different.
**Example 2** Simple Span T-Beam

The reinforcement using A615 rebars is shown below.

The stirrups are symmetrically spaced at 10” o.c. up to 5’ from the bearing center line and then @ 16” o.c. up to 25’.

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* Based on an example from “LRFD Design of Cast-in-Place Concrete Bridges,” 1st Ed., Schneider, E.F. and Bhide, S.B. Portland Cement Association, Skokie, IL, 2006, 156 pages.
Redesign by using A1035 bars with a specified yield strength of 100 ksi.

1. Girder – Flexure

Capacity

\[ M_u = 2052 \text{ k-ft.} \]

With 10 \# 8 bars in 2 layers, which fit within 22”, \( \phi M_n = 2204 \text{ k-ft.} \) O.K.

Maximum Reinforcement Requirement (§5.7.2.1)

\[ \frac{c}{d_t} = \frac{3.04}{39.5} = 0.077 < \frac{3}{8} \quad \therefore \text{Tension – controlled, O.K.} \]

Minimum Reinforcement Requirement (§5.7.3.2)

\[ 1.2M_{cr} = 1.2 \times \frac{4691}{12} = 469 \text{ k-ft} \quad \text{Controls} \]
\[ 1.33M_u = 1.33 \times 2052 = 2729 \text{ k-ft} \]
\[ \phi M_n = 2204 \text{ k-ft} > 1.2M_{cr} \quad \text{Hence, minimum reinforcement requirements are met.} \]

Maximum Spacing of Tension Reinforcement (§5.7.3.4)

\[ I_g = 262874 \text{ in.}^4 \]
\[ f_c = \frac{My}{I} = \frac{(1336 \times 12)(42 - 15.42)}{262874} = 1.62 \text{ksi} \]

§5.4.2.6: \[ f_c = 0.24\sqrt{f_c^*} = 0.24\sqrt{4} = 0.48 \text{ksi} \]

\( f_c \) > 80% of \( f_r \); hence, §5.7.3.4 needs to be checked.

\[ M_{\text{service}} = 1336 \text{ k-ft.} > M_{cr} = 391 \text{ k-ft.} \] Use cracked transformed section properties

\[ I_{cr} = 63200 \text{ in.}^4 \]

\[ y^- = 6.21 \text{ in.} \) (Measured from compression face)

\[ d_c = 1.5 + 0.5 + 1/2(1) = 2.5" \]

\[ f_s = n \frac{My}{I} = \frac{29000}{57\sqrt{4000}} \times \frac{(1336 \times 12)(42 - 2.5 - 6.21)}{63200} = 67.9 \text{ksi} \] (Approximately \( 0.6f_y = 0.6 \times 100 = 60 \text{ ksi} \))

\[ \gamma_c = 0.75 \]

\[ \beta_s = 1 + \frac{d_c}{0.7(h - d_c)} = 1 + \frac{2.5}{0.7(42 - 2.5)} = 1.09 \]

\[ s \leq \frac{700\gamma_c}{\beta_s f_s} - 2d_c = \frac{700 \times 0.75}{1.09 \times 67.9} - 2 \times 2.5 = 2.09" \]
The actual spacing of 5 # 8 bars, in each layer, is \(1 + \frac{22 - 2(1.5 + 0.5) - 5 \times 1}{4} = 4.25" > 2.09\),
which is not acceptable.
Revise the design by using 12 # 8 bars in 2 layers.

**Capacity**

\[ M_u = 2052 \text{ k-ft.} \]
\[ \phi M_u = 2672 \text{ k-ft. O.K.} \]

**Maximum Reinforcement Requirement (§5.7.2.1)**

\[ \frac{c}{d_t} = \frac{3.64}{39.5} = 0.109 < \frac{3}{8} \quad \therefore \text{Tension - controlled, O.K.} \]

**Minimum Reinforcement Requirement (§5.7.3.3.2)**

1.2\(M_u\) = 1.2 (4803/12) = 480 k-ft Controls

1.33\(M_u\) = 1.33 (2052) = 2729 k-ft

\(\phi M_n = 2672 \text{ k-ft} > 1.2M_{cr}\) Hence, minimum reinforcement requirements are met.

**Maximum Spacing of Tension Reinforcement (§5.7.3.4)**

\[ I_g = 267731 \text{ in.}^4 \]

\[ f_c = \frac{M_y}{I} = \frac{(1336 \times 12)(42 - 15.56)}{267731} = 1.58 ksi \]

§5.4.2.6: \(f_c = 0.24 \sqrt{f_y} = 0.24 \sqrt{4} = 0.48 ksi \)

\(f_c > 80\% \text{ of } f_y; \quad \text{hence, } \text{§5.7.3.4 needs to be checked.} \)

\(M_{\text{service}} = 1336 \text{ k-ft} > M_{cr} = 400 \text{ k-ft. Use cracked transformed section properties}\)

\[ I_{cr} = 74218 \text{ in.}^4 \]

\(y = 6.74 \text{ in. (Measured from compression face)}\)

\[ d_c = 1.5 + 0.5 + 1/2(1) = 2.5" \]

\[ f_s = n \frac{M_y}{I} = \frac{29000}{57 \sqrt{40000}} \times \frac{(1336 \times 12)(42 - 2.5 - 6.74)}{74218} = 56.9 ksi \]  
(Approximately 0.6\(f_y = 0.6 \times 100 = 60 \text{ ksi}\))

\[ s \leq \frac{700 \gamma_e}{\beta_s f_s} - 2d_c \]

\(\gamma_e = 0.75\)

\[ \beta_s = 1 + \frac{d_c}{0.7(h - d_c)} = 1 + \frac{2.5}{0.7(42 - 2.5)} = 1.09 \]
\[ s \leq \frac{700\gamma_e}{\beta_s f_s} - 2d_e = \frac{700 \times 0.75}{1.09 \times 56.9} - 2 \times 2.5 = 3.46" \]

The actual spacing of 12 # 8 bars, in each layer, is
\[ 1 + \frac{22 - 2(1.5 + 0.5) - 6 \times 1}{5} = 3.4" < 3.4" \]
which is acceptable.

**Skin Reinforcement (§5.7.3.4)**

\[ d_e = 39.6" > 36" \]; skin reinforcement needs to be provided. However, for consistency with the original example, skin reinforcement is not provided in the redesign with A1035 reinforcing bars.

**Fatigue Limit State**

\[ M_f = 278 \text{ k-ft} \]

Cracked transformed section properties: \( I_{cr} = 74218 \text{ in.}^4 \) and \( y^* = 6.74 \text{ in.} \)

\[ f_s = n \frac{M_y}{I_{cr}} = 8 \frac{(278 \times 12)(42 - 2.5 - 6.74)}{74218} = 11.8 \text{ksi} \]

\[ f_f = 21 - 0.33 f_{\text{min}} + 8(r/h) \]

\( f_{\text{min}} = \) stress under dead load moment, which is 597 k-ft. > \( M_{cr} = 400 \text{ k-ft.} \)

\[ f_{\text{min}} = n \frac{M_y}{I_{cr}} = 8 \frac{(597 \times 12)(42 - 2.5 - 6.74)}{74218} = 25.3 \text{ksi} \]

\[ f_f = 21 - 0.33 f_{\text{min}} + 8(r/h) = 21 - 0.33 \times 25.3 + 8 \times 0.3 = 15.1 \text{ksi} \]

11.8 ksi < 15.1 ksi O.K.

**Summary:** From a strength point of view, 10 #8 A1035 bars (\( A_s = 7.9 \text{ in.}^2 \)) provide adequate flexural capacity. However, the requirements related to spacing of mild reinforcement (§5.7.3.4) result in additional area of steel. The use of A1035 bars as flexural reinforcement saves about 49% in terms of the amount of steel (10 #11 A615 (\( A_s = 18.72 \text{ in.}^2 \)) vs. 12 #8 A1035 (\( A_s = 9.48 \text{ in.}^2 \))).

**Girder – Shear**

\[ \begin{align*}
    h & = 42.0 \text{ in.} \\
    d_s & = d_e = 38.5 \text{ in.} \\
    a & = 3.09 \text{ in.} \\
    d_v & = d_s - a/2 = 37.0 \text{ in.} \\
    0.72h & = 30.2 \text{ in.} \\
    0.9d_e & = 34.7 \text{ in.} \\
    \text{Final } d_v & = 37.0 \text{ in.}
\end{align*} \]
Assume 1’-4” wide support.

The critical section is at \( x = 37 + 8 = 45’ = 3.75’ \)

<table>
<thead>
<tr>
<th>Distance (ft.)</th>
<th>Point along span</th>
<th>( V_u )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>( x/L )</td>
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<tr>
<td>0</td>
<td>0</td>
<td>189</td>
</tr>
<tr>
<td>0.67</td>
<td>0.0134</td>
<td>185</td>
</tr>
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<td>0.07</td>
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<td>0.1</td>
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<tr>
<td>15</td>
<td>0.3</td>
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</tr>
<tr>
<td>20</td>
<td>0.4</td>
<td>75</td>
</tr>
<tr>
<td>25</td>
<td>0.5</td>
<td>46</td>
</tr>
</tbody>
</table>

Use U shaped # 4 A1035 stirrups

\[
\frac{A_v}{f_y} \leq \frac{f_c}{b_v} = \frac{0.4}{22} = 0.018 \text{ in.}^2
\]

\[
s_{\text{max}} = \frac{A_v f_y}{0.0316 \sqrt{f_c b_v}} = \frac{0.4 \times 100}{0.0316 \times 22} = 28.8 \text{ in.}
\]

The simplified procedure for the determination of \( \beta \) and \( \theta \) may be used if the spacing of the stirrups does not exceed \( s_{\text{max}} = 28.8 \text{ in.} \). For the simplified procedure \( \beta = 2.0 \) and \( \theta = 45^\circ \).

<table>
<thead>
<tr>
<th>Distance (ft.)</th>
<th>( v_u ) (ksi)</th>
<th>Is ( v_u &lt; 0.125 f_c )?</th>
<th>( s_{\text{max}} ) (in.)</th>
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</tr>
<tr>
<td>25</td>
<td>0.063</td>
<td>Yes</td>
<td>24</td>
</tr>
</tbody>
</table>

The maximum shear resistance, \( V_n \), is given by the lesser of \( V_n = V_c + V_s + V_p \) and \( V_n = 0.25 f_c b_v d_v \).

\[
V_n = 0.25 f_c b_v d_v = 0.25(4)(22)(37) = 814 \text{ kips}
\]
$V_u @ the critical section = 168 < \phi V_u = 0.9(814) = 733 \text{ kips}$

$V_c = 0.0316 \beta \sqrt{f_y' b_y d_y} = 0.0316(2) \sqrt{4(22)(37)} = 103 \text{ kips}$

$V_s = \frac{A_y f_y' d_y}{s} \cot \theta + \cot \alpha (\sin \alpha) = \frac{A_y f_y' d_y (\cot 45 + \cot 90)(\sin 90)}{s} = \frac{A_y f_y' d_y}{s}$

At the critical section, $V_u = 168 \text{ kips}$

$V_u = \phi(V_c + V_p + V_s)$

$168 = 0.9 \left(103 + 0 + \frac{A_y f_y' d_y}{s}\right) = 0.9 \left(103 + 0 + \frac{0.4 \times 100 \times 37}{s}\right); s = 17.69''$

Stirrup layout (symmetrically placed) of U-shaped #4 A1035 stirrups:

- Start the first stirrup at 9'' from the support.
- Provide 3 spaces @ 17'' o.c.
- Provide 10 spaces @ 24'' o.c.

**Tensile Capacity of Longitudinal Reinforcement (§5.8.3.5)**

The following equation needs to be satisfied.

$A_p f_p + A_y f_y \geq \frac{M_u}{\phi d_y} + 0.5 \frac{N_u}{\phi} + \left(\frac{V_u}{\phi} - V_p\right) - 0.5 V_s \right) \cot \theta \quad 5.8.3.5-1$

For this case, the above equation is simplified to

$A_y f_y \geq \frac{M_u}{\phi d_y} + \left[\left(\frac{V_u}{\phi}\right) - 0.5 V_s \right] \cot \theta$

(i) **Critical Section**

$M_u = 633 \text{ k-ft } \& \ V_u = 168 \text{ kips}$

For # 4 stirrups @ 17” o.c., $V_s = \frac{A_y f_y' d_y}{s} = \frac{0.4 \times 100 \times 37}{17} = 87.1 \text{ kips}$

$V_s$ in Eq. 5.8.3.5-1 cannot be $V_u/\phi = 168/0.9 = 187 \text{ kips}$ Hence, use $V_s = 87.1 \text{ kips}$.  

$T = \frac{M_u}{\phi d_y} + \left[\left(\frac{V_u}{\phi}\right) - 0.5 V_s \right] \cot \theta = \frac{633 \times 12}{0.9 \times 37} + \left[\left(\frac{168}{0.9}\right) - 0.5 \times 87.1 \right] \cot (45) = 371 \text{ kips}$

$l_{db} = \max\left(\frac{1.25 A_y f_y'}{\sqrt{f_y'}}, 0.4 d_y f_y'\right) = \max\left(\frac{1.25 \times 0.60 \times 100}{\sqrt{4}}, 0.4 \times 0.875 \times 100\right) = 37.5''$

No modification factors are necessary; hence, $l_d = l_{db} = 37.5'' > 12'' \therefore l_d = 38''$

As seen below, the available distance to develop the bar is 55'', which is larger than $l_d = 38''$. Therefore, $f_{ax} = f_y$

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\[ T_{\text{provided}} = A_s f_y = (16 \times 0.79)100 = 1264 \text{kips} > T = 371 \text{kips} \quad O.K. \]

(ii) **Midspan**

\[ M_u = 2052 \text{ k-ft and } V_u = 46 \text{ kips} \]

For # 4 stirrups @ 24” o.c., \[ V_s = \frac{A_s f_y d_v}{s} = \frac{0.4 \times 100 \times 37}{24} = 61.7 \text{kips} \]

\( V_s \) in Eq. 5.8.3.5-1 cannot be taken greater than \( V_u/\phi = 46/0.9 = 51 \text{ kips} \); hence, use \( V_s = 51 \text{ kips} \).

\[ T = \frac{M_u}{\phi d_v} + \left[ \left( \frac{V_u}{\phi} \right) - 0.5V_s \right] \cot \theta = \frac{2052 \times 12}{0.9 \times 37} + \left[ \left( \frac{46}{0.9} \right) - 0.5 \times 51 \right] \cot(45) = 765 \text{kips} \]

At midspan, there is no concern about development length; hence, \( f_{sx} = f_y \).

\[ T_{\text{provided}} = A_s f_y = (16 \times 0.79)100 = 1264 \text{kips} > T = 765 \text{kips} \quad O.K. \]

(iii) **Face of Bearing**

\[ M_u = 0 \text{ and } V_u = 168 \text{ kips} \quad (Note: \( V_u \) is taken as the shear at \( d_v \) from the face of support.) \]

For # 4 stirrups @ 17” o.c., \[ V_s = \frac{A_s f_y d_v}{s} = \frac{0.4 \times 100 \times 37}{17} = 87.1 \text{kips} \]

\( V_s \) in Eq. 5.8.3.5-1 cannot be taken greater than \( V_u/\phi = 168/0.9 = 187 \text{ kips} \); hence, use \( V_s = 87.1 \text{ kips} \).

\[ T = \frac{M_u}{\phi d_v} + \left[ \left( \frac{V_u}{\phi} \right) - 0.5V_s \right] \cot \theta = 0 + \left[ \left( \frac{168}{0.9} \right) - 0.5 \times 87.1 \right] \cot(45) = 143 \text{kips} \]
\( l_d = 38'' \)

As seen below, the available distance to develop the bar is 21”.

\[
\begin{align*}
  f_{sx} &= \left( \frac{l_{d, available}}{l_d} \right) f_y = \left( \frac{21}{38} \right) 100 = 55.2 \text{ksi} \\
  T_{provided} &= A_s f_{sx} = (16 \times 0.79) 55.2 = 698 \text{kips} > T = 143 \text{kips} \quad O.K.
\end{align*}
\]